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Chess and Mathematical thinking
Cognitive, Epistemological and Historical issues

Dottorando: Giuliano D'Eredità

Tutor: Prof. Claudio Fazio

Co-Tutor: Prof. Luis Radford

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Acknowledgments

This work is dedicated to our Maestro Filippo Spagnolo, who has recently passed away. He believed in me and in my research project, and he gave me several opportunities for a cultural and scientific enrichment.

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Introduction

This work deals with chess and mathematical thinking. The reasons of my interest in this topic are basically two.

The first reason is the increased interest in chess activity by educational agencies in the last years, especially in Italy. Not only a lot of Italian schools decided to start chess practice, but also several academic projects involved chess and learning, in particular by Turin University, in collaboration with national research council (Rome) and one time in collaboration with our University too. Recently The National Institute for Evaluation of Scholastic System (INVALSI), that is a governmental agency, performed an inquire named SAM, that is an acronym for Chess and Math Learning (Scacchi e Apprendimento Della Matematica in Italian), realized over entire Italy with the collaboration of Italian Chess Federation (coordination by Piemonte Committee), and in which we collaborated too.

Secondly, my personal activity as Chess Instructor and my role in the Italian Chess Federation (at the moment member of the National Council), together with my scientific background, produced in me a great interest and passion in deepening possible relationships between chess and education, in particular with mathematics education. Hence, I formulated the general primordial research question as follows:

Is chess a useful tool for Education, in particular for Mathematics Education? Notice that I used the word primordial because, in the course of my doctoral work, as a consequence of reflections about the nature of the teaching/learning processes and about my experimentations, I shifted my vision. Now I state that it is more correct to reformulate the research question in a different way:

What conditions, methods and approaches are advisable to make chess a useful practice for Education, in particular for mathematics Education?

Besides, during my PhD course, quickly I became aware that chess and math education are two very large, almost never-ending, worlds. Hence, my work in looking for relationships between chess and mathematical thinking became more similar to an exploration. Then, I realized that for a better understanding of these relationships, I had to perform first a historical-epistemological analysis of chess, according to the theoretical framework of my Maestro Filippo Spagnolo. (Spagnolo 2009). This analysis is presented in Chapter I. It begins with a section dedicated to a comparison between the different conceptions of strategy and tactics in East and West, through the most representative logic game for each area: Wei-ch'i (Go) for East and chess for west. I decided to insert this section to remark the importance of social and cultural factors in thinking and cognition (and consequently in teaching/learning processes), according to the approach adopted by Luis Radford (2008) in his theoretical framework in math Education, in which the Vygotskian point of view on cognition is coherently developed in Math Education. I met Prof. Radford in March 2010 and from that moment on, I followed his theoretical framework in my research, adapting it to the various contexts I deal with.

Within this framework, cognition and the learning/teaching processes are seen as processes strongly dependant on cultural and social contexts.

Chapter 1 comprehends also a section dedicated to the history of chess thinking, in which I remark the relevance of social and cultural factors in an apparently isolated, abstract world like chess strategic thought. In the subsequent section, are presented the noticeable, reciprocal collaborations between chess and computer science. Chess is a perfect arena to test thinking machines and software, and some of the most important mathematicians working in Artificial Intelligence deal with chess, like Von Neumann and Shannon did since the 40's.

The last section deals with *freestyle*, a new form of playing chess, in which men and machines are fully integrated forming the Centaurs. The centaurs' cognitive and choice-taking style resulted very interesting and innovative.

Chapter 2 deals with the nature of human chess thinking. The first section deals with psychological frameworks, related most to chess cognition, in which also some important results from neurosciences are discussed. In the second section is presented my pilot study on pupils' chess thinking, realized in a middle school in Palermo in 2008. It revolves around a chess position to be studied by the students. To analyze the students' protocols I performed an a priori analysis of the expected behaviours.

Chapter 3 is dedicated to chess and math. First I pay attention to relations between chess and skills proficiency, and in the subsequent section I discuss and comment the main studies on chess and mathematics, introducing some possible links between the two disciplines. In the third section I consider some theoretical topics in Philosophy of Mathematics and in Math Education, and several argument from Radford's math education theory, as discussed by Radford (2003, 2005). Relationships with chess conceptions and chess education are discussed, and the topic of the epistemological obstacles in chess education is presented. In the fourth section I present my first experimentation, held in Palermo in 2008-2009. I looked for improvements in math skills, submitting to students a pre and a post test, selecting an experimental group participating in a chess course, and a control one not participating in. As part of an ongoing exploration, the statistics turned out to be relatively poor; also I did not take information about other activities performed in non-curricular time, and no information about students' scholastic trend, including the math curriculum and the teachers' methodology and approach. Besides, I did not realize a fine analysis about the structure of items with respect to the contents, competences involved and contexts proposed. Yet, this experimentation proved to be useful to help me refine the next one, as I shall discuss later. Then, in fifth section, is presented my subsequent experimentation, Agrigento 2009. The set-up was similar, i.e. pre and post test, and experimental and control group, but I performed a slightly more refined analysis. According to the PISA framework, questions were grouped by content and competence, and the analysis was performed taking into account this grouping.

Chapter 4 is concerned most with the INVALSI SAM project, in which our research group collaborated, in particular in writing some contents of the chess protocol to be used in the project, by all the experimental classes (60). As a consequence of the shift of perspectives I quoted in the former, I and Mario Ferro inserted in the chess protocol some contents in which several mathematical concepts and symbols were used. At the end of the first section the early, unofficial results of SAM project are commented. Also, we followed the chess and math activity of a class participating in the project, and we recorded in that class a video concerning students engaged in a math task. In the last section I presented briefly this activity and a short analysis of the video, performed by highlighting some crucial episodes and commenting the learning processes, with particular reference to key words, gestures, and writings occurred. In the conclusion, all main outputs obtained, and the various considerations made in the course of my work are summarized, with particular reference to math education. I did not answer the primordial research question, but the reformulated one, according to the beliefs and vision I constructed myself in the course of my work. Last but not least, some open problems are proposed to further research.

Chapter 1
Historical and epistemological aspects of chess

Summary

This chapter deals with chess, considered through important aspects like the historical and epistemological ones First, being a strategic and tactic game, an analysis of pure concepts of strategy and tactics is performed in section 1.a in which these concepts are faced with sharp reference to the culture, comparing east and West conceptions through the traditional games, i.e. chess and wei-ch'i (Go). Significant cultural differences emerged, with a predilection for analytic-deductive reasoning for West, and for a synthetic-holistic approach for East, and it is very important in today's multicultural world. After, in section 1.b, an historical review of chess thinking is realized, discussing the main theoretical developments, and highlightingthe influence of social and cultural factors on chess thinking, coherently with the theoretical framework outlined in the introduction, i.e. conceiving the train of thought as a process historically and culturally based; in section 2.c are reported the important, fruitful cooperation between chess and AI. This cooperation acted in both directions, allowing important theoretical advances in AI, obtained by famous scientists like Shannon and Von Neumann, and improving chess knowledge at every level. Today modern technologies are fundamental in chess; as a consequence of this strong connection, nowadays a new format of chess is arising, *freestyle*, that is chess with any kind of help. In the section 1.d, the basic aspects of freestyle are showed, discovering very interesting way of reasoning employed in a full man-machine integration that can become relevant in Education too.

Riassunto

Questo capitolo tratta degli scacchi, considerati attraverso alcuni importanti aspetti come quelli storici ed epistemologici; Per prima cosa, essendo gli scacchi un gioco strategico e tattico, nella sezione 1.a si svolge una analisi dei concetti puri di strategia e tattica, e questi concetti sono affrontati con chiaro riferimento alla Cultura, comparando l'Oriente e l'Occidente attraverso i giochi tradizionali, gli scacchi ed il wei-ch'i (Go). Sono emerse significative differenze culturali, in particolare riguardo alla predilezione per il ragionamento analitico-deduttivo (occidente) e per un approccio sintetico-olistico (oriente), e questo è molto importante nel mondo multiculturale di oggi. Quindi, nella sezione 1.b, è stata effettuata una rivisitazione storica del pensiero scacchistico, discutendo i principali sviluppi teorici ed evidenziando l'influenza di fattori sociali e culturali nel pensiero scacchistico, coerentemente con il quadro teorico delineato nell'introduzione, cioè la concezione che il corso del pensiero è un processo basato culturalmente e socialmente. Nella sezione 2.c sono riportate le importanti collaborazioni tra scacchi e Intelligenza artificiale.

Questa collaborazione ha agito in entrambe le direzioni, consentendo importanti progressi teorici in AI, ottenuti da famosi scienziati come Shannon e Von Neumann, e migliorando la conoscenza scacchistica, a tutti i livelli. Oggi le moderne tecnologie sono fondamentali negli scacchi; Come conseguenza di questa forte connessione, oggigiorno sta sorgendo un nuovo format per gli scacchi, il freestyle, cioè il gioco con qualunque ausilio. Nella sezione 1.d, gli aspetti basilari del freestyle sono esaminati, scoprendo modi di ragionamento molto interessanti, impiegati in una completa integrazione uomo-macchina, che può assumere rilevanza in Didattica.

1.a Cultural diversities through the traditional games: chess and wei-ch'i

Summary

In this section the different ways of reasoning are observed through traditional games like chess for the western world and Wei-ch'i (Go) for the eastern one (Chinese) Arguing and conjecturing are not independent on the social and cultural context, and the success of chess and Go as strategic games in different parts of the world is not a case. In fact, the rules and metarules themselves of the two games are generated by cultural and social factors. Western way of reasoning prefers hypothetic-deductive analysis, Eastern way of reasoning prefers instead a more synthetic one. In fact, in chess it is often used a deep if/then reasoning, and frequently it is necessary to concentrate the efforts on a unique goal, on a single zone in which the battle is at a crucial point, while in Go to consider only a zone of the board is very dangerous, and to gain space in a given zone implies inevitably to give up other space to the opponent, and it is a sharp cultural influence (Yin/Yang approach). I think that these reflections may be useful not only to better understanding the Nature of chess, but also in Education, because of the increasing of cultural interchanging in the today's world. This section is based on a paper published by Filippo Spagnolo and I (D'Eredità & Spagnolo 2009), and on a chapter written by me in a recent book (Spagnolo & Di Paola eds. 2010)

Riassunto

In questa sezione i differenti modi di ragionare sono esaminati attraverso giochi tradizionali come gli scacchi per il mondo occidentale e il Wei-ch'i (Go) per quello orientale (cinese). L'argomentare ed il congetturare non sono indipendenti dal contesto culturale e sociale, ed il successo di scacchi e Go come giochi strategici in parti differenti del mondo non è un caso. Infatti, le regole e le meta regole stesse dei due giochi sono generate da fattori sociali e culturali. Il modo occidentale di ragionare predilige l'analisi ipotetico-deduttiva, mentre quello orientale preferisce invece un approccio più sintetico. Infatti, negli scacchi è spesso usato un profondo ragionamento di tipo se/allora, e frequentemente è necessario concentrare gli sforzi su un unico scopo, in una singola zona dove la battaglia è in un punto cruciale, mentre nel Go considerare solo una zona del goban è molto pericoloso, e guadagnare spazio in una data zona implica inevitabilmente cederne dell'altro all'avversario, e questa è una chiara influenza culturale (approccio Yin/Yang). Penso che queste riflessioni possano essere utili non solo per meglio comprendere la natura degli scacchi, ma anche in Didattica, per l'incremento degli scambi interculturali nel mondo d'oggi. Questa sezione è basata su un articolo pubblicato da me e Filippo Spagnolo (D'Eredità & Spagnolo 2009) e su un capitolo scritto da me in (Spagnolo & Di Paola eds. 2010).

It was observed that hypothetic – deductive reasoning may represent in Western culture a fundamental reference for "strategic" choices. It is concerned with maths and also with other cultural contexts including economy. What about other cultures?

We stress that in the Chinese classical classification of the most important arts Wei-ch'ì (an ancient Chinese strategy game, known also by the japanese name "Go") is considered a very important art. We believe that Wei-ch'i plays an important role for arguing, conjecturing and proofing in Chinese thinking. We glimpse in some experimental work significant differences, confirmed by historical-epistemological references (Chemla, Shuchun & Lloyd, 2007), but we are not in a position to say more on this topic.

We believe that learning to conjecture and argue are not independent from the social, cultural and specific educational context.

In the modern world, where the exchange of information has increased quite a lot, and where one must deal with continuous questions and issues relative to the integration of cultures, we hold it useful and constructive to propose a reflection on the different approach to the problematic of choice and decisions, which different sectors embrace and place themselves as common denominator for the setting up itself of some human activities. Strategy and tactics are some of the generalisations of behaviours and of choices which can be very useful for the comprehension of the same.

Games of strategy, in particular chess in the occidental world and Wei-ch'i (Go) in the orient are paradigmatic environments for the mentioned themes. In these (as probably also in other games) the abstraction is such that it consents to an analysis which is independent enough from contingent bonds.

In particular, the examination of the different approaches to the themes of the strategic and tactical type builds interest both for the understanding of determined behaviours, also on a macroscopic scale, and in the environment of Research in Didactics of the Sciences. In this field, these deeper studies have value as a tool for the understanding of teaching/learning phenomena and for possible aimed didactic proposals, which can also consider the use itself of the games of strategy. Let us consider tactics and strategy in more depth.

"*Tactics is knowing what to do when you have something to do; strategy is knowing what to do when you don't have anything to do.*"(Tartakower).

Strategy and tactics are moments of the activity in a determinate context, and in general are deeply interconnected, even if the respective phases of intervention are normally quite separate one from the other.

One **strategy** is the formulation, even in an implicit form, of a plan of action which, even long term, is taken as a reference for the coordination of the activities addressed to the reaching of a predetermined goal. The sectors within which one can speak about strategy can be the most disparate, and the strategy is formulated, also through different phases of actuation, in that area where, to reach the objective, there is not only a single choice and the outcome is, in general, uncertain. The word 'strategy' is derived from ancient Greek and meant 'general' (**στρατηγός)**. The first necessary option is precisely the determination of the goals, that is, the explicit identification of the objectives on the basis of an evaluation of the situation. The evaluation phase is perhaps the most delicate and depends on the data at hand, on the decision making ability of the subject in terms of aptitude, on experience, and on knowledge. This often poses some important bonds on the successive strategic planning operations. The true and real strategic operations are based on objective considerations and also on psychological considerations. The explicit identification of the objectives, or better yet, more in general, the choice of the aspects of the situations worth consideration and/or passably important developments, depends on the situation and on the decision making subject, in the terms expressed above. It is interesting to underline that the decision making subject can be a person, a group of people, a machine, a group of machines, and even an animal or group of them. The possibility has often been discussed that machines or animals can have a true and real strategic ability, in any case that is outside of our aims here. One important aspect of strategy is the frequent placing in hierarchy of the objectives, that is, the fact that often useful or indispensable partial or intermediate objectives for the successive goals are often defined. This frequently implies the establishing of a chain of implications which, in its details, often resembles tactical type operations. It is often stated "it is better to have an incorrect strategy than to not have one at all", in the same way it is highlighted that those subjects are destined to fail which have an absolutely unequivocal strategy and are devoid of flexibility and adaptation to reality, meaning both the evaluation and the decision *in progress* and the fact that the initial conditions very rarely are always the same.

We must walk a thin line between flexibility and consistency (Kasparov, 2007)

Strategy has its natural complementary action in tactics.

Tactics comprise the methods used for achieving established objectives. Tactics are the means, real or logical, used to obtain a goal, be it partial or total.

A tactical operation has the goal of realising a single action within the strategy, or also for gathering possibilities offered by an adversary or from the physical or logical environment in which it is found. Strategy refers to operations done to reach a long term objective and is put into effect on a wide geographical scale (more in general contexts). Tactics, instead, refer to actions done to reach a short term objective and generally is put into effect on a reduced geographical scale (more in particular contexts). In tactical operations, by means of concrete actions, one aims at obtaining an advantage, recovering from a disadvantage, or maintaining the status quo which is held to be satisfactory. An advantageous tactical operation which stands out can convince one to decisively change his formulated strategic plans if necessary. Not only, but also the evaluations asserted in the strategic phase can turn out to be erroneous, or much less evidently absolute, following the realization of a tactical operation. This can also happen in Physics or in Science in general, where determinate mathematical or applicative difficulties can convince one to abandon a previously formulated model.

In particular, in science, a theory provides a set of relationships between properties and quantity of the real world, while the models determine the levels of accuracy, select the details and areas of interest, the variables to consider, and specify initial conditions and constraints. Theories and models, together, allow us to make predictions, and the consequences of results and deepening lead us back to refine the model or to reject it.

This may cause to question the whole theory, or reconsider it as a special case of a broader theory; In mathematical logic, we say a theory model of any structure that respects the axioms, is this sense of the word that does the so-called T**heory of models.**

The first meaning is local and covers classes of problems and has no pretensions of generalization. The second meaning is related to the organization of knowledge in Theories. In Mathematics, a Theory concerns in particular the « Theory of models ». In the twentieth century this idea had different interpretations as that named « Abstracts Models », or also « Syntactic Models ». In any case it concerned a formal, axiomatic systematization, with a well-defined and sufficiently rigorous language. Both meanings concern also with different ways to represent real phenomena. First meaning is « tactic », analyzing local situations. The second one is « strategic », analyzing broader problem classes, and making long-term predictions.

Given the complexity of science today, it requires a systemic approach to modelling in different fields, as discussed by Morin (2001) and De Le Moigne (1999).

It requires a general theory that encompasses both the formal and the modelling of complex systems. It seems that modelling is the only tool to interpret the reality at the beginning of the twenty-first century.

Coming back to East and West, in advance we notice that the terms East and West are obviously very general, involving almost entire hemispheres, with many regions and ethnic groups.

It should be noted that by West we mean *western thought*, in some features that distinguish it from the *eastern.* We refer to the culturally *Western identity*, distinct from the *culturally Eastern* one**.** In particular, we consider, for present purposes, a general framework that can refer to terns Confucius / Tao / Buddha for the South East Asian (Chinese) thought and Socrates / Plato / Aristotle to the West (Italian).

For the first treatise on strategy, one can certainly go back to Sun Zu, a general who lived in China (VI-V century B.C.) entitled *The Art of War* (Sūnzǐ Bīngfǎ). The text is explicitly in reference to war, but it is still considered one of the basic texts for the learning of strategy in every field of human action. Sun Zu, besides establishing several fundamental principles of a strategic character as well as political, morale, and practical, often stops himself to underline how success can depend completely on the quality of the execution of an operation (tactics). He goes beyond this and supplies some useful indications for understanding that success, seen as the obtaining of a goal, brings with it losses and, more generally, that the actions undertaken have some consequences; with his own words *Obtaining one hundred victories out of one hundred battles is not the epitome of ability. Beating the enemy without having to do battle, that is the epitome of triumph* (Sun Zu, 2008). Make visible a rational layout where the risk must absolutely be minimised. "*Therefore, the victories obtained by the Masters of the Art of War do not distinguish themselves either for the use of force or for their audacity. Their successes in war do no depend on good luck. Because to win it is enough not to commit errors. "Don't commit errors" means placing oneself in a condition to win with certainty. In this way*, an enemy already beaten is subdued… in that way, a victorious army wins first, and then goes to battle. An army destined for defeat first goes to battle and then hopes to win".

In short *Total control*.

It is very interesting to note how audacity is considered a virtue which is not strictly basic for a strategy. In this, we note a contrast with the aggressive conceptions which are encountered in the western world, above all in the field of economics, where strictly connected to the concept of risk. This is fanatic in a world where innovation is decisive and coming in second in the development of a sector is like not finishing at all. Audacity, the attack, the aggressiveness contain in themselves the advantage often of being able to orientate things on one's own preferred terrain.. The classical western approach to a conflict can be summarized as follows by D. Lai (Web ref.3):

The Greeks developed what has been called the Western way of war a collision of soldiers on an open plain in a magnificent display of courage, skill, physical prowess, honour, and fair play, and a concomitant repugnance for decoy, ambush, sneak attacks, and the involvement of non-combatants.

It is interesting to underline how in the first chapter "Evaluation", Sun Zu traces the five guiding principles for the evaluation of the real situation: *The first of the fundamental elements is the Tao, the second is the Heavens* (climate), *the third is the ground, the fourth is the command, and the fifth is the doctrine* (Rule, organisation). Not by chance, the first place is reserved for the element "Tao", the word, difficult to translate, here it is placed to indicate that people, army, and sovereign have the very same intent, that the Chiefs have great moral strength, that they are united in reaching their goal, and that the reciprocal trust and esteem are total. This is present in a very similar way in Von Clausewitz (1832). This has a lot to do with motivation, and also finds an easy comparison in the didactic sphere. It implies a clear definition of the goal, and in general the profuse spending of the respective resources and abilities in a synergetic way. Also, generalizing about more limited group or individual activities, commitment and Concentration are in the first place. Nevertheless, and this is very interesting, Sun Zu does not hesitate to declare that in some cases it is necessary to disobey the orders of the sovereign …the *ninth is: there can be circumstances in which the sovereign's orders must not be obeyed"*. In synthesis, the decisions must be taken by who is competent and has the elements to do so, and not on the basis of pure hierarchy, if this is necessary for the supreme common good; a concept also present in Von Clausewitz, but criticised in a strongly hierarchic vision of the military and government organisation.

Frequently the strategic skills have had a decisive role in the history of the human race: is assumed that after a genetic modification modern Homo sapiens has acquired "... The ability to devise and implement action plans in the long term, a feat which the Neanderthal could not ever enjoy.. The distinction was small, but ultimately had profound consequences: *Our species survived and prospered, while Neanderthals died out*. (Wynn & Coolidge, 2009). Let us now to consider more closely chess and Go.

The world is a game of Go, whose rules were unnecessarily complicated (Chinese proverb).

Life is Chess (Bobby Fischer)

A **game of strategy** is typically a <u>board game</u> or a <u>videogame</u> in which the ability of the player to take decisions has a great impact in determining the result. Many games include this element to a greater or lesser degree, making it difficult to establish a demarcation. It is therefore more appropriate to speak of degree of strategy of a game, rather than of the fact that it is or isn't a game of strategy. A game of strategy is a game in which the rules are well defined and clear and are known by the players. Von Neumann demonstrated that chess, in that it is a game of complete information, is described by a matrix (endless, the possible games are of the order of 10 to the 50th) which contain a saddle point, and therefore, the game is solvable, that is, there exists a perfect match where both of the players, if they were able to evaluate *everything*, play their best (how this is defined is not known, the chess player's experience makes him inclined to a draw). The same is true for Wei-ch'i, with the difference that there are many more possible matches (10 to the 172nd).

In chess, the goal of the game is to capture the adversary's King, the checkmate. I will deepen evolution of chess thinking in the next section; here I just note that chess strategy is a complex of player's activities which bring one to consider peculiar aspects of the position, to establish priorities, and to carry out some forecasts on the advancement of the match, to plan a series of operations.

Tactics in chess are concrete operations, normally aimed at altering the existing position by means of a sequence of one or more moves, which are often obligatory. A single error in tactics, always if the adversary plays without errors, can be fatal for the player. The ability to calculate, of a strictly deductive type, in a given position, becomes the balance keeper of a match.

With the passage of time, the mastery of the player is no longer represented by the mere ability to calculate the possible moves, but also by the selective ability of orientation of his thought and of his attention by means of the recognition of visual or abstract patterns, as deepened in chapter2. This has rendered fascinating, starting with the 60s-70s of the XX century, the competition between human players and artificial players, the latter of these which today are practically unbeatable on the tactical plane. It will be deepened in a next section. From the second half of the XX century, the concept of initiative and dynamic game has taken the upper hand over a static conception based *only* on the classical strategic canons.

Wei-ch'i, in the Occident better known by its Japanese name 'go', probably began in China about 4000 years ago and its complete development dates from the VII to the V centuries B.C. It was introduced in Japan and Korea around 700 A.D. It spread amongst the imperial Chinese functionaries and in a period of continuous war also represented, besides being a philosophical type discipline, an important exercise in military strategy. The aim of the game is the control of the territory on the goban (board/checkered board of the game) and represents an elevated form of abstraction of thought, at which point, in traditional China, it rises to a second discipline amongst the four held to be basic for the instruction of a person of elevated rank. The knowledge of Wei-ch'i is a necessary condition for holding high government positions. It summarises, in itself, several fundamental characteristics of Chinese culture, in line with the tradition of the I-King, with some symbolic meaning tied to spatial and temporal representations of the Universe. Different from chess, where often the actions are completely aimed at reaching an identified objective and probably the adversary will impede it at any cost, in Wei-ch'i it is implicit that the choice of an area of influence or of operation brings with it an analogous adversarial action and this is part of a conception that aims at avoiding the absolute identification of an element in favour of a dynamic vision and of interchange between the various elements themselves. In traditional Wei-ch'i, the players were, more than adversaries, two parts of a cause for creating something intrinsically valid. The basic strategic Rules in Wei-ch'i are, in a schematic form:

1. If you have a weak group of stones, reinforce it.
2. If your opponent has a weak group of stones, planned to attack
3. If you can make a move that has a broad territorial effect, make it
4. If the opponent can make a move that has a broad territorial effect, try to prevent, destroy it or reduce it with appropriate measures (web references 4)

Everything is based on the techniques of encirclement or counter-encirclement of the adversary.

"According to his model of rational behaviour, rationality consists <u>not in the optimum</u> (in Wei-ch'i theoretically optimal strategy exists as a principle, but is and will remain unknown even for the best player, because of the quantity of possible variants which one should take into account in calculating it) but in the <u>satisfying</u>. (Boorman 2004)

Chess and Wei-ch'i are profoundly different, and not by chance the areas of its diffusion were, for a long time, clearly distinct between the Occident and the Orient. Obviously, at an elevated level of thought processes, we find in both some common denominators in the strategic and tactical elements. But it is quite obvious a connection with some basic rules of social life. The so called "basic virtues" of western world, rising from the double helix of Western civilization (Greek-Roman tradition and Judeo-Christian religion) are almost always focused on individuals as a part of social environment and also as an isolated subject. We can define this approach as the "perfect isolated" harmonized at the" Right in the middle" as we find in Aristotle (Di Paola, 2009). The Eastern approach to Ethics is quite different, and is focused more on social organization than on the individual, and this is connected with Chinese Philosophical traditions, that have not a similar in Western world. *Tao*, as we mentioned earlier, is one of the most high and deep concept the human mind ever produced, and it's intrinsically unexplained, and for a long time incomprehensible to the Western world. Tao direct applications are found in various sectors like Calligraphy, Tai Chi Chuan, Medicine, and in Wei-ch'i itself. This is a strongly cultural item for Chinese people. We refer to a three parts- braid composed by *Tao, Confucianism and Buddhism* (Di Paola, 2009). Coming back to Chess and Wei-ch'i, the chess player uses patterns as tools for conjecturing and arguing in a typically deductive framework, in which moreover valuation assessments are not always sharp but can be *fuzzy* (e.g. “white is slightly better”). Tactics is fundamental in chess; it deals with concrete developments that require an exhaustive analysis at the most efficient level. Instead the big strategic frameworks conceived by the greatest theoretician of chess are very general and abstract Theories. Even after the criticism of the classical principles by hypermoderns like Reti, Tartakower, Nimzowitch (Nimzowitch, 1975) (Reti, 1932).

Strategic principles fail to have an absolute validity, so much so that are considered also like epistemological obstacles (Bartolotta, 1997), and today are used as available *elements to evaluate a position* in a non-dogmatic way.

In Wei-ch’i, according to Chinese thinking tradition, to consider or to select only a feature, even in an exhaustive way, is not useful but is extremely dangerous. *Nothing makes sense except in context*. In the following “Ten Commandments” (“meta-rules”) we find always the reference to the context, and it is recommended to avoid focusing too much on particular.

Go Ten Commandments (Otake Hideo, 9º dan) (web references 5)

1. «Gluttony does not lead to victory »
2. «To penetrate the opponent’s zone gently and simply»
3. «If you attack your opponent, pay attention to your shoulders»
4. «Abandon the easy gain, and fight for the initiative»
5. «Let the little falls, concentrate on large»
6. «If you are in danger, abandon something»
7. «Be careful, do not wander randomly on goban»
8. «If necessary, blow by blow»
9. «If your opponent is strong, protect yourself»
10. «If your group is isolated in the middle of an opponent’s zone, choose the peaceful way»

In Chess the game can be decided by a single move, and in the most evident case checkmate will be or will be not (bivalent logic), and in a more general way, in general the presence of more or less important pieces aligned addresses clearly the game toward the final target by displaying pieces toward vital points and by destroying opponent’s defence. This approach is related with western way to combat a fight, or a conflict, or any situation (web references 3). Instead, in Wei-ch’i success is represented by a gradual series (Boorman, 2004), somehow more *fuzzy (*Kosko 1995). The target is consider not to defeat opponent completely, but to maximize your benefits. It’s not a duel, but an economic competition to obtain an asset of little value (Boorman, 2004). It’s interesting to stress that in Chess a little advantage must be converted in a tangible advantage to win the game, while in Wei-ch’i it is sufficient to retain an imperceptible advantage to the end. The following table, within the limits of any schematization, may be useful to focus some interesting items:

Discipline	Purpose of the game	Social function	Strategic elements	Function of tactics
Chess	Checkmate (capture of the Opponent's King).	None in particular–an exception is the Soviet Union from 1925 to the 80s	Identification of Partial Objectives and evaluation Of the position. Optimisation of the action of one's own pieces and limitation of the opponent's ones. Recognition of visual and abstract patterns.	Determining. A single tactical action, well carried out, becomes the main and often conclusive one of the match.
Wei-ch'i or Go	Control of the largest area possible of the territory on the goban.	Very important; second place amongst the traditional arts and necessary for the education and the instruction of functionaries and dignitaries in imperial China and imperial Japan.	***Concept of control*, in the sense of a Continuous evaluation of the total situation. Choice of the Elements can be Considered satisfactory. Strategic items: e.g. defence of own groups, attack to opponent's groups, territorial gain.**	**The knowledge of the tactical themes is very important, but a local victory risks to become counter-productive Tactics are not tied to strategy. The concept that a tactical success leads to a strategic success is alien to the spirit of Wei ch'i.**

Table 1– Chess and Wei-ch'i

1.a.1 Connection to didactics and open problems

We maintain that several thematic explained above can have connections to didactics. The different approaches to strategy and tactics in different cultures, supply, in their entirety, some formidable tools for confronting the most disparate thematic of life and are very interesting in a didactic context.

We limit ourselves to giving some schematic notes on these possible connections, providing that a more specific analysis of some of these requires dedicated research, with quantitative and qualitative methods.

1. First point, the full awareness that without dedication, unity of intent, clear definition of the goal, attention and motivation it is rare that one reaches an objective whatever it is. This point is of a general character, but precisely for this it is often neglected in Didactics

2. The aptitude of *Evaluation*: the consideration of the elements with an elevated degree of objectivity and selectivity of one's attention allows the formulating of strategies, and sometimes also methods and tools capable of the solution of problems and of the definition of a situation. That in line with a modern vision of didactics which aims at an increase of competences, that is, of the use of one's knowledge and abilities in different contexts and situations; this vision has to be taken into account particularly in Italy, where there is a proven lack of it.

3. Hypothetic-deductive thinking, also at a high deepening level, it's an important item in strategy games and useful in all fields. Hypothetic-deductive thinking is found in the western Culture since from Euclide's Elements, a Model of Aristotle Logic, as highlighted by Spagnolo (2005). Hence, in the Western culture strategic thinking may be addressed by Aristotle Logic (inherent to Natural Language), while in the Eastern Culture, as we mentioned earlier, by Wei-ch'i "meta-rules"

4. Visual or abstract patterns recognition and continuous adaptation to the reality is the normal practice in chess and in Wei-ch'i, typical of high order competences. A strategy adopted in a repetitive way, without adaptation to reality, can be proofed wrong by practice. This is related with Epistemological Obstacles Theory in Education (Bartolotta, 1997). This feature is found in both cultures; chess and Wei-ch'i support it

5. Education to strategic and tactical features recognition could be important in social fields. It could become, as mentioned earlier, a useful tool to understanding different cultural approaches. We can imagine a more Hypothetic-deductive approach by western students, and a sharp separation between strategy and tactics. In eastern culture this separation is not so sharp. Probably this approach depend also on Wei-ch'ì meta-rules. These meta-rules maybe clarify the differences between strategy and tactics in a more subtle way with respect to western approach (Ajello, Spagnolo & Zhang Xiaogu, 2005).

6. The self-concept being separated from others as a reference for decisions and constant search for best solutions are useful topics in Education.

There is no doubt that the different conceptions of strategy and tactics between the Orient and the Occident have deep cultural roots and highlight a different approach to reality. The same course of history was influenced by these tactical and strategic conceptions and today different experts see, in the economic development of China and in its behaviour in the world market, a strategic approach similar to some concepts of Wei-ch'i). In any case, this lies outside of our aims of deepening the study of said connections. Also, the flourishing of chess in the occident and Wei-ch'i in the Orient, even if in different epochs, is not casual considering the nature of the two games. However, both can give great richness of themes and suggestions in the sphere of didactics and in the training of people.

The use itself of games of strategy, particularly chess, in a scholastic environment has been the subject of scientific research, as widely discussed in the following, and even of government interventions (a notable example is in France, in the municipality of Cannes). Certainly cultural diversities emerge more at the moment in which the necessity of a multicultural integration is posed in a class where there are students present who come from different nations and these thematic absolutely cannot be undervalued at any level. Chess displays a universal language that helps integration and communication among people from every country and social contexts. The opportunity of such integration, involving students in an intellectual activity, makes chess an advisable activity in scholastic context, as will be deepened in the following. Also, chess offers a wide range of links with Art and literature, as fairly described by Antonio Maestri in his Thesis work (Maestri, 2011).

1.b Historical Elements of Chess Thinking

Summary

In this section I present a historical review of chess thinking. All the phases of development of chess are summarized, quoting the major players and chess thinkers. It is highlighted the influence of social and cultural factors on chess thinking, coherently with the theoretical framework outlined in the introduction, i.e. conceiving the train of thought as a process historically and culturally based; in chess, the role of theoreticians and players may be compared to the roles of theoretician scientists and experimental ones. Without practice theory loses sense and vice versa. On the board, like in science and in sport, results are *tangible,* so novelties and new ideas or conceptions have to pass the hard test of competition. It is supposed a link between some new tendencies in Art and Literature and some new ideas in chess thinking.(Dadaism-Futurism/hypermoderns in chess) Some fundamental, epistemological aspects of chess emerged from this historical review, like the relevance of strategic thought connected with pattern recognition, a basic cognitive element psychologists very effectively studied. In today's chess, the use of modern technologies became fundamental, and this is the topic of the subsequent sections

Riassunto

In questa sezione presento una rivisitazione storica del pensiero scacchistico. Tutte le fasi dello sviluppo degli scacchi sono sintetizzate, citando i principali giocatori e pensatori scacchistici. E' stata evidenziata l'influenza di fattori sociali e culturali sul pensiero scacchistico, coerentemente con il quadro teorico delineato nell'introduzione, cioè la concezione che l'andamento del pensiero sia un processo basato storicamente e culturalmente, negli scacchi, il ruolo dei teorici e dei giocatori può essere comparato con quello degli scienziati teorici e di quelli sperimentali. Senza la pratica la teoria perde senso e viceversa. Sulla scacchiera, come nella scienza e nello sport, i risultati sono tangibili, così le novità e le nuove idee devono superare il difficile test della competizione. E' supposto un legame tra qualche nuova tendenza in Arte e letteratura e alcune nuove idee negli scacchi (dadaismo-Futurismo/scacchi ipermoderni). Alcuni aspetti epistemologici fondamentali degli scacchi emergono da questa rivisitazione storica, come la rilevanza del pensiero strategico connesso con il riconoscimento di configurazioni, un elemento cognitivo base che gli psicologi hanno utilmente studiato. Negli scacchi di oggi, l'uso delle moderne tecnologie è diventato fondamentale, e questo è l'argomento dei successivi paragrafi.

Chess is a very ancient game. Even if it is not in the scope of this work to deepen questions about the place(s) and time of its origin, it is useful to note that the most likely first appearance, in a similar form as we play it nowadays, is supposed to have occurred in India on 570 A.D. (chaturanga). The game spread quickly across Persia (shatranj) and Middle East, coming to Europe mainly via the Arab invasions, in the Iberian Peninsula first. We also know that ancient board games were played in China many centuries before Christ, as better specified hereinafter. Anyhow, we shall focus on modern chess, defined by standard rules as established nowadays over the world. According the approach outlined in the introduction, the train of thought is a process historically and culturally based. We believe that there is a sharp confirmation of it in chess thinking. In the course of time the style of playing changed dramatically, because of many reasons. There is no correspondence between “philogenesis” and “ontogenesis”, i.e. a beginner does not play as a XVI century player, but his/her style will (shall) depend most on his/her teachers and on chess environment (milieu) he/she attended. More generally, the chess thinking and the style of playing has been influenced by the styles of the most important players, and in general of the C*ulture* in which players, and chess lovers in general, were embedded. In this sense, it is relevant the role played by *tradition* and *communication*, i.e. frequently new ideas and achievements moved slowly by a lack of communication means. Still today it is not infrequent in Italy to observe amateurs playing according to old rules, abolished since centuries.

To better understand the reasons for changes in chess thinking, it is necessary to go into more depth. Changes in chess occurred because both players and theoreticians achieved higher knowledge levels compared to the past. The role of theoreticians and players may be compared to the roles of theoretician scientists and experimental ones. Without practice theory loses sense and vice versa. On the board, like in science and in sport, results are *tangible*. A player without results does not set any example to be imitated. This is the reason because the influence of the most important players, especially the world champions, is so relevant in chess thinking. As a consequence, our short review concerns most the great players, but does not ignore theoreticians and others. At the beginning of development of modern theory, in the XVII century, almost all people that gave contributions to the theory were players, but slowly there was a change. In fact the complexity and richness of the game stimulated some people to focus on a more restricted field of analysis, like basic endgames, producing extensive analysis without being competitive players. Until XVIII century there was not a general theory of chess, just some positions were accurately analyzed, and some rough principles were declared. The field in which the first general rules arose was the endgame. It occurred because in certain endgames it was simpler to calculate completely any possible development. In some endgames, a general rule was stated, often inductively, and to proof it by an exhaustive analysis was manageable. On the other hand, the intrinsic beauty and interest of chess is based mainly on the fact that chess is a finite game from a theoretical point of view (10 exp 50 possible positions, see in the following), but from a practical point of view this is not the case. As a consequence, the player, to make a choice, is unable to generate a complete analysis, therefore has to stop at a certain point and evaluate the reached position.

So, the required skills for a player are of a double nature: concrete move-by-move analysis and position evaluation. In modern chess, ability to calculate deeply move-by move is often called “tactical” ability, whilst the ability to bring the game to a target position, considered favorable by the player on the basis of holistic / pattern recognition mechanisms, is defined as “strategic” ability. These skills are not easily clear cut demarcated, and the distinction between them is not sharp, because the strategic skill influences tactic thinking and vice versa, as will be better explained hereinafter. The key practical elements the player has to consider to optimize the evaluation are both quantitative and qualitative. The quantitative element is the simple counting of the pieces remaining on the board, or in a particular sector of it. The setting of the qualitative elements, and their *specific Weight* goes somehow straight into the deepest nature of chess and is the driver for the change of style in playing chess in the course of time.

In the late Middle Age and in the first centuries of Modern Age chess was played most by nobles and in religious contexts, mainly in Italy and Iberian Peninsula. The first books and reports (Damiano, Ruy Lopez and Gioacchino Greco are well known) appeared, in which various games and positions were analyzed, and some elements were highlighted, but without the intent of creating a unified theory.

The XVI and XVII century’s style was characterized by a daring and fighting style, without care of material balance, trying to hunt out the enemy’s king by a rapid mobilization of the own forces, attacking the opponent’s defenses, straight to the goal; the checkmate.

The drawback of this approach is that against a good defender very often the attack could have extinguished and the attacker heavily punished. Nevertheless, in those times, the way you played a game, bravely and in the name of the honor, was more important to win it.

This approach can be explained by the fact that chess is what people think of, a metaphor of the war. In this sense the style of early players, and a long tradition of playing during the course of time, has been heavily influenced by the western conception of war.

> *the Greeks developed what has been called the Western way of war—a collision of soldiers on an open plain in a magnificent display of courage, skill, physical prowess, honor, and fair play, and a concomitant repugnance for decoy, ambush, sneak attacks, and the involvement of noncombatants*. (web references 3)

As we will deepen in a next section, the Eastern approach, the Chinese one in particular, is quite different.

Anyhow, at one point the importance of material balance became clear. In fact at the end of the XVII century, games analyses do report frequently this issue. Here the influence of the deepening of the basic endings can be identified, where the *crude quantity* of pieces is of the essence. In the XVIII century the *Scuola Modenese* flourished, with a consistent production of fine studies mainly about endings and openings. The *Modenese* scholars searched for general laws, but the first "general" theory was formulated by Francois-Andrè Danican Philidor. His book *Analyse du jeu des Échecs,* printed in 1749, became a milestone in the chess theory. Some basic principles are still valid, in particular the role of the pawn. It is the weakest piece on the board and as a consequence of it an exchange involving a pawn and a piece is disadvantageous for the player exchanging the piece. Therefore, the control on the squares exerted by the pawns is more effective with respect to the other pieces. This concept is really general, and it is also applicable in other fields (i.e. minimum resources employed to get maximum result). Besides, the role of the center of the board was outlined. Setting the pawns in the center of the board implies a sharp advantage. Philidor's most famous advice is"The *pawns are the soul of chess".* Some experts relate the relevance of pawns stated by Philidor to the political background in France during the XVIII century (pawns like "The Third Estate of Chess"), but this interpretation would require further support. Nevertheless, in the Philidor's work emerges the spirit of Enlightenment. A more rational approach arose, opposed to the old *heroic Italian style.* Again, of the influence of the dominant culture is felt in chess as well. In the course of XIX century, the first, historical chess clubs were founded, and also specialized magazines appeared, and international tournaments organized. So, the diffusion of knowledge increased, and a chess *community* arose. In the chess scene the XIX century is synonym for Romantic style of playing, meaning a brilliant and attacking style. A sort of coming back to *the Italian Style*. The German Adolf Anderssen was the most famous player of that period (1850 ca.). It was the preferred style for generations of chess amateurs, and some of the games by Anderssen gained the glory and named as "the Immortal" and "the Evergreen". These kind of games represent a sort of paradigm in the mind of the chess players, also because are frequently showed in chess courses for beginners as beautiful samples of chess mastery. The word *romantic* is used in the context of Art and Literature of the XIX century (*Romanticism*), and in chess it can be described by two features. First, the importance of creativity. The player has a sort of duty/pleasure in searching in the game for somewhat creative move sequence. Secondly, material sacrifices frequently appeared, according to an approach in which the matter has much less importance compared with ideas and concepts.

These concepts have to be taken with caution. The influence of the artistic and literary trend is definitely a fact, as well as the features described above. But Anderssen playing was very different with respect to the XVI century players' style. Anderssen attacks were well calculated against *any* defence, and he was able to conceive subtle manoeuvres to better place his forces.

We would say that the end of the XVIII Century saw chess expressing a form of 'calculated romanticism', where the initial signs of modern thinking were already in there; on the other hand, the french revolution had already occurred, and the signs of it were spread throughout Europe.

A very particular role in the history of chess, and even more so in chess thinking, is played by Paul Morphy. The short career of the American was amazing and abrupt. *Child prodigy*, in a few years (1857-1859) he defeated in America and Europe the best players of that period. The Morphy style was characterized by a complete sight of the board, and by the understanding of the importance of tempo in the development of the forces; in general he reached full understanding that every action in chess requires sound basis and execution speed to start with. Almost always, his attacks and brilliant moves had deep reasons founded on gaining time advantage in mobilizing his pieces, a concept largely under evaluated at that time. This approach, together with his extraordinary analytic skill, and his direct, energetic and fresh style, made him a really far ahead player with respect to his contemporaries.

Morphy did not produce any theoretical, general work, but in his direction moved Wilhelm Steinitz, who became the first world champion in 1886.

He begun as all-attacking player, but from the '70 of the XIX century he developed a new style, the *positional* style, posing the basis for modern chess. Not in contrast with Morphy, Steinitz proceeded with a complete analysis, stating some important principles. He searched for the basic laws of chess, in such a way that is possible to compare his work as the turning point from romantic period to the *positivistic* one, and in general the point in which chess became *a scientific* object of inquires. I believe that Steinitz's approach is in accordance with the scientific *allure* of the period, including the unconditioned faith in the humans' possibility to discover objectives and deterministic nature's laws.

More in detail, let us consider some of Steinitz's theoretical statements.

First, chess is a game of equilibrium, i.e, the game has an intrinsic equilibrium that players may try to change. In general, White, moving first, has the duty/right to take and keep initiative from the opening.

To try to unbalance the game, the player has to perform an analysis of the structure of the position, considering strong and weak points of the own and opponent's position. This evaluation deals with the *positional elements* as pieces' activity, weak squares, open columns, king's position, centre's structure and so on. To create weaknesses in the opponent's position, and to improve his/her position, the player conceives specific manoeuvres. So, the play can assume a slow trend, and the closer target may not be the checkmate, but just a small, positional or material advantage.

Some people, according to a sort *of code of honour* of playing, as quoted in the former, still criticized the Austrian chess player, calling his style “coward”, but results and general appreciation show that Steinitz was moving toward the right direction, slowly to the goal. It is the modern chess thinking, and it is noteworthy that concepts such as ‘reversibility’ in thermodynamics, and the mass balance in chemistry are somehow similar, meaning that one of this world principles is that ‘equilibrium’ is a key concept and the description of the world is a matter of describing how it moves from an equilibrium to another. From this point of view, Steinitz was slightly ahead of his own culture.

The development of positional elements is due also to the introduction of the game time limit through appropriate clock systems.

In the course of XIX century, the introduction of the clock to set a time limit for the games, and in general the setting of the maximum time for every player, depending on the typology of the tournament, played a fundamental role in the changing of style. In fact players cannot count on endless time, and as a consequence of it, they have to be more ‘pragmatic’ in searching for the best move. Between XIX and XX century emerged the figures of Siegbert Tarrasch and Emanuel Lasker. They were proud rivals, with a bad reputation of each other. The battle between them was on the board but also on chess concept. Tarrasch, medical doctor by profession, somewhat representing XIX century chess, continued along the Steinitz’s path, and deepened strategic concepts. He was a prolific writer, and gave great contribution to the diffusion of modern ideas to the average chess people. He was called *Praeceptor Germaniae.* He was criticized for the high level of dogmatism in his opinions, in a period (the end of XIX century and beginning of the XX) in which several certainness begun to fail, and it was not by case that a man like Emanuel Lasker challenged him. Lasker became world champion in 1896, defeating Steinitz. Lasker somewhat represents the rising century. He also was a fine mathematician and philosopher, and a friend of Einstein. His conception of chess is quite *relativistic* with respect to the *absolute* ones of his predecessors. For Lasker, chess is a fight, a confrontation between minds. Not the *best* move, but the *most troublesome* to the opponent has to be played. Hence, Lasker employed Psychology, i.e. knowledge of the opponent, individuating his weaknesses and as a consequence of it choosing the best strategy. Besides, Lasker generalized his concept of fight, the “machia” (from Greek), as a general philosophy, as widely discussed in his most famous philosophical work. This conception became quickly a powerful weapon for competitive players, and nowadays this aspect is very important, as will be showed in the following. Chess is a competition between persons, and this has psychological and educational consequences. This aspect has not to be forgotten when chess is introduced in scholastic and educational contexts.

After World War I, the most significant ideas in chess came from the Hypermodern School, meaning a group of players and theoreticians like Nimzowitsch, Reti and Tartakower, all from the Central Europe Their ideas, paradoxical in some aspects, were connected with new tendencies in Art and Literature like Dadaism and Futurisme. Like Dadaists, the Hypermoderns challenged the orthodoxy, in particulat about the way to play for the center. They also stated new, fantastic concepts like superprotection, block, outpost (Nimzowitch, 1975). I guess that these concepts are also available in other fields, e.g. superprotection. If we have to check a weak point, if we employ one of our resources, we block this resource. If we use two or more (*Superprotection),* after we can employ for other purposes *all* these resources, depending on which I need, because at least another still remain checking our weak point. This is applicable in various fields like military matters, economics, and so on. The Hypermoderns critiziced the dogmatism in chess, searching for the exceptions to the principles stated by Tarrasch and Steinitz, representing the Orthodoxy. The influence of the culture of that period for such a movement in chess shows out really impressively. In the second and third decades of the XX century arose the stars of Josè Raul Capablanca, the *Mozart of chess*, and Aleksandr Aljechin, The *Beethoven of Chess* . They were fantastic players, known over the world and still heroes for the chess people nowadays. Capablanca showed an incredible, apparently simplistic style. Very talented, he almost did not study theory, his playing flowed spontaneously, in a harmonic way. Like Mozart's music, Capablanca did not lose a game for almost 10 years, in the World War I period, winning by playing 'simple' moves that, in reality, were made possible by his profound, albeit illiterate, knowledge in endgames; even the smallest, apparently negligible advantage in an endgame allowed him to win the game. He was not entirely dedicated to chess, perhaps the last great player who did not pay an enormous price in terms of hard work to become a world-class player. On the contrary, Aljechin worked very hard to reach the world title, putting together a great passion and strong willing. He charged himself of an enormous analytical work, and created everlasting masterworks. He used to say "to beat me, they have to beat me three times: in opening, in middle game, and in endgame. From about the 1930, a new fact happened, and changed the chess history: in The Soviet Union, the authorities begun to consider chess as an important activity for Russian people, establishing structures and procedures to diffuse chess among the Country, to improve general level, and to foster talents. In this atmosphere, shortly Soviet players' level reached the highest levels. The most important player from the Soviet school, and in my opinion fundamental figure in chess, was Mikhail Botvinnik. He became world champion in 1948. Botvinnik, perfectly integrated in the Soviet system, applied a scientific approach to chess. He was also a high level electrical Engineer, and was the pioneer of AI in chess, as deepened in a next section. First, as a player, he applied entirely the Lasker's approach, studying in depth games and styles of his various opponents, individuating weaknesses and preparing accurately the choice of the openings, at best for a specific opponent. Second, he understood the importance of psychological and physical preparation to the main chess events. He used to move to an isolated *dacia* before matches, where he trained himself with chess and physical practice like swimming or long walks. These concepts were far ahead with respect to his contemporaries. Nowadays all top players train themselves with physical preparation, because the benefit of general fitness and mind performance is recognised. The study of the style of the opponents is usual practice now, too, especially with the aid of computers and web databases. But even today the average chess player does not entirely believe in the importance of mind and physical fitness for better performing in chess. Also, Botvinnik founded his famous school, attended by many top soviet and Russian players for as long as twenty years. Karpov, Kasparov and Kramnik, all future world champions, are some of the most famous Botvinnik's students. Botvinnik understood that in modern chess talent is not enough, as he believed in engagement and organization, promoting in his school a hard but exciting work, stimulated by expert and motivating teachers. It was the key for success. Botvinnik and soviet school established the basis for contemporary chess. From a technical point of view, soviet players since 1950's improved dynamic style, and fostered technical skills. Nevertheless, among them arose some legendary players, famous for brilliancy and creativity like Keres, Bronstein, Smyslov, Tal, Petrosjan, Spassky. Russia dominated along forty years and only a single man succeeded against such a Country team effort: Bobby Fischer.

The American left school when he was young, engaging himself in a titanic, almost lonely work. He learnt by himself several languages including Russian to better read chess books, and quickly reached the world class, becoming Grand Master at the age of fourteen. A fantastic talent, an enormous passion together with a strong will, IQ rated 190, allowed Fischer to become world champion in 1972 defeating Spassky in *the match of the century.* Mass media and politics charged that match with political meaning, depicting Fischer representing the free world and the free market against the Soviet system.

Fischer’s bizarre and often misanthropic character stoked up the legend. Fischer was seen as the unpredictable genius fighting against a whole iper-technical system, from a technical point of view this statement is incorrect, because Fischer’s style was also very technical and he learnt a lot from soviet players ‘games. Besides, Spassky is a quite creative player. But these reasons were not relevant for media. What is more interesting for our purposes is *the querelle* concerning the Fischer’s way to reach the highest level without a relevant, external teaching and coaching. Perhaps the environment, the dominant culture, is not so important? I still believe not. Fischer, as quoted in the former, studied in a very deep way the contemporary best players’ game, especially Russians. He participated in the most important international tournaments, confronting himself with other players and ideas which embedded entirely his way of thinking.. Paradoxically as it may appear, but his style was a result of the Soviet school, too, as it is demonstrated by the fact that he did not refuse to play the most advanced opening choices found out by Soviet theorists; simply, his superior mind was able to introduce theoretical novelties at some key point, therefore reverting the weapon against his creator. It is interesting to note that Fischer retired after the conquest of the world title, isolating himself from the rest of the world, coming back only after more than twenty years for a revenge match with Spassky. He won too, but the game level, according to the experts, is not comparable with his old standard. Surely he was twenty years older, but the lack of a continuous practice in world-class tournaments played a fundamental role in the bad quality of his games. In this sense we find the role of Culture.

He may have suffered some kind of mind disease in the late period of his life, but certainly it appears interesting that, in the century of the ‘uncertainty’ (quantum mechanics, Woodstock and crisis of conventional values) the best chess player loses his path to ‘normality’ and comes out with bizarre theories on the world control by Jewish and on US arrogance.

It is also interesting to note that Fischer was the inventor of a new chess clock model, giving a time bonus after every move that is the standard today.

During the ‘70’s of the XX century, Russian school continued to dominate the scene, so Anatoly Karpov retained the world title for ten years. Karpov is a highly sophisticated player, skilled in strategic field but also author of wonderful attacking games. Like all the contemporary players, and along the tradition initiated by Aljechin, Karpov is very prepared to face all the phases of the game. This aspect is very interesting for my work, because it deals with the utilisation of resources in a task that is a topic closely connected with modern education and with the concepts of cognitive and metacognitive skills that will be deepened in the following. In the 80’s, from a Russian-jewish family living in Azerbaijan, arose the star of

Garry Kasparov. *Child prodigy*, enormously talented he reached the title of Grandmaster at 14, like Bobby Fischer, and through a hard work in Botvinnik's school and participation to world-class tournaments, became contender for the world title in 1984, when he was 21. The chess tradition, and the external perception of chess, reports a paradigm in which the top players are adult, not so young. Chess is frequently twinned with Intelligence, in particular with analytical and deductive thinking, and as consequence of it, a very young player becoming world champion could seem somewhat surprising, but, it is not, as the similarity with the young age of mathematicians at the top of their creative career is clear.

Here we have another sign of the culture: chess, like analytic science, nowadays require young talented minds to absorb the mass of existing knowledge and to create on the 'shoulder of the giants'.

Again, time is of the essence, and nobody can reasonably think to become a strong chess player when more than twenty years old.

Kasparov, at the age of 22, after a hard fighting with Karpov, became world champion. Kasparov, in the tradition of Aljechin, played with a fantastic, amazing creative energy, and these qualities, together with top-class technique and a astonishing memory, make him a good candidate to be the strongest chess player ever. After reigning about 15 years, Kasparov retired from competitions, becoming a chess writer and a political man, involved in opposition against the current Russian government. He heavily influenced the chess world, founding the professional's syndicate, and also proposing new possible variations to the *immutable* rules of chess, i.e. the *Advanced Chess*, in which the player can use a computer support. Advanced Chess is the father of Freestyle that will be discussed in depth in a next section. Also in this case, the Purists cannot conceive such a variation, but this is the destiny of every proposal modifying the *status quo*. Today all players, from amateurs to professionals, use computers to prepare themselves for tournaments. In fact wide databases are available on line, in which is possible to find games of everyone. It is the today *Culture.*

Also, computers and on-line platforms are widely adopted for chess teaching-learning. It is a very interesting subject to evaluate the effects and possible advantages and/or disadvantages of using computer for educational purposes, which will be dealt with later in this work.

1.c The role of Artificial intelligence and computer Science in chess

Summary

In this section I am going to deal with connections and interchanges between chess and Artificial Intelligence - Computer Science. Chess is a sort of ideal arena in which testing theories and algorithms, it because of several reasons: Firstly, chess is a rational game with simple rules, but its complexity is exponential; after, expertise in chess is sharply rated (Elo rating system). In chess there is a large literature available and many professionals players; besides, there is a big potential for software market and sponsorships. Last but not least, chess is a complex game based on choices (also metaphor of war), and many techniques working in chess are used also in different disciplines (expert systems, theorems proofing, automatic learning, etc.). Cooperation between chess and AI acted in both directions, allowing important theoretical advances in AI, obtained by famous scientists like Shannon and Von Neumann, and improving chess knowledge at every level. Today modern technologies are fundamental in chess.

Riassunto

In questa sezione mi occuperò delle connessioni e interscambi tra scacchi ed Intelligenza Artificiale- Computer Science. Gli scacchi sono una specie di terreno ideale nel quale testare teorie e algoritmi, e questo per diverse ragioni: Per prima cosa, gli scacchi sono un gioco razionale con regole semplici, ma con complessità esponenziale; poi, l'abilità negli scacchi è facilmente classificabile (sistema di classificazione ELO). Esiste una vasta letteratura disponibile negli scacchi, e molti giocatori professionisti. Inoltre, c'è un grande potenziale di mercato per i software e per sponsorizzazioni. In ultimo, ma non per importanza, gli scacchi sono un gioco complesso basato sulle scelte (anche metafora della Guerra), e molte delle tecniche utilizzate negli scacchi sono usate anche in alter discipline (sistemi esperti, dimostrazioni di teoremi, apprendimento automatico. La collaborazione tra scacchi e AI ha agito in entrambe le direzioni, consentendo importanti progressi teorici in AI, ottenuti da famosi scienziati come Shannon e Von Neumann, e migliorando la conoscenza scacchistica a tutti i livelli. Oggi le moderne tecnologie sono fondamentali negli scacchi.

Chess is a game between two players, and it is at the same time, competition, fight, science, sport and creativity.

During the Ages, the cultural meaning and the approach to the game changed, and the rules changed as a consequence of it, too. Several individuals deepened these aspects, with sharp and meaningful references to Psychology, Psychoanalysis, and Social sciences (Leoncini, 2010). Frequently chess was seen as metaphor of the war, and approaching chess was like approaching a war. War is conceived in various forms, depending on historical period and populations; interestingly, we note that the Christian people in the middle Age used to interpret chess as "no more a representation of war fight by opposite armies, but slowly a symbol of sophisticated education, indispensable tool for every noble man who wants to promote himself in civil and military society” (Leoncini 2010, my translation). As mentioned earlier, chess became the intellectual game *per antonomasia*, representing the highest level of abstraction of the human thinking, then the interest in chess of scientific world arose, in particular by the scientists using quantitative approaches. In fact intelligence is very hard to define, and chess offers a wonderful domain to explore. Philosophical and technical studies on intelligence are old as the human race. Artificial Intelligence arose as a discipline in the XX century, but we know that constructing a *thinking machine* was a dream for many people, from Raimondo Lullo’s *Ars combinatoria* to the various *proofing machines* realized since XIII century. About chess, the most famous machine before XX century was “The Turk”, a fake automaton realized in 1769 by Wolfgang Von Kempelen, an automaton maker and later carried to North America by Maelzel. Inside the automaton stayed, well hidden, a strong chess player. The hoax was very effective, surprising people during exhibitions, and in the main European courts. “The Turk” defeated many leaders of that time, such as Frederic the Great and Napoleon.

What is somehow interesting in relation to the subject of this work is the debate developed on it, involving Edgar Allan Poe. Poe wrote an essay named “Maelzel’s chess player” (Poe 1836), in which he reached the right conclusions, i.e. a chess player was hidden in The Turk, but his theory was not convincing at all (Cardellicchio 2002). Poe never accessed the machine, and his hypotheses were based on the following elements: The first element deals with the size of the automaton, and the ritual adopted by Maelzel, which could lead to the right conclusion, but is not a proof.. After, Poe considered the most important automata realized ever, including the last one, the famous Babbage’s calculator. Poe tried to proof that no machine can play chess. According to Poe, in normal calculation we have input data and a unique, determined solution resulting by them. A calculating machine has a mechanism for determining solutions, but in chess, Poe claimed, the solution does not come straight from the analysis of the situation on the board, because of the ‘absolute’ uncertainty related to the best move to play. Poe argues that even the best chess player sometimes does not agree about the best move to play and therefore, it would be impossible to set up a machine to play chess. As a confirmation of it, the Turk did not win all games, so, according to Poe, it means that the Turk is not a machine, because a suitable machine finds the *solutions, always*. I remark these last words, sharply showing the Culture of XIX century on reasoning and machines in a deterministic manner. In the XX century an extensive development of both theoretical and technical aspects on Artificial Intelligence took place and chess played an important role, because of the following topics (web references 6):

• Metaphor of the war and conflict situations
• Rules are simple but the game complexity is exponential
• A lot of literature available
• Many humans chess professionals
• An objective rating system for chess strength evaluation
• Potential for software market
• Potential for sponsorship (IBM, Intel)
• Many techniques working in chess are used also in different disciplines (expert systems, theorems proofing, automatic learning, etc.)

The first relevant theoretical result in Theory of Games was by Zermelo (1913), who proofed that in a game like chess, in every position, assuming players acting in a rational way then exists a determined conclusion, i.e., white or black wins, or draw. It seems trivial, but it is not the case. It is a clear refutation of Poe's hypotheses, meaning that there is not (theoretical) uncertainty on the best move. In parallel (1914) Torres y Quevedo succeeded to implement a simple machine able to find the checkmate in a basic ending (King+rook vs. King). Theory of Games grew largely during the XX century, involving concepts like *decision and choice*. It had remarkable applications in Economics, in Politics, in Social sciences, and in general in all disciplines involving rational choices. The milestone in this field is the work of Von Neumann and Morgenstern (Von Neumann & Morgenstern, 1944). This work was of the essence not only for his high-level content, but also because supplied concrete proofing about the importance of mathematics with respect to *real world*, in opposite with a common, simplistic approach to modern mathematics. Besides, the inquires by Von Neumann and Morgenstern brought light in the dark representing the human thinking and decisions, often represented as unpredictable and depending on psychological factors. The basic idea by Von Neumann and Morgenstern is that economic and political situations can be simulated by a game. As a consequence, theoretical researches on chess algorithms developed a lot after World War II. In fact, another champion of science, Claude E. Shannon got interest in chess, publishing a paper named "Programming computer for playing chess" (Cardellicchio, 2002) Shannon faced the chess player's choice problem using cybernetic concepts like tree structures, first developed by Von Neumann and Morgenstern in their studies on Games of perfect information, in which every leaf is associated to a numerical value representing the rating of the resulting position. (Fig. 1)

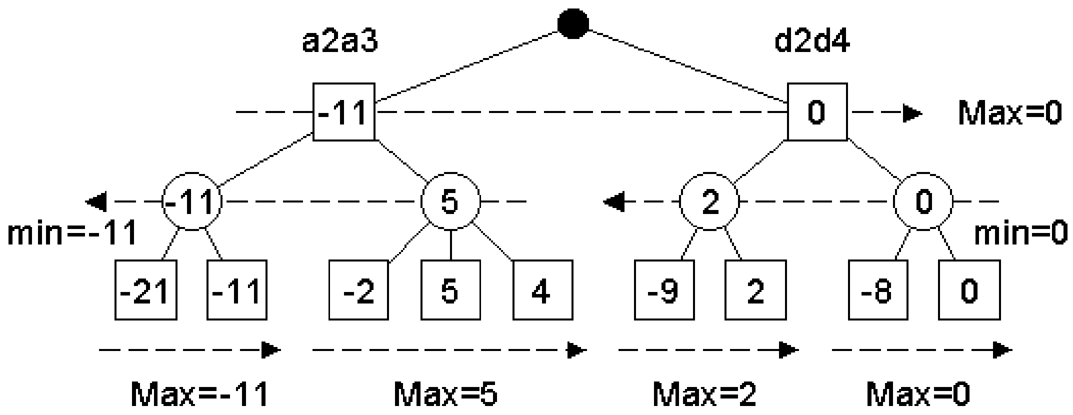

Fig. 1

Every ply (identifiable with horizontal dot-lines) represents a new position occurred.
Every move played leads to the ply below. In this simplified example, the possible choices are just two. Of course every (rational) player will choice the move containing the best rating for the own side. Using white as reference, white has to maximize the value, black to minimize. The value in every leaf is assigned considering the values of the lower leaves, and the upper leaf assumes the maximum value if is white to play, the minimum otherwise. The analysis starts with the deepest ply, giving values to the leaves according to an evaluation function, and these values determine the values to the upper ply, and so on step by step to the top. This is the chess implementation of the well known MINIMAX algorithm, discovered by Von Neumann and representing a basic concept in Theory of Games. MINIMAX is *adopted, consciously or not, by every chess player* (Ciancarini, 1992). It introduces two important topics: the first one is the *combinatory explosion*, i.e. in a given position the alternatives are normally more than two, and for every ply the number of leaves grow exponentially. Calculations made show in 10^{50} the number of possible *different* chess games, eliminating redundancies and repetitions. On average, a chess game lasts about 40 moves (80 semi-moves, 80 plies), and for every move there is an average of 33 alternatives, then the complete analysis of a tree representing a game requires the analysis (on average) of 33^{88} positions (leaves). Eliminating repetitions and redundancies, the estimated number of different chess games is about 10^{50}, anyway enormous (Allis, 1994). It implies that the exhaustive analysis of a chess game cannot be completed by computers by now in a reasonable time. In fact a good CPU makes about 3 billion operations per second (3 GHz), a rough estimation gives more than 10^{40} seconds, i.e. about 1034 years to evaluate all possible moves in a chess game. Parallelization of computing, developing nowadays more and more, is lowering this time, but by now a complete, exhaustive analysis of chess is not realizable. So, computers and humans

have the same problem: they are not able to calculate exhaustively the variations, and they have to cut the tree at a certain point. It implies an evaluation, and it introduces the second topic: the problem to assess what value has to be inserted in a leaf. At a first glance it seems a chess expert dedicated problem, but anyway the translation of chess contents from chess language to numbers (i.e. values comparable by the machine) implies the construction of a function in which the independent variables are the features of position, and the output is a real number. Besides, every chess feature itself has to be translated in quantities to insert in the function. So, the dominion of such a function becomes a n-dimensional real space. Hence, from the 50’s of XX century, a strong collaboration among hardware designers, AI experts and chess experts, commenced and it is still working.

It is worth to remark that this kind of problem applies in general in human activities, and the interest of the major AI scientists in chess is due to this generality. Besides Shannon, other outstanding scientists like Von Neumann and Turing were interested in chess. In fact, some features of chess, as mentioned above, allow concrete testing of the improvements in calculation methods (AI), hardware architecture (HW experts), and evaluation functions (chess experts). First, computers used a *brute-force* (blind expansion) approach, i.e. an analysis (visit of a tree-structure) the deeper the more computing power was available. It was called by Shannon approach “A”.

This approach has evident failures, because of lack of any “strategy” in cutting the tree, and also because of non-optimal use of the computing power, as will be detailed in the following. To avoid an early explosion in the tree, people begun to adopt heuristics, i.e. general rules driving the choice of the alternatives, and reducing the alternatives to the reasonable ones (approach “B”). A remarkable progress in *pure calculations* was occurred by introducing pruning of the tree, the ALFA-BETA algorithm (Allis, 1994), as illustrated in (Fig. 2):

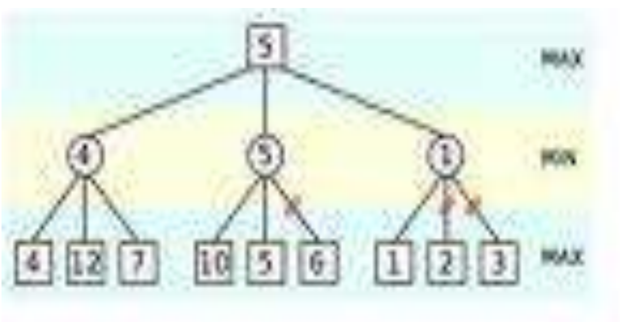

Fig. 2

Analysis starts from the left, in the third ply, the part of the tree developing from the right leaves (values inside 2 and 3) is useless, because in any case you shall obtain a worst value with respect to another one computed already. In other words, it is useless to analyze a position descending from a move that your opponent will not play at all, because a better move is there. The words alpha and beta represent the two types of cuts (for lower values or higher); of course if in a ply an alpha cut occur, in the next ply only a beta cut is possible. The gain depends on specific game, but on average allows to halve the exponent in the number of leaves (positions) to explore (see reference for wider explanation). This procedure was described first by Newell, Shaw and Simon (Newell et al. 1955), and is very effective. Despite these improvements, computer's performances remained poor for decades. It depended both on non-refined evaluation functions and on poor computing power. To support the machine, entire books on openings were memorized in the machines, avoiding possible errors in the opening, and the evaluation function were refined more and more by collaboration among scientists and chess experts, as in the case of Russian project "pioneer", conducted by Mikhail Botvinnik, former world champion and an electric engineer. A standard euristic item is based on "killer move", i.e. if a move is very good in a given position, there is a high probability that it will occur also in similar positions, very near in the tree. So, it is very convenient to consider first this move to enhance the probability of alfa–beta pruning, diminishing the time consumed. The tree is "ordered" at the best to maximize the efficiency. The most typical blunders done by the old machines were generated mainly by bad cutting, e.g. consider a change of queens: white captures first, and if the cut is done just after the white's move, the evaluation function outputs an enormous advantage for white. Computer does not see the next move! A sharp case of *blind* expansion! To prevent it, techniques aiming to avoid cuts in non-quiet positions were developed. The *Deep Blue* team adopted a new method, called "singular extension visit". Evaluation function's values are considered, comparing the value of a leaf to the generating leaf. If the difference is high, it is clear that the position is not quiet, so it will be necessary to deepen the visit, and so on, till reaching a quiet situation. Using this approach, the machine shall never cut the tree at the wrong moment, preventing disasters. This procedure is called *Singular extension*, because the visit can become very deep for a given path representing a crucial variation (Anantharaman, Murray & Feng-Hsiung, 1988). This concept was adopted by the well known IBM program *Deep Blue*, that challenged the former world champion Kasparov, defeating him in the second, dramatic match held in Ney York in 1997. That match represented a milestone in AI history: a machine defeated the chess world champion! The humanity can claim no more a sort of superiority on machines, not even in a game in which pure calculations are not enough to reach mastery! The subsequent developments dealt on various topics: first, the computing power, by introducing parallel computations, more powerful processors and more efficient algorithms; second, more and more chess knowledge was supplied to machines, both to refine evaluation function and to improve heuristic selection of the alternatives. Dedicated machines were built, obtaining a sharp enhancement of machines' strength of playing. Thousands of chess positions were well assessed and became *weapons* of the machines, most in endings. In summary, nor A neither B approach prevailed, instead a mix of them. For our purposes, a very fascinating field of research is the ideation and implementation of pure knowledge-based software. It is a pure B approach, strongly linked with learning and cognitive topics. Various approaches were developed, most based on cognitive items. As shown later, items like pattern, chunk, and template emerge from the analysis of human chess thinking developed by well known Cognitive Psychologists like De Groot, Gobet, Simon and others, as will be dealt in chapter 2. These concepts were applied in the planning of software, and programs like MACH, MORPH, TAL, SUPREM, PAL were produced (Di Sario, 2002), giving interesting results mostly as supporting software, like in the famous Hitech by H.Berliner. These models help machines in recognition of well known patterns, influencing and improving their choices. Also, they tried to *teach* the machine to *learn,* i.e. using previous outputs to better planning the future, in the sense of suggesting plausible moves, or in some cases, to avoid transpositions, i.e. analyze twice the same position. (Hash tables).

Besides, several scientists deal with simulation of cognitive processes to implement chess software; they started from fundamental psychological studies in which the processes of human chess thinking were described.

From important theories like *Chunking Theory, Template theory, Mind's eye,* and others, emerged several interesting cognitive models like CHREST and others (Di Sario, 2002); they are process models simulating human approach to chess (see reference for a deeper details). Modern chess machines use a combination of the techniques quoted above, a powerful mix of brute force and very subtle heuristics. During the years machines reached and overcame the top class Grandmasters. Endings with less than 5 pieces on the board are exhaustively analyzed. Nowadays, the PC is an irreplaceable companion for every tournament-chess player, fundamental for opening preparation and very useful to analyze games.

Having said that, surprisingly enough, at the moment, the best player in the world is nor man or machine, is a *Centaur*.

1.d The Freestyle in chess: a sample of AI evolution

Summary

In this section it is presented a new format of chess, the freestyle, i.e. chess with any help. This kind of playing chess arose some years ago, in the 2000's, and some of its features resulted of interest for my work. Freestyle is played on-line, and the reflection time to finish the game allowed for every player is one hour (in standard tournaments every player has 2 hours or more). The early freestyle tournaments yielded shocking results, because not the best players and not the most powerful software won, but non-professional people! This people reached a full integration man-machine, becoming modern centaurs. The reasons of it are deepened in the section, using a former work of mine as reference point.

I think that the topics outlined below are interesting in chess and in today Education, most for the careful use of statistics and for a responsible use of machines (hardware and/or software) in making choices. In fact the use of multimedia means increased more and more in Education, and we all are candidates to become centaurs as well

Riassunto

In questa sezione si presenta un nuovo tipo di scacchi, il freestyle, cioè gli scacchi con qualunque ausilio. Questo modo di giocare è sorto alcuni anni fa, negli anni 2000, e alcune delle sue caratteristiche sono d'interesse per il mio lavoro. Il freestyle è giocato on-line, con un tempo di riflessione di circa un'ora per giocatore (nei tornei standard ogni giocatore ha 2 ore o più). I primi tornei di freestyle risultarono scioccanti, perché non vinsero né i migliori giocatori, né i più potenti software, bensì non professionisti! Queste persone raggiunsero una piena integrazione uomo-macchina, divenendo moderni centauri. Le ragioni di questo sono approfondite in questa sezione, dove uso un mio precedente lavoro come punto di riferimento. Penso che gli argomenti delineati nel seguito siano interessanti nell'ambito degli scacchi ed in Didattica, particolarmente per l'uso della statistica e per l'uso responsabile delle macchine (software e/o hardware) per prendere decisioni. Infatti l'uso di mezzi multimediali aumenta sempre di più in Didattica, e tutti noi siamo candidati parimenti a diventare centauri.

In the previous section the role of artificial play was highlighted, with particular reference to the support to the player intensively using software and hardware commercially available. The higher is the level, the more unavoidable is the use of the computers. Computers' computational power increases more and more, and parallel computing will allow further improvements. Besides, the most refined analysis engines have working features somewhat comparable with humans. In fact they retain into a non-volatile memory some information classified as important, and this file increases more and more as the software works through.

This information is utilized for a better evaluation when a position similar to the one stored is reached on the board. Also in choosing openings this software uses statistics of its own games. These features seem to diminish the distance between man and machine.

In current market plenty of smart chess software is available, and more is required. Also e-books, educational CD, e-platforms and wide databases are currently available. But the most revolutionary item for modern chess is the web. Internet changed our lives, information is available almost immediately, contacts are in real time, and distances virtually vanish.

Chess is very supported by the net. The basic point is that chess is a logic game, then its nature is not altered if players are physically distant each other. This allows a wide opportunity to compete, and exchange of experiences, studies and so on, because of high speed to collect and change information with respect to about fifteen years ago. Nowadays a lot of on-line playing platforms operate worldwide, as well as teaching/learning chess-dedicated sites. Courses, lessons on-line and remote tutoring spread over the world. Today we can watch live on the net the most important national and international tournaments.

As a consequence of it, hit counters of chess sites give high values. On average, every time in a day there are hundreds of thousands playing chess on-line, e.g. the most accessed Italian chess site counts in the evening more than 2000 users. Note that using software is normally forbidden, and very refined programmes can check players about it. The net became a familiar environment for chess players, in which they play, study, research, and so on. New ideas and proposals are continuously tested and commented. Official chess can ignore no more such a *scenario.* Kasparov already became aware of it, and proposed the *Advanced Chess*, i.e. a normal game between humans with a *codified help* by a computer. Every player uses her/his own PC. *Codified help* means that using PC during the game is allowed in pre-determined modes only. *Advanced chess* did not explode, maybe because of the birth, almost in the same period (about 2005), of *Freestyle*. Freestyle is chess with **any** help. It means that any help by machines, or by humans, or both, is allowed in a total freedom-like fashion. Tournaments are played on line, with short thinking time allowed to the players (very important feature). Results from early freestyle tournaments were shocking. Why? Because not the best players, and not the most powerful software won, but non-professional people! From (web references 7):

> *in the first Freestyle tournament in 2005 not the favoured Russian GM's asserted themselves, but two chess computer freaks from the US with the legendary handle ZackS. This sensation was worth even Garry Kasparov a column which he put in NewInChess under the futuristic title "Chess 2.0*

As normal players, they are just good *amateurs*, but in freestyle they represent a sort of centaurs, half man and half machine, using at the best their *double* nature. The reasons of this success are, in my opinion, very interesting, and matter of investigation involving various fields, including maths and science education.

Going more in depth about the *modus operandi* of the centaur, it is noteworthy that he/she has plenty of software available, the best on the market. But software are different each other. A strong Grandmaster knows the game, while a strong freestyler knows how software works at best.

As reported in a my interview with a well-known world-class *centaur,* nicknamed *Spaghetti chess* (D'Eredità, 2009), frequently to be a strong player is counterproductive. It especially if the player has not a deep knowledge of chess engines' strong and weak points, risking a conflict among machines. Freestyler's action is not using computers just to avoid tactical blunders, like in the early Advanced Chess tournaments, is much more. An expert freestyler is able to use a complex interaction system composed by human and artificial resources, decisional processes, and managing information.

He does not execute a pre-ordered hierarchy of thoughts, but a flexible one assuming more or less importance depending on the concrete position on the board, and on opponent and time remaining. A professional player, normally, tends to *use* machines, to *drive* them. Instead, in *the Freestyle, one has to enter into a grey area in which it is no longer clear who would be the horse or the rider in the course of a chess game* (web references 7). Really very interesting! This kind of approach probably will become common more and more in our future and a precise awareness of it can be very important in Education. An expert centaur uses databases containing millions of games, and also table bases (databases of thousands of similar positions) of endings. In the case in which there are less than six pieces on the board, table bases are exhaustive and the game is over, i.e. exists (and it is known) a perfect move sequence conducting to the end. Although statistics is an useful weapon for centaurs, they have a great care in using it. Using the words of *Spaghetti chess*:

> *Statistics sometimes lie! If you trust in it, you have to check games one by one, i.e.....don't use it! It is hard to explain without a specific, technical insight, but I try… imagine I play as white, and in a given position I found in my database 200 games, with 2 reasonable moves (A and B), statistically equally distributed. Move A shows a winning percentage for white of 75%, while B shows a 60%. According these percentages "A" seems preferable, but going forward by one step (ply), I discover that black countermoves to "A" are "C" (75 wins for white) and "D" (25 losses). So, if I, confident in statistics, play "A", my opponent plays "D" and .I am lost. Maybe I would be better satisfied by 60 of "B"* (D'Eredità, 2009).

In my opinion, these words are a shining example about caution in using statistics. Statistics may lead to wrong decisions if used without care. Centaurs use statistics when they meet a critical situation, when they miss sharp reference points. In fact statistics frequently supply much summarized information, losing the specificity, and the quality of information itself. In Education, most in scientific and technological field, often statistics is invoked to make right choices and decisions. More awareness of potential and limits of statistics would be requested. Besides, centaurs point out that using software performances statistics is dangerous. In fact, statistics can answer to general questions like: how many wins, how many draws or losses scored a given programme, but can seldom tell us who has the better position, or which are the weak points, when is better to be aggressive and when cautious, and so on. Also in this case we loss the quality of information. Another amazing point of interest is about power calculation and work in team. A spontaneous thought is that the more large is the team (or processors available) the more powerful is the centaur, but it is not exactly the case. To gain a semi-move (ply) the centaur has to spend the double of the elapsed time to reach the previous ply as explained when we took into account the tree of variations (combinatorial explosion). On this, freestylers give an unanimous view (D'Eredità, 2009). It implies a double calculation power employed, but a ply is a poor gain, taking into account all the freestyler's *tricks of the trade,* who knows weak and strong point of software, and knows *when and how* to launch his/her engines at their best, in a very deep analysis. So, freestylers frequently aggregate themselves in team, but team too large are counterproductive, because consultations have to be quick and effective and final decision on a move has to be done in a short time. Of course, all this makes sense till the calculation power will be so high to make position evaluations almost useless.

Freestylers almost do not take into account some typical aspects of human competition as choosing move with respect to the *character* of the opponent, i.e. searching *the most troublesome* move to the opponent. This approach is noticed also in other sport or, in general, in other human contexts. Centaur searches for the best. He/She is much more similar to Fischer than to Lasker. The relation man-machine is complicated, and becomes complex and structured more and more. In cultural environment, a very sensitive point is the man's frustration with respect to the machines' performances. Machines are programmed by men, but it is not enough. We all feel a sort of impotence, inferiority.

It is very interesting observing human role in a highly sophisticated and technological context like freestyle. It may be a sort of anticipation of the future. We all will be assisted more and more by machines and it is necessary to accept it and feeling confident. Machines are resources, but we have to become well aware of their limits and performances. Full integration requires deep knowledge. These topics have to be faced in Education, because students in our country are most digital native and it will be impossible not to deal with. It implies also a serious matter of communication between teacher and students, and among students. In 2010 the Torino University and the National Research Agency of Rome, with the collaboration of our University, conducted an interesting inquire about the digital learning of chess. The research highlighted that using a well-planned software (10 hrs training) for learning chess (beginners) can give better results with respect to traditional methods, unless the chess courses were conducted by a couple of expert chess instructors (web references 11). Another point of interest for Education is teamwork, most about integration and reciprocal confidence. Finally, we have to take into account also the self-learning ability of machines.

Summarizing, I think that probably we have to reset educational paradigms in the light of the new technologies available with sharp reference to the Theory of Complexity. Once again, chess is a fascinating, *objective* field of experimentation.

Chapter 2
Nature of chess thinking

Summary

In this chapter is tackled mainly the question of what is chess thinking and how it works. In the first section I deal with psychological frameworks and results of neurosciences about chess thinking. In the second one, I present my first pilot experimentation, realized in 2008, in which some theoretical items were used to drive the trial. In first section the main psychological theoretical frameworks concerning the mechanisms of chess thinking are critically presented. Cognitive psychologists faced the question, deepening cognitive aspects and studying perception in chess, performing various experimentations. Chess was called the drosophila of the Psychology, because of its specific characteristics, especially the rules worldwide accepted, and a sharp rating of expertise, the ELO system, used over the world. Various theories were developed, and a sort of standard model was stated in which pattern recognition plays an important role to define the chess expertise (chunk theory, template theory). In fact is not so clear that masters analyze so much more than club players, meaning the deep of thought. Besides, the reasoning of chess players is characterized by a strong visuo-spatial component, as confirmed by neuroscience outputs. This component acts both in a classical, hypothetic- deductive way and in an automatic (based on previous knowledge) way, mainly in a non verbal modality. It is noted that there is not, at the moment, a complete theory of chess thinking, including cognitive and functional items that in my opinion are strongly depending by cultural and social factors, as highlighted in the former. In the second section, my first experimental study is presented. Students were asked to analyze a chess position, writing all procedures followed. The goal of the trial was mainly to analyze the way of reasoning of the students, most about using pattern recognition. An a priori analysis was performed, and results were analyzed by the CHIC software (implicative analysis). Outputs were interesting, confirming that young chess players use pattern recognition as a tool to make the choice. Students (11 years old) showed also a firm willing to reach a conclusion in the reasoning. All the outcomes of this chapter supported me to a better understanding of chess thinking, and to better select the aspects more useful in Education.

Riassunto

In questo capitolo si è affrontato principalmente la questione di cosa sia il pensiero scacchistico e come funziona. Nella prima sezione mi sono occupato di impostazioni di tipo psicologico e di risultati tratti dalle neuroscienze riguardo il pensiero scacchistico, nel secondo ho presentato la mia prima sperimentazione, realizzata nel 2008, nella quale alcuni elementi teorici sono stati usati per condurre il test. Nella prima sezione sono stati presentati criticamente i principali quadri teorici di Psicologia inerenti i meccanismi del pensiero scacchistico. Gli psicologi cognitivi hanno affrontato la questione, approfondendo aspetti cognitivi e la percezione negli scacchi, realizzando varie sperimentazioni. Gli scacchi sono chiamati la drosofila della Psicologia, a causa delle sue specifiche caratteristiche, in particolare le regole uniformi in tutto il mondo,ed una chiara classificazione dell'expertise, che è il sistema ELO, usato universalmente. Sono state sviluppate varie teorie, ed una sorta di modello standard è stato stabilito, nel quale il riconoscimento di configurazioni gioca un ruolo importante per definire l'expertise scacchistico (chunk theory, template theory). Infatti non è così certo che i maestri analizzino

così tanto più dei giocatori di club, riferendosi alla profondità di pensiero. Inoltre, il ragionamento degli scacchisti è caratterizzato da una forte componente visuo-spaziale, come confermato da risultati dalle neuroscienze. Questa componente agisce sia in un modo classico, ipotetico deduttivo, sia in un modo automatico (basato su precedenti conoscenze). E' rilevato che non c'è, al momento, una teoria completa del pensiero scacchistico, che comprenda elementi cognitivi e funzionali, che secondo me sono fortemente dipendenti da fattori sociali e culturali, come evidenziato in precedenza. Nella seconda sezione, è presentato il mio primo studio sperimentale. E' stato chiesto agli studenti di analizzare una posizione scacchistica, scrivendo tutte le procedure attuate. Lo scopo del test era principalmente di analizzare il modo di ragionare degli studenti, per lo più riguardo l'utilizzo del riconoscimento di configurazioni. I risultati sono stati interessanti, confermando che i giovani giocatori usano il riconoscimento di configurazioni come strumento per prendere una decisione. Gli studenti (11 anni) hanno anche mostrato una ferma volontà di raggiungere comunque una conclusione nel ragionamento. Tutte le risultanze di questo capitolo mi sono state utili per una migliore comprensione del pensiero scacchistico, e per una migliore selezione di aspetti utili in Didattica.

2.a Psychological frameworks and results from neurosciences

Summary

This section deals with the researches and theories developed about the human chess thinking, with particular reference to cognitive aspects. Psychologists most were involved in, and the various results and theoretical hypotheses are critically presented. Many experimental inquires were performed since 1894, when Binet pay attention to chess. Also results from neurosciences performed in the last 20 years are presented. Chess was called the drosophila of the Psychology, because of its specific characteristics, especially the rules worldwide accepted, and a sharp rating of expertise, the ELO system, used over the world. The worldwide diffusion of chess allowed the game to become a clear reference for various kind of scientific inquires. Besides, chess was often considered by the people as a synonymous for intelligence, so the inquires on chess skills were matched to the ones concerning intelligence. Anyway, this relationship it is not so evident. Various theories were developed, and a sort of standard model was stated, in which pattern recognition plays an important role to define the chess expertise. (chunk theory, template theory). In fact is not so clear that masters analyze so much more than club players, meaning the deep of thought. Besides,the reasoning of chess players is characterized by a strong visuo-spatial component, as confirmed by neuroscience outputs. This component acts both in a classical, hypothetic-deductive way and in an automatic (based on previous knowledge) way, mainly in a non verbal modality. Chess players use mental imagery to perform an effective calculation, and they seem to have more attitude to pay attention in a very selective way. It is noted that there is not, at the moment, a complete theory of chess thinking, including cognitive and functional items that in my opinion are strongly depending by cultural and social factors, as highlighted in the former. All the outcomes of this section supported me to a better understanding of chess thinking, and to better select the aspects more useful in Education.

Riassunto

La sezione tratta delle ricerche e delle teorie sviluppate riguardo il pensiero scacchistico umano, con particolare riferimento ad aspetti cognitivi. A queste si sono dedicate soprattutto gli psicologi, ed i vari risultati ed ipotesi teoriche sono presentate criticamente. Molte indagini sperimentali sono state realizzate sin dal 1894, quando Binet prestò attenzione agli scacchi. Sono presentati anche risultati ottenuti dalle neuroscienze negli ultimi venti anni. Gli scacchi sono stati chiamati la drosofila della Psicologia, per le sue specifiche caratteristiche, principalmente le regole accettate in tutto il mondo, e una chiara classificazione dell’expertise, il sistema ELO, utilizzato universalmente. La diffusione in tutto il mondo degli scacchi ha fatto sì che il Gioco potesse diventare un chiaro riferimento per vari tipi di indagini scientifiche. Inoltre, gli scacchi sono stati spesso considerati dalla gente come sinonimo dell’intelligenza, e per questo motivo le indagini sulle abilità scacchistiche sono state confrontate con quelle sull’intelligenza. In ogni caso, il nesso non è così evidente. Sono state sviluppate varie teorie, ed una sorta di modello standard è stato stabilito, nel quale il riconoscimento di configurazioni gioca un ruolo importante per definire l’expertise scacchistico (chunk theory, template theory). Infatti non è così certo che i maestri analizzino così tanto più dei giocatori di club, riferendosi alla profondità di pensiero. Inoltre, il ragionamento degli scacchisti è caratterizzato da una forte componente visuo-spaziale, come confermato da risultati dalle neuroscienze. Questa componente agisce sia in un modo classico, ipotetico deduttivo, sia in un

modo automatico (basato su precedenti conoscenze). Gli scacchisti usano immagini mentali per fare un calcolo efficace, e sembrano avere più attitudine a concentrarsi in un modo molto selettivo. E' rilevato che non c'è, al momento, una teoria completa del pensiero scacchistico, che comprenda elementi cognitivi e funzionali, che secondo me sono fortemente dipendenti da fattori sociali e culturali, come evidenziato in precedenza. Tutti i risultati di questo sezione mi hanno supportato per una migliore comprensione del pensiero scacchistico e per meglio selezionare gli aspetti utili in Didattica.

Let us go into more depth to the core of human chess thinking, i.e. which cognitive resources are employed by chess players, and which are the fundamental mechanisms involved.

In other words, it is of the essence to explore what chess-players do while playing. Chess players' thinking process was studied by several Cognitive Psychologists, since 1893, when the well known psychologist Binet, a pioneer in modern psychological intelligence testing, analyzed chess thinking mechanisms in a study on blindfold chess.

Chess was the subject of several, important psychological studies in the second part of XX century and in the beginning of XXI.Chess was defined "The drosophila of the Psychology" (The term was used first by Chase and Simon in 1973) , in fact it is an ideal environment for cognitive mechanisms because of

- Rules worldwide accepted
- Notation and symbol system worldwide accepted
- A precise worldwide rating scale to quantify players' expertise (rating ELO)
- Many samples and tests available for all levels
- Diffusion in educational contexts
- Relationship with Artificial Intelligence

Besides, another important factor pushed researches to chess. It is the common vision that chess is representing *cleverness* or, more precisely, that to be good at chess somewhat implies to be clever.

Psychologists dedicated their efforts on two basic segments: The first one deals with analysis on which cognitive resources the chess players use, in particular the skilled ones; the second one deals with the links between chess skills and other skills, including, in general, intelligence. Of course this implies a theoretical framework defining intelligence, which is a double-edged matter, and chess again offers tools to better framing the question. Also, scholars worked on the role of talent in chess skill and performance.

In this section, cognitive researches and results from neurosciences are analyzed; whilst the following chapter will be tackle other viewpoints.

It is very difficult to summarize works and theories exhaustively, because some studies faced with fundamental aspects of human thinking, elaborating complex models, sometimes in a very formal and accurate way. Frequently studies are well supported by experimental data, and in the last twenty years there are relevant references to neurosciences.

Binet first dealt with chess in a scientific work. He studied blindfold chess and was interested in how the blindfold player recalls in mind the chess positions, and how is the structure of this representation (De Groot 1946, 1965). It is an interesting topic, because blindfold players have to use heavily mental imagery, crucial element in our subject. Summarizing, Binet highlighted three fundamental *qualities* in the blindfold player:

- Knowledge and experience in chess (*l'érudition*)

-Imagination (l*'imagination*)

-Memory (*la mémoire*)

Binet understood that masters perceived position on the board in a meaningful way because of their knowledge; Binet call it a *mémoire des idées*, somewhat abstract,contrasting with la *mémoire des sensations*, a more mechanical one (Binet 1894). About how blindfold players represent the board in mind, reports were not clear and/or uniform, and *Binet concluded from his material that the blind player does not, in general, have a fully complete picture of the position before him, but rather only a rough Gestalt which he searches step by step, or rather: He continually recalls (the details of) the position* (De Groot, 1965).

In blindfold playing, there is no visual representation, meaning a picture of the board reproduced in mind. Players speak of features of position and main relations among pieces, something more abstract than pictorial.

Also in normal playing is possible to set this framework, although is useful to clarify that these representations are often realized by imagination of physical gestures like to capture a piece or pawn, or to move a piece along the board before to put it in a square, as this was stated (or better assumed) by De Groot and it is in accordance with important, subsequent works on cognition and learning, as it will be deepened in the following.

A crucial point arose: visual perception and representation in mind are highly linked with knowledge. This was deepened at most by other important psychologists.

This aspect became clearer considering the outputs of the well known experiment carried out by Djakow, Petrovsky and Rudikh (1927), three psychologists from Moscow University. They were interested in the factors underlying the chess skill, and tested the participants of 1926 Moscow International Tournament. The methods adopted were not quite effective with respect to the modern ones, but a sharp output emerged: professional, high-skilled chess players did not perform better than comparable subjects. The tests were carried out according to a debatable framework, probably a consequence of the common vision about chess in that period, but outputs resulted highly surprising compared with the common view considering chess players representing excellence in intelligence.

The three Russians tested also visual abilities, according to a more acceptable experimental scheme. Again there were not better performances by the chess players, but in tests including specific abilities having *resemblances* with chess (Djakow, Petrovsky & Rudikh, 1927). The latter result, in our opinion is very important, and was basically confirmed by subsequent studies. It will be a reference point for my work.

A fundamental study, quoted almost everywhere, was carried out by Adrian De Groot in 1946. He was the first scholar *to carry out an experimentally based psychological analysis of chess thinking* (De Groot, 1946). An important topic in his study dealt with the ability to recall a position. Masters performed significantly better only if positions made sense, i.e. representing a chess position that could occur in a standard game. When pieces are set randomly there was no difference between masters and beginners. De Groot understood the role of perceiving *complex of pieces*, a key item that will be studied by important scholars in the subsequent decades. Another aspect investigated by De Groot was the structure of chess thought, searching for the features underlying skills and talent. Somewhat surprisingly, chess masters did not show a clear superiority in depth of analysis, or in number of variations analyzed. More recent studies show that experts analyze slightly deeper than non experts (Gobet, 1998), anyway depth of analysis seems not to be the reason for being skilled in chess.

Chess skill reveals in early finding/troubleshooting and in selecting the right variations to analyze. It recalls the way minimax and alfabeta algorithms proceed. This finding and selection are enhanced by players' chess "knowledge".

De Groot identified the critical role of perception, which allows quick access to information stored in long-term memory (Gobet & Campitelli, 2002). Perception in chess seems *driven* by expertise. We know anyway that similar results about perception were found also in different disciplines. Again, more than visualization, chess experts seem to use a continue abstraction-and- reconstruction (Di Sario, 2002).

More in general, protocols in De Groot's study show some typical features in chess player's thinking, summarized as follows (Di Sario, 2002):

- Identification of the problem(s)
- Various levels of depth, according to a classical tree-structure
- Players always look for a subjective reason of choice

- Players apply a continuous feedback mechanism (goal feedback)
- The feedback mechanism may cause a radical change of the problem itself,

and of the ways to tackle it

Another milestone in understanding chess thinking was set by Chase and Simon (1973). They proposed their famous *chunking theory* to explain De Groot’s results, also using Jongman’s results on chess players’ ocular movements (Jongman 1968). The chunking theory is a model of chess players’ perception; The chunk is a typical piece ensemble, immediately recognized by the skilled player, and perceived as a whole. In the following figure (Fig. 3) an example of chunk: the typical king’s fianchetto (white pieces in the bottom right corner).

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Fig. 3 example of chunk

For the skilled chess player the white pieces’ ensemble occupies an elementary unit of memory. Then the player skimps on own memory space. It does not happen to beginner, who uses more bits of memory to recall the chunk; Chunking is a cognitive resource for chess skill, in particular in tasks of recalling a position

This is the first concept for understanding the processes of pattern recognition in chess.

Pattern recognition is considered the most important cognitive resource for chess skill, as confirmed in almost all relevant studies.

It is clear again that chunking is strictly connected with knowledge and practice, otherwise test results about recalling a position would be hard to explain. Chunks are supposed to be stored in long term memory (LTM).

> *Through years of practice and study, masters have learnt several hundred thousands of perceptual patterns, which, once recognized in a particular position, give rapid access to information such as potential moves or move sequences, tactics, strategies, and so on. Simon and his colleagues proposed that pattern recognition explains a number of important phenomena, such as highly selective search (even chess grandmasters rarely search through more than one hundred moves before selecting a move), automatic and*

> *"intuitive" discovery of good moves, and extraordinary memory for game-like chess positions. Simon and Chase (1973) suggested that at least ten years of practice and study were necessary to acquire the minimum knowledge required to become a grandmaster* (Gobet & Campitelli, 2002)

This big amount of time and efforts necessary to reach high level in chess will be put in relation with educational aspects in the following. After this claim by Simon and Chase, other scholars tackled the question concerning the role of innate attitude to reach high levels. Of course, this debate does not deal with only chess, but *more solito* chess is the *drosophila*.

Another, very interesting, theoretical item proposed by Chase and Simon (Chase & Simon, 1973) is the so-called *mind's eye*. In short, mind's eye is a model of chess problem solving. The player solves the problems making in mind visuo-spatial operations. It's an *active* mental imagery, in fact often players use the typical sentence "I saw it", meaning the discovery of a move , but they do not refer to a "real" visual operation, whilst to a kind of perception similar to the one of the mathematician who, walking after hours of work on problem, suddenly "sees" the solution. By the mind's eye theory, chess players solve problems in a visual-perceptive way. Is it a rational, deductive approach? I believe that it is, but certainly not a classical one. The mind's eye is a very interesting theoretical structure with potential links with other disciplines, including maths, and useful to better understanding chess thinking.

In general, psychologists are not so interested in performing a complete analysis of the game; such an analysis has to include historical, epistemological, psychological and functional issues. I think that there is not still a general theory of chess, including all the quoted components of the analysis, and it is the scope of this work to provide a contribution, although small, about it. In this framework, my research work may be considered as a pilot study.

Coming back to the basic structure of chess thinking as summarized by De Groot, skilled players use to select the more plausible options, and then analyze them trough and trough. It is a deep visit of the tree, performed in a classical, deductive manner. This way of operation, which I believe is reasonable, is supported by players' reports after any tournament games, and by tests from the most important and classical chess books. The chess players like to remember the paragraph on analysis in Kotov's *Think like a grandmaster* (Kotov, 1983), in which are reported in a very detailed way the thought processes of an amateur player when he have to tackle a complex chess position involving strategy and tactics. Moreover, chess players are able to reconstruct verbally their thoughts, even if their reports are sometimes not so clear as their moves on the board. This item is highlighted also by Montero and Evans (2011), in arguing versus Herbert Dreyfus' theory of expertise, assuming that expert action generally occurs automatically and unreflectively.

Dreyfus (2005) argues that experts' actions are based on intuitions and, although proceeding in a very effective way, are without full consciousness of deliberation, like a pilot driving a race car, or like whoever of us climbing home's stairs.

The argument is subtle, because the great players' moves seem often part of an automatic, spontaneous flow, especially during rapid games, albeit this is true only in some phases of the game.

On the other hand, in different phases of the game, a logic, deliberate, and sometimes very deep analysis occurs in tournament practice, as well highlighted by Montero and Evans quoting Larry Evans' report of an his own game; the Evans report is logic, verbal and, indeed, reconstructs apparently correctly his thoughts during the game, showing a full awareness and not any kind of 'automatic pilot'. I concur with Montero & Evans' opinion; it is not acceptable reducing chess reasoning to a sort of automatic retrieval and application of information. The player recognizes configurations and considers his/her experiences, but the analysis proceeds in an analytical way, and only eventually euristic and synthetic considerations are made. Just in specific standardized positions, playing is somewhat automatic. More on the ocular movements was stated by De Groot and Gobet in a study in 1996 (De Groot & Gobet, 1996); summarizing, results show on average that experts have a larger visual span of the board ; experts also "observe" single squares less frequently than a beginner , and they cover more space by an ocular movement. Skilled players observe important squares for more time compared to non skilled, and also they engage more along the edges of squares, probably referring to the relationship among pieces (Di Sario 2002). Their ocular movements are more fluid too. Similar results were obtained in more detailed studies by Rheingold, Charness, Pomplun, and Stampe (2001) and by Rheingold, Charness, Schultetus, and Stampe (2001). These experimental outputs give strength to the interconnection between perception and knowledge.

Knowledge and culture do not come after perception, intervening *a posteriori*, but in my opinion these items interact at the same time, in a complex and dynamic occurrence. It is in accordance with Psychological theoretical framework of Gestalt, in which humans' perception of reality it is not a sensorial mosaic, but a synthesis, a structural unit (Devoti, 1997). This form, the *Gestalt*, is the main human model of reality. In this framework, the *Gestalt is* *depicted by perception, and learning consists in the perception of a situation, in recognition of its troubling aspects, and in solution obtained by perceptive insight* (Devoti, 1997).

More on perception:

> *rather than being a purely biological act, human perception is a social process through and through. It is, as Wartofsky put it, "a cultural artifact shaped by our own historically changing practices" (1984, p. 865)* (Radford, 2010, pag.2).

In the case of vision, using again Radford's words *"It is the process that converts the eye (and other human senses) into a sophisticated intellectual organ – a "theoretician"* (Radford, 2010, pag.4).

Visual aspects are fundamental in chess, so we can speak *of chess domestication of the eye.*

A logical evolution of Chunking theory was the Template Theory by Gobet and Simon. (1996). Basically, a template is a position in which several, known chunks are present. Hence, the template becomes a sort of super-chunk, i.e. a recognizable pattern, in which options and features are suggested by the configuration itself. It is useful to pass the typical limit of seven chunks for short term memory (a similar mechanism we use to remember a phone number, grouping the digits in groups –chunks- of two or three). An example of template is represented in the following figure.

Fig. 4 Example of template

The very aim of psychologists is to understand human mind mechanisms tackling a problem, so they developed computational models applicable also to other domains than chess (Di Sario, 2002). As quoted in the former chapter, these models have theoretical basis and also a learning procedure, like in CHREST (Chunk Hierarchy and Retrieval Structure) model, by Gobet and Lane that was the computation model arising from the Template Theory. It is a cognitive architecture that models human perception, learning, memory, and problem solving. CHREST has its theoretical basis on EPAM Elementary Perceiver and Memorizer), developed by Feigenbaum and Simon (1964). It is beyond our purposes to investigate computational models, albeit it is important to remark that chunking and template theories represent the actual standard model for chess expertise, but several scholars criticized them. (Holding 1985, 1992).

The theories above, although well supported by experimental data, do not seem to be exhaustive. Like in the example of the mathematician "seeing" the solution, chunks and patterns could be the basic "brick" on which the mind works, but the rise of a complex solution to the mind should heavily involve computational ability, knowledge; Holding proposed an alternative approach, in which skill does not rely on pattern recognition, but on more depth and size in exploring the tree of variations, i.e. the expert analyzes more variations and in a more depth. This approach was called SEEK (Search Evaluation and Knowledge) (Holding, 1985). These three factors are present also in the standard model; just it is a matter of *specific weight*. Experimental data seem not to confirm differences in size of searching, and differences in depth seem not so relevant to justify giving up chunking theory (Di Sario, 2002) (Gobet, 2008). Very relevant are the studies by Saariluoma, who studied chess thinking by a theoretical framework of content-oriented psychology of thinking. According to Saariluoma, *attention and memory psychology does not provide much information to answer this kind of strongly content-oriented problems. The basic notions of capacity and format are not sufficiently powerful in expression to allow one to discuss problems of contents integration in representations (Saariluoma, 1997). This also means that they provide only partial answers to the problems of selectivity in thinking* (Saariluoma, 2001).

For Saariluoma, to (skilled) chess players attending a chess position occurs not *seeing*, but *apperception*. *It assimilates the perceptual stimulus and conceptual memory information into a semantically self-consistent representation that is characteristic of the human mind.*

Apperception simplifies the variations tree, otherwise too wide, reducing the problem to smaller problem subspaces, that Saariluoma called *mental spaces.* So he proposed a new concept, the *thought model*, referring to a characteristic piece configurations and a set of possible, reasonable moves.

It is clear that it is very similar to a template, and anyway strictly connected with chunking Theory and template Theory, but the shift to a sort of holistic approach is embedded. It is somewhat like "all chess *things* connected with a given pattern", moving from a visual-perceptive dimension to an almost-abstract one. *Thought models are by nature complex sets of associated elementary actions, which people have learned. Large parts of our knowledge used in thinking are organized around such wholes. An architect planning a house, for example,* *knows that he must have walls, windows, parking lots etc. He or she has a scheme of the required elements. However, to adapt his original model to the reality, he must follow principles that make sense and these rules are functional in nature* (Saariluoma, 2001)

This theoretical framework it is not contradictory with standard model, has been developed in the environment of cognitive Psychology and takes into account the role of knowledge and culture in the concept of apperception, because content-oriented thinking is based on mental contents that are basically cultural objects. Another theoretical framework for expertise, the LT-WM (Long Term-working memory) Theory, was proposed by Ericsson and Kintsch in 1995. They stated that experts encode information into LTM so that *"cognitive processes are viewed as a sequence of stable states representing end products of processing" and that "acquired memory skills allow these end products to be stored in long-term memory and kept directly accessible by means of retrieval cues in short-term memory (...)* (Ericsson & Kintsch, 1995)

They *s*uggest that strong chess players use a hierarchical retrieval structure corresponding to the 64 squares of the chess board, although they were criticized, most for not full explaining the experiment of recalling random positions. Also, this theory seems not suitable for computational models (Gobet, 2000), which makes more difficult to progress on it.

In an interesting study Horgan (1987) concluded that children could perform a highly complex cognitive task as well as most adults. Horgan found *that while adults progress to expertise from a focus on details to a more global focus, children seem to begin with a more global, intuitive emphasis* (Horgan, 1987). This is the reason because many non expert adults lost chess games by children. Frequently the non expert adult focuses his/her attention on a limited part of the board, overlooking a crucial feature on another side. It happens typically in chess clubs. Also, (Chi, 1978) demonstrated that young player can remember pieces' position on the board better than non-player adults. It is an interesting topic for in Education, in fact, using Horgan's words

> "*this may be a more efficient route to expertise as evidenced by the ability of of pre-formal operational children to learn chess well enough to compete successfully with adults .Educators, rather trying to "stamp out" the intuitive, quick judgments, would do well to encourage these judgments as well encouraging careful, analytic thought. Many pet phrases of teachers discourage quick judgments:"look before you leap", "neatness counts", "go slow". It may be that practice in making fast judgments forces the integration of a child's rapidly expanding knowledge base. The combination of forcing quick judgments and encouraging analytic processes may speed the*

acquisition and revision of schemas: Complex problems should be approached from both the intuitive and the reflective modes.

It is a very interesting approach, and I think may become very proficient for children.

Sound evidence for standard model comes from studies using methods from neurosciences, like magnetic resonance imaging of the brain.

In an important study Atherton, Jiancheng Zhuang, Bart, Xiaoping Hu and Sheng He (2003), applied FMRI (functional magnetic resonance imaging) to several chess players involved in specific test (game position, random, empty board), obtaining interesting results. Using authors' words

The high degree of activation in the parietal areas and the lack of activation in the left lateral frontal lobe, normally associated with traditional measures of intelligence and logical reasoning, are surprising and may suggest that chess cognition is primarily spatial. This inference is also supported by the high degree of activation in the occipital/parietal lobes, which may indicate preliminary spatial and visual processing.

It is in accordance with standard model's framework about retrieving information; in fact Atherton et al. hypothesized also that

Much of the activation in the parietal lobes could be related to the mental imagery involved in checking plausible moves, and the superior frontal areas may be involved in the maintenance and possibly the selection of spatial patterns within the posterior regions.

The study applied the same technique to 'go' players. Results were similar, not surprisingly because of the similar nature of chess and go, being both board games with high level strategy requested. The only differences were in activation, in some Go players, of the brain area 44 that is normally related to speech operations. It may be *explained because Go players maybe more familiar with the names of strategic positions than chess player* (Atherton et. al., 2003).

Amidzic, Riehle, Fehr, Wienbruch and Elbert in 2001 presented some interesting experimental data. They found that expert players show more activity in frontal and parietal cortices, amateurs in medial temporal lobes. It is consistent with standard model, because the more γ bursts[1] in frontal and parietal cortices indicate retrieving information (chunks) from the LTM. Besides, *Lesions in structures that are activated in amateur players impair recent memory while leaving remote memory intact. Grandmasters seem to rely more on remote than on recent memory* (Amidzic, Riehle, Fehr, Wienbruch & Elbert, 2001).

[1] A gamma wave is a pattern of neural oscillation in humans with a frequency between 25 to 100 Hz, probably associated with conscious perception

These studies, and other important ones like by Onofrj, Curatola, Valentini, Antonelli, Thomas and Fulgente (1995), Nichelli, Grafman, Pietrini, Alway, Carton and Miletich (1994) and by Campitelli, Gobet, and Parker (2005) tend to converge on some general results, summarized as follows:

- Tasks like solving a chess problem or playing chess seem to activate brain frontal and parietal areas
- Tasks concerning the mere retrieval of information seem to activate brain temporal zones
- Experimental results during chess tasks seem to indicate a poor activation of brain areas devoted to verbal activity with respect to the visuo-spatial devoted brain areas
- Experts and novices activate different zones of brain, probably because experts refer more to information stored in LTM

These important outputs from neurosciences are in full accordance with the standard model.

From cognitive psychologists' and neuroscientists' studies emerge some aspects of chess cognition that, in addition to the other aspects of chess thinking outlined before, are to be taken into account in searching for useful links with math education.

Firstly, the relevance of spatial component in chess thinking; secondly, in chess there is a strong attitude to problem solving reasoning, using cognitive resources based both on knowledge and calculation; chess players use mental imagery to perform an effective calculation, which, for our purposes, generates an interest in deepening how these skills can be useful in education, in particular in mathematics. Thirdly, as emerging from the psychology studies, the difficulty to state a strict equivalence between chess skill and general intelligence, as will be deepened in chapter 3.

These aspects of chess are very important in an educational environment, as experimented and practiced in a massive way all over the world since the 30's of XX century, with a positive outcome both from a didactic point of view and from a scientific point of view in the field of understanding the mechanisms of human mind.

In the following we will go in depth on educational aspects of chess and relationships with skill acquisition in various fields, especially mathematics.

2.b My Pilot Study

Summary

In this section I will explain my pilot study, performed in 2008 in a middle school in Palermo with a group of 28 sixth grade students attending the chess laboratory at school. I deal with the same topic in my paper of 2008 (D'Eredità, 2008). The aim of the test was to analyze the way of conjecturing and arguing of the students considering a chess position. In particular, the research hypotheses were expressed in terms of using notation, framing the problem, and using pattern recognition as basic tools for conjecturing and arguing to make a choice. A situation-problem, a chess position, was submitted to students, asking them to analyze it, writing all procedures followed. An a-priori-analysis was performed, listing the expected behaviors and assigning to each of them a variable. It was performed a quantitative analysis supported by Software CHIC, useful especially for implicative analysis. Binary variables were adopted. The research hypotheses were explained, where possible, as binary variables occurrences and as relations among them.

Summarizing the results, I obtained good evidence to the hypothesis of the pattern recognition as guide and inspiration in chess thinking, and leading to better opportunities in framing and concluding a problem. In fact arguing and conjecturing were carried out with a clear reference to expertise. It is difficult for players without a basic chess instruction to reach concrete results by exhaustive analysis only. This is similar to what happens in mathematics or in physics when students hardly solve problems without a form of recognition of a known configuration or a tool.

In conclusion, I obtained a confirmation of the importance of the expertise in chess players' thinking, with precise reference to pattern recognition, in accordance with some important findings in literature. Last but not least, in this study I noticed also that students-chess players show strong inclination to reach a conclusion in any case. They don't give up!

Riassunto

In questa sezione è presentato il mio studio pilota, realizzato nel 2008 in una scuola media a Palermo, con un gruppo di 28 studenti di prima media del laboratorio di scacchi della scuola. Lo stesso argomento è stato trattato in un mio articolo del 2008 (D'Eredità 2008). Lo scopo del test era analizzare il modo di congetturare e argomentare da parte di studenti nel considerare una posizione di scacchi. In particolare, le ipotesi di ricerca sono state espresse in termini di uso della notazione, inquadramento del problema, e uso del pattern recognition come strumento base per congetturare e argomentare nel prendere una decisione. Una situazione – problema, una posizione scacchistica, è stata sottoposta agli studenti, chiedendo loro di analizzarla, scrivendo tutte le procedure seguite. E' stata fatta un'analisi a priori, elencando i comportamenti attesi e assegnando ad ognuno di questi una variabile. E' stata realizzata una analisi quantitativa supportata dal software CHIC, utile soprattutto per l'analisi implicativa. Sono state utilizzate variabili binarie. Le ipotesi di ricerca, ove possibile, sono state descritte come occorrenze delle variabili binarie e come relazioni tra loro. Riassumendo i risultati, ho ottenuto un buon impulso all'ipotesi secondo la quale il pattern recognition funge da guida e ispirazione nel pensiero scacchistico, e

conduce a migliori possibilità nell'inquadrare e concludere un problema. Infatti, l'argomentare e il congetturare sono stati realizzati con chiari riferimenti all'expertise. E' difficile per giocatori senza un'istruzione scacchistica di base raggiungere concreti risultati solo per mezzo di un'analisi esaustiva. Ciò è connesso con quanto avviene in matematica e in fisica quando difficilmente gli studenti risolvono problemi senza forme di riconoscimento di una configurazione nota o di uno strumento. In conclusione, ho ottenuto una conferma dell'importanza dell'expertise nel pensiero dei giocatori di scacchi, con precisi riferimenti al pattern recognition, in accordo con alcuni importanti risultati in letteratura. In ultimo ma non per importanza, ho notato anche che gli studenti-scacchisti mostrano una forte inclinazione a raggiungere comunque una conclusione.
Non abbandonano!

2.b.1 The set-up

My first pilot study concerned some aspects of cognitive resources mobilized by chess players during their thinking. An experimental test (a situation-problem, a chess position) was submitted to a group of students. This test was realized on May 14, 2008 in a middle school in Palermo (Italy) with a group of 28 students, and an a-priori analysis was carried out. Within this topic, the aim of this work is to explicit some logic and metacognitive skills used in conjecturing and arguing of the choice in chess, and relations among them. It was stressed in literature that chess mastery manifests itself not only in the logic-analytic capacity to explore a tree (more or less deep and/or branched), but also in recognizing already-known structures, like chunk, template and patterns, and it allows a quicker and safer evaluation of position. In particular, pattern recognition not only suggests what to do, but also orients in strong way chess players' choices. This is pointed out as a content-oriented selective procedure (Saariluoma, 2001). The pattern recognized represents a target in chess player's reasoning. In our theoretical framework, the reasoning, the analysis, the opponent's last moves and style, and other various available means represent the way to objectification, and at the end of the process the appearance of a known pattern in the player's mind represents the objectification. The process of objectification has an important role in Radford's theoretical framework:
That is, etymologically speaking, a process aimed at bringing something in front of someone's attention or view (Radford 2002, pag.14).

This process of objectification may appear in various, specific contexts. In fact many concepts in chess have a geometrical intrinsic nature, related both to pictorial and logical aspects (nature of the pieces and formal rules of movement), very easy to recognize (factual thinking), and comparable to Fischbein's figural concepts, as will be specified in chapter 3. On the other hand, chess elements make sense in the context as practical, dynamic tools much more difficult to understand. In fact it is requested first an understanding of the dynamic potentiality of the configuration (e.g. as performed in chess lessons, in which typically it is learned also the name of the configuration), and in a higher level to recognize the possible relevance of the configuration itself in a given, even very complex position (contextual thinking). These elements can be called *configural* concepts. (Ferro, 2011). This theoretical issue will be discussed in chapter 3.

Coming back to experimentation, now I am aware that trials are not independent on specific educational context in which they are tested (see section2.a). As a consequence, the choice of a sample is always reductive and dangerous, and also our sample is not so large (28). Keeping in mind these considerations, we can consider data and results from this test as my first attempt to obtain some data in my research. Therefore we will refer to a sample of students of a secondary Junior school, as specified in the following.

2.b 2. Methodology

We have to carry out an experimental test to identify using of specific skills and items adopted in chess thinking to conjecture and arguing, and relations among them. We chose appropriately the test (a chess position, of course), and submitted it to students of the sample, specifying what they had to do exactly (in an open-answer questionnaire). I needed to examine the various outputs from the students' protocol, connecting the various behaviours and trying to match them to corroborate my research hypotheses. Because of the former reasons, considering the nature of the trial (open-answer questionnaire) and the quantity of students involved, I chose to employ implicative statistical analysis, using specific mathematical tools as described by Gras (2000) and Gras, Suzuki, Guillet &. Spagnolo (2008). According to the Theory of Didactical situations in mathematics of Guy Brousseau (1997), I realized an a-priori analysis, performing an epistemological analysis of the task based on my chess knowledge and on chess literature.

In the a-priori analysis I have explained the expected behaviours. As a consequence, I identified several binary variables, to be used for protocol-analysis, in such a way that an expected behaviour corresponds to a binary variable. The research hypotheses were explained as binary variables occurrences and as relationships among them, in particular using the implication index. This index allows us to quantify how much the occurrence of a variable implies the occurrence of another one. Given two variables a and b, representing two of the expected behaviours, the implication index q(a,b') of a and <u>non b</u> is defined as follows (Gras, 2000):

$$q(a,\bar{b}) = \frac{n_{a\wedge\bar{b}} - \frac{n_a n_{\bar{b}}}{n}}{\sqrt{\frac{n_a n_{\bar{b}}}{n}}} \quad \forall n_{\bar{b}} \neq 0$$

Where n_a is the number of students that put into action exp. behavior strategy a, $n_{\bar{b}}$ is the number of students not putting into exp. behavior b, n is the total number of students (28, in our case), and $n_{a\wedge\bar{b}}$ is the number of students both showing expected behaviors. *a* and not showing exp. behaviors *b*.

In this study, I call v_n the variables associated with a given expected behavior, as specified in the following.

The calculation of indexes was done by using the CHIC software (Classification Hiérarchique Implicative et Cohésitive)

The open questionnaire allows us to realize also qualitative analysis with some details.

2.b.3 Choice of the test

To carry out such an experimental test, we decided to submit to the sample a chess position (Fig. 5), containing few pieces (An endgame with King + 2 pawns Vs. King + pawn). In this position initial options are not so many, and variables' tree is not so impossible to explore. Especially in this position students can recognize several important topics (patterns), already being carried out in their chess laboratory.

Fig.5 – The test (Black to move)

Text of the protocol: BLACK TO MOVE. YOU AS WHITE. ANALYSE THIS POSITION, CONSIDERING ALL INITIAL OPTIONS.YOU CAN MOVE PIECES ON THE BOARD. WRITE ALL PROCEDURES FOLLOWED.

We can identify 3 pattern levels, from the simpler (pawn promotion), to the intermediate (concept of opposition and practical application), and to the hardest one (corresponding squares and triangular manoeuvre). In the protocol text we ask to students to consider all initial opportunities, playing as white, and to write all procedures followed. It was allowed to move the pieces, because the position is not trivial. The work is individual. There is a winning strategy for white, achievable from recognition of the hardest pattern. It is also possible to obtain the same result by exhaustive analysis, but is more complicated if they recognize the simplest patterns only.

2.b.4. Choice of the sample

We chose to submit the questionnaire to students of the State secondary junior school “Leonardo da Vinci” in Palermo, aged 11 to 14. These students attended the chess laboratory of the schools, first or second year. The school is located in Palermo,, and the catchment area covers more social groups. In the school there is a strong chess tradition, the chess laboratory works since 1990 ca. and the school obtained also important wins in Students’ Italian Championships (2 times Italian Champion). So, we are dealing with a good youth chess level, but of course students have not a big tournament practice and have not a so high level of play. We believe the sample suitable for our goals.

2.b.5 Expected behaviours

We have identified several types of logic and metacognitive skills, used in conjecturing and arguing of the choice:

- Kind of language used;
- Spatial orientation;
- Use of IF/THEN (deduction);
- Abduction
- Induction
- Visual pattern recognition;
- objectification of an abstract pattern (classes of patterns), also not displayed, referred to previous own knowledge, or acquired from literature, or acquired from other persons or groups.

In particular, as above, and according to consolidated experiences of chess didactics, expected behaviours summarized

As following:

- Use of algebraic notation[2] and/or natural language
- Spatial Orientation and preliminary considerations about framing of a problem
- Conjecturing and arguing for the choice, also in an implicit form
- Pattern recognition and using it as basic elements of conjecturing and arguing, for preliminary and/or concluding framing of problem.

2.b.6. Research hypothesis

1. Using symbolic language leads to better results
2. Preliminary framing and correct spatial orientation lead to better results
3. The chess player argues and conjectures to make a choice
4. Discipline and consistency of thought lead to better results
5. In making a choice the chess player uses pattern recognition as basic element of

IF/THEN reasoning

6. Using patterns allows correct arguing and leads to better results

2.b.7 Definition of variables

VARIABLE 1: USE OF ALGEBRAIC NOTATION AS PRIMARY TOOL IN ARGUING

(Using a symbolic language and not the natural one allows a better chance of communication and a better development of arguing, that is displayed in a more efficient and less ambiguous way

VARIABLE 2: CORRECT IDENTIFICATION OF THE SQUARES AND CORRECT SPATIAL ORIENTATION

(To face the problem in a well-defined spatial context)

[2] For algebraic notation means identification of the squares by 2 coordinates: columns (vertical) are identified by a letter from A (left) to H (right), ranks (horizontal) by numbers from 1 (below) to 8 (above). It is very interesting to note that until 1970 ca. Anglo-Saxons and Spanish countries used mostly descriptive notation, which identifies the squares describing its position with respect to a reference. For example the white move Bb5 (Bishop moves to b5) becomes in descriptive notation BQN5 , i.e. Bishop moves to the fifth square of the Queen Knight (referred to the starting square of the Queen Knight). The biggest difficulty in descriptive notation is that in the black's moves squares are identified in the same manner as white does. In summary, different squares are the same identification depending on who moves. (for example: Algebraic notation.: White move Bb5 followed by black move Bb4; in descriptive notation it becomes BQN5 (W) and BQN5 (B) again).

VARIABLE 3: PRELIMINARY CONSIDERATIONS AND CONJECTURING ABOUT FRAMING OF A PROBLEM

(Considerations about the type of the problem and conjecturing on methods to use and expected results)

VARIABLE 4: IN THE INITIAL POSITION CONSIDERS ALL OPTIONS

(Correct starting of analysis)

VARIABLE 5: DURING THE ANALYSIS CONSIDERS CORRECTLY PLAUSIBLE OPTIONS BY RECOGNITION OF THE PATTERN ”PAWN PROMOTION”

(Correct arguing by acquired knowledge)

VARIABLE 6: DURING THE ANALYSIS CONSIDERS CORRECTLY PLAUSIBLE OPTIONS BY RECOGNITION OF THE PATTERN “OPPOSITION”

(Correct arguing by acquired knowledge)

VARIABLE 7: DURING THE ANALYSIS CONSIDERS CORRECTLY PLAUSIBLE OPTIONS BY RECOGNITION OF THE PATTERN “ CORRESPONDANT SQUARES – TRIANGULAR MANOEUVRE”

(Correct arguing by acquired knowledge)

VARIABILE 8: INTERNAL CONSISTENCY IN FOLLOWING CORRECTLY OWN ASSUMPTIONS ALREADY MADE, AND DISCIPLINE OF THOUGHT DURING THE ANALYSIS (THAT IS, THERE IS AN ALSO IMPLICIT REASON IN THE CHOICE)

(Internal consistency in arguing)

VARIABLE 9 : REACHES FINAL DEFINITION OF A PROBLEM

(To make a conclusion in arguing)

VARIABLE 10 : MAKES A CORRECT FORECAST OF FINAL DEFINITION OF A PROBLEM BY CORRECT ARGUING

(To make a conclusion about problem by correct arguing)

VARIABLE 11: MAKES A CORRECT FORECAST OF FINAL DEFINITION OF A PROBLEM BY INTUITIVE CONSIDERATIONS

(To make a conclusion about problem by intuition)

VARIABLE 12: RECOGNITION OF PATTERN “PAWN PROMOTION”

(Recognition of plausible using of an acquired knowledge)

VARIABLE 13: RECOGNITION OF PATTERN “OPPOSITION”

(Recognition of plausible using of an acquired knowledge)

VARIABLE 14: RECOGNITION OF PATTERN “CORRESPONDANT SQUARES – TRIANGULAR MANOEUVRE”

(Recognition of plausible using of an acquired knowledge)

2.b.8 Research hypothesis as occurrences and relation among variables

Hyp. 1: variables v1 and v2 should have big or total occurrence ; anyway it should be a strong implications with the variables representing reaching of a result or final definition of problem, i.e. v5,v6, v7, v8,v9,v10,v11

Hyp.2: Preliminary framing and correct spatial orientation should imply better results, i.e. v2 and v3 should show an implication with v5- v6-v7-v9-v10-v11

Hyp. 3 : Occurrence of variables v3, v4, v5, v6, v7, v8, v9, v10, v11, v12,v13,v14 , all connected with conjecturing and arguing, of course depending on expertise: in particular we expect a high occurrence for the variables v5,v12

We expect low or no occurrence for v7-v10-v14 (high expertise level)

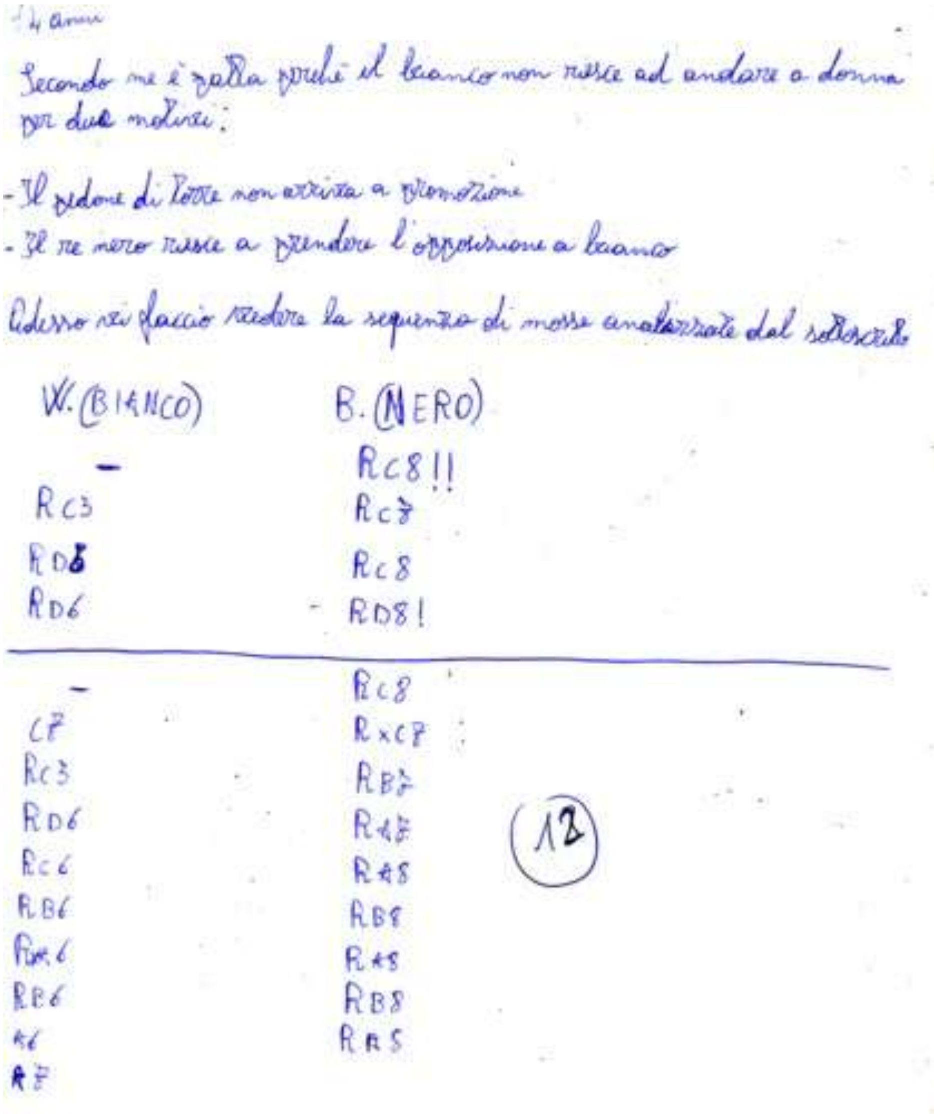
Hyp.4: Discipline and consistency of thought lead to better results, and in general to correct arguing, i.e. implications of v4-v8 with v9-v10-v11, and with v5-v6-v7

Hyp.5: We expect that using of pattern recognition during the analysis, i.e. occurrence of v12-v13, and the implication with v5-v6 (possibly also v7-v14, high level expertise); Besides, according to a normal, linear acquisition of knowledge , v14 should imply v12 -v13, and v13 should imply v12

Hyp.6: Using pattern recognition leads to correct arguing and to better results in final definition of a problem, i.e. the implication between v5-v6-v7 and v8-v9-v10

2.b.9 Analysis of students' protocols

Values "0" or "1" for variables have been assigned examining students' protocols. To better specify this methodology and the use of the above theoretical considerations in practice; let us observe in detail a protocol (n. 12 in the trial) in Fig. 6:


14 anni

Secondo me è patta perché il bianco non riesce ad andare a donna per due motivi:

- Il pedone di Torre non arriva a promozione
- Il re nero riesce a prendere l'opposizione al bianco

Adesso vi faccio vedere la sequenza di mosse analizzate dal sottoscritto

W. (BIANCO)	B. (NERO)
–	Rc8!!
Rc3	Rc7
RD8	Rc8
RD6	RD8!
–	Rc8
c7	Rxc7
Rc3	RB7
RD6	RA7
Rc6	RA8
RB6	RB8
RA6	RA8
RB6	RB8
A6	RA8
A7	

12

Fig. 6 – Protocol n. 12 (14 years old, boy)

I noticed in advance that the student was using correctly algebraic notation, and was framing the problem, then I assigned the value 1 in variables v.1, v2, and v3. Besides, he gave a sharp answer to the "hidden question" concerning the ultimate assessment about the position, typical of a chess player. The student says "*in my opinion the game is draw because white cannot promote a pawn to Queen, because of two reasons*:

- *The rook pawn cannot obtain promotion*
- *Black king succeeds in taking opposition to White*

Now I show you the move sequence analyzed by me."

Very interesting phrases! The student recognizes the two basic patterns “pawn promotion” and “opposition” hence I assigned value 1 in variables v5 and v6, but did not consider all options. Just a short comment about this: he did not consider the other options by black probably because... they are less effective! Like the majority of chess players, he selected immediately the crucial variation, in which Black has serious chances of drawing, and he did not care of the sharply losing options! But the text of the protocol was clear, so I assigned 0 in v4.

After the phrases above, the student wrote two variations, attributing a double exclamation mark (very strong move in chess symbolic language) to the move Kc8, without doubt the best for Black.

Like all his mates, the student did not recognize the pattern “triangular manoeuvre”, that is difficult also for good club players. Then I assigned 0 in v7. Concerning v8, it is a sharp 1. In fact the coherence of the reasoning is clear. This clearness about the final definition led me to assign 1 in v8 and v9. Because of the reasons discussed above, it is a 0 for v10 and v11 (correct forecast), 1 for v12 and v13 (patterns recognized), and 0 for v14.

2.b.10 Data

In the following table are reported occurrences and percentage of the variables

Variable	Occurrence	Percentage
V1	24	0.86
V2	27	0.96
V3	5	0.18
V4	7	0.25
V5	24	0.86
V6	15	0.54
V7	0	0.00
V8	21	0.75

V9	26	0.93
V10	1	0.04
V11	0	0.00
V12	28	1.00
V13	15	0.54
V14	0	0.00

Table.2 – Occurrences and percentage of the variables

In the following table is reported the Implication index $q(v_n, v_m)$, among variables v_n, $v_m$where v_n is on vertical and v_m on horizontal (percentage)
(Poisson law):

variable	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14
V1	100	21	33	53	45	56	0	55	25	46	0	0	56	0
V2	34	100	46	46	54	49	0	51	30	45	0	0	49	0
V3	4	16	100	32	51	67	0	36	30	35	0	0	67	0
V4	63	22	35	100	63	63	0	52	9	36	0	0	63	0
V5	45	58	50	53	100	56	0	71	25	46	0	0	56	0
V6	63	41	57	57	63	100	0	89	66	48	0	0	100	0
V7	0	0	0	0	0	0	100	0	0	0	0	0	0	0
V8	58	53	46	51	80	76	0	100	78	46	0	0	76	0
V9	32	24	47	40	32	55	0	63	100	45	0	0	55	0
V10	13	4	20	17	13	37	0	22	7	100	0	0	37	0
V11	0	0	0	0	0	0	0	0	0	0	100	0	0	0
V12	37	26	44	44	37	43	0	40	32	45	0	100	43	0
V13	63	41	57	57	63	100	0	89	66	48	0	0	100	0
V14	0	0	0	0	0	0	0	0	0	0	0	0	0	100

Table.3 –Implication indexes for variables (percentage)

2.b.11. Data analysis

We proceed with data analysis using as references the above research hypotheses, integrating quantitative analysis with some qualitative considerations.

In advance we notice that in the implicative analysis we have to eliminate variables at 100% (v12) and at 0% (v7, v11, v14). Also v2 (27/28) is almost total, so we don’t consider it for implicative analysis as well.

Hyp. 1 : v1 occurred a lot as expected (24/28, we expected also100%..). Not so high implications with v5 (ind. 45) v6 (ind. 56) , v8 (55) e v 10 (46) resulted, and weaker with v9 (25). This makes us think about connection between used language and final definition of a problem or correct arguing; maybe this connection is not so strong and natural as we expected, at least to the requested level of thinking.

Hyp. 2 : v2 occurred almost 100%, as we expected. v3 occurred poorly (18%), but there is a certain implication with v5 (ind. 51), and better with v6 (ind. 67) but not with v9 and v10. We must be cautious about it because v9 occurred very high and v10 almost 0. Anyway this makes us think about poor using of preliminary framing by students. It is an interesting topic in didactics.

Hyp.3 : As expected, considering the soundness of the basic level of students, we have a high occurrence of v5 (86%), and total occurrence of v12 (100%.)
Sound level is confirmed by good occurrence of v13 and of v6 also (54%). It is connected to the recognition and the use of a classical chess didactics’ item, the opposition (often not known by habitual players without specific chess instruction) . Nobody has recognized the hardest pattern (v14 and v7 0%), but that is explainable considering the level of students.

Not many students considered all initial opportunities (v4 25%); probably because some opportunities were trivial (but in the text we told them to do it!).
V9 occurred highly (93%), and this makes us think that chess player uses to reach a conclusion in analysis anyway.

Hyp.4: The good implication of v4 with v5 and v6 (ind. 63) corroborates the hypothesis, but this is not the case with v9 and v10, while at the above perplexity about v9 and v 10’s occurrences. Much stronger is the implication of v8 with v5 and v6 (ind. 80 and 76) and with v9 also (78), in line with the hypothesis, but it lower resulted with v10.

Hyp. 5: As expected we found total implication of v13 with his “twinned” v6 and vice versa (ind. 100). No comments about v12 and about v7-v14.

Hyp. 6: There was a high implication of v5 with v8 (ind. 71), but not with v9 and v10, see the above considerations. About v6, there is a very strong implication with v8 (ind.89), in a clear line with the hypothesis, and consistently with the expected expertise, represented by v6.
v6 shows implication with v9 (ind.66), but less with v10 (ind.48)

2.b 12. Remarks on the pilot study

In advance we notice that the experimental test was held in a very quiet and serious way thanks to the professionalism of Prof. Rao and thanks to his diligent students. Boys and girls displayed sound chess bases and good ability of exposure. No many students framed first the problem, and this shows no connection with better results. We obtained good support to the hypothesis of the pattern recognition as guide and inspiration in chess thinking, and leading to better opportunities in framing and concluding a problem. In fact arguing and conjecturing were carried out with a clear reference to expertise. It is difficult for players without a basic chess instruction to reach concrete results by exhaustive analysis only. It can be connected to what happens in mathematics or in physics when students hardly solve problems without a form of recognition of a known configuration or a tool.
Summarizing, we obtained a confirmation of the importance of the expertise in chess players' thinking, with precise reference to pattern recognition, according some important outputs in literature.
It encouraged me to focus my work on visual tools and visual imagery as the principal skills connecting maths and chess.
Finally, in this study I noticed also that students-chess player show strong inclination to reach a conclusion in any case.
They don't give up!

Chapter 3
Chess and Mathematics Education

Summary

This chapter deals with chess and math education. It is divided in 5 sections; the first three sections are dedicated to theoretical topics, the last ones to two experimentations realized in 2008 and 2009. The first section deal with chess and skills proficiency. Chess is often defined as the game representing the intelligence, but it is not so easy. In fact some intellectual abilities result stimulated by the chess practice, but this is not always true. Skills like planning and visuo-spatial abilities may improve, especially in children, but most the engagement and motivation in a gaming, intellectual activity like chess product noticeable outputs. In the second section were discussed, more in detail, the relationships between chess and math skills, through important studies, and by analyzing some possible overlapping abilities. Chess may help math learning, but under some conditions, regarding the ways in which the educational activity is performed. In the third section, more complex theoretical topics in Philosophy of mathematics and Education are discussed, and some relationships with chess thinking and chess education are outlined. The fourth section deal with my experimentation held in Palermo in 2008-2009. I considered a group of 52 students in total, whose 16 participated in a chess course from October to May. To observe possible effects of chess practice, I submitted to students a pre-test and a post-test, respectively before and at the end of chess course. Chess players performed better than the average in both test, but not better than the control group in post-test, with no particular difference with respect to the content. The statistical data was not compelling, and I did not take information about other activities performed in non-curricular time, and no information about students' scholastic trend, including the math curriculum and the teachers' methodology and approach. In the last section, is presented the experimentation performed in Agrigento in 2009. The set-up was similar, and also in this case with a very poor statistics. Just the analysis is performed in a slightly more accurate way, i.e. I considered the performance with respect to the contents of test items and with respect to the structure of the item themselves, according to PISA framework. The experimental group performed better in "form" (geometrical) and "uncertainty" items in the area of content, and in "connection" items in the area of competence. On the other hand, no particular improvement occurred in "quantity" and "reproduction" items. It is in good accordance with my beliefs and forecasts, but once again these outputs have to be considered with great caution because of poor statistics and the lack of information about scholastic and chess activity during the trial. These reflections, and other considerations, led me to a shift of perspectives, starting from 2010.

Riassunto

Questo capitolo si occupa di scacchi e didattica della matematica. E' diviso in cinque paragafi, i primi tre paragrafi sono dedicati ad argomenti teorici, gli ultimi due a due sperimentazioni realizzate nel 2008 e 2009. La prima sezione tratta di scacchi e sviluppo delle competenze. Gli scacchi sono spesso definiti come il gioco che rappresenta l'intelligenza, ma non è così facile. Infatti alcune abilità intellettuali risultano stimolate dalla pratica scacchistica, ma questo non è sempre vero. Abilità come la pianificazione e le abilità visuo-spaziali possono migliorare, specie nei bambini, ma soprattutto è l'impegno e la motivazione in una attività ludica e intellettuale come gli scacchi che può produrre notevoli risultati. Nella seconda sezione sono discusse, più in dettaglio, le

relazioni tra scacchi e competenze matematiche, attraverso studi importanti, e analizzando alcune possibili abilità che si sovrappongono. Gli scacchi possono aiutare l'apprendimento della matematica, ma a certe condizioni, in considerazione dei modi in cui l'attività didattica è realizzata. Nella terza sezione, sono discusse più complesse tematiche di Filosofia della matematica e didattica, e si sono delineate alcune relazioni con il pensiero scacchistico e la didattica degli scacchi. Il quarto La quarta sezione tratta della mia sperimentazione tenuta a Palermo nel 2008-2009. Ho considerato un gruppo di 52 studenti in totale, dei quali 16 hanno partecipato ad un corso di scacchi da ottobre a maggio. Per osservare possibili effetti della pratica scacchistica sulla abilità matematica, ho somministrato agli studenti un pre test and un post test, rispettivamente prima ed alla fine del corso di scacchi. I giocatori di scacchi hanno avuto una migliore prestazione rispetto alla media in entrambi i test, ma non meglio del gruppo di controllo nel post-test, e senza particolari differenze rispetto al contenuto. I dati statistici non sono convincenti, e non ho assunto informazioni riguardo l'andamento scolastico degli studenti, incluso il curriculum di matematica e l'approccio e la metodologia degli insegnanti. Nell'ultimo sezione, è presentata la sperimentazione realizzata ad Agrigento nel 2009. L'impostazione era simile, ed anche in questo caso con una statistica bassa. Giusto l'analisi è stata realizzata in un modo leggermente più accurato, cioè ho considerato la prestazione rispetto ai contenuti degli item dei test e rispetto alla struttura degli item stessi, secondo lo schema PISA. Il gruppo sperimentale ha avuto una prestazione migliore negli item di "forma" (geometrici) ed "incertezza", e negli item relativi alla "connessione" riguardo l'area della competenza. D'altro canto, nessun particolare miglioramento si è avuto negli item relativi a "quantità" e "riproduzione". Questo è in buon accordo con le mie previsioni e convinzioni, ma una volta ancora questi risultati devono essere considerati con grande cautela per la bassa statistica e per la mancanza di informazioni riguardo la attività scolastica e scacchistica durante l'indagine. Queste ed altre riflessioni mi hanno portato, a partire dal 2010, ad un cambio di prospettive.

3.a Relations between chess and skills proficiency

Summary

This section deals with relations between chess and skills proficiency. Chess is commonly considered as the intellectual discipline *par excellence*. But it is not so easy. I am interested most in Chess and Education, specifically in Math education, so it is necessary to consider carefully the role of chess with respect to well-defined skills we are interested in, and also distinguish if the same role can be played by another intellectual discipline, or any extracurricular activity performed by the students. In the course of the section several studies and examples concerning chess, intelligence and various skills are critically examined, and some theoretical positions are discussed. Quoting Schoenfeld, the effect of a curriculum it is not independent of the context in which this curriculum is proposed in a class or students group. So, in my work there was a shift of perspectives in this direction. Chess may be related with problem solving skills like planning and visuo-spatial abilities, but most the engagement in an intellectual, gaming, motivating activity may product wonderful outputs. Chess is perfect for this purpose, like occurred also in particular social contexts like New York City.

Riassunto

Questo sezione tratta delle relazioni tra scacchi e sviluppo delle competenze. Gli scacchi sono comunemente ritenuti la disciplina intellettuale per eccellenza. Ma non è così semplice. Io sono per lo più interessato a scacchi e Didattica, specificatamente didattica della matematica, così è necessario considerare attentamente il ruolo degli scacchi rispetto a competenze ben definite cui siamo interessati, ed anche distinguere se lo stesso ruolo può essere giocato da un'altra disciplina intellettuale, o da qualunque attività extra curriculare svolta dagli studenti. Nel corso del sezione diversi studi ed esempi riguardanti gli scacchi, l'intelligenza e varie abilità sono esaminati criticamente, e discusse alcune posizioni teoriche. Citando Schoenfeld, l'effetto di un curriculum non è indipendente dal contesto nel quale questo curriculum è proposto in una classe o gruppo di studenti. Così, nel mio lavoro c'è stato uno spostamento di prospettive. Gli scacchi possono essere connessi con abilità di problem solving come la pianificazione e le abilità visuo-spaziali, ma per lo più l'impegno in una disciplina intellettuale, ludica e motivante può produrre meravigliosi risultati. Gli scacchi sono ideali per questo, come è avvenuto anche in particolari contesti sociali come la città di New York.

In the common vision, chess is a paradigm of intelligence; we often hear of sentences like: “You need to be intelligent to play chess”, “Chess fosters intelligence”, “Chess fosters maths”.

But such statements are very hard to support!

First, we know that intelligence is a word with which we intend to several meanings, including skills, ability, knowledge, relational attitudes, and as a consequence of it will be necessary to specify sharply what we want to consider, e.g. see (Gardner 1999)

Secondly, what chess? Tournament practice? A scholastic course? Or other? These questions are not trivial, and require attention.

In fact, we have to consider also the time required to acquire chess skills, that is a not negligible factor, especially in education.

If a correlation may be established for high level chess skills and other abilities, probably it will be not relevant in educational contexts, in which chess are practiced mainly at basic level.

According to the elements emerged from the historical epistemological analysis and cognitive aspects of chess made in the previous chapters, in my opinion better shaped research questions may be formulated in this way: Is it possible to find sustainable arguments bringing evidence of links between specified chess ability and other specified skills? Also, at what age a chess course and practice may become proficient in educational context, especially in mathematics? And how are relevant the contents and the development of the chess course?

In this section I consider critically the principal scientific works published on this topic. I focused mostly on studies dealing with students, because it is my principal interest, but also general studies are considered, by their intrinsic value and by possible generalization of the outputs in educational contexts.

In 1976 J. Christiaen in a Belgian school studied the effects of a one year and half chess course on cognitive development of fifth grade students, aged about 10 (Christiaen, 1976).

Students were divided randomly in two groups. Cognitive items were tested according to Piaget's framework on the appearance of stages of development, in particular the transition from *concrete operational* stage to the *formal-operation* one. No pre-test was submitted. Post test consisted of two standard Piagetian tests, and also school results were considered in the study. The results were not sharp, but chess group anyway performed better in all tests, with a significant improvement in school results.

As noted by Gobet and Campitelli (2009) in their fair review of Chess & Education studies, this study has several points of strength but weaknesses as well. In fact the correct random allocation of the groups, and test variety represented points of strength, but teachers' awareness in participating to a complex trial may be a non negligible effect.

Gobet & Campitelli noted also that in this and in other studies there is not a placebo group. Using the word *placebo* in this context means that the awareness of participating to a trial is considered relevant.

I agree, but it is not all. In fact, also the engagement in an intellectual activity may be relevant. E.g. in Christiaen experimental setting, control group students went home when exp. Group students attended chess course. So, also the quality and quantity of the activity of a possible placebo group would be studied. This kind of engagement was treated in (Radford & Roth, 2011), in which the authors resort to the concept of *togethering* to capture the ethical commitment participants make to engage in and produce activity.

It results because of not trivial question about transfer of skills (Gobet & Campitelli, 2009), that is possible only when there is an effective overlapping of abilities. These abilities are to be specified if developed or not during the activity of an experimental group, a control or a possible placebo group. Then the effectiveness of chess as an educational curriculum may be found if the specific overlapping abilities are developed, becoming matter of pre and post test.

More in general, as fairly described by Schoenfeld (2007):

There are two fundamental conceptual issue related to curriculum implementation is what one considers an "implemented curriculum" to be (...)

Perspective 1. A curriculum is the set of instructional materials and preparation to use them that teachers are given. Whatever the teachers do with those materials in the classroom is the "implemented curriculum."In this case, what counts as the simplest measure of the curriculum's effectiveness is the average performance of all those students who were in classrooms where that curriculum was used. Another perspective is as follows.

Perspective 2. There is a strong degree of interaction between curriculum and context. Given different contexts or different degrees of support, there may be more or less fidelity of curriculum implementation. "Degree of fidelity of implementation" (in conformity with the intention of the designers) matters and should be taken into account in analyses of curriculum impact.

In our case, chess is the curriculum. In the first two years of my doctoral course, my approach was mainly as described in Schoenfeld's Perspective 1. I was looking for effects of chess on math learning, conceiving chess, math learning, and trials adopted like separate, closed worlds. I did not take care of specific context in which I realized trials, meaning the contents of chess course and of math curriculum of the classes. During the third year of my course, I shifted my perspectives, first moving from Schoenfeld's perspective 2 to 1, and also convincing myself that only a fine historical-epistemological analysis of chess may allow an effective search for overlapping of chess and math skills. I think that learning is a complex phenomenon in which act many factors, like cognitive, psycho-pedagogic, logical, epistemological, and semiotic ones. They interact specifically in a given context, and may bring to *objectification*. Then, become determinant how the chess course is performed, in terms of contents and proficiency, and how is tested the possible effects on math learning.

Coming back to studies on chess and skills, in 1973-74 Dr. Albert Frank performed in Kisangani, Zaire, a study called Chess and Aptitudes (Frank & d'Hondt, 1979)

The study dealt with links between learning chess and some *aptitudes*:

- Spatial attitude
- Perceptive speed
- Reasoning
- Creativity
- General intelligence

Two groups of students, aged 16-18 were selected, the experimental group and the control one.

Three 30 students classes for every group

In the experimental group, students dedicated 2/7 of their maths hours in learning chess.

Frank administered pre and post test using the following tests:

- The Belgian version of the G.A.T.B. (General Aptitude Test Battery)
- The P.M.A. (Primary mental abilities by Thurstone)
- The D.A.T. (Differential Aptitude Test) by Bennet, Seashore and Wesman)

- The D2 (Brieckenkamp – test of attention)
- The Rorschach (projective test). (Web ref. 2)

In pre test groups performed equally.

In the post test, among tested aptitudes, two show significant differences in favour of the experimental group: the arithmetical aptitude, with a threshold of .05 and erbal logic?most often measured by the identification of synonyms or antonyms) with a threshold of .01. (Web ref. 2)

The result in verbal logic?sounds strange, but somewhat similar will occur in other studies.

I have no exact information about how was measured the arithmetical aptitude, anyway Gobet and Campitelli (2009) comment on numerical aptitude and on possible statistical effects, and conclude that results are not completely sharp, considering the poor result of control group. In fact, Frank e D'Hondt themselves refer about a lack of motivation of some students included in the trial.

R. Ferguson realized in 1979-83 a study named *Developing Critical and Creative Thinking Through Chess*, (Ferguson, undated) in which participated gifted students from Bradford, Pennsylvania, in grades 7 to 9 (aged about 12 to 15). Students participated in various activities including chess, computer, and other. Alternate forms of the Watson-Glaser Critical Thinking Appraisal test (CTA) and of the Torrance test of creative thinking were used for pre and post testing (Ferguson, undated). Chess group performed significantly better in Critical Thinking, fluency, and most in originality. Weaknesses in this study are highlighted by (Gobet & Campitelli, 2009): small sample (15 students a group), and mostly the fact that chess group students attended also other activities included in the trial. Lastly, the experiment deals only with gifted students, with obvious lack of generality. Anyway, two very important characters of chess thinking are critical and creative thinking. As mentioned earlier, the impossibility to perform an exhaustive analysis (in most cases) stimulates players to consider the basic strategic factors, they learned before, in a non automatic way. In fact the situations the players encounter on the board are different each other, and the strategic factors (heuristics) have to be considered case by case. Good chess trainers teach fairly which factors are to be considered and how to avoid dogmatism. These factors can be also in contrast each other, so a realistic approach is needed, nevertheless based on sound theoretical fundamentals. The continuous matching with the real situation and the creative options the chess player can discover in his/her thinking represent, in my opinion, a very useful practice for education.

The second study by Ferguson, Developing of reasoning and memory through chess (Ferguson, undated) tested students from a Pennsylvania school attending a chess course from

September 1987 to may 1988. California Achievement Test battery in “memory” and “verbal reasoning” were used. The differences between the post test and the pre test were confronted to the national norms. The chess group improved more than the general population in the “memory” subtest ($p < 0.001$). In “verbal reasoning,” the chess group performed better than the general population in a non relevant way ($p < 0.10$) (Gobet & Campitelli, 2009). It was noted (Gobet & Campitelli, 2009) that the study uses well standardized measure and pre and post test, but *There was no random allocation of participants to groups, the sample was very small, and there was no control group carrying out an activity different than chess.* More in general, it is well known that to engage in an intellectual activity gets benefits almost in all cases, so caution is recommended at this juncture.

Another, historical trial on chess and Education was the “Learning to think Project”, that was realized in 1979-83 in Venezuela. The general aim was to test if chess can improve intelligence of children as measured by the *Wechsler Intelligence Scale for Children (WISC).* In the project 100,000 teachers were involved to teach thinking skills and a sample of 4,266 second grade students was considered, reached a general conclusion that chess, methodologically taught, is an incentive system sufficient to accelerate the increase of IQ in elementary age children of both sexes at all socio-economic level, see FIDE Report 1984 (Tudela, 1984)

B.F. Skinner, the famous psychologist, wrote: “There is no doubt that this project in its total form will be considered as one of the greatest social experiments of this century” (Linder, 1990).

Because of the success of the study, the chess program was greatly expanded. Starting with the 1988-89 school year, chess lessons were conducted in all of Venezuela’s schools (Linder, 1990).

This study is frequently quoted in documents and web sites concerning the relevance of chess for Education, but not so much in scientific works, because of incompleteness of its experimental setting. Anyway, the large-scale of the trial, and the subsequent massive introduction of chess in Venezuela’s schools make it a non negligible event.

(Horgan & Morgan, 1990) studied the scoring of a group of young chess players (113 students from a little primary school in Memphis, Tennessee), in tests like Raven’s progressive matrices (concerning perceptions of relationships in geometric figures) and a Piagietian task (a plant task designed analogously to the “colourless liquids problem to measure combinatorial logic in formal operations) (Horgan & Morgan, 1990). A chess task was submitted too. The best chess players performed better in all tests. These outputs seem to find evidence about the proficiency of chess practice for improving visuo-spatial abilities for children, but seem that these results are not to be generalized easily for adults. In fact, (Djakow, Petrowsky & Rudik, 1927) study apart, an important study by (Waters, Gobet & Leyden, 2002) *found no evidence for a correlation between chess skill and visual memory ability in a group of adult chess players.* As fairly noted by the authors, *visual-memory and visuo-spatial intelligence may be unimportant factors in long-term acquisition of chess skill.* It is somewhat confirmed by the important study conducted by (Unterrainer, Kaller, Halsband & Rahm, 2006) about the planning abilities. They tested 25 adult chess players (exp. group) and 25 non chess-players (control group). The groups were recruited by strong similarities for age and level of education. The trial compared the groups’ performance using a standard psychometric planning task, The Tower of London test. They also studied fluid intelligence, using the Raven test, and verbal and visuospatial working memory. The chess players performed sharply better in planning abilities, but not in fluid intelligence and verbal-visuospatial working memory. It is a very interesting result. The attitude of chess player to tackle a problem and to pursue a solution, whatever it is, emerges.

Again in Belgium, (Frydman & Lynn, 1992) conducted an inquiry on young gifted Belgian chess players’ IQ, using the French version of the Wechsler Intelligence Scale for children. The group scored a general higher IQ than the population mean both in performance and verbal IQ (especially in performance). The stronger players performed better than the weaker players.

Another well-known study is from Liptrap (Liptrap 1998). The purpose of the study was to document the effect of participation in a chess club upon the standardized test scores of elementary students. 571 from third to fifth-grade students of elementary school were tested, using TAAS (The Texas Assessment of Academic Skills). Chess students and non-chess students were compared. *In fifth grade, regular track chess players scored 4.3 TLI points higher in Reading ($p<.01$) and 6.4 points higher in Math ($p<.00001$) than non-chess players.* (Liptrap, 1998).

The study relies on a sound and significant sample, but, as observed by (Gobet & Campitelli, 2009) too, students chose to participate in chess club or not, so a selection effect could be relevant. Again, in this case, and more in general, the benefits of an intellectual and recreational activity on Education seem to be evident. Students use to be engaged and this custom has transversal proficiency. Chess often represents a fantastic opportunity in contexts suffering social and youth distresses. The most famous example is without doubt the Chess-in-the schools project. This big project is active in New York City since more than twenty years, involving thousands of students, especially in ghettoes. It was activated by a non-profit educational organization, and results are excellent. Using Buky and Ho’s words (web references 8):

> *... The ACF (American Chess Foundation) embarked on the Chess in Schools Program which focused on New York's Harlem School district. Initially the program was focused on improving math skills for adolescents through improved critical thinking and problem solving skills. This was achieved as "test scores improved by 17.3% for students regularly engaged in chess classes, compared with only 4.56% for children participating in other forms of enriched activities.". Also noted was that many students social habits improved when playing chess. The game allows for students of dissimilar backgrounds to integrate with others. Many disadvantaged or special education students are becoming actively involved in chess programs as the value of chess as a social tool is further explored. Advocates of chess are hoping that some of New York's gang related problems will be solved as children and students play chess in their spare time instead of becoming involved with gang related activities. Thus chess steers youth away from trouble by keeping them off the streets as well as being a useful learning.*

These outputs are impressive but do not surprise teachers and chess trainers working in the schools.

In my personal experience in some chess courses, held in Palermo working class districts, I noticed the wonderful changes occurred to several boys and girls. They changed their characters and approach, becoming more reflexive and less violent. These attitude changes in students are frequently noticed by teachers during chess course, most in attention and concentration. It happened also in our experience in SAM project, as specified in the next chapter. Coming back to New York, a fair report about a particular experience in Harlem can be found in (Coudert 1989), and an interesting inquire was performed by (Margulies, undated), in which also reading skills were related to chess. 53 students participating in a chess course in South Bronx, New York, were compared with 1118 students’ non participants. It was evaluated the performance in Reading power, tested by the “Degree of Reading Power Test”,

(DRP test). A pre and a post test were realized. As control group was considered first the non participants as a whole, and also a subset of them obtaining the same result in pre-test, to avoid selection effect. The results were very encouraging for the chess group. It can be excepted that in the experimental design there is not a group engaged in another intellectual activity, anyway the results are sharp.

Another interesting study is from (Fried & Ginsburg, undated).

Expanding Christiaen's study, they deal with the effects of chess instruction on the development of perceptual ability, visuo-spatial ability, and attitude towards school.

The sample consisted of children with learning and behavior problems, referred for counseling by their teachers because of behavior problems. Thirty New Yorker students from 4^{th} and 5^{th} grade were assigned randomly in one of three group: do-nothing (control), counseling (considered as placebo groups for the trial purpose), and chess (receiving chess instruction 2 hours a week for 18 weeks). After the course, three tests were submitted: (a) the "picture completion" subtest of the revised version of the Wechsler Intelligence Scale for Children, as a measure of visual ability, in particular visual awareness to detail; (b) the "block design" subtest of the same test,

Measuring visuo-spatial ability; and (c) a survey of school attitudes, to measure the regard for the school. The study failed to identify relevant differences among the groups; Just picture-completion task showed a slight advantage for the chess group. In the chess group also a gender difference was noticed.

Other studies, like Doll and Mayr (1987), Waters, Gobet and Laiden (2002), Grabner, Neubauer and Stern (2006) and Unterrainer, Kaller, Halsband and Rahm (2006) did not found any sound evidence of correlation between chess skill and general intellectual abilities, using standard intelligence tests, as clearly reported in (Bilalic, McLeod & Gobet, 2006). It is very important to notice, for my purposes, that the quoted studies deal with adult chess players, most expert players, as will be discussed in the following. In fact, as noted by (Bilalic et al.), one of the reasons of no sharp correlation chess skill-intelligence in adults, is the selection drop-out effect, due to the fact that most people give up chess, so the chess-skilled population tend to restrict to individuals with similar characteristics (e.g. intelligence, motivation) (Bilalic et al.), and it may be a reason for the outputs from the studies, quoted in the former, that conversely show some evidence of influence of practicing chess on school proficiency and general skills of students. It is a fair confirmation of what is noticed by thousands of teachers in the world about the effectiveness of chess practice in students, in particular concerning the attitude to face a problem and pursuing a solution. It makes reference to Unterrainer, Kaller, Halsband and Rahm (2006) and to my first pilot study. I confirm it on the basis of my experience in schools and in chess clubs. There is a general accordance, in literature, about the reasons for chess skill, that is mainly deliberate practice as observed daily in the chess clubs, and about the lack of a sharp correlation between chess skills and intelligence This assessment, of course, is to be considered on average. I am completely agree, my experience as a chess teacher proves that any well motivated, enthusiast guy can become a very good player. The question about the rising of *chess phenomena* like Capablanca or Kasparov is quite different, and it is not inherent in *normal* Education, so I don't want deep this topic by now. In their very accurate study, (Bilalic & al. 2006) considered the possible correlation between intelligence, as measured by WISC III, and chess skill, as measured by tests and taking into account the ELO ratings. The trial was performed with a group of 57 young players; taking into account also the amount of practice and the years of experience as possible other effective factors for chess skill. Results show clearly a lack of positive correlation, between intelligence and chess skill, except for an elite sub-sample. It is in contrast with the common vision conceiving intelligence as a basic request for chess skill. Again, it is confirmed that practice is the main reason for expertise. It was also found a *moderately positive* relationship between intelligence and skill for pupils whom just started to play chess, and this relationship disappear for higher levels of skill. It may be depend on various factors, the authors suggest that *While more intelligent children seemed to spend more time on chess than their less intelligent peers, this was not the case in the elite subsample — more intelligent children in the elite subsample invested less time in chess.* (Bilalic et al). Again, (Bilalic et al.) pointed out very appropriately that in chess, as in real life, factors like practice, age, gender and intelligence itself are in mutual, continue relationships into each other, so to separate them as isolated factors is very difficult and, I think, frequently makes no sense. It happened to me in the first part of my work, conducting me to a shift of perspectives, as will be better specified in the following.

3.b Chess and Mathematics: analysis of the possible mutual links

Summary

This section deals with chess and maths. In the first part, I discuss the main works on the topic, taking into account also my personal experiences and beliefs. Many scholars looked for correlations between maths and chess, and various interesting outputs occurred. These correlations depend strongly on how is performed the chess practice, on contents proposed, and of course on the kind of math activity and curriculum at school. So, a full generalization is not possible, anyway some features emerged sharply. The ordinary chess practice in school

seems produce no improvements in calculation ability, but some specific chess courses, as developed by Ho & Buky, and after, independently, by me and Mario Ferro in SAM project, involved calculation items and, as a consequence, this may produce improvements in calculation ability too. Chess practice resulted proficient in math problem solving skills and for the attitude to engage in an intellectual effort, as emerged in several studies and inquires. This is confirmed by various reports by teachers I experienced during my personal activities dealing with schools. In Italy, an accurate study, carried out by Trinchero & Piscopo, gave significant outputs. These results can be summarized as follows: duration of chess course (min. 30 h), gaming approach, absence of any pressure on students, motivating and valid work methodology. Besides, cognitive abilities' improvements seem depend more on learning game logic (values, moves, positions, strategies), than on the practice hours. Practice hours, however, are important to better fix these concepts. Other factors, like gender, kind of activities in leisure time, preferred games and scholastic judgments seem non influential. About visuo-spatial abilities, they seem improved by the chess practice in children, but this trend tends to disappear in adults. In particular, the chess player uses visuo-spatial abilities, ,in somewhat similar way to geometrical mental imagery, as noticed by (Presmeg 2006). It is performed a parallel between geometrical objects, as depicted by Fischbein, and chess concepts, considering the figural concept in the Fischbein framework as concepts driven by their definition, and the configural concepts in chess as concepts that make sense by the relationships with the other ones in various patterns.

The correlation of chess practice with skill in geometrical demonstrations and proofs has to be still developed.

Riassunto

Questa sezione tratta di scacchi e matematica. Nella prima parte, discuto i principali lavori sull'argomento, prendendo in considerazione anche le mie esperienze personali e convinzioni. Molti studiosi hanno cercato correlazioni tra scacchi e matematica, e vari interessanti risultati sono emersi. Queste correlazioni dipendono fortemente su com'?sviluppata la pratica scacchistica, dai contenuti proposti, e naturalmente dal tipo di attività matematica e dal curriculo a scuola. Cos? una piena generalizzazione non ?possibile, comunque alcuni particolari sono emersi chiaramente. La pratica scacchistica ordinaria a scuola sembra non dare alcun miglioramento in abilità di calcolo, ma alcuni specifici corsi di scacchi, come sviluppati da Ho e Buky, e dopo, indipendentemente, da me e Mario Ferro nel progetto SAM, coinvolgono argomenti di calcolo, e conseguentemente, questo può produrre anche miglioramenti in capacità di calcolo. La pratica scacchistica è risultata efficace per l'abilità nel problem solving matematico e per l'attitudine ad impegnarsi in uno sforzo intellettuale, come emerso in vari studi ed indagini. Questo ?confermato da vari report di insegnanti di cui ho avuto esperienza durante le mie personali attività con le scuole. In Italia uno studio accurato, condotto da Trinchero e Piscopo, ha dato risultati significativi. Questi risultati possono essere sintetizzati come segue. Durata del corso (minimo trenta h), approccio ludico, assenza di pressione sugli studenti, e metodologie di lavoro valide e motivanti. Inoltre, i miglioramenti delle abilità cognitive sembrano dipendere dall'imparare la logica di gioco (valori, mosse, posizioni, strategie), piuttosto che dalle ore di pratica. Le ore di pratica, comunque, sono importanti per fissare meglio i concetti. Altri fattori, come il genere, il tipo di attività nel tempo libero, i giochi preferiti e i giudizi scolastici sembrano non influenti. Riguardo le abilità visuo-spaziali, queste sembrano incrementate dalla pratica scacchistica nei bambini, ma questa tendenza tende a scomparire negli adulti. In particolare, il giocatore di scacchi utilizza le abilità visuo-spaziali, con relazione al mental imagery geometrico, come rilevato da N. Presmeg. E' stato realizzato un parallelo tra oggetti geometrici come descritti da Fischbein e concetti scacchistici, considerando i concetti figurali nella impostazione di Fischbein come guidati dalla loro definizione, ed i concetti configurali negli scacchi, cioè concetti che prendono senso dalle relazioni con gli altri, in varie configurazioni. La correlazione della pratica scacchistica con l'abilità in dimostrazioni geometriche deve essere ancora sviluppata.

3.b.1 General issues and studies

Many people say that between chess and math there are sound correlations, but it is necessary to proceed with caution. I am interested most in correlations concerning Math Education, and in matters regarding using chess in educational contexts. In the following I will consider the various possible links, considering the main works on the topic, commenting them and taking into account my experiences and beliefs.

A very quoted study is by (Gaudreau 1992), in which 437 fifth-graders students of the province of New Brunswick, Canada were tested during almost 3 years, divided in three groups. Group A received a traditional math curriculum; Group B received a traditional math curriculum in the first year, and after an integrated curriculum with chess and problem solving. Group C received the integrated curriculum since the first year. There were no appreciable differences among groups in calculation, but a sharp advantage for groups b and C in comprehension, and most in problem solving, resulted. For Group C emerged also an impressive improvement in problem solving score (from 62% to 81%). These outputs are surely encouraging for chess practicing in schools, but a great care is requested for a correct interpretation of them. In fact, this experimental design did not plan any group doing another intellectual activity. There is a general accordance in considering chess as a proficient integrated activity in school curriculum, but as stated already in the former, probably also another structured, intellectual activity may be a valid stimulus for children. Anyway results from (Gaudreau, 1992) are interesting and encouraging, most because the assignment of students to groups seemed to be random, avoiding selection effects. I want to remark one of the Gaudreau's output, the lack of improvement in calculations, that is an output I and Mario Ferro noticed also in the Agrigento trial presented in the following. To calculate in a arithmetic sense is not an ability used most in chess practice, unless calculations exercise were inserted in chess course using chess icons or symbols, as fairly explained by Buky & Ho in (web references 8), and as realized (independently) in the Italian SAM project chess protocol in 2011, discussed in the following. Buky & Ho studied the outputs of an enriched chess-math integrated instruction, adopting for curricular courses a textbook by Ho, in which several math topics are developed using chess icons and symbols. They selected an experimental group attending the chess-math enriched instruction and a control group. Tests of TONF (The Compass Learning Explorer Online Diagnostic Tool was used for both the pre-test and post-test. The results were very sharp showing a clear advantage for the experimental group. I notice that Buky and Ho speak about improvements in calculation, because the Ho book develops arithmetic calculations by means of chess. Ho and Buky highlight:

> *That while students playing chess learn concepts through physical and visual stimuli and correlate these concepts to cognitive patterns, mathematics in the classroom usually involves only pure symbolic manipulation. Thus there seems to be some evidence to suggest that chess acts as a sort of link in connecting form (symbolic) with understanding (physical and visual).* Ho also states (Ho 2006) that *Dr. Montessori observed that younger children were intensely attracted to sensory development apparatus. Chess being hands-on and multi-sensory, involves coordination between eyes, brain and hands in multi-direction, and embodies concepts that are non-linear when compared to most video and computer games.*

In this sense, there are sharp similarities with those math education theories involving embodiment. Chess pieces, bridging from sensorial to conceptual, work as pure means of objectification.

In 2007-2008, on the occasion of a big project involving thousands of students in the Italian Region of Piemonte, Trinchero & Piscopo (web references 9) conducted an important research on the effects of introducing chess in the Primary school on cognitive skills, named *Scacchi gioco per crescere* (chess: a game to grow up with). Trinchero & Piscopo in the introduction of their work sharply identify those skills used in problem solving processes that are stimulated by chess practice.

The experimentation was held separately during two scholastic years: 2006-2007 and 2007-2008

The inquire concerned 290 3th grade primary school students (about 8 years old) in 2006-2007 and 289 in 2007-2008. The main difference between the periods concerned the duration of the chess course: 10 hours in 2006-2007 and 30 hours in the following, and it will be a very important factor.

The basic schema of the research plan was the following:

- A pilot test submitted to a limited sample to better refine the definitive test
- A pre test was submitted to students before they attend a 30 hours chess course
- (in the second year) Experimental and control group was set depending on the

pre test results (similarities)

- 30 hours chess course, monitoring the development of the course itself, the Teachers involved in, etc.
- Post test

Going more into depth, the test items comprehended a closed answer and an open answer: The close answer pointed to observe the product of the child's problem solving activity, while the open one to the process (web references 9). The double answer allowed the researchers to reconstruct the resources used by the children, the approach to the items, and the action strategies employed to solve items and in reflecting about. The items concern mainly logical and mathematical matters, in non scholastic contexts, in such a way to avoid mnemonic answers and stimulate independent reasoning. Students never participated in chess activities before. For a better analysis, the authors considered, during all the trial, several factors like:

- *Gender*
- *Activities performed by the children in non curriculum hours (accomplishing homework, watching television, practicing sports, reading, playing, attending parish recreation centers)*
- *Playing any game*
- *School assessments (Italian language, mathematics, Science)*
- *Satisfaction in the chess course (Almost all students liked very much to attend the chess course)*
- *Playing chess after the course*
- *Playing chess outside the school*

(my translation from Italian language) (Web references 9)

The chess protocol used in the chess course was the same in all classes.

It was realized a monitoring of the chess courses, i.e. were reported the experience of the chess instructors, and the various logistic problems that happened.

In my opinion, in realizing a trial to observe possible effects of chess practice on education, the consideration of all these factors quoted above it is fundamental. It is not enough practicing chess. To perform a well-based analysis of an experimentation, we need to know how this treatment is realized, which contents are presented to students, what happened during the chess course, referred to students, teachers and general scholastic context. It is in according with our general theoretical framework on teaching/learning, as specified in the introduction.

Therefore, I think that Trinchero & Piscopo's research is very relevant.

The outputs of the inquire are summarized by the authors as follows:

a) Experimental groups improvements are concentrated on items concerning math abilities and extracting abstract rules from a situation and apply them.

b) Chess may be a valid support for those cognitive abilities important in Education, provided that:

- b1) Courses performed for 30 h at least
- b2) Motivating and valid didactic methods and contents adopted
- b3) Chess presented as a mere game and nothing more
- b4) the setting of the course and the environment in which the course is realized have to allow students to learn chess, without pressure or dependence on results; it implies a good approach by the teacher, and a good atmosphere in the classroom.

c) Cognitive abilities' improvements seem depend more on learning game logic (values, moves, positions, strategies), than on the practice hours. Practice hours, however, are important to better fix these concepts. Other factors, like gender, kind of activities in leisure time, preferred games and scholastic judgments seem non influential.

In the former points we find some key points related to the research question as outlined in the introduction of my thesis. The basic point is that chess practice allow students to grow up several cognitive resources (like those specified in the former chapter), that are in principle proficient for scholastic curriculum, especially in mathematics. But chess is not a medicine, and in this sense I want to remark two fundamental things: first, the capacities and the approach of the teacher are fundamental factors in educational context, including chess. Besides, like happens for other disciplines, the recreational approach seems to be more proficient for learning, according with many pedagogical and educational studies. Secondly, as previous research teaches us, the skill and abilities empowered are strictly connected with the contents and methodologies adopted in the chess courses. It implies that the overlapping abilities are not so general, and to generalize hypothetic benefits is extremely dangerous. The benefits are relevant most for child in primary school, and these benefits may represent a launching-pad for the child and not the target itself (if anyone exists). Besides, we have to consider with a great care the time necessary to reach high skill in chess, which may be counter-productive for the education of the student. It is remarkable also that chess, like other competitive activities, may give cause troubles connected with anxiety. On this topic, recently appeared in Italy two interesting works, Martinengo and Sgrò (2010) and Sgrò (2009). It may be interesting to go into more deep on the role of emotions in game thinking in general and its effect on learning. For this purpose may be significant to cooperate with neuroscientists. This aspect, even if relevant, could move away me from the main aspect of my work.

3.b.2 Visual concepts and ability in math and chess

In the last decades there was a strong interest in Math education for visual thinking. This most for geometry, but also for other branches of math. Using Presmeg's words, *The importance of visual processing and external manifestations of this cognition in mathematics was increasingly recognized. After all, mathematics is a subject that has diagrams, tables, spatial arrangements of signifiers such as symbols, and other inscriptions as essential components* (Presmeg, 2006).

As frequently reported in this work, it is clear that chess has a strong component in visual thinking; Let me discuss some features of these relationships. I highlighted in the former chapter several cognitive aspects of chess practice. The visuo-spatial component plays in chess an important role, considering the active processes related to visual perception, as outlined by the Gestalt framework., in fact the mental imagery adopted in chess may be considered most as a dynamical one, because is strongly depending on the relationships with other pieces (context), and on the possible, and plausible, moves on the board. This dynamical aspect was noticed also by N. Presmeg, quoting chess thinking in (Presmeg 2006):

> *For instance, pattern imagery, which was a strong source of generalization for the learners who used it in Presmeg's research, might also involve elements of Dörfler's figurative image schemata because it is perceptual, without transformations. However, pattern imagery by its nature is capable of depicting relations (e.g., in the "lines of force" described by master chess players in describing a game on the board), thus it also incorporates elements of relational image schemata. Further, the categories may overlap, e.g., pattern imagery may also be dynamic.*

Dorfler (1991) remarks the role of imagery as "inductor" of meaning, using his words: *"Meaning is viewed here to be induced by concrete 'mental images' as opposed to propositional approaches".* In chess the meaning is related to the plausible advantages or, in general, changes caused by the mutual relationships among pieces, both in the position in which they are and in a possible one.

There is a strong parallel with geometrical concepts. The logical and figural aspect of geometrical entities was well defined and studied by Fischbein (1993). Dealing with geometrical entities, he states: "*They are of a conceptual nature. At the same time, they have* *an intrinsic figural nature: only while referring to images one may consider operations like detaching, reversing or superposing. ...* Fischbein, among the properties listed (1993), points out that a square *..." is a shape controlled by its definition (though it may be inspired by a real object).*

It is the same referred to chess pieces, in which the physical, pictorial aspect is twinned with the logical properties. Once again, as specified in chapter 2, sensorial, mental and logical aspects contribute to cognition as a whole. Fischbein (1993) defines the mental entities used in geometry *figural concepts, which reflect spatial properties (shape, position, magnitude), and at the same time, possess conceptual qualities (like ideality, abstractness, generality, perfection).* But in chess an element (a chess concept, inducted by the real position) makes sense because of its dynamic potentialities, as depicted in section 2.b, becoming a *configural* concept (see section 2.b), as discussed by Ferro (2011). The objectification is represented by the appearance of the element in player's mind, at the end of an anticipatory process that can be very complex, depending on the position and context. Then, the word configural is composed by the words context and figural.

The meaning of the chess element is represented by the dynamic potentiality itself. Without this anticipatory and exploratory process (means, way of objectification) the element remains static, limited to its geometrical features. In this sense, the former ideas are quite in accordance with Saariluoma's thought models (Saariluoma, 2001).In Educational context, it is natural to suppose that practising chess may be useful in Geometry, but a great caution is necessary, as widely discussed in the former. Some evidence resulted in improved children visuo-spatial ability, but this seems to disappear in adults. It is not a surprise, because of the multiplicity of stimuli and studies occurring during an adult's life and concurring in building personal skills, abilities and culture. The overlap of abilities may concern spatial orienteering and anticipatory spatial thinking. About spatial orienteering, I convinced myself that chess can improve it, especially in pupils. About anticipatory spatial thinking may be interesting to study in particular the possible correlation between chess practice and ability in geometrical thinking. It is an open problem.

3.c Mathematics education: theoretical framework and relationships with chess education

Summary

This section deals with theoretical arguments in mathematics education and their relationships with chess, in particular with chess education. Starting from a basic topic in philosophy of mathematics, i.e. the conception of the mathematical objects, and passing through the topic of the semiotic representations of these objects, some basic themes in math education are outlined. The two main points of view in approaching the mathematical objects, the realistic and the pragmatic one, are synthetically presented, as well as several arguments from Radford's mathematics education theory (2003, 2005). Relationships with chess conceptions and chess education are discussed, and the topic of the epistemological obstacles in chess education is presented, with reference to the critique of the principles as outlined in chapter I, and as a useful tool in education. The relevance of historical-epistemological approach is highlighted, as discussed in the theoretical works of F. Spagnolo (2009)

Riassunto

Questa sezione tratta di argomenti teorici in didattica della matematica e delle sue relazioni con gli scacchi, in particolare con la didattica scacchistica. Partendo da un argomento base in filosofia della matematica, cioè la concezione degli oggetti matematici, e passando attraverso l'argomento della rappresentazione semiotica di questi oggetti, si sono esposti alcuni argomenti base in didattica della matematica. Si sono sinteticamente presentati i due punti di vista principali nell'approccio agli oggetti matematici, il realistico e il pragmatico, così come diversi argomenti dalla teoria della didattica della matematica di Radford (2003, 2005e alcune relazioni tra le concezioni scacchistiche e la didattica scacchistica, ed è presentato l'argomento degli ostacoli epistemologici in didattica scacchistica, con riferimento alla critica dei principi come descritta nel capitolo I, e come utile strumento in didattica. E' sottolineata la rilevanza dell'approccio storico-epistemologico, come discusso nei lavori teorici di F. Spagnolo (2009).

In math learning, the students have to face abstract objects, having not physical reality. Then, the activity the students are engaged in, deals with a complex of mathematic/semiotic activities that allow them a proficient knowledge of the mathematical object by its representations and depictions.

The object has for the speaker a definite sense and a sharp semiotic representation, and operating semiotic changes, a new object resulted, and not always it is the same (D'Amore, 2006). It introduces a basic question, which is the possibility to know an abstract object we consider by its representations only (Duval paradox) (Duval, 1993)

Then, in math education we have the serious risk to confuse the object with its representation; it most for students, because their experience and knowledge are still in formation, and they do not approach the math object with a sharp awareness of the multiple, possible meanings and applications the object is involved in.

As a consequence, in math education we cannot avoid to tackle the problem of the nature of mathematical objects.

Of course the problem has an enormous philosophical and scientific relevance, anyway I am interested most in its educational aspects.

In Mathematics Education, two basic frameworks emerged: The realistic framework and the pragmatic one.

According to the realistic approach, the math objects have an intrinsic existence, depending neither on the operations in which they are involved, nor on various philosophical approaches do people adopt dealing with them.

The mathematical objects, in this framework, exist *in se ipsis*, the mathematicians have just the task to identify these objects and deal with them.

This platonic approach in Philosophy of mathematics is well described by Panza and Sereni (2010): (the mathematical objects) are so defined *not because they satisfy a given general mathematical criteria, but simply because it will be stated that a certain theory, generally considered as mathematical, deal with them.*

The pragmatic framework considers the problem under an anthropological point of view.

As fairly stated by D'Amore and Godino (2006):

> *The mathematical objects have to be considered as symbols of cultural units, emerging from a custom system related to the mathematical activities some people perform, and then they* (the math obj.) *evolve by the passing of time. In our conception, the reason of emerging of the math objects is the fact that, within some institutions, given practices are performed, and that the meaning of these objects is strictly connected to the problems faced and to the activities realized by the humans, since is not possible to reduce the meaning of the mathematical object to its mere mathematical definition* (D'Amore & Godino, 2006) (my translation from Italian language).

As stated by Radford (2005), *in the semiotic- anthropological approach, the concept of the object is related with the historical-cultural context, and the ideality of the math objects is completely due to the human activity.*

I adopted this approach also dealing with chess concepts, as developed in the course of my work, and performing a historical- epistemological review of chess theory too. It occurred in coincidence with the shift of perspectives of my work depending on a reflection on my methods and on the outputs and methods employed in my experimentations. This reflection involved also the mathematical targets I was planning and, then, inevitably, the nature of math learning.

In fact, coming back to the basic problem in education dealing with math objects, we have to consider carefully the mathematical objects as they appear to students.
This situation can be summarized by the semiotic triangle.
The first form of the semiotic triangle was introduced by Peirce (1931-1958) whose vertexes are the representamen, the interpretant and the object.

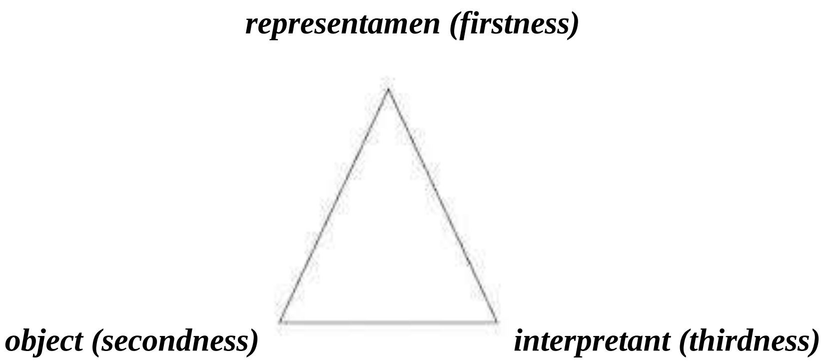


Fig. 7 – Peirce triangle

The philosophy that gave rise to triangles is explained by Peirce as follows*:*

A sign... (in the form of a representamen) is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes called the ground of the representamen (Peirce 1931-58, 2, 228).

In 1892 Frege introduces a triangle whose vertexes are Sinn (sense), Bedeutung (meaning/denotation) and Zeichen (expression).

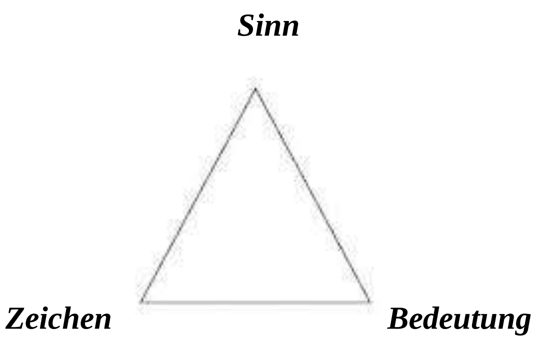


Fig. 8 – Frege triangle

This deep argument, that has philosophical implications, was treated by D'Amore (1999). Using his words,

> *Now, the most naïve and immediate interpretation is that signified of the signifier is the object itself to which it refers. This stand leads to a fallacy ("extensional fallacy"); although it throws into crisis every code theory that needs objectual extensions to a real state of the world, it doesn't disturb mathematics whose objects can be defined in an extensional form, but without the need of any reference to an empiric objective state of the world. (It is not by chance that that the mathematical logician Frege can allow himself to consider Bedeutung in a strictly extensional sense, since above all he thought of mathematics and not of natural language*

From an educational point of view, the problem of approaching abstract objects is crucial. In the educational practice, we pass through the activity to construct the knowledge. Hence, using Radford's words: *The students have to actively engage in mathematical activities not to "construct" the object (for the object is already there, in the culture) but to make sense of it.*) (Radford 2005). This awareness is obtained through an activity complex, concerning mobilization of several means like, words, writings, gestures, concrete objects, sounds, that interact in the cultural context leading to the *objectification.* Then, the above means are called *semiotic means of objectification* (Radford 2005). In this way, the cultural object is known by the students. *To learn, then, is to objectify something* (Radford 2003). Changing the semiotic register, consequently the semiotic representation changes, but the contrary it is not always true. i.e. the semiotic representation changes in the same semiotic register; In this sense, it is useful to recall the graph reported in (Radford 2004)

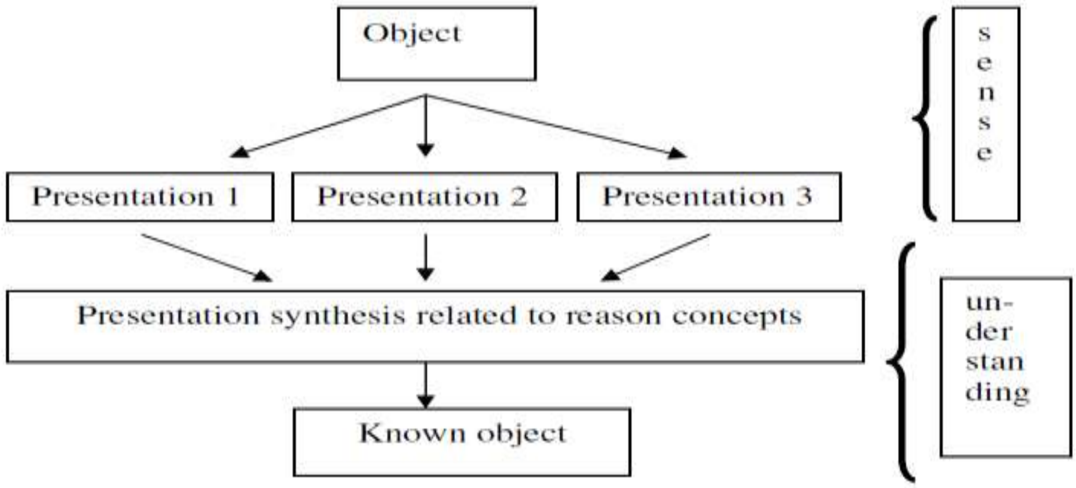


Fig. 9 – Semiotic representation changes

In chess the meaning of an element, as discussed in the former, lies in the array of operations in the context the player puts in action, using the element itself. In this sense the player objectifies the element that is a cultural object too, as discussed in chapter 1. Then, the meaning has its foundation in the relationships with other elements, it is the configural concept, as already presented and discussed in chapter 2 and in the previous section. There is a strong analogy with the understanding of a concept as presented by (Sierpinska, 1990):

> *Understanding a concept will be therefore conceived as the act of acquiring its meaning. Such an act will probably be an act of generalization and synthesis of meanings, in relation with particular elements of the "structure" of the concept (the structure of the concept is the net of meanings of the of statements we have considered).*

Then, the chess player synthesizes the experiences and relationships to define the concept. Of course, the more rich and well done are the operations, the more refined will be the understanding, depending on various factors as the chess instructor, the practice, and the engagement profused. In principle, this factors are those intervening in Vygotsky's ZPD (Zone of proximal development), as fairly specified by (Radford, 2010):

This is why, rather than an absolute concept, the ZPD is a relational one (see also Schneuwly, 2008). In particular, it is forged out of the interaction between students, and between the students and their teacher. The ZPD is not a kind of well-delimited and rigid region that belongs to one particular student but asocial, complex system in motion with evolving tensions.

The chess player, also using mental imagery, comes in touch with the relationships among various elements, enriching continuously his/her knowledge, and becoming aware of new situations. There is an analogy with scientific thinking and modelling, regarding the continuous matching with reality, represented in chess by new games and positions. The set of players' beliefs is tested by the practice, and typically he/she encounters also some (apparently) paradoxical situations in which a given element shows behaviour completely different with respect to the known one. It is the typical appearance of an epistemological obstacle, as fairly discussed in math Education by Spagnolo and Ferreri (1994) and Brousseau (1983). In an interesting article, Bartolotta (1997) point out that in chess education some concepts are introduced by the Instructors as useful and applicable, but the more the player increases his/her level, the more he/she becomes aware of the limits of a strategic item or play system. This because almost all chess strategic principles have no absolute value and several counter-examples is encountered. The mechanic employment of the strategic rules in chess can be assimilated as an epistemological obstacle in Math Education, in the sense of Brousseau (1983) and Spagnolo and Ferreri (1994). Bartolotta (1997) suggested introducing these concepts in Chess Education, in particular highlighting the role of the classic strategic principles as stated by Steinitz and Tarrasch (see chapter 1). This suggestion is more applicable, of course, dealing with expert chess players whom already experienced themselves the exceptions quoted above. The critique of the principles, as occurred in the chess history, may be a proficient topic in chess education and very useful in Math and science education. In fact, I think that to analyze rigorously and in a complete way the sense of a rule, its area of employment, and the exceptions, is a good practice in Education. It allows also a proficient input to lateral and creative thinking. I think also, in accordance with the theoretical framework of the Experimental Epistemology, as outlined by Spagnolo (2009), that an historical deepening of the concept we are dealing allows a more complete understanding. Besides, I noticed also, in my experience with students, that the historical approach to a scientific concept succeeds in capturing students' attention and in stimulating curiosity. Then, I followed these ideas in the course of my work most in chapter I, in which an historical-epistemological analysis of chess is presented. This is also in full accordance with the approach to the teaching/learning in mathematics as social and cultural processes, as fairly depicted by Radford (2005), but the experimentations I realized in 2008-2009 (Palermo and Agrigento) were not performed using this approach, because they were performed before my shift of perspectives occurred in 2010-2011. In fact both the set up of the trials and the behaviours and skills I looked for, were planned not paying attention to processes and to class activities, both in chess and maths. It especially for the Palermo experimentation. The following sections are dedicated to the two quoted experimentations.

3.d My first experimentation- Palermo 2008-2009

Summary

My first experimentation was held in Palermo middle school "Leonardo da Vinci" in the scholastic year 2008-2009. I considered a group of 52 sixth grade students in total, whose 16 participated in a chess course from October to May. To observe possible effects of chess practice, I submitted to students a pre-test and a post test, respectively before and at the end of chess course. The experimental group was composed by the students participating in the chess course and in both tests (15 students). The control group was selected searching for similar performance in pre-test among students non participating in chess activity. For both pre and post test I adopted an INVALSI (Italian Agency for Evaluation of the scholastic system) math test, using also the rough INVALSI item content classification, i.e. "number" geometry" and "measures/data". Chess players performed better than the average in both test, but not better than the control group in post-test, with no particular difference with respect to the content. The statistics was poor, and also I did not take information about other activities performed in non-curricular time, and no information about students' scholastic trend, including the math curriculum and the teachers' methodology and approach. Besides, I did not realize a fine analysis about the structure of items with respect to the contents, competences involved and contexts proposed. Also misses an analysis of the contents of the chess course, and of course misses a concrete comparison between item contents/processes requested and activity performed during the chess laboratory . Hence, I consider the trial not so proficient for the purpose it was planned. It not because the results were poor for the exp. group, in fact similar reasons I will represent in my next experimentation in which, on the contrary, exp group performed sharply better than the control one.

Riassunto

La mia prima sperimentazione si è tenuta nella scuola media Leonardo da Vinci di Palermo. nell'anno scolastico 2008-2009. Ho considerato un gruppo di 52 studenti di prima media in totale, dei quali 16 hanno partecipato ad un corso di scacchi da ottobre a maggio. Per osservare possibili effetti della pratica scacchistica sull'abilità matematica, ho somministrato agli studenti un pre test e un post test, rispettivamente prima ed alla fine del corso di scacchi. Il gruppo sperimentale era composto dagli studenti che hanno partecipato al corso di scacchi e ad entrambi i test (15 studenti). Il gruppo di controllo è stato selezionato usando i dati del pre test e cercando prestazioni simili al gruppo sperimentale tra gli studenti che non avrebbero partecipato la corso di scacchi. Sia per il pre test che per il post test ho adottato un test di matematica dell'INVALSI (Ist. Naz. Per la Valutazione del sistema scolastico), usando anche la rozza classificazione INVALSI riguardo al contenuto, cioè : "numero" "geometria", "dati e misure". Gli scacchisti hanno ottenuto prestazioni migliori della media sia nel pre sia nel post test, ma non meglio del gruppo di controllo nel post test, senza alcuna differenza rispetto al contenuto. Non ho acquisito informazioni riguardo le attività svolte in periodo non curriculare, né riguardo l'andamento scolastico degli studenti, compreso il curriculum di matematica e la metodologia e l'approccio degli insegnanti. Inoltre, non ho realizzato un'analisi fine riguardo la struttura dei quesiti rispetto ai contenuti, competenze coinvolte e contesti proposti. Manca anche una analisi dei contenuti del corso di scacchi, e naturalmente manca una comparazione concreta tra i processi/contenuti degli item e l'attività realizzata nel corso di scacchi. Quindi

considero la sperimentazione non efficace per gli scopi per i quali era stata pianificata. Questo non perché i risultati sono stati scarsi per il gruppo sperimentale, infatti argomenti simili li rappresenterò per la mia successiva sperimentazione, dove, di contro, il gruppo sperimentale ha avuto una prestazione chiaramente superiore al gruppo di controllo.

3.d.1 The set-up

The experimentation was held in the Middle school "Leonardo da Vinci" of Palermo.
In this school chess is practiced since 15 years, and the school obtained several good results in youth team Italian championships (2 times Italian champions). In the 2008-2009 scholastic year the chess laboratory was activated under the supervision of Prof. Cesare Rao, a valid math teacher and chess instructor. The experimentation deal with chess beginners students, who worked two times a week from October 2008 to may 2009, for a total amount of about 80 hrs. I considered two first classes (about 11 years' old pupils), 52 students in total, whose 16 participated in chess beginners' course (level I at school). I submitted to all students a pre test in October 2008 and a post test in May 2009. 15 students attending the chess course participated in both tests. The purpose of the trial regarded the effects of the chess course on math abilities.
The pupils tackled the tests seriously, in teacher's presence. I decided to submit to students an INVALSI test for surveying math learning in Italian school released for scholastic year 204-2005. INVALSI is the National Institute for the Evaluation of the Scholastic system (Istituto Nazionale per la Valutazione del Sistema Scolastico). I chose that test because it was a national test, and results for entire Italy were known. So, the pre-test was the INVALSI test 2004-2005, the post test the INVALSI test 2005-2006. The structure of tests was very similar. Every test was composed by 28 multiple closed-answered questions. Test is about math knowledge and abilities, and is planned not to evaluate a single student, but the complex of the scholastic system, as declared by INVALSI (web references 10). Hence, the tests are quite unsuitable to evaluate competences, as for example in PISA, and closed-answers give not good information on processes. Anyway the tests are structured to stimulate reflection and not to answer in a mechanic way. INVALSI divided the questions, in relation to content, in three subsets: "number", geometry", "measures and data". Items were equally distributed with respect to these subsets. I also analyzed data according to this grouping. The complete tests are in appendix (Appendix 1 and 2)
The experimental group was represented by the 15 students attending the chess course and participating in both tests, so I needed a control group. I selected 15 students not attending the

course whose performance in pre test resulted similar to the 15 future players. I looked for a "twin" for every component of the experimental group, as reported in Fig. 10 and 11.

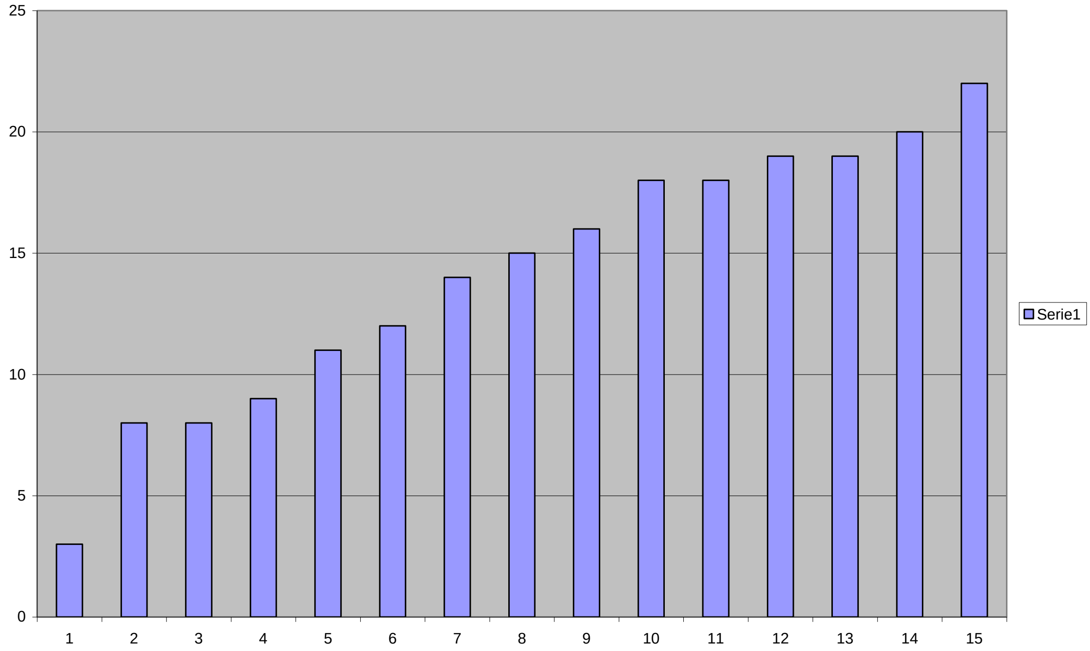


Fig. 10 Pre-test performance of the experimental group

The histogram report on abscissa the students, numbered from 1 to 15 (n. 1 for the poorest performance, n.15 for the best one), and on ordinate the performance (sum of the scores of the 28 items, 1 point for every correct answer, 0 otherwise). In fig. 2 the performance of the students chosen as control group

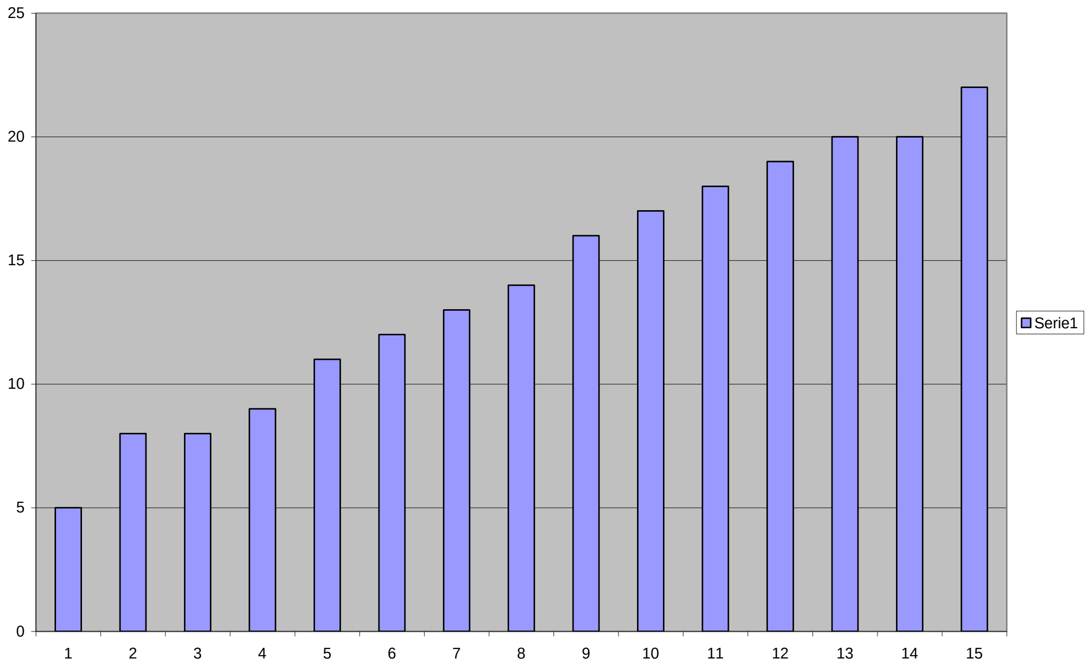


Fig. 11 pre test performance of the control group

The total performance of the two groups is the same, but the "twinning" is not perfect, i.e. the singular performances are not exactly the same.

Note that that exp. group performed better than the whole sample.

The overall performance is shown in fig. 12

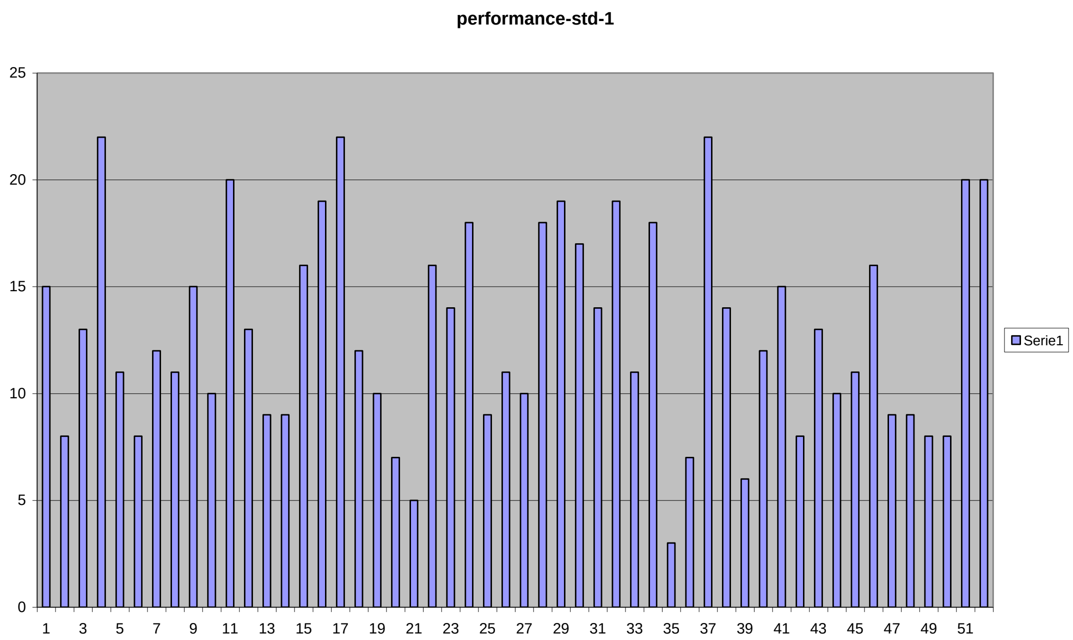


Fig.12 pre test overall performance

Students are numbered randomly.

The better performance by the future chess players is in accordance with forecasts, because the children who want to begin an intellectual activity like chess are most likely the more proficient. It is often reported in literature, as quoted in the former.

3.d.2 Analysis of the Pre-test

The early considerations on the trial concern the score of the whole sample. The general correct answers percentage is 46,1%, while the national value is 60% and the regional one 57%. Gender difference is +2,5% for females (better in a classroom, worst in the other one) . This difference with respect to the national average may be explained considering that pre test was submitted in October, while the national trial occurred in April. About the content, in the following table are reported the outputs:

	Number	**Geometry**	**Measures/data**	**General**	**Standard deviation**	**Coefficient of variation**
Whole sample	49%	41%	46%	46,1 %	5.04	0,4
Exp. group	53%	47%	50%	50%	5.46	0.39

Table 3 Summary of pre-test

These data confirm roughly the general weakness of Italian students in geometry. In fact, students performed better in "number" and worst in "geometry"

3.d.3 – Analysis of the post-test

50 out of 52 students participated in post – test. This time, the overall performance resulted about 56%, perfectly in accordance with regional values in this kind of test (57%). It confirmed the explanation given above about the lower performance with respect to the national and regional ones occurred in pre-test. In fact post test was submitted in May. The outputs are summarized in the table below.

	Number	Geometry	Measures/data	General	Standard deviation	Coefficient of variation
Whole sample	63%	35%	70%	56%	4,63	0,3
Exp. group	62%	34%	74%	58%	5,33	0.33
Control group	64%	44%	77%	61%	3,88	0,23

Table 4 Summary post-test

The results show that exp. and control group performed better than the whole sample, but a comparison between experimental and control group shows a better performance of the control group. It will be commented in the next section. About contents, this time students performed better in “data/measures” , and again worst in “geometry”.

3.d.4 Final remarks on the trial and critical review

A preliminary remark concerns the poor statistics available for the trial. An experimental and control group composed by 15 students give not sound statistical basis for any inference. Also, the entire design of the trial feels the effects of my primeval conception, as I described in a previous section, calling it “perspective 1”, as fairly outlined in (Schonefeld, 2007). In fact I had care about the choose of the sample and groups, but I had no care about all happened during the course, in particular about the activity in which the students of the whole sample engaged from October to May, in the chess course or everywhere. It is not trivial. In fact, I have no information about other activities performed in non-curricular time, and no information about students’ scholastic trend, including the math curriculum and the teachers’ methodology and approach. Besides, I did not realize a fine analysis about the structure of items with respect to the contents, competences involved and contexts proposed. Also misses an analysis of the contents of the chess course, and of course misses a concrete comparison between item contents/processes requested and activity performed during the chess laboratory. Hence, I consider the trial not so proficient for the purpose it was planned. It not because the results were poor for the exp. group, in fact similar reasons I will represent in my next experimentation in which, on the contrary, exp group performed sharply better than the control one.

3.e My second experimentation: Agrigento 2009

Summary

This section deals with an investigation between chess and mathematics education; we planned and executed in 2009 a test in a low secondary school in Agrigento (Sicily-Italy), in which a 30-hours chess course was planned; I and Mario Ferro published an article on this trial (D'Eredità & Ferro, 2011), on which is based this section. Our goal was to observe differences in students' performances in maths before and after the chess course. We submitted the students to a pre-test and a post-test. According to the PISA framework, questions were grouped by content and competence. We divided the students in an experimental group, formed by the students attending the chess course, and in a control group, formed by other students. By analyzing both pre and post test performances, we compared the performances by the students of experimental and control group, focusing on content and competence, too. Unfortunately, only 10 students attended the chess course, out of 45 students participating to the investigation, therefore the results should be taken as preliminary and without a sound statistic significance.

In Education in general, and for this low numbers in particular, results are to be considered with great caution. Also we have no information about the activity during the chess course, and about the mathematical curricular activity in the classes involved. Nevertheless we obtained outputs in a good concordance with our beliefs and with literature, in fact the experimental group performed better in "form" and "uncertainty" items in the area of content, and in "connection" items in the area of competence. On the other hand, no particular improvement occurred in "quantity" and "reproduction" items. In conclusion, we have found potential for mutual influence between chess practice and maths skills, to a certain extent. Some benefits of chess practice seem in relation with using of visuo-spatial abilities by the chess players. Concerning processes, chess seems effective in strenghtening problem-solving skills and a more proactive approach to new situations and tasks.

Riassunto

Questa sezione tratta di un'indagine su scacchi e didattica della matematica; abbiamo pianificato ed eseguito un test in una scuola media di Agrigento (Sicilia-Italia), nella quale era stato pianificato un corso di scacchi di 30 ore. Io e Mario Ferro abbiamo pubblicato un articolo su questa sperimentazione (D'Eredità & Ferro, 2011), sul quale è basato questa sezione. Il nostro scopo è stato di osservare differenze nelle prestazioni degli studenti in matematica prima e dopo il corso di scacchi. Abbiamo somministrato agli studenti un pre e un post-test. Secondo l'impostazione PISA, le domande sono state raggruppate per contenuto e competenza. Abbiamo diviso gli studenti in un gruppo sperimentale, formato dagli studenti che hanno seguito il corso di scacchi, e in un gruppo di controllo, formato dagli altri. Analizzando le prestazioni del pre e del post test, abbiamo confrontato le prestazioni del gruppo sperimentale e di quello di controllo, concentrandoci anche su contenuto e competenza. Sfortunatamente, solo 10 studenti hanno seguito il corso di scacchi su 45 che hanno partecipato ala sperimentazione, quindi i risultati devono essere presi come preliminari e senza una solida base statistica. In generale, in didattica ed a causa di questi numeri piccoli, il risultati devono essere considerati con grande cautela. Non abbiamo avuto anche alcuna informazione sull'attività durante il corso di scacchi, e riguardo l'attività curriculare di matematica nelle classi coinvolte.

Ciò non di meno, abbiamo ottenuto risultati in buona concordanza con le nostre convinzioni e con la letteratura, infatti il gruppo sperimentale ha reso meglio negli item riguardanti “forma” e “incertezza” per l’area dei contenuti, e negli item di “connessione” nell’area della competenza. D’altro canto, nessun particolare miglioramento si è avuto negli item riguardanti “quantità” e “riproduzione”. In conclusione, abbiamo trovato del potenziale per la mutua influenza tra scacchi e abilità matematiche, fino ad una certa quantità. Alcuni benefici della pratica scacchistica sembrano in relazione con l’uso di abilità visuo-spaziali da parte degli scacchisti. Riguardo i processi, gli scacchi sembrano efficaci per le abilità di problem-solving e per un approccio più proattivo a nuove situazioni e compiti.

3.e.1. The Set-up

The investigation was realized in 2009 in the low secondary school “Anna Frank” of Agrigento (Sicily-Italy), including 45 students about 11 years old. Ten of them followed a 30 hours chess activity (including students’ tournament). Data limitations are due to the fact that the participants to the chess course had different ages, and so we focused on 11 years old students only although we are aware of the limits of our sample. Our goal was to observe differences in students’ performances in maths before and after the chess course. We submitted students to a pre-test and a post-test, the items were extracted from INVALSI test, like in the former experimentation (Palermo2008-2009). According to the PISA framework, questions were grouped by content and competence. The pre-test consists of 28 questions, including 14 open-response items and 14 close-responses. Regarding content, 13 items were classified "quantity", 8 items “Form and Space”, and 7 "uncertainty "[3]. Regarding competence, we classified items according to PISA “competences clusters” (Reproduction, Connection, and Reflection). But PISA is planned for 15 years old students, while our set is composed by 11 years old students, so we decided to divide our items in 2 groups only: 13 "reproduction" and 15 "connection”.

Similarly, the post-test consists of 28 questions including 7 open-response and 21 close-response items. About content, 10 of them were classified "quantity", 10 “Form and Space”, and 8 "uncertainty ". Regarding competence, 10 questions were classified "reproduction" and 18 "connection".

The complete tests are in appendixes (appendix 3 and appendix 4)

We divided the students in an experimental group (formed by the students attending the chess course) and in a control group (formed by other students). By analyzing both pre and post test performances, we compared the performances by the students of experimental and control group, focusing also on content and competence. To be more rigorous, we should

[3] For a better insight of this classification, see OCSE-PISA 2003 and 2006 official reports

consider a control group with the same pre-test performance of the experimental group, but it was no possible in our experiment, so we considered all non chess players as control group.
We realized an a priori-analysis to study the students' protocols. Such an analysis allowed us to define binary variables referred to the expected behaviors of the students (answers present on single items of protocols). The analysis is not very complex. In many cases, as in all the closed answer items, the analysis is just a partition between right and wrong answers. Given the little number of students, we decided not to go into more depth in this analysis.

3.e.2 Pre test results

In the pre-test, on average each student answered correctly to 11.89 items of 28 (42%). About content, the best performance was obtained in "quantity" and "uncertainty" items. About competence, similar performances were obtained in "reproduction" and "connection" items.
In more detail, the students have correctly answered to 44% of the "quantity" items, 31% of the "form" and 53% of the "uncertainty" (Fig. 13). About competence, they answered right to 44% of the "reproduction" items and to 40% of the "connection" ones (Fig. 14).
Data are referred to the whole set of students; specific considerations about experimental and control group's performances are displayed in the following sections.

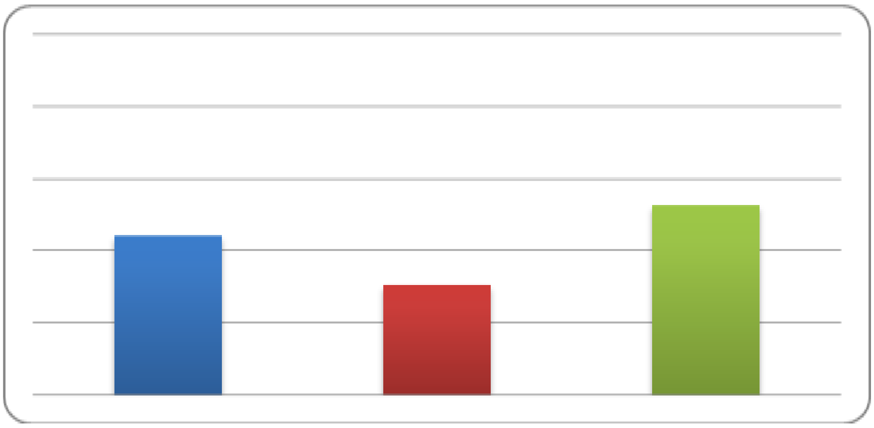


Fig. 13 Pre test performance by content

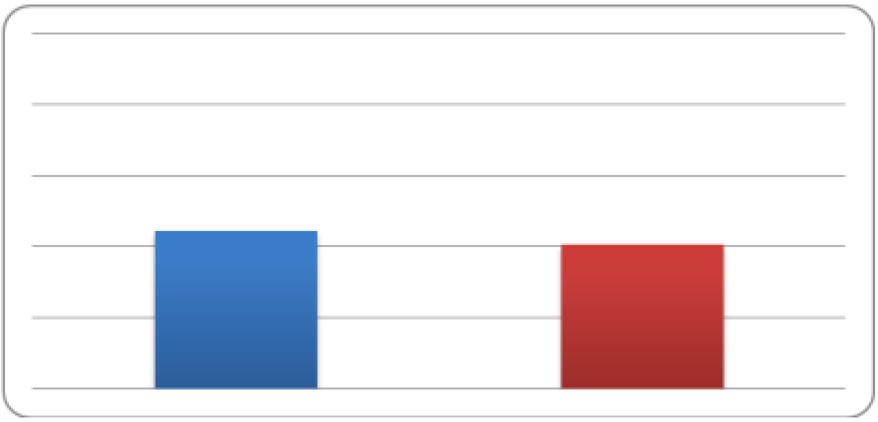


Fig. 14 Pre test performance by competence

3.e.3 Post results result and analysis

In the post-test, on average each student performed 12.5 exercises out of 28. The general performance improved by 0.6 i.e. about 5% with respect to the pre-test. The improvement is to be considered standard considering that two months elapsed between tests (and a chess course!). The experimental group performed 13.5 with respect to 12.5 obtained by the same students in the pre-test, i.e. 8% better.

On the other hand, the control group reached 12.2 with respect to 11.7 of the pre-test (+ 4%).

Regarding content, the best general performance was obtained in "quantity" and "uncertainty" items. About competence, the best general performance was obtained in “reproduction” items.

Going into more detail, all students have improved in "Quantity" and "Uncertainty" items. Considering the experimental group, in “Quantity” they performed to 51% (46% in the pre-test) and non-chess players to 53% (42 % in the pre-test). (Fig.15)

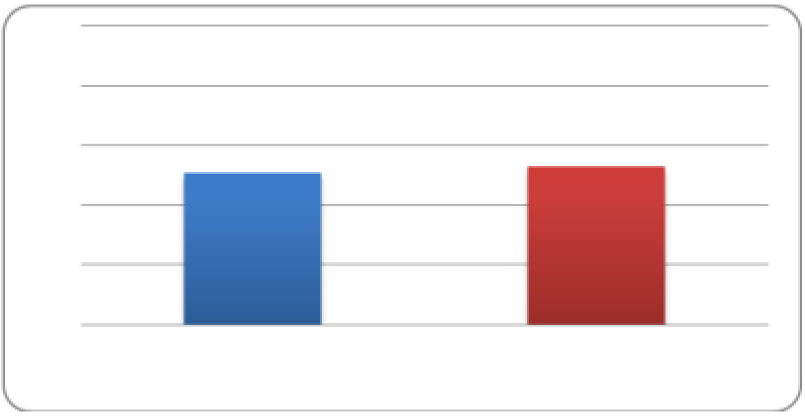


Fig. 15 Post test- Quantity

About “Uncertainty” items, the experimental group answered correctly to 59%, showing a sharp improvement with respect to the pre-test (51%), and control group to 55% (In the pre-test to 52%). (Fig. 16).

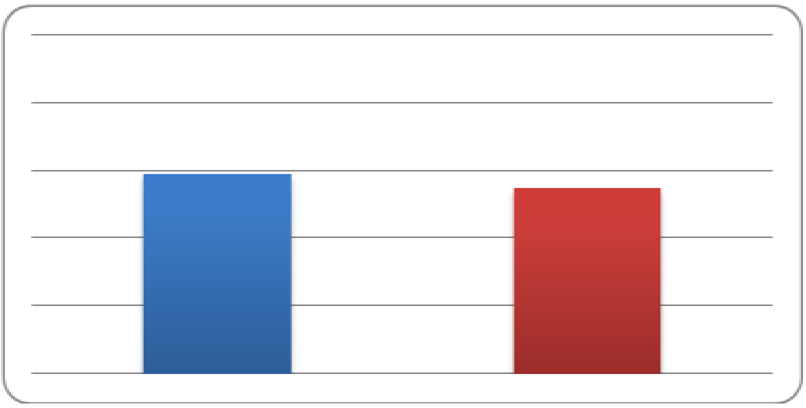


Fig. 16 Post test –Uncertainty

A substantial improvement was obtained by chess players in “Form” items, performing 37%, when in the pre-test they obtained a poor 27.5%, Control group went down to 26%, when in the pre-test 31% occurred for the same group . (Fig.17).

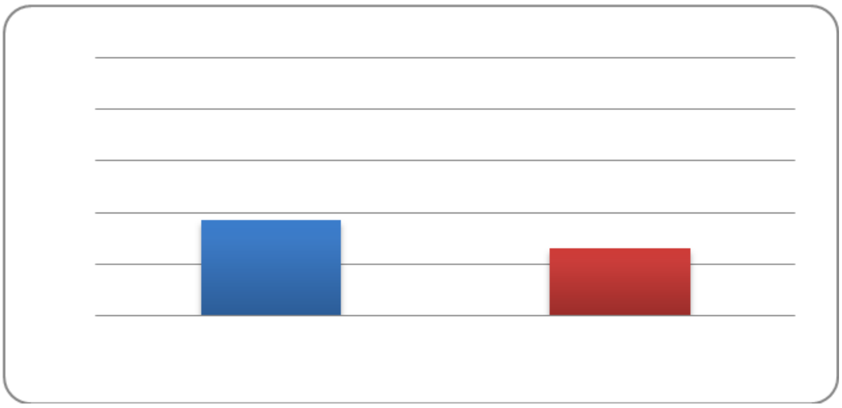


Fig. 17 Post test – Form

About competence, we found interesting results.

In “Reproduction” items, Experimental group improved just an edge, arriving to 48% starting from 47.7 % of the pre-test, while control group performed strongly to 51% of questions with respect to 42% of pre-test. (Fig. 18).

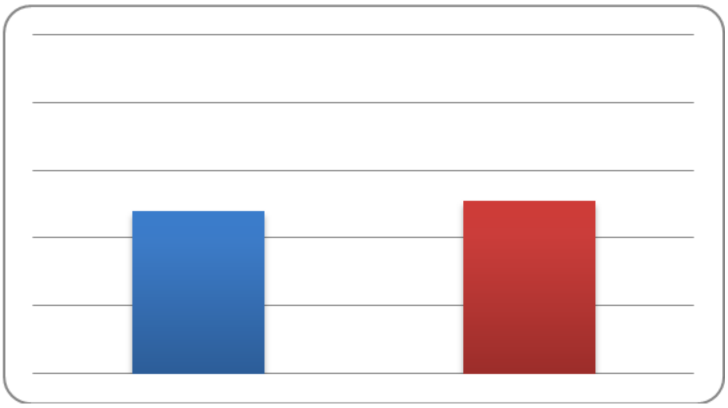


Fig. 18 Post test – Reproduction

But very significant outputs resulted in the analysis of “Connection” items, where experimental group performed to 48%, starting from the 37.3% of the pre-test. Instead, the control group obtained the same 39.6% performed in the pre-test. (Fig. 19)

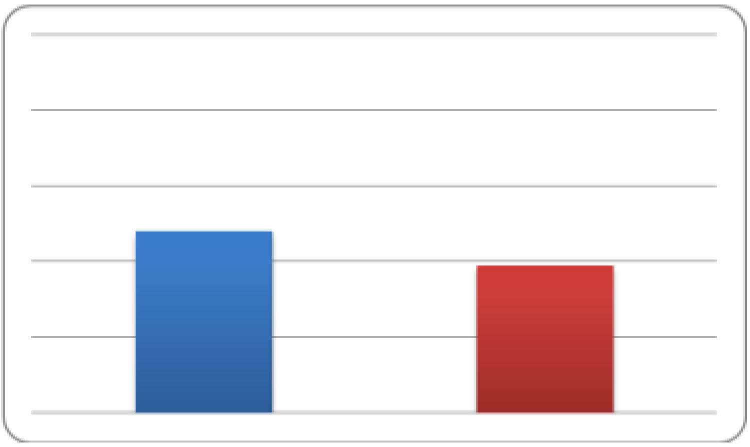


Fig. 19 Post test – Connection

3.e.4. Final remarks on the trial and critical review

Again, like in the previous section, I remark the poor statistics of the trial. The outputs of the investigation are, to some extent, coherent with the theoretical framework and with our beliefs about mutual influence of chess and maths skills, but also in this case we had no information on the activity performed by all students during the period of chess course, unless the report by the Chess instructor, in which she specifies the contents proposed in the course, and indicates that pupils participated in scholastic and local chess tournaments, that is anyway a relevant information. No information also we know about the scholastic trend and the mathematical activity in the classes involved. Then, like for the Palermo trial, I do not believe that the trial can represent a study with a strong scientific value.

Examining the general performance, the experimental group improved the 8% and control group the 4%. We do not consider this general improvement directly referred to chess practice, but probably it is due to the engagement in an intellectual, enjoying activity for boys and girls. To better reinforce this hypothesis, it should be suitable to consider one group more that is a group engaged with another intellectual and amusing activity. It requires an experimental set-up that was not possible in our investigation.

Regarding content, more subtle considerations are to be made. The performance of the two groups in “quantity” items suggests that chess practice do not lead to a better skill in calculations, and in general in activities in which the application of a procedure or algorithm is requested. The sharp improvement of experimental group in “form” items suggests that the visual-spatial abilities are relevant in chess practice. Besides, during a typical 30 hours chess course, including tournaments, the chess contents are at a beginner level, so the chess activity is strongly focused on elementary pattern recognition (basic endings, elementary checkmates). It would be interesting to consider various skills during the evolution of student’s chess expertise. The good results of experimental group occurred in “uncertainty” lead us to reinforce the belief that chess practice is very useful in approaching new situations and data, because chess players are used to face the reality finding with methods and strategies to understand it and to go ahead and proceed.

Concerning competences, the experimental group does not improve the performance in “reproduction” items, according to interpretation that chess practice is not very useful for calculation and/or mere application of a known procedure. Instead, the significant improvement in “connection” items by chess players group corroborates the idea that chess practice is very useful in problem-solving situations, stimulating both deductive thinking and the using of various cognitive or meta-cognitive resources.

In conclusion, we are aware of the limits of our statistical data and the use of a “non-perfect” test. In experimental work of the kind that we conducted we have to cope with so many constraints that is difficult to control everything as we would like. Yet, our result shows some interesting avenues. Let us start by noting that we did not realize an implicative analysis and/or a factorial analysis, because of the limited information about processes that the test produces. Nevertheless, the outputs of this investigation reinforce our beliefs about the using of chess practice as an integrative tool for maths education.

New and deeper researches would be made to give more extensive results, and to consider interactions among factors like different maths skills and increasing chess expertise.

Chapter 4
Shifts of perspectives in 2010-2011 and participation in SAM project

Summary

This chapter concerns most the participation in SAM project. (SAM is an acronym for Scacchi ed Apprendimento della Matematica , i.e. Chess and Math Learning). In the first section, I presented the project itself, specifying the set-up and clarifying the role of our work in the inquire. I think that is very important that a governmental agency paid attention to chess and looked for correlations between chess and math learning. The framework of the inquire consists in submission of a math pre test and post test to an experimental and to a control group. 30 schools were involved in the project, from various parts of Italy; then for every school INVALSI made a draw to decide what classes had to become experimental or control classes. About 2000 3th grade students (8 years old) were involved in. Then, experimental classes attended a 30 hours chess course, held by FSI instructors. Curricular teachers were always present during the activities. Over entire Italy, the chess course was based on a chess protocol written by some of the most relevant Italian chess instructors, with the collaboration of our University. We inserted in the protocol some item having a mathematical nature. It was in accordance with the shift of perspectives occurred during my work,

in fact I begun to consider the effects of chess practice no more as an absolute, independent thing, but as depending strongly on the context, intending contents proposed, didactical approaches, and motivational factors.

. In the second section, I presented the activity realized in a school participating in the SAM project, following a class in particular. We planned to follow the math activities of the classroom during the chess course, trying to make a parallel with the activities and contents treated in the chess course. During the course, the teacher reported on sharp improvements in math by the students, especially in concentration and in attitude to reasoning. We recorded a video about the accomplishing of math tasks by the students, choosing a math topic (numerical pyramids) the pupils faced in a similar form during the chess course. It is presented the analysis of the video, performed by highlighting some crucial episodes and commenting the processes, with particular reference to key words, gestures, and writings occurred.

In the last part I commented the data of the SAM inquire I deduced from the paper protocols of the students. The outputs show an improvement of 5% from pre test to post test, in accordance with the national trend. Both pre and post test performances resulted better than the national ones.

It is clear that it is not correct to generalize outputs obtained by a single trial. Anyway this output is in accordance with the general good level in mathematics of the class.

Riassunto

Questo capitolo riguarda soprattutto la partecipazione al progetto SAM. (SAM è un acronimo per Scacchi ed Apprendimento della Matematica). Nella prima sezione ho presentato il progetto stesso, specificando il set-up e chiarificando il ruolo del nostro lavoro nell'indagine. Penso che sia molto importante che una agenzia governativa abbia dato attenzione agli scacchi ed abbia cercato delle relazioni tra scacchi ed apprendimento della matematica. L'impostazione dell'indagine consiste nella somministrazione di un pre e di un post test di

matematica ad un gruppo sperimentale ed ad un gruppo di controllo. 30 scuole sono state coinvolte nel progetto, da varie parti d'Italia, quindi per ogni scuola l'INVALSI ha fatto un sorteggio per determinare quali classi dovessero diventare sperimentali e quali di controllo. Circa 2000 studenti di terza elementare (8 anni) sono stati coinvolti. Quindi, le classi sperimentali hanno seguito un corso di scacchi di 30 ore, tenuto da Istruttori FSI. Gli insegnanti curricolari erano sempre presenti durante le attività. In tutta Italia, i corsi di scacchi si sono basati su un protocollo scacchistico scritto da alcuni tra i più rilevanti istruttori di scacchi italiani, con la collaborazione della nostra Università. Abbiamo inserito nel protocollo qualche elemento di natura matematica. Questo è in accordo con il cambio di prospettive avvenuto nel corso del mio lavoro. Infatti ho cominciato a considerare gli effetti della pratica scacchistica non più come una cosa assoluta e indipendente, ma come dipendente fortemente dal contesto,intendendo i contenuti proposti, gli approcci didattici, i fattori motivazionali. Nella seconda sezione, ho presentato l'attività realizzata in una scuola che ha partecipato al progetto SAM, seguendo una classe in particolare. Abbiamo pianificato di seguire le attività di matematica della classe durante il corso di scacchi, provando a fare un parallelo con le attività e i contenuti trattati nel corso di scacchi. Durante il corso, l'insegnante ha riferito di progressi evidenti in matematica degli studenti, specialmente riguardo la concentrazione e l'attitudine al ragionamento. Abbiamo registrato un video riguardo all'esecuzione di un compito di matematica da parte degli studenti, scegliendo un argomento matematico (le piramidi numeriche) che i ragazzi avevano affrontato in modo simile durante il corso di scacchi. E' presentata l'analisi del video, realizzata evidenziando alcuni episodi cruciali e commentando i processi, con particolare riferimento a parole chiave, gesti e scritti che si sono realizzati. ho commentato i dati dell'indagine SAM che ho dedotto dai protocolli cartacei degli studenti. I risultati mostrano un miglioramento del 5% dal pre al post test, in linea con l'andamento nazionale. Sia i risultati del pre test che quelli del post test sono risultati migliori di quelli nazionali. E' chiaro che non è corretto generalizzare risultati ottenuti da un singolo tentativo. Comunque questo esito è in accordo col buon livello generale in matematica della classe, come confermato dall'insegnante e dall'Istruttore di scacchi, che anche un matematico.

4.a The SAM Project: a governmental inquire on math and chess: our role, the integrated protocol, early results.

Summary

This section is dedicated to the SAM project. (SAM is an acronym for Scacchi ed Apprendimento della Matematica, i.e. Chess and Math Learning). It is a governmental inquire promoted by INVALSI (Italian Governmental Agency for the evaluation of the scholastic system), in collaboration with Italian chess Federation – FSI (coordination by the FSI Piemonte Regional Committee). It was performed in 2011, and I have not yet the complete outputs, just information on the general trend. The first part of the section deal with the set up of the inquire. The framework of the inquire consists in submission of a math pre test and post test to an experimental and to a control group. 30 schools were involved in the project, from various parts of Italy; then for every school INVALSI made a draw to decide what classes had to become experimental or control classes. About 2000 3th grade students (8 years old) were involved in.

Then, experimental classes attended a 30 hours chess course, held by FSI instructors. Curricular teachers were always present during the activities. Over entire Italy, the chess course was based on a unique chess protocol . We inserted in the protocol some item having a mathematical nature. It was in accordance with the shift of perspectives occurred during my work. In fact I begun to consider the effects of chess practice no more as an absolute, independent thing, but as depending strongly on the context, intending contents proposed, didactical approaches, and motivational factors.

We have not yet the complete results of SAM, but from the early information I know that there are encouraging outputs for the chess practice occurred, in fact experimental classes performed sharply better in post test than the control ones, in a statistically significant way. Another important fact is that the improvements do not hold for whose already knew chess, and this result is not surprising because for children the benefits most concern the learning the game logic.

Riassunto

Questa sezione è dedicata al progetto SAM (SAM è un acronimo per Scacchi ed Apprendimento della Matematica). E' un'indagine governativa promossa dall'INVALSI (Istituto Nazionale per la valutazione del Sistema scolastico), in collaborazione con la Federazione Scacchistica Italiana – FSI (coordinamento da parte del Comitato Regionale Piemonte FSI). E' stata realizzata nel 2011, e non ho ancora i risultati completi, solo informazioni riguardo l'andamento generale. La prima parte della sezione riguarda il set up dell'indagine. L'impostazione dell'indagine consiste nella somministrazione di un pre e di un post test di matematica a un gruppo sperimentale ed ad un gruppo di controllo. 30 scuole sono state coinvolte nel progetto, da varie parti d'Italia, quindi per ogni scuola l'INVALSI ha fatto un sorteggio per determinare quali classi dovessero diventare sperimentali e quali di controllo. Circa 2000 studenti di terza elementare (8 anni) sono stati coinvolti. Quindi, le classi sperimentali hanno seguito un corso di scacchi di 30 ore, tenuto da Istruttori FSI. Gli insegnanti curricolari erano sempre presenti durante le attività. In tutta Italia, i corsi di scacchi si sono basati su un protocollo scacchistico scritto da alcuni tra i più rilevanti istruttori di scacchi italiani, con la collaborazione della nostra Università, col contributo mio e di Mario Ferro. Abbiamo inserito nel protocollo qualche elemento di natura matematica. Questo è in accordo con il cambio di prospettive avvenuto nel corso del mio lavoro. Infatti ho cominciato a considerare gli effetti della pratica scacchistica non più come una cosa assoluta e indipendente, ma come dipendente fortemente dal contesto,intendendo i contenuti proposti, gli approcci didattici, i fattori motivazionali. La seconda parte della sezione è dedicata ai commenti sui primi risultati che conosco. Questi risultati sono incoraggianti per la pratica scacchistica, infatti le classi sperimentali hanno avuto un risultato migliore nel post test rispetto alle classi di controllo, in modo statisticamente significativo. Un altro importante fatto è che i miglioramenti non ci sono stati per coloro che già conoscevano gli scacchi, e questo risultato non è sorprendente in quanto per i bambini i benefici riguardano principalmente l'apprendimento della logica del gioco.

4.a.1 The set up

The SAM (Scacchi e Apprendimento della Matematica – Chess and Math Learning) project is an activity promoted by INVALSI (Italian Governmental Agency for the evaluation of the scholastic system), in collaboration with Italian Chess Federation – FSI (coordination by the

FSI Piemonte regional committee). It was performed in 2011, from January to June, and concerned the search for the effects of chess practice in the learning of math in Italian primary school.

It is remarkable that a governmental agency pays attention to chess as an useful tool to empower mathematical abilities. In my opinion, it occurred as a consequence of both chess activity in schools and former scientific inquires in Italy. In fact, the Italian chess scholastic activity increased more and more in the last years, reaching an estimated value of 50.000 students involved. The image of chess as an educational tool is very good for many Italian teachers and school principals. About scientific inquires, recently the Torino University performed the research "Scacchi gioco per crescere (chess, a game to grow up with) discussed in a previous chapter, and , in collaboration with CNR (National Research Agency) and with our University, another inquire, realized in 2010, concerning the digital learning of chess, quoted in chapter 1-d.

The framework of the inquire consists in submission of a math pre test and post test to an experimental and to a control group. 30 schools were involved in the project, from various parts of Italy; then for every school INVALSI made a draw to decide what classes had to become experimental or control classes. About 2000 3th grade students (8 years old) were involved in.

Then, experimental classes attended a 30 hours chess course, held by FSI instructors. Curricular teachers were always present during the activities.

All chess courses of the project adopted a chess protocol written by some of the best Italian chess instructors in collaboration with our University (in particular me and Mario Ferro). The protocol consisted of two phases: 10 hours for chess formal rules and check mate recognition, and a 20 hrs phase in which students approach effective chess thinking. We inserted in the 2nd phase several items and tasks in which chess and math concepts are used together. This as a consequence of the shift of perspective~~s~~ occurred in the course of my work. In fact, I think that it is crucial not merely practicing chess, but how practicing it, and which contents are to be studied. This is the main sense of this shift of perspectives. The whole protocol proposed in the course is in the appendix 5.

In particular, we introduced some simple exercise on the balance of the material remaining on the board, like the following (taken from the protocol, original in Italian language):

(Item) *5) Exercise of material balancing:*

It is useful for exercises on sums and order relations, for calculation and for learning a rule (like the quantification of the material: Queen=9, Rook=5, Knight=Bishop=3, Pawn=1) and apply it in a concrete situation (position), and to train using of mathematical symbols >, <, =

Fig. 20Exercise of material balancing

White: 5 pawns, 1 rook, 1 bishop
Black: 4 pawns, 1 rook, 1 knight.
White: 5x1+1x5+1x3=13.
Black: 4x1+1x5+1x3=12.
White's score =13 > Black's score =12
White has material advantage
Note: After balancing, i.e. the matching between "white values" and "black values" remember always to not forget that it is concerned with material advantages/disadvantage/equilibrium.
A not correct understanding of it may become an obstacle when the "relative value of the pieces" item will be presented. (Typical advice to be followed by Instructors in chess beginners courses).

It is suggested to dedicate a theoretical lesson on it, submitting to the pupils the task consisting of writing on a sheet the calculation (pieces balancing) *obtained in various chess positions, arising from free games or given by the instructor.*

Pieces' counting is fundamental to take a choice in chess, at every level. Another interesting item proposed by us in the protocol is the chess pyramid. Inspired by an interesting work by Prof. Malara (Malara & Navarra, 2003), we inserted in the protocol the chess pyramids. Like in the numerical ones, in the chess pyramids every brick results as the sum of the two bricks below. The chess symbol substitutes the numbers, using the conventional value of pieces (Queen 9, Rook 5, Bishop and Knight 3, pawn 1), like this:

Fig. 21 Chess pyramid

In this example, the empty brick has to be filled with a pawn, meaning number 1, necessary together with 3 (bishop) and 1 (pawn), to reach 5 (rook).

The numerical pyramid will be the task proposed on may 25 in a classroom, recorded in the video that is the subject of next section. In the following table are reported the most interesting points concerning the possible connections between some contents of the chess protocol and mathematical and logical skills.

Contents of chess course	Mathematical and logical skills related
1st phase: 10 hours for chess formal rules, including tasks on recognition of a checkmate on the board	Fostering of pattern recognition; formal arguing on the occurrence of an event by if..then approach
2nd phase (point 4): in the task "invent a checkmate" students were asked to insert a piece on the board generating a checkmate	Fostering of anticipatory thinking, concerning a qualitative and spatial choice, depending on relationships deducible from the context. Verbal explanation of the choices (if..then arguing)
2nd phase: (point 5) Material balance exercises, counting the pieces on the board, establishing order relations, introducing also symbols >, < ,	Highlighting the emerging of the ordinal value from the numerical data and possible comparison between two numbers (subtraction)
Chess Pieces' pyramids tasks (point 6), balancing numerical pieces' values	recognition symbol/quantity, uniqueness of the solution on the mathematical structure of the pyramid
Point 8: using chess diagrams, maximize the material gain, individuating undefended and defended pieces	relationships between visuo-spatial and arithmetic registers, this time including or excluding choices depending on more complex relationships

Table 6 Connections Chess-Math

I noticed that some activities and exercises proposed in the protocol are similar to some topics proposed by Ho (Ho 2006), anyway I want to highlight that we prepared them independently. The items of the pre and post tests were chosen by INVALSI only.

22 items were submitted in the pre test, 21 in the post-test. The items concerned calculation tasks, visuo-spatial abilities, and simple problem-solving skills. Items were adequate for 8 years old pupils. The whole pre and post test are in appendixes (appendix 6 and appendix 7).
Before initiating the test, pupils were asked to furnish some information like age, gender, liking of math, self-estimated chess skill and other. Over entire Italy, it was possible to submit the tests to students on line using a dedicated platform, or after the Instructor had to charge the data from students' paper protocols on the platform. The school in which we worked did not have computers enough to work on-line, then students worked on paper protocols.
The SAM project has to be considered as a key point in my research, due to the following considerations: the project methodology, involving a specific chess curriculum, allows me to express some considerations on the relation between chess and math practice, on a sound statistical basis. In fact SAM involves a lot of students from various Italian regions. All students, in principle, follow the same chess course, albeit in different contexts. It is a condition very difficult to attain in other, similar projects. It was in accordance with the shift of perspectives occurred during my work. In fact I begun to consider the effects of chess practice no more as an absolute, independent thing, but as depending strongly on the context, intending contents proposed, didactical approaches, and motivational factors.

4.a.2 Early results and comments

We have not yet the complete results, probably the complete outputs will be released in the early months of 2012. We have got some information on the general trend over entire Italy. First, the experimental and the control group resulted absolutely homogeneous with respect to gender, age, origin, preliminary knowledge of chess and math skills. According to the early outputs, it seems that there is an appreciable difference in post test, i.e. experimental group performed sharply better than the control one, in a statistically significant way. Also, it seems that this difference does not hold for those pupils that already knew chess. It is necessary to consider this information about the general trend with great caution, and to wait the official release. Anyway these results are very interesting and quite reassuring for us. This not only because we collaborated in the project, but also because these early information seem in accordance with our beliefs and forecasts about the possible benefits of chess practice on math skills, and with some important results already obtained in literature. More in general, we cannot claim that the positive output of SAM depends on the structure of the protocol, or on the nature of the items proposed, or on both, and to explain it a more deep inquire is needed. But surely these results are not in contrast with a vision conceiving chess as an intellectual and gaming activity stimulating children to a better attitude for reasoning and engaging in problem solving. The data about the children that already knew chess corroborate the hypotheses that chess is more proficient in Education in the early phase, stimulating the full activation of cognitive resource and thinking attitude. This is in full accordance with Trinchero's assertions in (web references 9): *"Cognitive abilities' improvements seem depend more on learning game logic (values, moves, positions, strategies), than on the practice hours*"

As quoted in the former, the more age and chess skill increase, the more is difficult to compare the various factors intervening in the personal growth, not least the time consuming to reach high skill in chess.

4.b The experience at school, and Video Analysis on a math task at school

Summary

This section deal with the activity I experienced in a school participating in SAM project. I follow in particular a class, with the collaboration of the teacher and of the Chess instructor. In the first part of the section this activity is described.

We planned to follow the math activities of the classroom during the chess course, trying to make a parallel with the activities and contents treated in the chess course. During the course, the teacher reported on sharp improvements in math by the students, especially in concentration and in attitude to reasoning. We recorded a video about the accomplishing of math tasks by the students, choosing a math topic (numerical pyramids) the pupils faced in a similar form during the chess course. In the second part of the section is presented the analysis of the video, performed by highlighting some crucial episodes and commenting the processes, with particular reference to key words, gestures, and writings appeared. Also some interventions of the teacher and of a colleague were analyzed, and in a specific episode the process of objectification was presented. In the third part I commented the data of the SAM inquire I deduced from the paper protocols of the students. The outputs show an improvement of 5% from pre test to post test, in accordance with the national trend. Both pre and post test performances resulted better than the national ones.

It is clear that it is not correct to generalize outputs obtained by a single trial, having at disposal a very few data. Of course, just one casual accident (like a hard school day or other) can change the result of a single trial. Anyway this output is in accordance with the general good level in mathematics of the class, as confirmed by the teacher and by the chess instructor, also a mathematician.

Riassunto

Questa sezione tratta dell’attivit?che ho praticato in una scuola che partecipava al progetto SAM. Ho seguito in particolare una classe, con la collaborazione dell’insegnante e dell’Istruttore di scacchi. Nella prima parte della sezione si descrive quest’attività. Abbiamo pianificato di seguire le attività di matematica della classe durante il corso di scacchi, provando a fare un parallelo con le attività ed i contenuti trattati nel corso di scacchi. Durante il corso, l’insegnante ha riferito di progressi evidenti in matematica degli studenti, specialmente riguardo la concentrazione e l’attitudine al ragionamento. Abbiamo registrato un video riguardo all’esecuzione di un compito di matematica da parte degli studenti, scegliendo un argomento matematico (le piramidi numeriche) che i ragazzi avevano affrontato in modo simile durante il corso di scacchi. Nella seconda parte della sezione ?presentata l’analisi del video, realizzata evidenziando alcuni episodi cruciali e commentando i processi, con particolare riferimento a parole chiave, gesti e scritti che si sono realizzati. Anche alcuni interventi dell’insegnanti e di un collega sono stati analizzati, ed in uno specifico episodio ?stato presentato il processo di oggettificazione. Nella terza parte ho commentato i dati dell’indagine SAM che ho dedotto dai protocolli cartacei degli studenti. I risultati mostrano un miglioramento del 5% dal pre al post test, in linea con l’andamento nazionale. Sia i risultati del pre test che quelli del post test sono risultati migliori di quelli nazionali. E’ chiaro che non è corretto generalizzare risultati ottenuti da un singolo tentativo, avendo a disposizione pochi dati. Naturalmente, anche solo un accidente casuale (come un giorno di scuola pesante, o altro) può alterare il risultato

del singolo tentativo. Comunque questo esito è in accordo col buon livello generale in matematica della classe, come confermato dall'insegnante e dall'Istruttore di scacchi, che ?anche un matematico.

4.b.1 – The activity at school

In Palermo the primary school "Garzilli" accepted to participate in the SAM project, with 3 experimental and 3 control classes (as established by the INVALSI draw). In particular we follow the work of a class, because of the valid collaboration of the teacher and the presence of Mario Ferro (my PhD colleague) as Chess instructor in that classroom.

Mario Ferro followed also some math activity in the classroom, in full accordance with the teacher.

During the course, the teacher reported on sharp improvements in math by the students, especially in concentration and in attitude to reasoning, as will be remarked below.

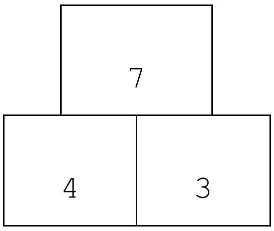
So, we planned to record students in accomplishing math tasks. We thought to record first students in a map task, a topic frequently adopted by the teacher, involving visuo-spatial orienteering and arguing. The try failed, because of technical problems. Then, we realized just the second idea, i.e. a mathematical task somewhat related to the contents proposed in the SAM chess course.

The video was realized in a experimental class participating in the project, at the end of the course, on may 25, 2011. It is a 3rd primary school class (8 years old pupils), and we submitted to the students some math tasks concerning numerical pyramids, similar to the chess pyramids proposed in the chess course (see the appendix 5). This kind of mathematical task was proposed first by Prof. N. Malara (Malara & Navarra 2003). There were 16 students in the classroom, divided in groups of 4, and we focused in particular on a group, formed by a girl and three boys (denoted A,B,C). The teacher and Mario were present in the classroom, but Mario did not intervene in the group's activity. The filming was performed by Munder Mohamed, another Phd colleague. He also did not intervene, unless in the fair episode depicted in the following.

Below the complete text of the task. (original in Italian language)

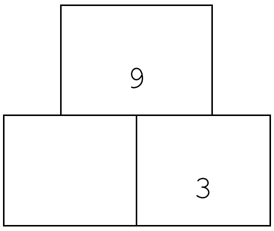
NUMERICAL PYRAMID: how is done? Every brick is the sum of the two bricks below, in this way:

1)



Invent some pyramids!

If I have the tip ...low is the base?



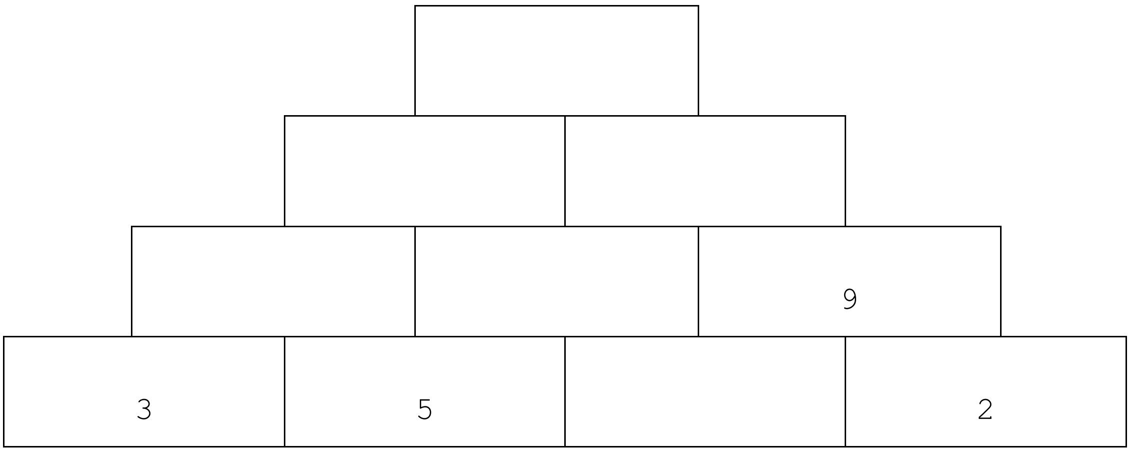
2) Build a pyramid having sum 8 – how many exist?

Can I build a pyramid having the same numbers but different?

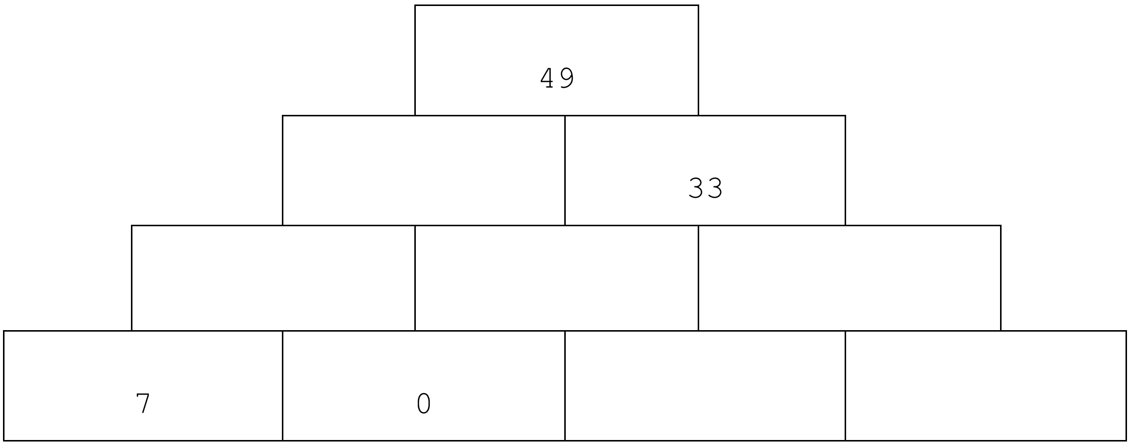
Can I build a pyramid changing rules?

Pyramids again!!

3) Complete the following 4-floor pyramids

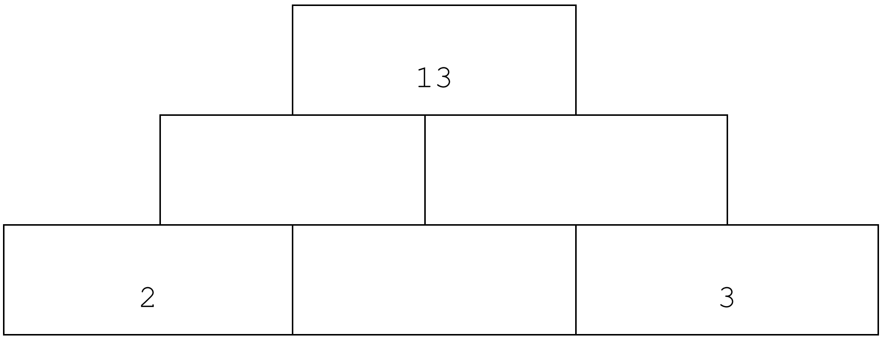


4)

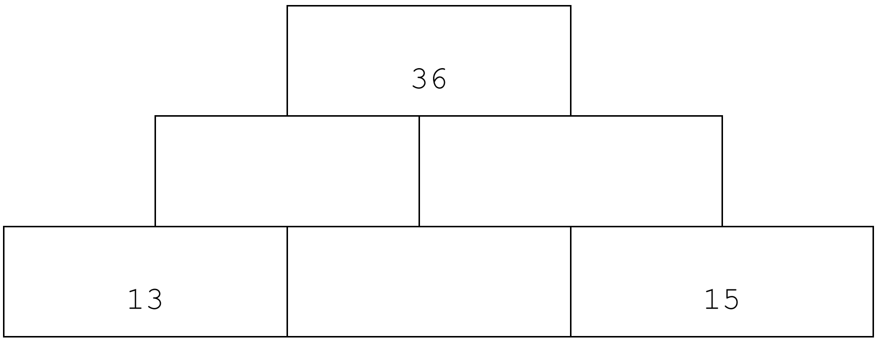


Coming back to 3-floor Pyramids......

5)



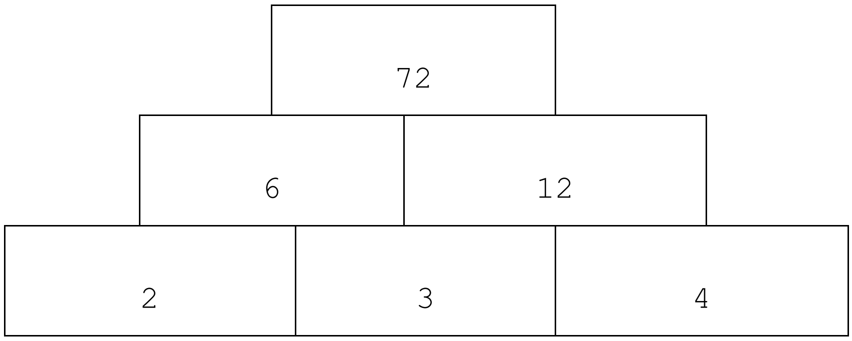
6)



Now another kind of pyramids....multiplication pyramids!

Every brick is the product of the two bricks below

7)



Try to complete this pyramid

n general, the initial tasks are quite easy for the students, but in the subsequent tasks complexity increases. Some items are already a good tool to introduce the first elements of algebraic thinking, because the numbers are unknown and to find them it is necessary to consider some logical properties deducible from the task context. Of course we deal with 3^{rd} grade students, so the passage from concrete/visual objects and operations to more abstract ones is very awkward.

To perform this passage, the role of the semiotic means of objectification, in the sense of Radford's theory, is fundamental. It is exactly what happened in the video. In the following I report some crucial points of the video, specifying the transcription on the left column and my comments/interpretations on the right one. The students involved in the video are three boys (named A, B, C) , and a girl, simply named the girl. Previously we asked students' parents for a written authorization for recording; parents gave it.

4.b.2 Qualitative analysis of the video

The analysis of the video was realized in a qualitative way. The interpretations proposed are performed by using different tools.

First, considering all words, gestures and writings by the students. I looked for key words like "if" "because", "right" "needed" to individuate conceptions and procedures adopted by the students, besides I considered case by case all writings, gestures, voice tone and rhythm (semiotic means of objectification). Secondly, the interpretations are made taking into account the cultural and mathematical background of the classroom, and the characters of the pupils involved, in fact the analysis of the video was performed after a deep discussion with the teacher on various episodes of the video itself.

Students' discourse	**Interpretations proposed**
4'26"-4'38" Girl: "but if it is a ladder, here it is needed……." (gestures indicating numerical series and/or regularity)	They don't use the requested rule to fill the spaces into the pyramid. They anyway search for a regularity. Use of If… then arguing
4'35" B boy : "6,9,12..5, 10, 15" (indicating a boxes' sequence)	Students try to recognize known structures
5'15-5'35 : A Boy: "no, no" , after indicating the zone in which the rule does not work	He notes that is not possible to fill all spaces using their supposed rule
7'55-8'10" they put all numbers from 1 to 10 in the pyramid	They changed rule (still not correct) and tried to fill spaces using every number from 1 to 10 like a Sudoku; the 10, (the largest one) on the top may be related to activities performed in the course, in which the piece put on the top was the most powerful
8'31" They seem not convinced	
9' 45" Teacher intervenes, stating again the rule and relations requested in the pyramid, using the word "to correspond"	Intervention of the teacher is fundamental, then students begin to apply correct rule
10 '25" -10' 55" girl quickly writes, in a correct way exercise 3	Till now they cooperated, suddenly they work alone. It seems that cooperation occur when they meet an obstacle
11'56"-12'55" (they start ex.4 from the top, girl writes on the desk operation 33+16, obtaining 49	No more trials and errors ; but operations
12' 54 A Boy : "correct the 9, because 9 e 7…"	If... then arguing
13 '28" "correct the 9, correct the 9 , because 9 e 7..."	
14'08 girl: "to arrive at 33" boy A "33-9 we have to do, 33-9!"	Subtraction appears, to perform numerical balance; what I need to? How can I obtain it?
14'30"-15'20" students solve correctly exercise 4	
15'30"-18'00" Students find difficulty in doing exercise 5; they understand the target ,i.e. to reach 13, try to compose 13, but they stop themselves.	Exercise 5 is really more difficult than the former. It is not a close, immediate contact among data. It is slightly more abstract
18'00" They move to ex. 7	

19’27”. Teacher remember to not act in an automatic way	
20’30”-21’30” boy A solves correctly aloud ex.7	
21’30” they come back to es. 5 and 6	
27’00”-28’00” teacher intervenes, indicating more and more spaces below 13 in ex.5, saying at 31’ 25” “do various compositions of the del 13”	
32’00-33’00” students try various composition of 13, deducing incorrectness	If..then arguing
33’00” C Boy and girl” 6 e 7” C boy: 6 and 7…2 and 4 ….“correct!” The boy indicates by fingers the spaces in the pyramid Students do correctly exercise 5	If …then arguing , C boy does not write, but imagines in mind number 2 and number 4 in the spaces, related by requested rule; it is performed by the gesture (pointing to the spaces)
35’25” students start ex.6	
41’15” colleague: “Look at this 10, you put a number plus and one minus”(indicating the number in the boxes of the pyramid)	It is a subtle suggestion about the quantities to be inserted
41’30” they try with 5 and 3 in the central, lower space	Trials and errors
42’22” Colleague: “plus than 3 o minus than 3?”	
43’04 B boy : “we’ll never do it”	
42’55”-43'14” C boy writes 17 and 19, and 4 below, and mentally adds 17 and 19 pointing to numbers with finger	C boy works alone, reflects, and maybe applying the former suggestion finds the right solution
43’14” C boy: “here is, I did it, results... 4…17 e 19”(then he looks at the colleague, smiling and standing up), writing correctly	
43’20-end students do correctly ex.6	

Table 7 - Video transcription and comments

A first, overall impression about the video concerns the strong engagement of the students. In fact, during the 44 minutes recorded, they worked hard almost without interruptions.

I know that the students knew they were recorded, anyway their attitude has confirmed by the teacher: although usually they work in a similar way no more than 20 minutes, during the chess course (about 2 months), the teacher noticed an overall development. In particular she noted sharp improvements in:

- concentration

- attention
- Deductive reasoning

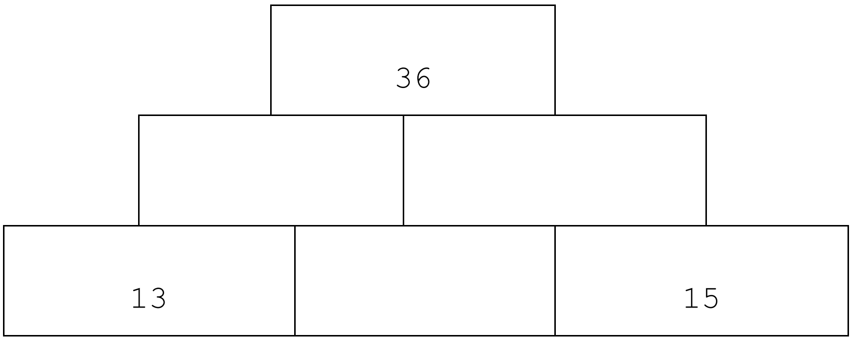
Concerning the latter, she noted the rising instances of if/then in the students' argumentations, albeit not always in a complete way, i.e. they often fail when they have to use deduction using more than one step.

Also, the chess course developed in the class cooperative learning. In fact in the class some students frequently disturbed the work because of their egocentrism (one of them is B boy); during the course this occurred less frequently, and this is observable also in the video, in fact Boy B's behaviour is quite cooperative. According to the teacher, in the chess course students used to verify classmates' choices, and this increased cooperative work.

The teacher told us also about a good result in a class activity concerning orienteering on a city map (the activity we failed to record). In fact the classroom worked very well on that task, and the teacher agrees to consider it as a possible consequence of chess practice, both for content and enhanced cooperative work.

More in detail, we asked students to work together, in groups of 4. It was a pity that the classroom was not large, so the quality of sound resulted poor. I focused on a group. In the first phase, they overlooked the correct rule, and of course they did not solve the task; we noticed a continuous research for regularity, trying to recognize known structures. This is also confirmed by students' gestures. After the teacher intervention, they quickly solved the exercise, at first they worked alone, subsequently they shared information. In solving another task, if...then arguing emerged, as well as operations needed to reach the goal.

Students found difficulty in solving exercises in which the deductions were slightly more complicated, although they were aware of rules and of the kind of logic mechanism needed. After another useful intervention of teacher, remarking the quantities to be used, students reached the target, using also a sort of mental imagery. This may be connected with chess practice, taking into account the age of students. In the last exercise they engaged but failed, and after an input by our colleague Munder Mohamed, directed to C boy, they start with trials and errors. Then C boy found the solutions after a moment of total isolation and concentration. Let me comment about this moment. The task to be accomplished was the following:



Our colleague gave a subtle input, pointing out a fair property, i.e. that any number put in the central lower box generates in the upper level two numbers differing by two (1 plus and 1 minus, he said). The suggestion was given observing an incorrect attempt of the pupils, indicating the numbers in the boxes, and showing the property on concrete numbers written by C boy. The numbers were wrong, but the property holds, of course! So, by means of that numbers and their relation, the C boy reached the solution. It's a fair example of use of means of objectification, i.e. the deictic words "guarda *qui* ("look at *this*") to written numbers, in the visual field of the boy (Radford 2002), and the gesture of the colleague. This semiotic mediation leads the pupil to the mathematical property concerned. In the picture below the gesture of the colleague (when he says "look at this").

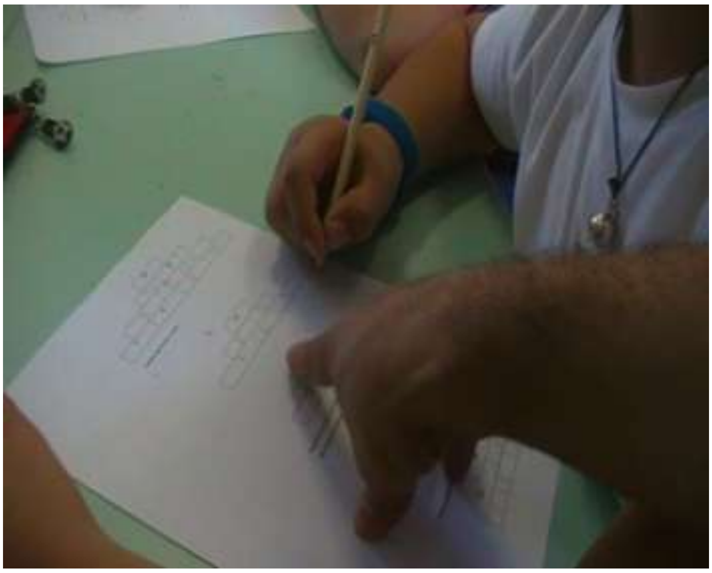


Fig. 22 – episode 41'15"

The objectification here is the understanding of the property (Radford 2005), fully applicable to accomplish the task.

As a final remark of the activity, I noted the regular, continuous use of if..then arguing by the students and their tenacious engagement, that may be related to chess practice. Also the search for regularity and the gestures indicating sort of sequences may be an interesting topic to study into more depth.

4.b.3 Outputs of INVALSI inquire for the class

Despite I have not yet the complete outputs; I extracted some information from the paper protocols I and Mario Ferro submitted to students.

In the following table are reported the students' test results (in percentage, i.e. number of correct answers/number of items).

Student	Pre-test performance	Post-test performance
s1	0,44	0,43
s2	0,81	0,82
s3	0,04	0,82
s4	0,89	0,93
s5	0,52	0,46
s6	0,70	0,68
s7	0,85	0,86
s8	0,74	0,71
s9	0,56	0,61
s10	0,52	0,68
s11	0,78	0,75
s12	0,78	0,64
s13	0,70	0,75
s14	0,96	0,93
s15	0,67	0,79
s16	0,96	0,82
mean	**0,68**	**0,73**

Table 8- Students' performances

The outputs show an improvement of 5% from pre test to post test, in accordance with the trend on entire Italy. Both pre and post test performances resulted better than the national ones.It is clear that it is not correct to generalize outputs obtained by a single trial, having at disposal a very few data. Of course, just one casual accident (like a hard school day or other) can change the result of a single trial.

Hence, I do not pay particular attention to these data, also because of the nature of the trial, i.e. closed-answers questionnaires. Dealing with this kind of tests, it is not possible to assume information useful about the solving strategies adopted by the students, consequently to make consideration and argue conclusions on these data is very dangerous, especially about specific skills and abilities. Just a comment on the performance mean, that resulted high with respect to the national values. It was not a surprise, considering the general good level in mathematics of the class, as confirmed by the teacher and by the chess instructor that is also a mathematician.

Final remarks

In this chapter I'll try to summarize the principal outputs of my work, trying also to unify the various considerations and evaluations made into a coherent framework. The vision of cognition as related to culture, and the awareness of different cultural approaches applied also in abstract topics like those of the strategic games, as discussed in the first section of chapter I, led me to consider the chess thinking and cognition under a sharp point of view. The analysis performed in chapter 1 about the historical elements in chess points in this direction, i.e. shows as the chess thinking changed in the course of time, depending on specific forms of culture, typical of the historical moment. This is related both to similar approaches in other disciplines and to the appearance of great players. Another very singular approach to chess, nowadays considered still unorthodox, is practiced in freestyle. The full integration man-machine (*The Centaur*) in a high-level making choice process and the necessary, but careful, use of statistics by the centaurs make this discipline very interesting and probably anticipatory of future cognitive styles. This may be a matter of relevance for education, especially in scientific context. Even in this case the thinking style is changing, depending most on the evolution of the computers and software and on the adaptability of the humans. These also are cultural forms. In Chapter 2, I analyzed and discussed the main psychological theories on chess cognition, including some important inquires from neurosciences. I consider valid the model I called standard model, in which pattern recognition plays an important role to define the chess expertise (chunk theory, template theory). In fact is not so clear that masters analyze so much more than club players, meaning the deep of thought. I think this is in accordance with Gestalt psychology vision of cognition. Besides, the reasoning of chess players is characterized by a strong visuo-spatial component, as confirmed by neuroscience outputs. This component acts both in a classical, hypothetic- deductive way and in an immediate (based on previous knowledge) way, mainly in a non verbal modality. I noted that there is not, at the moment, a complete theory of chess thinking, including cognitive and functional items that in my opinion are strongly depending by cultural and social factors. Anyway, it is not correct to reduce the role of the deductive thinking. Chess players, overall the more skilled, in their thinking processes, often base their analysis, in essence exhaustive, on pure deductions. This is not in contradiction with the role of pattern recognition, because the former acts as a tool for judgment and orientation in thinking, selecting the options within the tree of variations during the analysis, and supplying assessment criteria. The more a chess player is expert, the more he can use profitably his/her fund of experience (patterns recognizable) to orienting and judging in the analysis. Then, planning my pilot study, I looked for some evidence of this model, and I am quite satisfied of the results, albeit they are far to be considered conclusive.

The core of the work is represented by the searching for relations between chess and mathematical thinking, in particular chess and mathematical education.

As quoted in the introduction, the two worlds (Chess and mathematics) are really immense, and the relations are multiple.

First, I studied the relations between chess and skills in general. Chess is commonly considered as the intellectual discipline *par excellence*. But it is not so easy**.** It is not enough to be intelligent to play well chess, and vice versa. As widely discussed, chess skill is most dependent on experience. Besides, intelligence is a word meaning several skills and abilities, different among them, like the logical, relational, and practical ones**.** Chess may be related positively with problem solving skills like planning and visuo-spatial abilities, the former especially in children. But most the engagement in an intellectual, gaming, motivating activity may product wonderful outputs. Chess is perfect for this purpose, mostly in particular social contexts, like occurred in the famous New York City activity, and confirmed by many examples in the world, including my personal experience in Palermo. There is some evidence of influence of practicing chess on school proficiency and general skills of students. It is a fair confirmation of what is noticed by thousands of teachers in the world about the effectiveness of chess practice in students, in particular concerning the attitude to face a problem and pursuing a solution. Going into more deep, the visuo-spatial abilities seem to be improved by the chess practice in children, but this trend tends to disappear in adults. In particular, the chess player uses visuo-spatial abilities, in somewhat similar way to geometrical mental imagery, as noticed by Presmeg (2006). I performed a parallel between geometrical object, as depicted by Fischbein (1993) and chess concepts, considering the figural concept in the Fischbein framework as concepts driven by their definition, and the configural concepts in chess, as concepts that make sense by the relationships with the other ones in various patterns.

Coming back to the primeval research question, as I suggested in the introduction, I think that it is already clear that the question is not well-posed. This because of the outputs of my work, including my personal experience and beliefs, I constructed in these years. In fact the proficiency of a curriculum, in full accordance with Schonefeld (2007) makes sense only in a context. Then, the evaluations of the effects of a curriculum, in my case the chess practice, become effective if the way in which the curriculum itself is adopted at school is adequate and appropriate. A curriculum is not a medicine for learning, and chess is not a medicine for math learning. Any content you want to deal with, in educational context, (and not only in education) finds its definition in the culture, and the methodology, the approach and the motivational input given by the teacher, as developed in the students' community, are determinant factors. Of course the teacher is not the only responsible for a proficient scholastic course, the components involved are multiple and it is not appropriate to deepen now this complex topic. Hence, as I just declared before, I prefer to deal with the reformulated research question: What conditions, methods and approaches are advisable to make chess a useful practice for Education, in particular for mathematics Education? The question has many aspects. I know that "*there are no proofs in Mathematics education*", as said by Henry Pollak (Schoenfeld, 2000), but there are some sound evidences supporting the answers I present.

First, the **approach**, the *Tao* as Sun-Zu stated more than 2500 years ago (see section. 1.1). A valid, motivating approach can be determinant to make proficient an activity, chess in particular. A valid chess course requires 30 h at least, and the gaming approach, for students, is a very important factor.

Second, the **age**. It is clear that noticeable improvements in cognitive abilities are easier to obtain in children, including the visuo-spatial ones. In fact, as confirmed by important studies quoted in my work, adult chess players do not have better visuo-spatial abilities with respect to average adults, but in children the chess practice seems to allow them to reach a given level (in visuo-spatialabilities) before the average do.

Third, the **contents**: chess practice, especially under the above conditions, is excellent in improving problem solving abilities and attitude to reasoning. It is the learning of the logic of the game, (moves, values, strategies, techniques) that produces the improvements, more than the amount of practice. The amount of practice, however, helps to fix the concepts, and it is the main factor to improve in chess skill, as widely discussed in the former. Someone could make an objection considering that similar improvements can be obtained by introducing some other intellectual, gaming activities. As stated by Gobet and Campitelli (2009), there is not still an "ideal experiment" in which chess practice is evaluated considering the most important scientific criteria, in particular the matching with the practice of other intellectual activities. This is correct, but my counter-objection is: It is not so easy to find such an activity! This activity has to be intellectual, intriguing for pupils, and diffused among people in the context we are. In fact, without diffusion, we probably will meet difficulty to introduce this activity in the school, and probably we will not involve easily many instructors and teachers. Besides, choosing chess, the students will have the chance to play soon in competitions, playing and enjoying with students of the own city or Region. Further, apart from the “classical” contents and abilities developed in a chess course, it is possible to introduce mathematical contents in the chess course itself. It was experimented in U.S.A. by Ho, as reported in (web references 8). Ho inserted in his integrated math-chess handbook various math contents, obtaining good results also in calculation that is an ability normally not so improved by chess practice. We did somewhat similar in writing the contribution to the chess protocol used in SAM project. As I mentioned in chapter 4, we have got just early unofficial results from SAM project. They seem very positive; I am waiting for a complete analysis of the SAM project by INVALSI; even though the set-up of the inquire not was not provided for an evaluation of processes, the outputs of an inquire involving about 65 classes over entire Italy, performed very seriously, will represent a useful reference.

It is not correct attributing to our part of the protocol (math contents developed in chess environment) the merit of this (supposed) success, anyway I believe that it may be a valid strategy, and the idea of the integrated handbook may be a valid idea too, of course after a careful evaluation of the scholastic context and a full sharing by the teachers involved.

Our experience at school, in a class participating in the SAM project, resulted very proficient. During the course, the teacher reported on sharp improvements in math by the students, especially in concentration, attention, deductive reasoning and attitude to reasoning. Also the performance in the SAM project tests that I deduced from the students’ paper protocols, is quite satisfactory, in accordance with the teacher’s statements. The chess instructor in the class was Mario Ferro, my Phd colleague, a mathematician too. Then a special attention was devoted to the math contents, and we followed several math activities performed by the teacher, who collaborates with us very proficiently. The video recorded during a mathematical activity shows clearly that the students have a strong attitude to accomplish anyway the task we submitted. In the 44 minutes of recording quoted also before in the chapter 4, there is just a brief moment in which the students’ concentration goes down, an excellent result for 8 years old pupils. We (the teacher, Mario and I) think that this may be related with the chess course. In the video I focused an episode in which the process of objectification was presented, showing as the deictic, the gestures and the reference to an apparently unessential property act as means of objectification, allowing the student involved to accomplish the task.

In conclusion, I want to underlie some open problems and possible future research scenarios.

First, from a theoretical point of view, may be interesting to develop a more complete model of chess thinking, also in connection with mental imagery theories. About it, I think that a

strong cooperation with neurosciences become more and more advisable. This to deepen the role of working memory and long term memory, the role of the emotions, the anxiety related to the competition and the skills acquired by players.
Secondly, as suggested by Gobet and Campitelli (2009), it may be interesting to perform an "ideal experiment" regarding the effect of chess practice on education that has still to be planned and realized.
Thirdly, another interesting topic may be the study of postures and gestures of chess players during the games; this may be put in connection with the trend of the game, studying psychological, affective, and logical aspects. Another significant research topic I underlie is the relationship between chess practice and geometrical demonstration's skill. This because the deductive thinking within the mental imagery, as practised by the chess players, seems a very useful training for geometrical demonstrations.

Appendix 1
Palermo Pre test

1. Alessandro, Bianca, Carlo e Daniela abitano in diversi punti della città e devono raggiungere tutti la stazione per prendere il treno delle 17.05 per Torino. Alessandro esce alle 16.20 ed impiega 41 minuti; Bianca alle 16.25 ed arriva alla stazione in 29 minuti; a Carlo occorrono 32 minuti ed esce alle 16.36; Daniela lascia la sua casa alle 16.12 ed impiega 51 minuti. Chi di loro NON riuscirà a prendere il treno delle 17.05?

☐ A. Bianca.

☐ B. Carlo.

☐ C. Daniela.

☐ D. Nessuno.

---

2. Quale valore deve avere il ▲ perché l'uguaglianza sia vera?

$$\blacktriangle \times 8 = 63 - \blacktriangle$$

☐ A. 9

☐ B. 8

☐ C. 7

☐ D. 6

3. Osserva le seguenti figure.

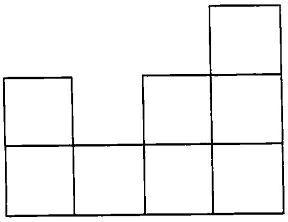


Figura 1

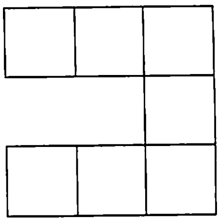


Figura 2

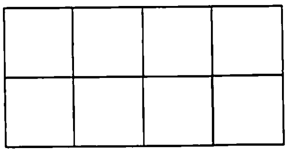


Figura 3

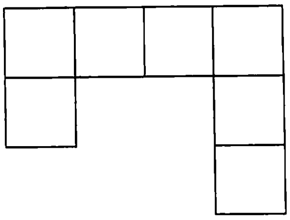
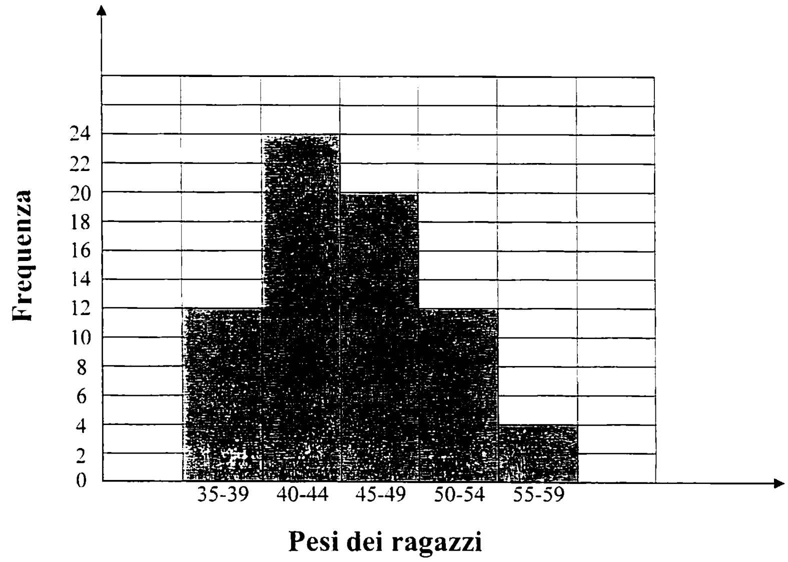


Figura 4

Quale affermazione sulle figure è vera?

- ☐ A. Solo 1, 2 e 3 hanno lo stesso perimetro.
- ☐ B. Solo 1, 3 e 4 hanno la stessa area.
- ☐ C. Tutte e quattro hanno lo stesso perimetro.
- ☐ D. Solo 2 e 4 hanno la stessa area e lo stesso perimetro.

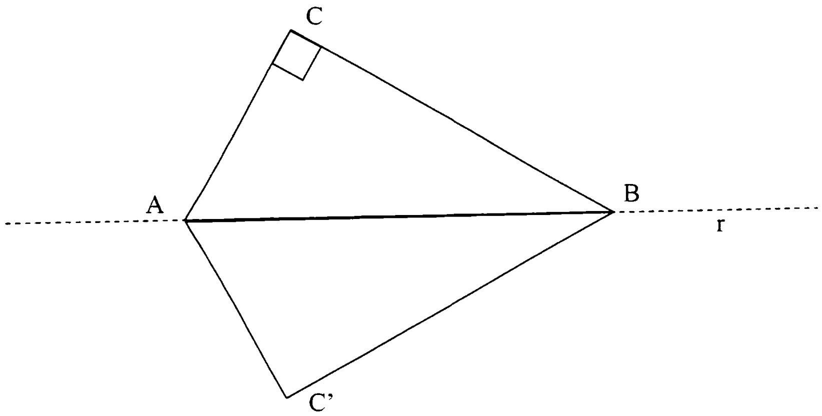
4. Il seguente grafico rappresenta i pesi, in chilogrammi, di ragazzi iscritti ad un gruppo sportivo.



Quanti ragazzi pesano MENO di 45 kg?

- ☐ A. 56
- ☐ B. 36
- ☐ C. 24
- ☐ D. 12

5. In figura è stato disegnato il triangolo AC'B simmetrico del triangolo rettangolo ACB rispetto alla retta r. Come si può chiamare la figura complessiva ACBC'?



- [ ] A. Trapezio rettangolo.
- [ ] B. Triangolo.
- [ ] C. Parallelogramma con due angoli retti.
- [ ] D. Quadrilatero con due angoli retti e due coppie di lati uguali.

---

6. Antonio ha eseguito nel trimestre 5 verifiche, la cui media aritmetica è 6. Quali possono essere i voti delle sue verifiche?

- [ ] A. 6, 7, 8, 8, 6
- [ ] B. 3, 6, 6, 6, 7
- [ ] C. 6, 7, 5, 5, 7
- [ ] D. 8, 7, 6, 6, 6

7. Su una scatola di spaghetti c'è scritto:

Spaghetti

cottura 12 minuti

500 grammi

Per cuocere, secondo le indicazioni, 250 g di spaghetti quanti minuti sono necessari?

☐ A. 6

☐ B. 9

☐ C. 12

☐ D. 24

---

8. Un quadrilatero ha le seguenti caratteristiche:
   - due coppie di lati paralleli;
   - le diagonali di diversa lunghezza e che si tagliano a metà;
   - i quattro lati uguali.

   Qual è il suo nome?

☐ A. Trapezio.

☐ B. Rettangolo.

☐ C. Rombo.

☐ D. Quadrato.

9. Quale valore deve avere il ▲ perché l'uguaglianza sia vera?

$$24{,}5 : 100 = 2{,}45 : \blacktriangle$$

☐ A. 10

☐ B. 1

☐ C. 0,1

☐ D. 0,01

---

10. Quanto può pesare un uovo di gallina?

☐ A. 250 g

☐ B. 1,5 hg

☐ C. 50 g

☐ D. 5 mg

---

11. Come si scrive in cifre il numero quattromilioniquarantamilaquattro?

☐ A. 4040004

☐ B. 4400004

☐ C. 40400004

☐ D. 400040004

12. Una fabbrica produce in un'ora 243 palline da tennis al costo totale di € 340,00. Le palline vengono messe in scatole che ne contengono 3 ciascuna.

Quale fra le seguenti espressioni permette di trovare il numero delle palline ancora da inscatolare, dopo aver riempito 56 scatole?

☐ A. $243 - 3 \times 56$

☐ B. $243 - 3 \times 56 + 340$

☐ C. $243 - 56$

☐ D. $243 : 3 - 56$

---

13. $12,45 + 3,4 + 1,32 + 6,8 =$

☐ A. 22,12

☐ B. 22,89

☐ C. 23,97

☐ D. 27,1

---

14. Con tre bastoncini lunghi 12 cm, 4 cm, 3 cm, che cosa è possibile ottenere?

☐ A. Un triangolo isoscele.

☐ B. Un triangolo scaleno.

☐ C. Un triangolo rettangolo.

☐ D. Nessun tipo di triangolo.

15. Osserva la seguente tabella, relativa alla temperatura di uno scolaro con l'influenza rilevata ogni 4 ore per tre giorni.

Giorno	*Ora*	**Temperatura**
Martedì	8	39,1
	12	37,4
	16	38,5
	20	39,2
Mercoledì	8	37,7
	12	38,0
	16	38,5
	20	39,5
Giovedì	8	37,3
	12	37,5
	16	37,5
	20	37,2

Quale delle seguenti affermazioni è vera?

La temperatura ...

☐ A. più bassa è stata registrata alle ore 12 di martedì.

☐ B. più alta è stata registrata alle ore 20 di mercoledì.

☐ C. non è mai scesa sotto i 37,3 gradi.

☐ D. ogni giorno ha avuto un andamento sempre crescente.

16. La mamma di Gianni va al supermercato e acquista

- un pacchetto di caffè: 250 g;
- un pacchetto di zucchero: 1 kg;
- prosciutto: 1,5 hg;
- un melone: 3,5 kg;
- pomodori: 1,5 kg.

Una borsetta di plastica può portare al massimo 4 kg senza rompersi. Di quante borsette ha bisogno, al minimo, per portare a casa gli acquisti?

☐ A. 1

☐ B. 2

☐ C. 3

☐ D. 4

---

17. Come si scrive in cifre il numero formato da 17 centinaia, 3 unità e 4 centesimi?

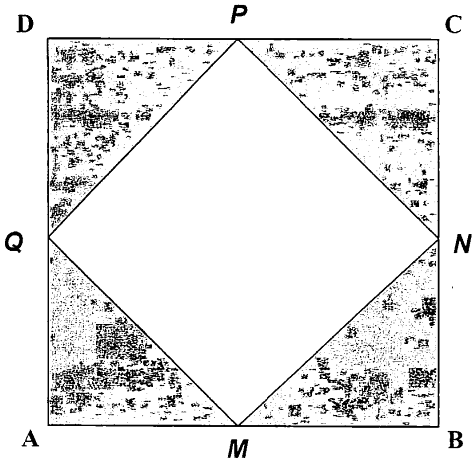
☐ A. 17,34

☐ B. 173,04

☐ C. 173,4

☐ D. 1703,04

18. Nel quadrato ABCD vengono segnati ed uniti tra loro i punti medi di ciascun lato.



Come sono tra loro le aree delle parti in bianco e in grigio?

L'area della parte...

☐ A. bianca è il doppio di quella grigia.

☐ B. bianca è un quarto di quella grigia.

☐ C. grigia è il doppio di quella bianca.

☐ D. bianca e quella grigia sono uguali.

23215

19. Quale affermazione tra le seguenti è vera per il numero 203,93 ?

- [ ] A. La cifra che ha il valore posizionale maggiore è 2.
- [ ] B. Le due cifre 3 hanno lo stesso valore posizionale.
- [ ] C. La cifra che ha il valore posizionale minore è 0.
- [ ] D. La cifra che ha il valore posizionale maggiore è 9.

---

20. La parte decimale dei fattori della moltiplicazione è stata coperta.

8,[redacted] × 25,[redacted] =

Quale può essere il risultato corretto?

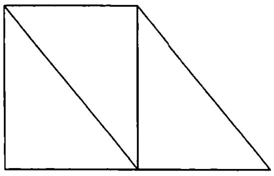
- [ ] A. 2,11328
- [ ] B. 21,1328
- [ ] C. 211,328
- [ ] D. 2113,28

---

21. Di quale unità di misura ti serviresti per esprimere l'altezza dell'edificio della tua scuola?

- [ ] A. dm
- [ ] B m
- [ ] C. m^2
- [ ] D. m^3

22. Quali figure geometriche riconosci nel disegno?



☐ A. Triangolo, parallelogrammo, trapezio, rettangolo.

☐ B. Rettangolo, pentagono, parallelogrammo, triangolo.

☐ C. Triangolo, trapezio, rettangolo, esagono.

☐ D. Quadrato, trapezio, triangolo, rettangolo.

---

23. Una penna, una matita ed una gomma costano complessivamente € 3,00. Se compri solo la matita e la penna spendi € 2,50. Quanti euro costano 5 gomme?

☐ A. 1,50

☐ B. 2,00

☐ C. 2,50

☐ D. 5,00

---

24. Quale tra le seguenti relazioni è vera?

☐ A. 54,061 > 5,4061

☐ B. 54,061 > 540,61

☐ C. 540.61 < 54,061

☐ D. 5406,1 < 540,61

25. Quale dei seguenti insiemi è composto solo da numeri primi?

☐ A. {0; 1; 2; 3; 4; 5}

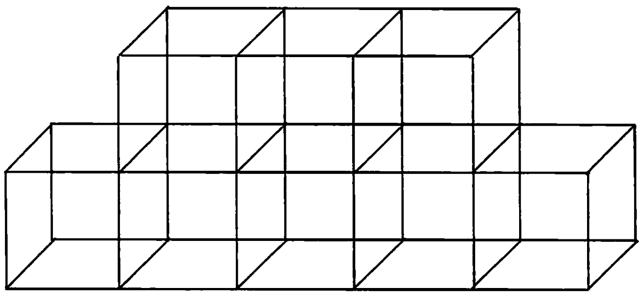
☐ B. {2; 3; 5; 7; 11; 13}

☐ C. {2; 4; 6; 8; 10; 12}

☐ D. {3; 5; 7; 9; 11; 13}

---

26. Osserva la figura.



Se ogni cubetto ha il volume di 8 cm^3, qual è la misura del volume del solido in centimetri cubi?

☐ A. 88

☐ B. 80

☐ C. 72

☐ D. 64

27. Quali tra le seguenti figure NON ha alcun asse di simmetria?

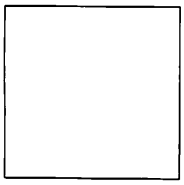


Figura 1

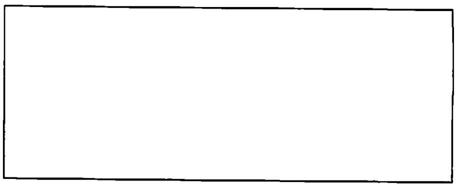


Figura 2

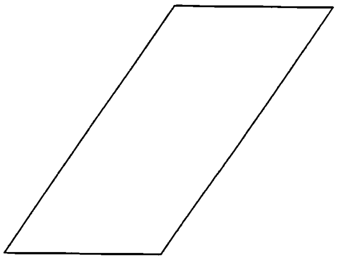


Figura 3

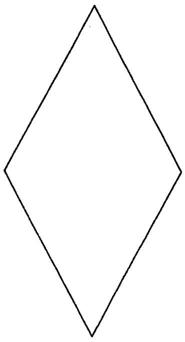
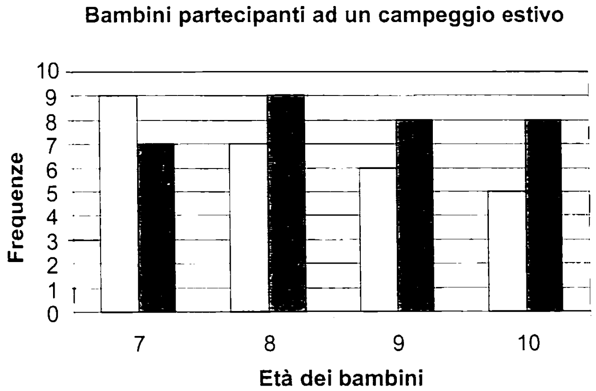


Figura 4

- ☐ A. Tutte.
- ☐ B. Solo la 3 e la 4.
- ☐ C. Solo la 2.
- ☐ D. Solo la 3.

28. Osserva il seguente grafico che rappresenta un gruppo di bambini partecipanti ad un campeggio estivo e divisi per femmine (rappresentate dalle colonne bianche) e maschi (rappresentati dalle colonne grigie).



Quanti sono, tra maschi e femmine, i bambini che hanno PIÙ di 8 anni?

- ☐ A. 45
- ☐ B. 27
- ☐ C. 16
- ☐ D. 14

Appendix 2
Palermo Post test

1. Quale tra i seguenti numeri:

0,07 0,08 0,008 0,0072

è il più grande?

☐ A. 0,07

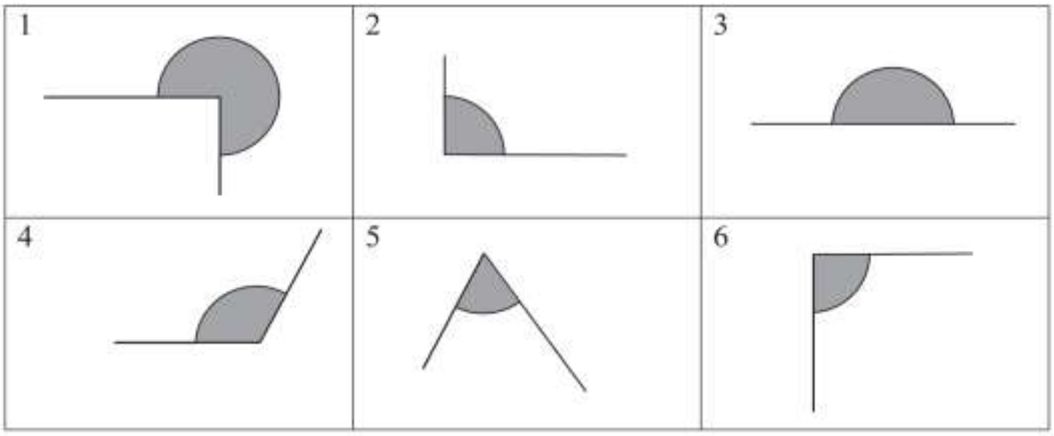
☐ B. 0,08

☐ C. 0,008

☐ D. 0,0072

---

2. Osserva gli angoli disegnati.



Quale dei seguenti gruppi è costituito solo da angoli NON retti?

☐ A. Tutti gli angoli.

☐ B. 2, 3, 4, 6.

☐ C. 3, 4, 6.

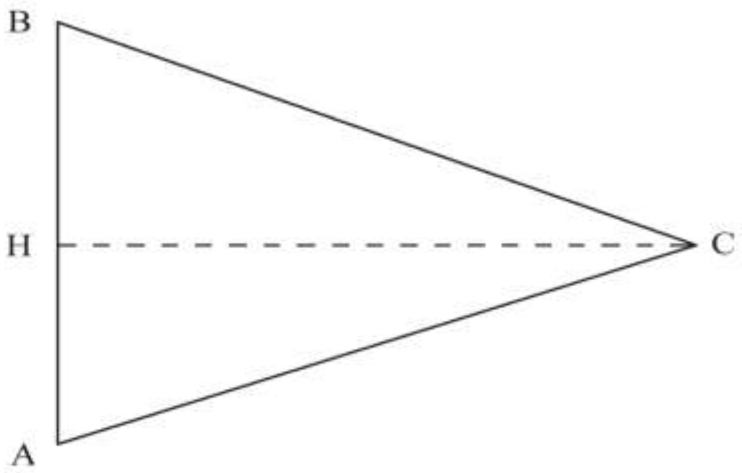
☐ D. 1, 3, 4, 5.

3. Nicola si è addormentato alle ore 22:15. Alle ore 7:30 suona la sveglia. Quante ore ha dormito?

- ☐ A. 7 ore e 45 minuti.
- ☐ B. 8 ore e 15 minuti.
- ☐ C. 9 ore e 15 minuti.
- ☐ D. 9 ore e 45 minuti.

---

4. Un triangolo isoscele ABC ha le seguenti misure: AB = 10 cm, AC = 13 cm, CH = 12 cm.



Qual è il suo perimetro?

- ☐ A. 34 cm
- ☐ B. 35 cm
- ☐ C. 36 cm
- ☐ D. 48 cm

5. La seguente tabella rappresenta le età di 14 ragazzi frequentanti un gruppo sportivo.

Età in anni	**Frequenze**
10	5
11	3
12	4
13	2
Totale	14

Qual è la serie delle età che corrisponde a quelle riportate in tabella?

☐ A. 13, 11, 13, 13, 12, 10, 10, 12, 14, 12, 10, 14, 10, 12

☐ B. 13, 10, 10, 13, 12, 12, 10, 11, 10, 11, 10, 11, 12, 10

☐ C. 12, 12, 10, 10, 11, 13, 10, 11, 10, 11, 12, 12, 10, 10

☐ D. 10, 10, 13, 10, 12, 11, 12, 11, 10, 10, 11, 12, 12, 13

---

6. A quale fra i seguenti numeri decimali corrisponde la frazione $\frac{4}{10}$?

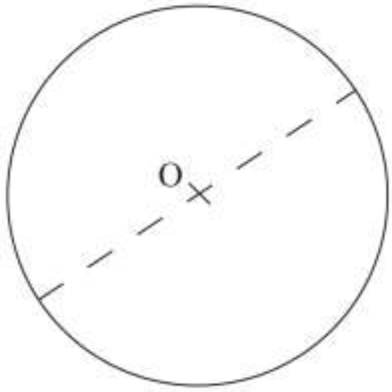
☐ A. 10,4

☐ B. 1,04

☐ C. 0,4

☐ D. 0,04

7. Osserva attentamente la figura.



Come si chiama il segmento tratteggiato in figura?

☐ A. Diagonale.

☐ B. Apotema.

☐ C. Raggio.

☐ D. Diametro.

---

8. Qual è l'unità di misura più appropriata per esprimere lo spessore di un foglio di cartoncino?

☐ A. Metri.

☐ B. Decimetri.

☐ C. Centimetri.

☐ D. Millimetri.

9. Quale delle seguenti terne di numeri è formata da multipli di 4?

☐ A. 12, 26, 48

☐ B. 20, 36, 92

☐ C. 32, 44, 62

☐ D. 36, 52, 66

---

10. Un triangolo che ha gli angoli che misurano 30°, 60° e 90° a quale dei seguenti insiemi appartiene?

All'insieme dei triangoli…

☐ A. rettangoli.

☐ B. equilateri.

☐ C. isosceli.

☐ D. acutangoli.

---

11. Il parallelogramma ABCD ha il lato AB di 15 cm, il lato BC di 7 cm, l'altezza DH di 5 cm. Quale sarà la misura della sua area?

☐ A. 27 cm^2

☐ B. 75 cm^2

☐ C. 105 cm^2

☐ D. 515 cm^2

12. In questo prodotto è stata coperta una parte dei fattori:

$$50,[redacted] \times 8,[redacted] =$$

Quale può essere il risultato corretto?

☐ A. 4,2867

☐ B. 42,867

☐ C. 428,67

☐ D. 4286,7

---

13. La tabella seguente rappresenta il peso di 300 alunni di una scuola, espresso in intervalli di 5 kg.

Peso in kg	Numero alunni
40-44	0
45-49	12
50-54	34
55-59	85
60-64	92
65-69	60
70-74	12
75-79	5
Totale alunni	300

Quanti alunni pesano PIÙ di 64 kg?

☐ A. 77

☐ B. 169

☐ C. 223

☐ D. 300

14. Come si scrive in cifre il numero novemilionisettecentododicimilatredici?

☐ A. 971213

☐ B. 9702013

☐ C. 9712013

☐ D. 97001213

---

15. A quale delle seguenti frazioni corrisponde il numero decimale 0,5?

☐ A. $\frac{5}{100}$

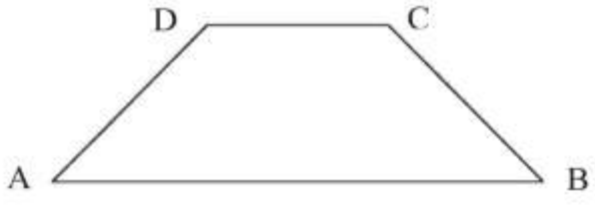
☐ B. $\frac{1}{5}$

☐ C. $\frac{5}{10}$

☐ D. $\frac{15}{10}$

---

16. Nel trapezio isoscele ABCD l'angolo acuto di vertice B misura 45°.



Quanto misura l'angolo di vertice C?

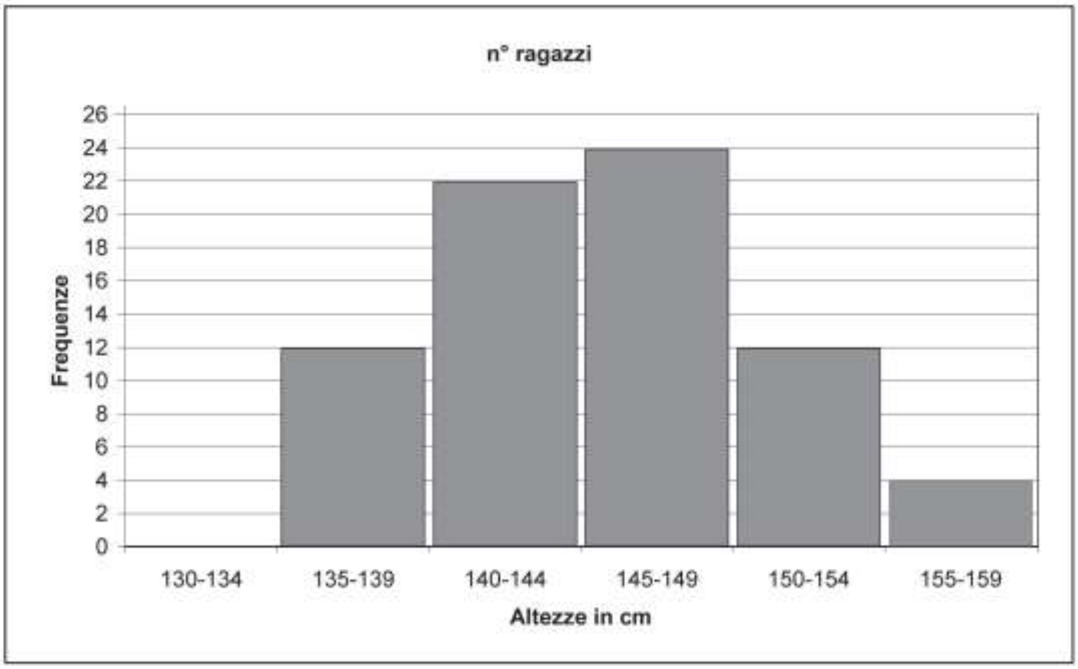
☐ A. 270°

☐ B. 135°

☐ C. 90°

☐ D. 45°

17. Il seguente grafico rappresenta le altezze, in centimetri, dei ragazzi delle classi prime.



Quale delle seguenti tabelle corrisponde al grafico?

☐ A.

Altezze in cm	Frequenze
135-139	12
140-144	22
145-149	24
150-154	12
155-159	4

☐ B.

Altezze in cm	Frequenze
135-139	12
140-144	22
145-149	24
150-154	4
155-159	12

☐ C.

Altezze in cm	Frequenze
135-139	12
140-144	24
145-149	20
150-154	12
155-159	4

☐ D.

Altezze in cm	Frequenze
135-139	4
140-144	12
145-149	22
150-154	24
155-159	12

18. Un quadrato ha la diagonale di 6 cm. Quanto misura la sua area?

☐ A. 36

☐ B. 24

☐ C. 18

☐ D. Non posso calcolarla solo con questo dato.

---

19. 1,85 + 6,3 + 32,236 + 0,564 + 2,1 =

☐ A. 31,940

☐ B. 42,950

☐ C. 43,040

☐ D. 43,050

---

20. La lancetta delle ore di un orologio è passata dalle 3 alle 12. Qual è l'ampiezza dell'angolo descritto?

☐ A. 270°

☐ B. 180°

☐ C. 120°

☐ D. 90°

21. Come si scrive in cifre il numero costituito da 26 migliaia, 31 decine e 17 unità?

☐ A. 2631017

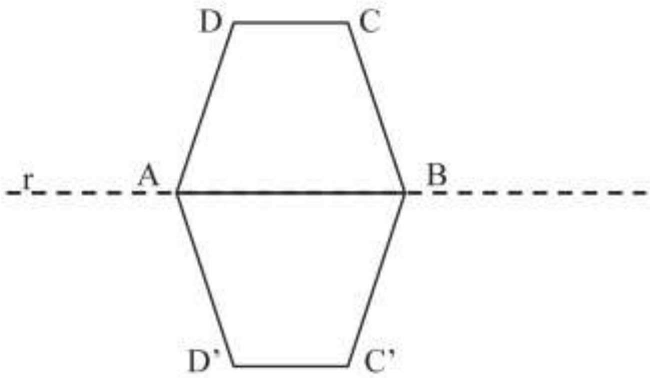
☐ B. 2603117

☐ C. 263117

☐ D. 26327

---

22. Nel trapezio ABCD la misura degli angoli è: A = 60° e D = 120°. In figura è stato disegnato il simmetrico ABC'D' del trapezio ABCD rispetto alla retta r.



Quale tra le seguenti affermazioni è FALSA?

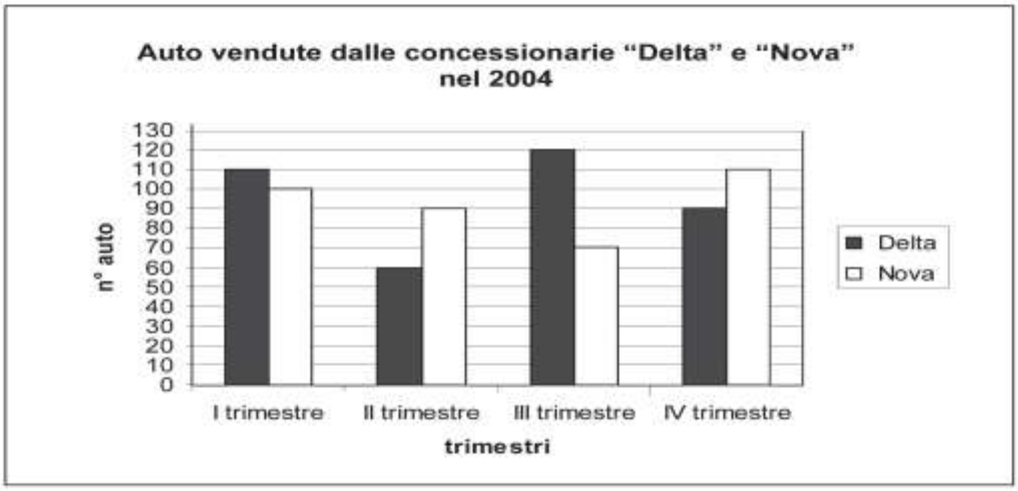
☐ A. Il perimetro della figura complessiva ADCBC'D' è il doppio del perimetro del trapezio ABCD.

☐ B. L'area della figura complessiva ADCBC'D' è il doppio dell'area del trapezio ABCD.

☐ C. L'angolo DAD' è un angolo ottuso.

☐ D. L'angolo AD'C' misura 120°.

23. Il grafico rappresenta le vendite degli autosaloni "Delta" e "Nova" nell'anno 2004, rilevate per trimestre.



Quale delle seguenti affermazioni è vera?

☐ A. Negli ultimi tre trimestri i due autosaloni hanno venduto complessivamente lo stesso numero di auto.

☐ B. Nei primi due trimestri la "Nova" ha venduto complessivamente meno auto della "Delta".

☐ C. In ogni trimestre la "Delta" ha venduto più auto della "Nova".

☐ D. La "Nova" ha venduto nell'anno 2004 più auto della "Delta".

24. Quattro alunni devono eseguire la seguente operazione:

$$475 \times 19$$

Ognuno ha svolto i calcoli in maniera diversa. Quale delle seguenti procedure NON è corretta?

- ☐ A. $475 \times 19 = (400 \times 19) + (70 \times 19) + (5 \times 19)$
- ☐ B. $475 \times 19 = (475 \times 20) - 1$
- ☐ C. $475 \times 19 = (475 \times 20) - 475$
- ☐ D. $475 \times 19 = (475 \times 10) + (475 \times 9)$

---

25. Una figura ha: due lati uguali, una sola coppia di lati paralleli e due angoli ottusi. Quale può essere tra le seguenti figure?

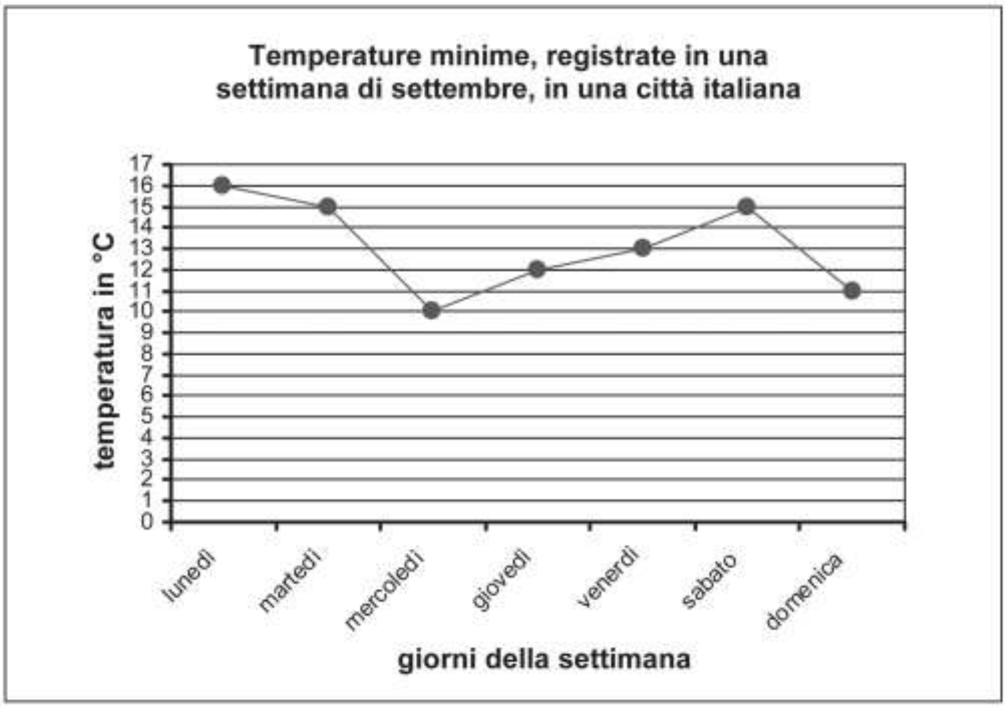
- ☐ A. Triangolo ottusangolo.
- ☐ B. Trapezio isoscele.
- ☐ C. Trapezio rettangolo.
- ☐ D. Parallelogramma.

---

26. Quanto può distare il piano di un tavolo dal pavimento?

- ☐ A. 78 cm
- ☐ B. 78 dm
- ☐ C. 78 m
- ☐ D. 78 dam

27. Il seguente grafico riporta le temperature minime registrate, in una settimana di settembre, in una città italiana.



Quale delle seguenti affermazioni è vera?

La temperatura minima…

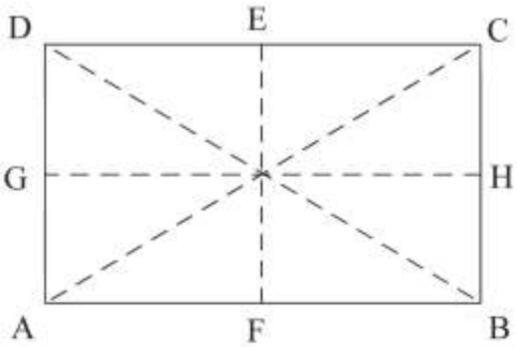
☐ A. più bassa della settimana è stata registrata domenica.

☐ B. di lunedì e di sabato è stata la stessa.

☐ C. registrata giovedì è di 12 °C.

☐ D. registrata sabato è di 14 °C.

28. Osserva attentamente la figura.



Quali tra i segmenti tratteggiati sono assi di simmetria del rettangolo ABCD?

☐ A. Tutti e quattro.

☐ B. Solo EF.

☐ C. Solo AC e BD.

☐ D. Solo EF e GH.

Appendix 3
Agrigento Pre test

1. Alessandro, Bianca, Carlo e Daniela abitano in diversi punti della città e devono raggiungere tutti la stazione per prendere il treno delle 17.05 per Torino. Alessandro esce alle 16.20 ed impiega 41 minuti; Bianca alle 16.25 ed arriva alla stazione in 29 minuti; a Carlo occorrono 32 minuti ed esce alle 16.36; Daniela lascia la sua casa alle 16.12 ed impiega 51 minuti. Chi di loro NON riuscirà a prendere il treno delle 17.05 ?

☐ A. Bianca

☐ B. Carlo

☐ C. Daniela

☐ D. Nessuno

---

2. Quale valore deve avere il ■ perché la seguente uguaglianza sia vera ?

$$■ \times 8 = 63 - ■$$

Motiva la tua risposta

---

1. Osserva le seguenti figure.

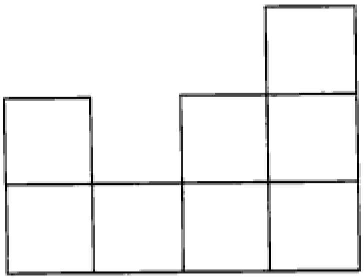


Figura 1

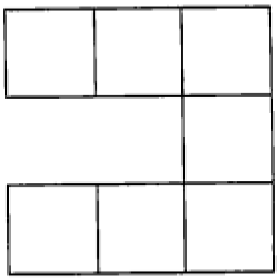


Figura 2

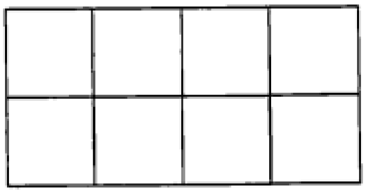


Figura 3

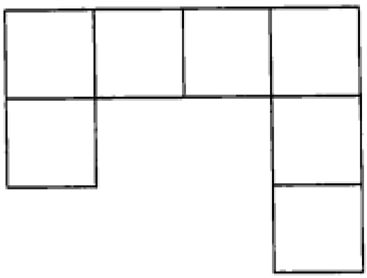


Figura 4

Indicare se le seguenti affermazioni sono vere o false

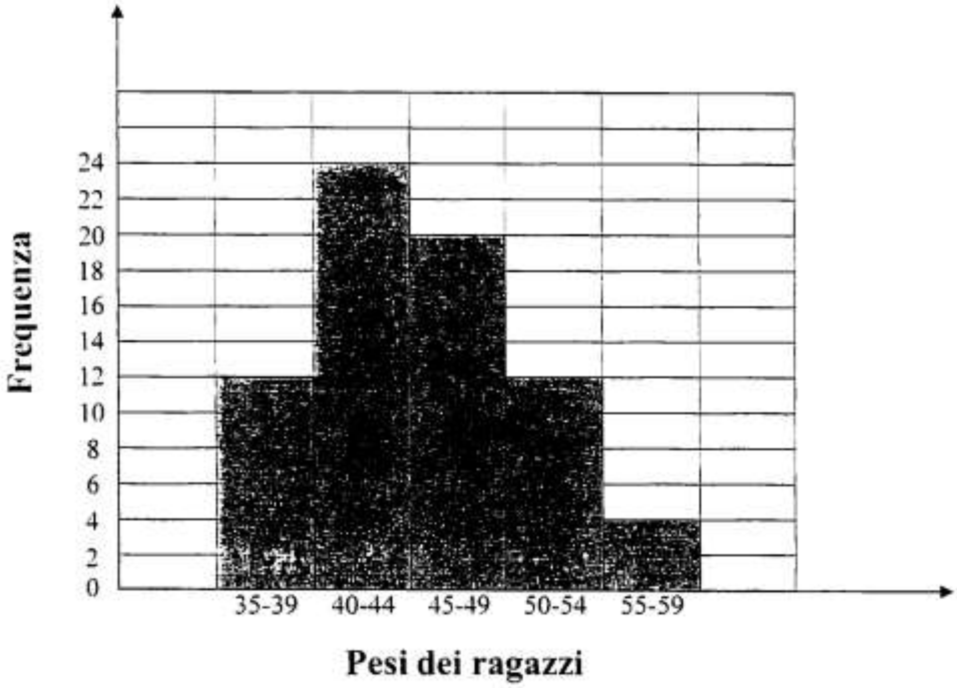
A. Le figure 1-2-3 hanno lo stesso perimetro V F

B. Le figure 1-3 hanno la stessa area V F

C. Le figure 2-4 hanno la stessa area V F

D. le figure 2-4 hanno lo stesso perimetro V F

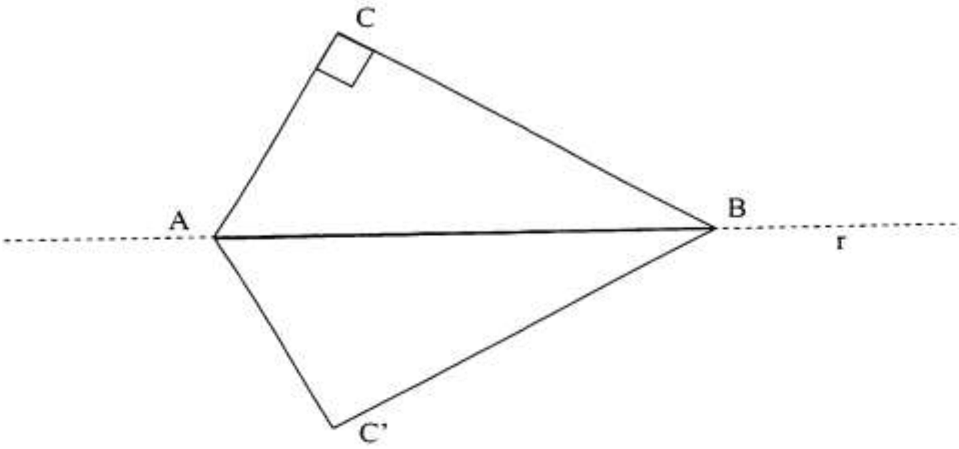
4. Il seguente grafico rappresenta i pesi, in chilogrammi, di ragazzi iscritti ad un gruppo sportivo.



Quanti ragazzi pesano MENO di 45 kg?

Motiva la tua risposta

5. In figura è stato disegnato il triangolo AC'B simmetrico del triangolo rettangolo ACB rispetto alla retta r. Come si può chiamare la figura complessiva ACBC'?



- [ ] A. Trapezio rettangolo.
- [ ] B. Triangolo.
- [ ] C. Parallelogramma con due angoli retti.
- [ ] D. Quadrilatero con due angoli retti e due coppie di lati uguali.

6. Antonio ha eseguito nel trimestre 5 verifiche, la cui media aritmetica è 6.

Per ognuna delle seguenti serie di voti indicare se possibile (P) o impossibile (I):

A.	6,7,8,8,6	P	I
B.	3,6,6,6,7	P	I
C.	6,7,5,5,7	P	I
D.	8,8,4,4,6	P	I

7. Su una scatola di spaghetti c'è scritto:

Per cuocere, secondo le indicazioni, 250 g di spaghetti quanti minuti sono necessari?

Motiva la risposta

8. Un quadrilatero ha le seguenti caratteristiche:

- due coppie di lati paralleli;
- le diagonali di diversa lunghezza e che si tagliano a metà;
- i quattro lati uguali.

Qual è il suo nome?

☐ A. Trapezio.

☐ B. Rettangolo.

☐ C. Rombo.

☐ D. Quadrato.

9. Quale valore deve avere il █ perché la seguente uguaglianza sia vera ?

24,5 : 100 = 2,45 : █

Motiva la tua risposta

10. Quanto può pesare un uovo di gallina?

- ☐ A. 250 g
- ☐ B. 1,5 hg
- ☐ C. 50 g
- ☐ D. 5 mg

11. Come si scrive in cifre il numero quattromilioniquarantamilaquattro?

13. Quattro fratelli vanno al supermercato: Mario compra un uovo di pasqua che costa € 12,45 , Giulia acquista delle caramelle per € 3,40, Valeria una bevanda che costa € 1,32, Luca prende dei biscotti che costano € 6,88. Quanto hanno speso in tutto?

14. L’insegnante di matematica dà ad ogni ragazzo in classe tre bastoncini, lunghi 12 cm, 4 cm e 3 cm. Giulia sostiene di poter costruire un triangolo rettangolo, Giovanni un triangolo isoscele e Carmela un triangolo scaleno. Chi ha ragione e perché?

15. Osserva la seguente tabella, relativa alla temperatura di uno scolaro con l'influenza rilevata ogni 4 ore per tre giorni.

Giorno	*Ora*	**Temperatura**
Martedì	8	39,1
	12	37,4
	16	38,5
	20	39,2
Mercoledì	8	37,7
	12	38,0
	16	38,5
	20	39,5
Giovedì	8	37,3
	12	37,5
	16	37,5
	20	37,2

Indicare se le seguenti affermazioni sono vere o false :

A La temperatura più bassa si è registrata alle ore 12 di martedi V F

B La temperatura più alta si è registrata alle ora 20 di mercoledi V F

C La temperatura non è mai scesa sotto i 37,2 gradi V F

D Ogni giorno ha avuto un andamento sempre crescente V F

16. La mamma di Gianni va al supermercato ed acquista:

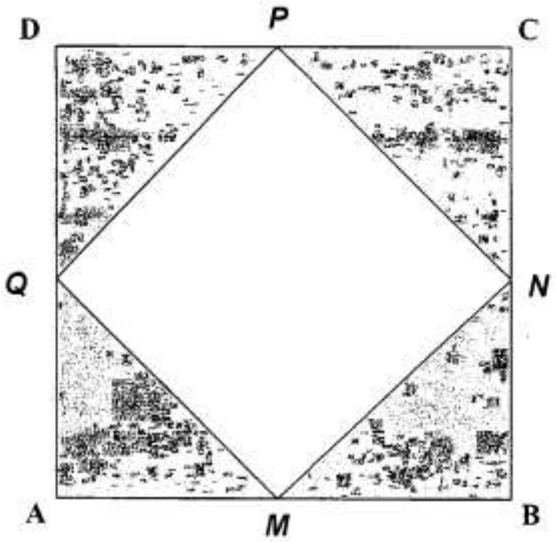
- Un pacchetto di caffè : 250 g
- Un pacchetto di zucchero: 1 Kg
- Prosciutto : 1,5 hg
- Un melone : 3,5 Kg
- Pomodori : 1,5 Kg

Una borsetta di plastica può portare al massimo 4 Kg senza rompersi. Di quante borsette ha bisogno, al minimo, per portare a casa gli acquisti?

Motiva la risposta

17. Come si scrive in cifre il numero formato da 17 centinaia, 3 unità e 4 centesimi ?

18. Nel quadrato ABCD vengono segnati ed uniti tra loro i punti medi di ciascun lato.



Come sono tra loro le aree delle parti in bianco e in grigio?

L'area della parte…

- ☐ A. bianca è il doppio di quella grigia.
- ☐ B. bianca è un quarto di quella grigia.
- ☐ C. grigia è il doppio di quella bianca.
- ☐ D. bianca e quella grigia sono uguali.

19. Quali affermazioni sono vere o false per il numero 203,93 ?

A. La cifra che ha valore posizionale maggiore è 2	V	F
B. Le due cifre 3 hanno lo stesso valore posizionale	V	F
C. La cifra che ha il valore posizionale minore è 0	V	F
D. La cifra 0 ha valore posizionale maggiore del 9	V	F

---

20. La parte decimale dei fattori della moltiplicazione è stata coperta.

8,[redacted] × 25,[redacted] =

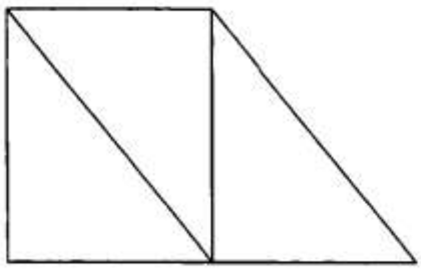
Quale può essere il risultato corretto?

- ☐ A. 2,11328
- ☐ B. 21,1328
- ☐ C. 211,328
- ☐ D. 2113,28

21. Di quale unità di misura ti serviresti per esprimere l'altezza dell'edificio della tua scuola?

- ☐ A. dm
- ☐ B m
- ☐ C. m^2
- ☐ D. m^3

22. Quali figure geometriche riconosci nel disegno?



23. Una penna, una matita ed una gomma costano complessivamente € 3,00. Se compri solo la matita e la penna spendi € 2,50. Quanti euro costano 5 gomme?

24. Quale tra le seguenti relazioni è vera?

☐ A. 54,061 > 5,4061

☐ B. 54,061 > 540,61

☐ C. 540.61 < 54,061

☐ D. 5406.1 < 540.61

25. Quale dei seguenti insiemi è composto solo da numeri primi?

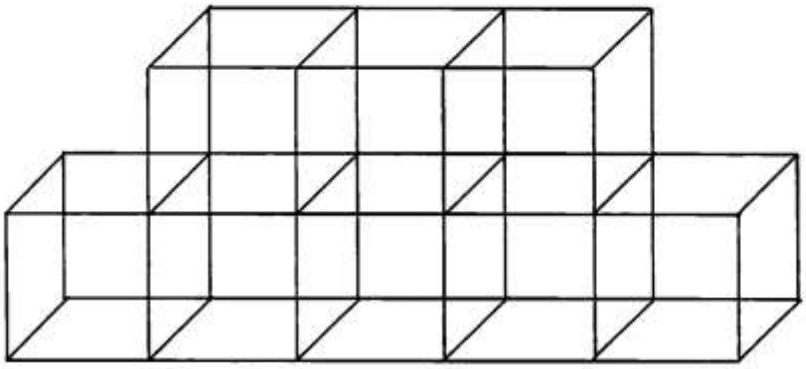
☐ A. {0; 1; 2; 3; 4; 5}

☐ B. {2; 3; 5; 7; 11; 13}

☐ C. {2; 4; 6; 8; 10; 12}

☐ D. {3; 5; 7; 9; 11; 13}

26. Osserva la figura.



Se ogni cubetto ha il volume di 8 cm^3, qual è la misura del volume del solido in centimetri cubi?

27. Quali tra le seguenti figure NON ha alcun asse di simmetria?

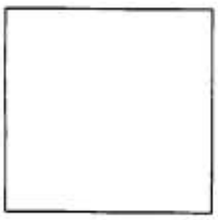


Figura 1

Figura 2

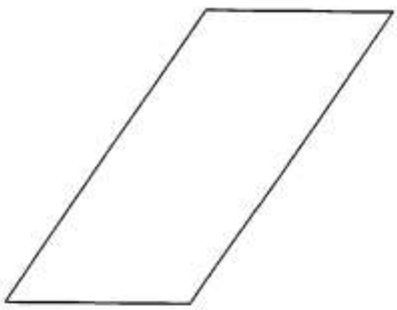


Figura 3

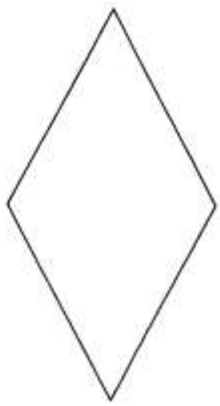
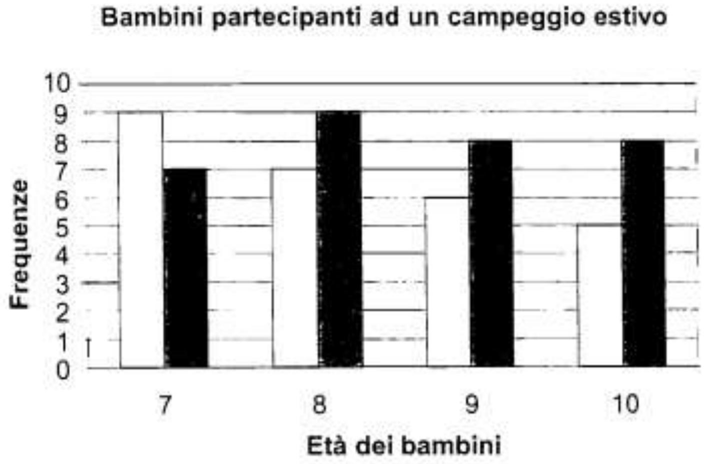


Figura 4

28. Osserva il seguente grafico che rappresenta un gruppo di bambini partecipanti ad un campeggio estivo e divisi per femmine (rappresentate dalle colonne bianche) e maschi (rappresentati dalle colonne grigie).



Quanti sono. tra maschi e femmine, i bambini che hanno PIÙ di 8 anni?

Appendix 4
Agrigento Post test

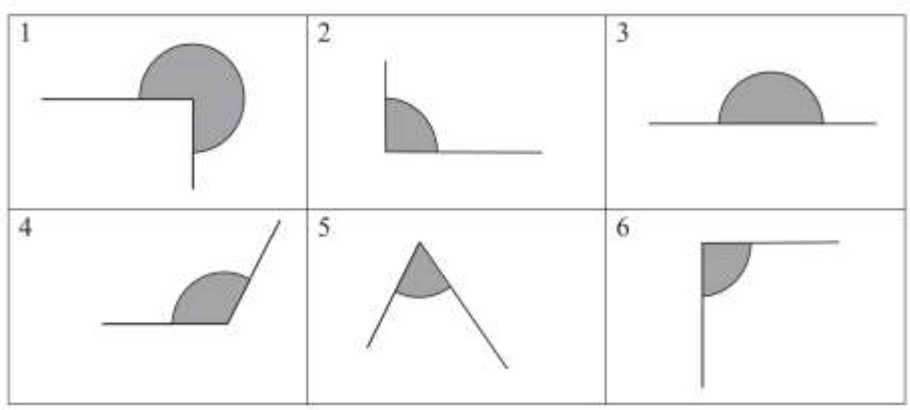
1. Quale tra i seguenti numeri:

   0,07 0,08 0,008 0,0072

   È il più grande?

O A. 0,07
O B. 0,08
O C. 0,008
O D. 0,0072

2. Osserva gli angoli disegnati.



Indica se le seguenti affermazioni sono vere o false:

A) Gli angoli 1 e 2 sono entrambi retti V F

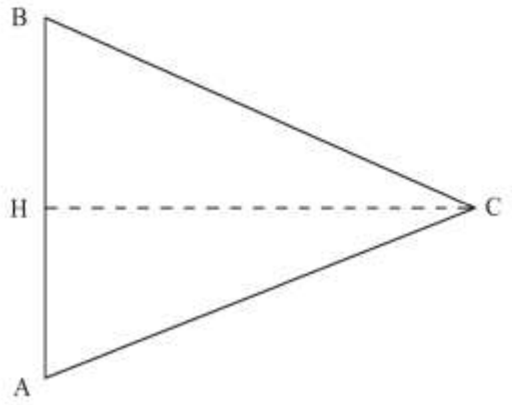
B) Gli angoli 2 e 6 sono entrambi retti V F

C) Gli angoli 3 e 5 non sono retti V F

D) L’angolo 4 è un angolo retto V F

3. Nicola si è addormentato alle ore 22:15. Alle ore 7:30 suona la sveglia. Quante ore ha dormito?

4. Un triangolo isoscele ABC ha le seguenti misure: AB = 10 cm, AC = 13 cm, CH = 12 cm.



Qual è il suo perimetro?

5. La seguente tabella rappresenta le età di 14 ragazzi frequentanti un gruppo sportivo.

Età in anni	Frequenze
10	5
11	3
12	4
13	2
Totale	14

Qual è la serie delle età che corrisponde a quelle riportate in tabella?

☐ A. 13, 11, 13, 13, 12, 10, 10, 12, 14, 12, 10, 14, 10, 12

☐ B. 13, 10, 10, 13, 12, 12, 10, 11, 10, 11, 10, 11, 12, 10

☐ C. 12, 12, 10, 10, 11, 13, 10, 11, 10, 11, 12, 12, 10, 10

☐ D. 10, 10, 13, 10, 12, 11, 12, 11, 10, 10, 11, 12, 12, 13

6. A quale fra i seguenti numeri decimali corrisponde la frazione $\frac{4}{10}$?

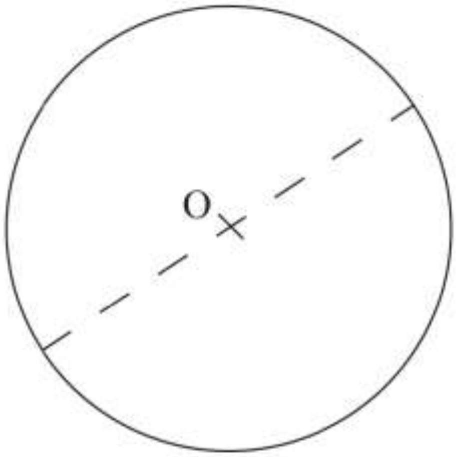
☐ A. 10,4

☐ B. 1,04

☐ C. 0,4

☐ D. 0,04

7. Osserva attentamente la figura.



Come si chiama il segmento tratteggiato in figura?

8. Qual è l'unità di misura più appropriata per esprimere lo spessore di un foglio di cartoncino?

- ☐ A. Metri.
- ☐ B. Decimetri.
- ☐ C. Centimetri.
- ☐ D. Millimetri.

-

9. Per ognuna delle seguenti terne di numeri dire se è vero o falso che sono composte da multipli di 4

A) 12, 26, 48 V F

B) 20, 36 ,92 V F

C) 32, 44, 62 V F

D) 36, 52, 66 V F

10.
Un triangolo che ha gli angoli che misurano 30° , 60° e 90° che tipo di triangolo è?

---

11. Il parallelogramma ABCD ha il lato AB di 15 cm, il lato BC di 7 cm, l'altezza DH di 5 cm. Quale sarà la misura della sua area?

12. In questo prodotto è stata coperta una parte dei fattori:

50,[redacted] × 8,[redacted] =

Quale può essere il risultato corretto?

- ☐ A. 4,2867
- ☐ B. 42,867
- ☐ C. 428,67
- ☐ D. 4286,7

13. La tabella seguente rappresenta il peso di 300 alunni di una scuola, espresso in intervalli di 5 kg.

Peso in kg	Numero alunni
40-44	0
45-49	12
50-54	34
55-59	85
60-64	92
65-69	60
70-74	12
75-79	5
Totale alunni	300

Quanti alunni pesano PIÙ di 64 kg?

14. Come si scrive in cifre il numero novemilionisettecentododicimilatredici?

- [ ] A. 971213
- [ ] B. 9702013
- [ ] C. 9712013
- [ ] D. 97001213

15. A quale delle seguenti frazioni corrisponde il numero decimale 0,5?

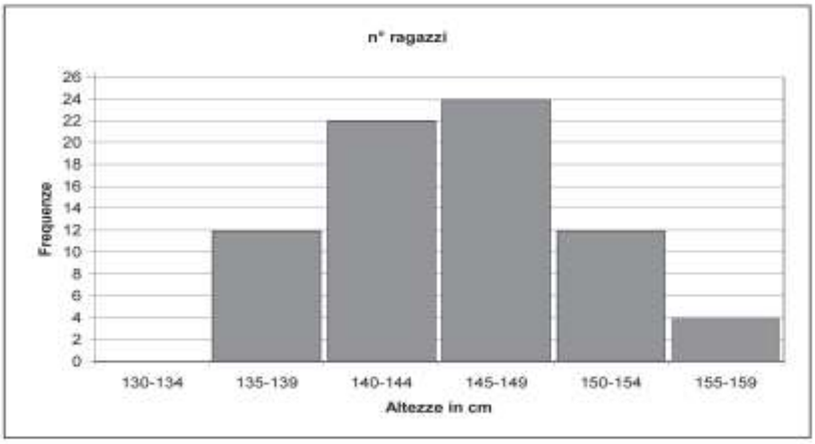
- [ ] A. $\frac{5}{100}$
- [ ] B. $\frac{1}{5}$
- [ ] C. $\frac{5}{10}$
- [ ] D. $\frac{15}{10}$

16. Nel trapezio isoscele ABCD l'angolo acuto di vertice B misura 45°.

Quanto misura l'angolo di vertice C?

- [ ] A. 270°
- [ ] B. 135°
- [ ] C. 90°
- [ ] D. 45°

17. Il seguente grafico rappresenta le altezze, in centimetri, dei ragazzi delle classi prime.



Quale delle seguenti tabelle corrisponde al grafico?

☐ A.

Altezze in cm	Frequenze
135-139	12
140-144	22
145-149	24
150-154	12
155-159	4

☐ B.

Altezze in cm	Frequenze
135-139	12
140-144	22
145-149	24
150-154	4
155-159	12

☐ C.

Altezze in cm	Frequenze
135-139	12
140-144	24
145-149	20
150-154	12
155-159	4

☐ D.

Altezze in cm	Frequenze
135-139	4
140-144	12
145-149	22
150-154	24
155-159	12

18. Un quadrato ha la diagonale di 6 cm. Quanto misura la sua area?

☐ A. 36

☐ B. 24

☐ C. 18

☐ D. Non posso calcolarla solo con questo dato.

19. $1{,}85 + 6{,}3 + 32{,}236 + 0{,}564 + 2{,}1 =$

☐ A. 31,940

☐ B. 42,950

☐ C. 43,040

☐ D. 43,050

20. La lancetta delle ore di un orologio è passata dalle 3 alle 12. Qual è l'ampiezza dell'angolo descritto?

---

21. Come si scrive in cifre il numero costituito da 26 migliaia, 31 decine e 17 unità?

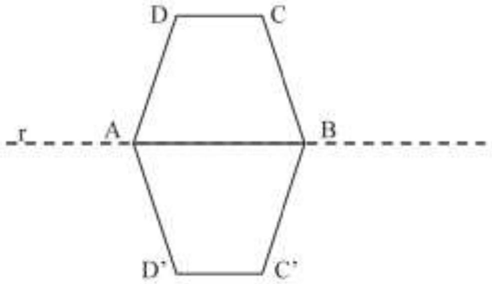
☐ A. 2631017

☐ B. 2603117

☐ C. 263117

☐ D. 26327

22. Nel trapezio ABCD la misura degli angoli è: A = 60° e D = 120°. In figura è stato disegnato il simmetrico ABC'D' del trapezio ABCD rispetto alla retta r.



-

Indicare per ognuna delle seguenti affermazioni se è vera o falsa

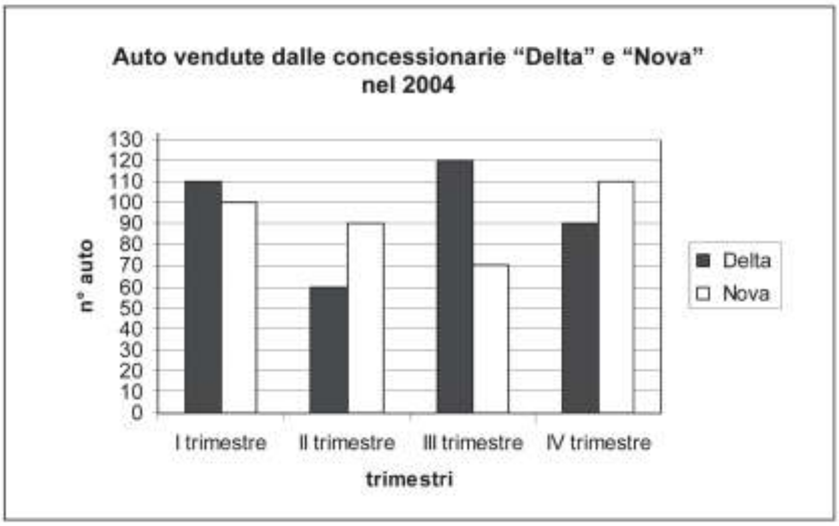
a) Il perimetro della figura complessiva ADCBC'D' è il doppio del perimetro del trapezio ABCD V F

b) L'area della figura complessiva ADCBC'D' è il doppio dell'area del trapezio ABCD V F

c) L'angolo DAD' è un angolo ottuso V F

d) L'angolo A D' C' misura 90 ° V F

23. Il grafico rappresenta le vendite degli autosaloni "Delta" e "Nova" nell'anno 2004, rilevate per trimestre.



Indicare per ognuna delle seguenti affermazioni se è vera o falsa

a) Negli ultimi tre trimestri i due autosaloni hanno venduto complessivamente lo stesso numero di auto V F

b) Nei primi due trimestri la NOVA ha venduto complessivamente meno auto della DELTA V F

c) In ogni trimestre la DELTA ha venduto più auto della NOVA V F

d) La NOVA ha venduto nel 2004 più auto della DELTA V F

25. Una figura ha: due lati uguali, una sola coppia di lati paralleli e due angoli ottusi. Quale può essere tra le seguenti figure?

☐ A. Triangolo ottusangolo.

☐ B. Trapezio isoscele.

☐ C. Trapezio rettangolo.

☐ D. Parallelogramma.

24. Quattro alunni devono eseguire la seguente operazione:

$$475 \times 19$$

Ognuno ha svolto i calcoli in maniera diversa. Quale delle seguenti procedure NON è corretta?

☐ A. $475 \times 19 = (400 \times 19) + (70 \times 19) + (5 \times 19)$

☐ B. $475 \times 19 = (475 \times 20) - 1$

☐ C. $475 \times 19 = (475 \times 20) - 475$

☐ D. $475 \times 19 = (475 \times 10) + (475 \times 9)$

26. Quanto può distare il piano di un tavolo dal pavimento?

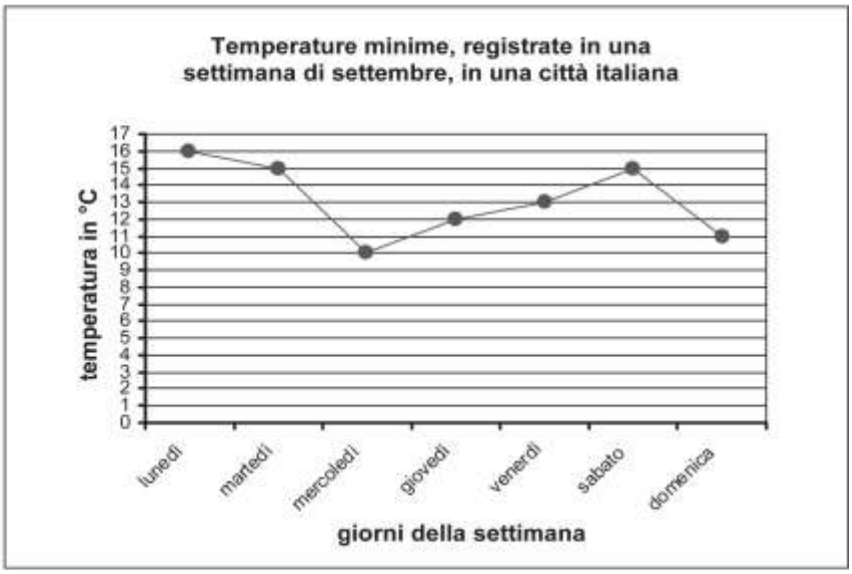
☐ A. 78 cm

☐ B. 78 dm

☐ C. 78 m

☐ D. 78 dam

27. Il seguente grafico riporta le temperature minime registrate, in una settimana di settembre, in una città italiana.



Indicare per ognuna delle seguenti affermazioni se è vera o falsa

a) La temperatura minima più bassa della settimana è stata registrata domenica

V F

b) La temperatura minima di lunedi e sabato è stata la stessa

V F

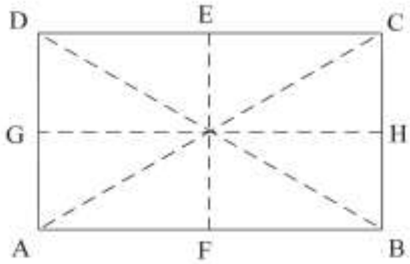
c) la temperatura minima di giovedi è stata di 12° C

V F

d) la temperatura minima di sabato è stata di 15°C

V F

28. Osserva attentamente la figura.



Quali tra i segmenti tratteggiati sono assi di simmetria del rettangolo ABCD?

- ☐ A. Tutti e quattro.
- ☐ B. Solo EF.
- ☐ C. Solo AC e BD.
- ☐ D. Solo EF e GH.

Appendix 5
INVALSI Chess protocol

Comitato regionale Piemonte F.S.I.

Progetto SAM - Protocollo delle 20 ore

Il protocollo ha 16 lezioni fisse di un'ora, ognuna composta da una parte teorica con esercizi, della durata massima di 30 minuti effettivi, a cui segue sempre la partita libera assistita. L'aspetto ludico può essere rinforzato, saltuariamente, dall'intervento dell' istruttore con proposte alternative alla semplice partita (simultanea, mini torneo, giochi scacchistici di vario genere).
Le 4 lezioni finali sono a libera interpretazione dell'istruttore per la parte teorica, da mantenersi fissa la parte di gioco libero. La partita libera è un momento puramente ludico, durante il quale l'istruttore interviene su esplicita richiesta degli allievi per chiarire dubbi e dare consigli (vedi protocollo delle 10 ore). La maggior parte delle lezioni prevedono, per la parte teorica iniziale, l'uso dei diagrammi. La presenza dei diagrammi è segnalata, ma sono inseriti a parte per facilitare la fotocopiatura. Tutte le lezioni sono numerate, il medesimo numero è riportato sulla pagina di diagrammi corrispondente.
Nella parte finale del documento, con il proposito di suggerire e fornire ulteriori spunti didattici, sono stati riportati gli scritti prodotti da Carlo Alberto Cavazzoni e Sebastiano Paulesu, che ringraziamo per il contributo dato.

Considerazioni di fondo

1) Il protocollo è necessario per la unificazione della didattica di tutti gli istruttori inseriti nel progetto SAM. Questa necessità è legata alla individuazione della componente METODO DIDATTICO, in fase di analisi dei dati provenienti dai test. Di fronte ad un eventuale risultato positivo, eventualità che ci auguriamo tutti, questa variabile potrebbe essere considerata determinante. Se i metodi didattici fossero molteplici ed estremamente differenziati tra loro non si potrebbe considerarla una unica variante bensì molte, rendendo impossibile la sua identificazione e collocazione e indebolendo così il risultato di tutta la ricerca.
L'individuazione dell'apporto dato dal METODO DIDATTICO, quindi, può solo avvenire se ogni istruttore adotta fondamentalmente lo stesso metodo.

2) L'utilizzo pedagogico del gioco degli scacchi è realizzato in orario scolastico, è eticamente corretto considerare che questa attività debba essere utile e coinvolgente per ogni alunno, senza discriminazioni. Lo stesso metodo didattico deve assolutamente prevedere che questo diritto dei bambini sia rispettato, escludendo a priori ciò che potrebbe non essere compreso, e quindi non adatto per definizione, da una parte di loro (vi sono casi particolari, che vengono però gestiti autonomamente dagli insegnanti).
Una seconda considerazione, più specificatamente legata al progetto di ricerca, e rivolta ai singoli istruttori, è che le classi inserite nel progetto vanno considerate interamente, poiché il risultato dell'attività scacchistica sarà indagato tramite i test che faranno tutti i loro componenti.
Questo significa che le classi devono essere affrontate nelle loro totalità, senza tralasciare neanche minimamente il coinvolgimento di tutti i ragazzi, perché sarà la risposta della classe a determinare, in senso positivo o negativo, l'esito della ricerca, non di certo i singoli allievi più spiccatamente dotati in ambito scacchistico che SONO SEMPRE LA MINORANZA, IN OGNI CLASSE.
Questa premessa va intesa profondamente, perché il seguente protocollo didattico è finalizzato al mantenimento della partecipazione integrale e costante di tutti gli allievi all'attività, anche di quelli inizialmente meno portati o motivati, sino alla fine del corso.
3) Nelle precedenti 10 ore si è lavorato fondamentalmente sulla comprensione dello scacco matto, che comporta l'acquisizione di tutte le regole di base del gioco (posizionamento iniziale e movimento dei pezzi, cattura, scacco al re, scacco matto, patta) senza indicazioni troppo specifiche da parte dell' istrut-

tore su come comportarsi (a parte arrocco e grossolano sviluppo per i motivi menzionati) durante i gioco libero. Questa libertà nel gioco va mantenuta il più possibile, poiché la capacità tecnica degli al lievi non è sufficientemente evoluta per permettere un approccio sistematico in qualsiasi settore dell par-tita (apertura, medio gioco e finale) né soprattutto la loro motivazione può essere sufficiente pe comprendere e applicare un qualunque tipo di consiglio strategico.
Ad esempio: possono anche acconsentire se suggerite loro di sviluppare il cavallo in f3 anziché in h3, m a loro non resterà nulla di concreto. Questo avverrà poiché la comprensione delle maggiori possibilit di cattura e di spostamento da f3 del cavallo non fa parte della loro esperienza (nemmeno il controll del "centro").
Dovranno capirlo da soli, condotti da voi, facendo esperienza innanzitutto su ciò che significa "cattura re". La possibilità di analizzare il fatto che da f3 si possono controllare più case è una conseguenza de fatto che si sappia che si può catturare un pezzo, o che si può manovrare per riuscire successivament a catturare un pezzo.
Ma questo non lo sanno ancora, poiché fino ad ora la loro attenzione è stata rivolta alla comprension del corretto movimento di quel pezzo (la famosa elle!) e voi avete verificato che ci siano arrivati. Quest è ciò che è stato fatto, ora si tratta di imparare quali sono le cose che si possono fare con quel cavallo partendo dalle sue funzioni più elementari, che sono le variabili di spostamento (controllo delle case) di conseguenza la cattura, la difesa e l'attacco.

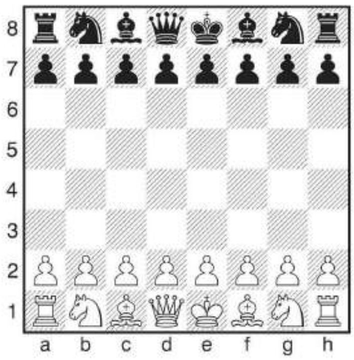
Lezioni

1) Scrivere e leggere gli scacchi

Prima di procedere con le lezioni dedicate all'attività dei pezzi insegnate loro, in modo molto concis come scrivere le mosse. Potranno anche imparare la simbologia base, se lo ritenete opportuno, e qual cuno potrebbe anche voler leggere una breve partita riportandola sulla scacchiera. Questa fase è però necessaria soprattutto perché possano lavorare con i diagrammi durante le lezioni successive.

Scrivere e leggere gli scacchi

Per poter annotare le nostre partite usiamo la notazione scacchistica. Per questo motivo usiamo l'esatta denominazione delle caselle e dei pezzi.



PEZZI	NOTAZIONE	SIMBOLI	VALORE
Re	R	♔ ♚	nessuno
Pedone	P	♙ ♟	1
Alfiere	A	♗ ♝	3
Cavallo	C	♘ ♞	3
Torre	T	♖ ♜	5
Donna	D	♕ ♛	9

Se volessimo scrivere la posizione di partenza faremmo così:
Bianco: Re1 Dd1 Ta1 Th1 Ac1 Af1 Cb1 Cg1 a2 b2 c2 d2 e2 f2 g2 h2
Nero: Re8 Dd8 Ta8 Th8 Ac8 Af8 Cb8 Cg1 a7 b7 c7 d7 e7 f7 g7 h7
Scriviamo sempre i pezzi in rapporto alla loro importanza (punti), in questo modo troviamo subito eventuali errori di comprensione o scrittura.

LA NOTAZIONE ABBREVIATA

Ogni mossa viene abbreviata, si scrive la prima lettera del pezzo e la casa d'arrivo. Se ui pezzo prende un altro pezzo si interpone una x tra l'abbreviazione del pezzo e la cas: d'arrivo. Se un pedone cattura un altro pezzo si scrive anche la colonna da dove è partit dato che il pedone non si scrive.

Se due pezzi uguali possono raggiungere la stessa casa bisogna definire esattamente qual pezzo muove (la notazione completa).

I segni più importanti

x	= presa/cattura	+	= scacco
0-0	= arrocco corto	#	= scacco matto
0-0-0	= arrocco lungo	=	= promozione del pedone

2) Matto in 1 - diagrammi

Per valutare se le prime 10 ore hanno avuto l'effetto desiderato (la comprensione dello scacco matto) in modo sufficiente, iniziate con i diagrammi di matto in 1. Se la media della classe ha almeno un 6 su 12 di risposte positive va bene. Considerate sempre che i casi particolari non devono tenere ferma tutta la classe, eventualmente farete qualcosa per rinforzare la comprensione dello scacco matto in modo individuale durante il gioco libero.

Come lavorare con i diagrammi

Quasi tutte le lezioni prevedono. nella prima parte, un esercizio scritto con 12 posizioni da risolvere, su un foglio A4 che consegnerete ad ogni alunno e che ad esercizio terminato (quando l'allievo avrà risposto a tutti i quesiti) vi verrà riconsegnato.
Cercate di correggere i risultati man mano che vi vengono dati, direttamente su ogni foglio, mandando subito a giocare insieme i ragazzi che hanno terminato prima (giocheranno di più). Fate mettere il loro nome sul foglio all'inizio dell'esercizio, inventatevi 2 simboli che significheranno "risposta giusta" e "risposta sbagliata" che riporterete di fianco ad ogni diagramma, durante la correzione. In tutti i diagrammi la mossa è al bianco, l'alunno scriverà la mossa di fianco al numero del diagramma.
Se non riuscissero a risolvere un diagramma meglio suggerire di proseguire con gli altri, per poi ritornare successivamente e riprovare.
E' importante badare al fatto che non si suggeriscano le risposte, tenendo distanti i ragazzi che avranno terminato prima dagli altri.

3) Impara a dare matto (T+T, D+T, corridoio)

In questa lezione non si useranno i diagrammi, si spiegherà come usare i pezzi pesanti per dare matto. Dare evidenza anche al matto del corridoio. Dopo la spiegazione i ragazzi proveranno, quando entrambi ci saranno riusciti (una volta per uno avranno solo il re) e ve lo avranno fatto vedere giocheranno la partita libera.

4) Inventa un matto - diagrammi

Scrivere il pezzo e la casella dove darà matto, come se fosse una mossa.

2) Il bianco muove e dà scacco matto in una mossa

1)

2)

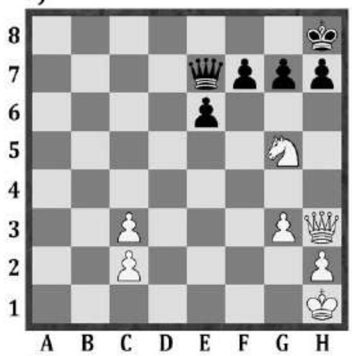
3)

4)

5)

6)

7)

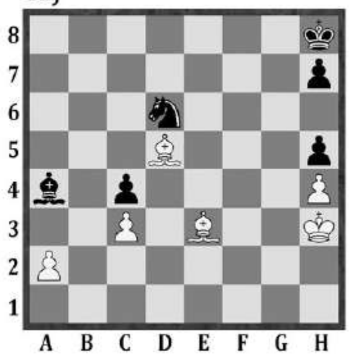


8)

9)

10)

11)



12)

4) Inventa uno scacco matto

1)

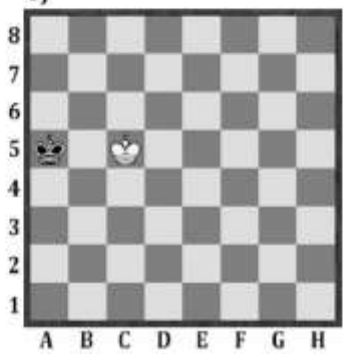
2)

3)

4)

5)

6)



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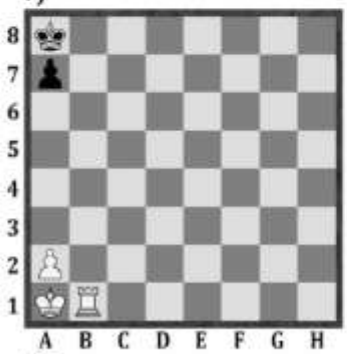

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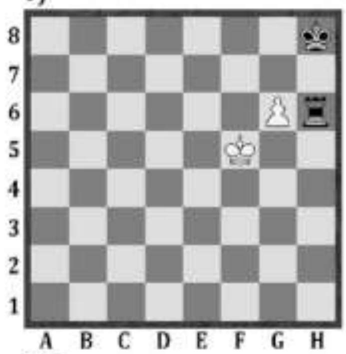

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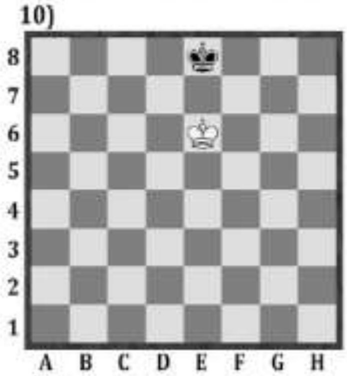

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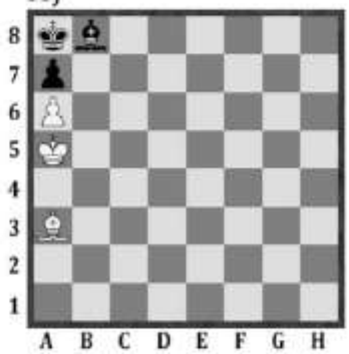


12)

Inizialmente la maggior parte delle partite che termineranno con scacco matto scaturiranno probabilmente da una situazione di enorme vantaggio materiale di uno dei due giocatori.
La loro prima esperienza di scacco matto in partita è una conseguenza del vantaggio materiale (non consideriamo qui i bambini che hanno già una precedente pratica scacchistica o che conoscono già lo scacco matto del barbiere etc.).
Il lavoro da fare con loro è quindi quello di valorizzare il vantaggio materiale, rendendolo il principale obiettivo di ogni partita, insieme alla comprensione e quindi alla conseguente capacità di dare scacco matto.
Tutti i diagrammi degli esercizi sono finalizzati alla stimolazione dell'abilità visuopercettiva degli allievi, senza la quale nessun progresso potrà essere ottenuto, perché è la qualità indispensabile per ottenere la padronanza dei propri pezzi, mettendoli successivamente in relazione con quelli dell'avversario.
Questo processo viene proposto inizialmente a livello di una mossa, quando si tratta di catturare direttamente o spostare il pezzo attaccato per salvarlo 1) scappo, per passare al livello di due mosse quando si considera la difesa del pezzo attaccato con l'impiego di un secondo pezzo 1) difendo...mangi 2) mangio.
Per valorizzare il concetto d vantaggio materiale, infine, può essere utile insegnare ai ragazzi il conteggio dei punti sulla scacchiera per stabilire chi ha vinto, ma solo quando il tempo della partita è terminato e lo scacco matto non si è verificato. Con 2 o più punti di vantaggio si vince (patte teoriche escluse ovviamente), con uno solo è patta. In questo modo ogni partita avrà una conclusione, e come in ogni gioco che si rispetti si saprà come è andata a finire.
La riflessione che dovete proporre è che dalla somma di ogni piccola mossa, quando appunto possiamo perdere materiale senza considerazione di quello che sta avvenendo, scaturisce il risultato finale.

5) Esercizio di bilancio del materiale

E' utile per esercizi sulla somma e relazioni d'ordine, per le capacità di calcolo, per apprendere una regola (come quella di quantificazione del materiale: D=9, T=5, C=A=3, P=1) ed applicarla in una situazione concreta (posizione) e per esercitare all' utilizzo delle simbologie matematiche "<,>,=".
N.B. Dopo aver effettuato il bilanciamento, e quindi confrontato i "valori bianchi" e i "valori neri", si ricorda di non omettere mai che si sta parlando di vantaggi/svantaggi/parità materiale.
Una scorretta comprensione di questo elemento potrebbe risultare da ostacolo quando si tratterà il "valore relativo dei pezzi".
Si consiglia di dedicare una lezione teorica su questo argomento, assegnando agli alunni il compito di scrivere il calcolo su un foglio (come nell'esempio a fianco), a fronte di una o più posizioni giocate in partita libera o proposte dall'istruttore,

Bianco: 5 Pedoni, 1 Torre, 1 Alfiere.
Nero: 4 Pedoni, 1 Torre, 1 Cavallo.

Bianco: 5x1+1x5+1x3=13.
Nero:4x1+1x5+1x3=12.

Punteggio Bianco=13 > Punteggio Nero=12
Il Bianco ha vantaggio MATERIALE.

6) Esercizio con la piramide dei numeri/pezzi - diagrammi

Segnaliamo anche una attività di matematica che ha dei chiari punti in comune con questo tipo di processi: cosa mi serve per fare una cosa, che effetto avrebbe una possibile scelta.
E' la Piramide dei numeri, molto adatta per attività aritmetiche e di avviamento al pensiero algebrico nella Scuola Primaria. Questa attività può essere eventualmente proposta ai docenti della classe.
L'idea è tratta dai materiali del progetto ArAL (http://www.aralweb.unimore.it/on-line/Home.html) - Università di Modena e Reggio Emilia.
L'idea base è molto semplice: con dei "mattoni" si possono fare "costruzioni" a forma di piramide. Su ogni coppia di mattoni affiancati si appoggia un altro mattone. La 'Regola' per sviluppare la costruzione è: il mattone in alto contiene la somma dei due numeri che figurano nei mattoni che lo sostengono. La minima piramide è di due piani, come questa:

 5
2 3

Per dare una connotazione scacchistica all'esercizio e metterlo in relazione alla lezione precedente sono stati creati dei gradini di 3 + 1, con un utilizzo simile alla piramide matematica.
L'allievo dovrà scrivere il simbolo del pezzo mancante per equlibrare le somme.

7) Catture semplici (che pezzo indifeso puoi catturare?) - diagrammi

12 posizioni per la cattura semplice di un pezzo indifeso, con questo esercizio inizia la fase di stimolazione visuopercettiva.

8) Catture semplici 2 (quale pezzo indifeso è meglio catturare?) - diagrammi

Come il precedente, con la differenza che dovranno operare una scelta catturando il pezzo di maggio valore.

9) Difesa semplice (hai un pezzo che sta per essere catturato, dove lo sposti per non fartelo mangiare?)

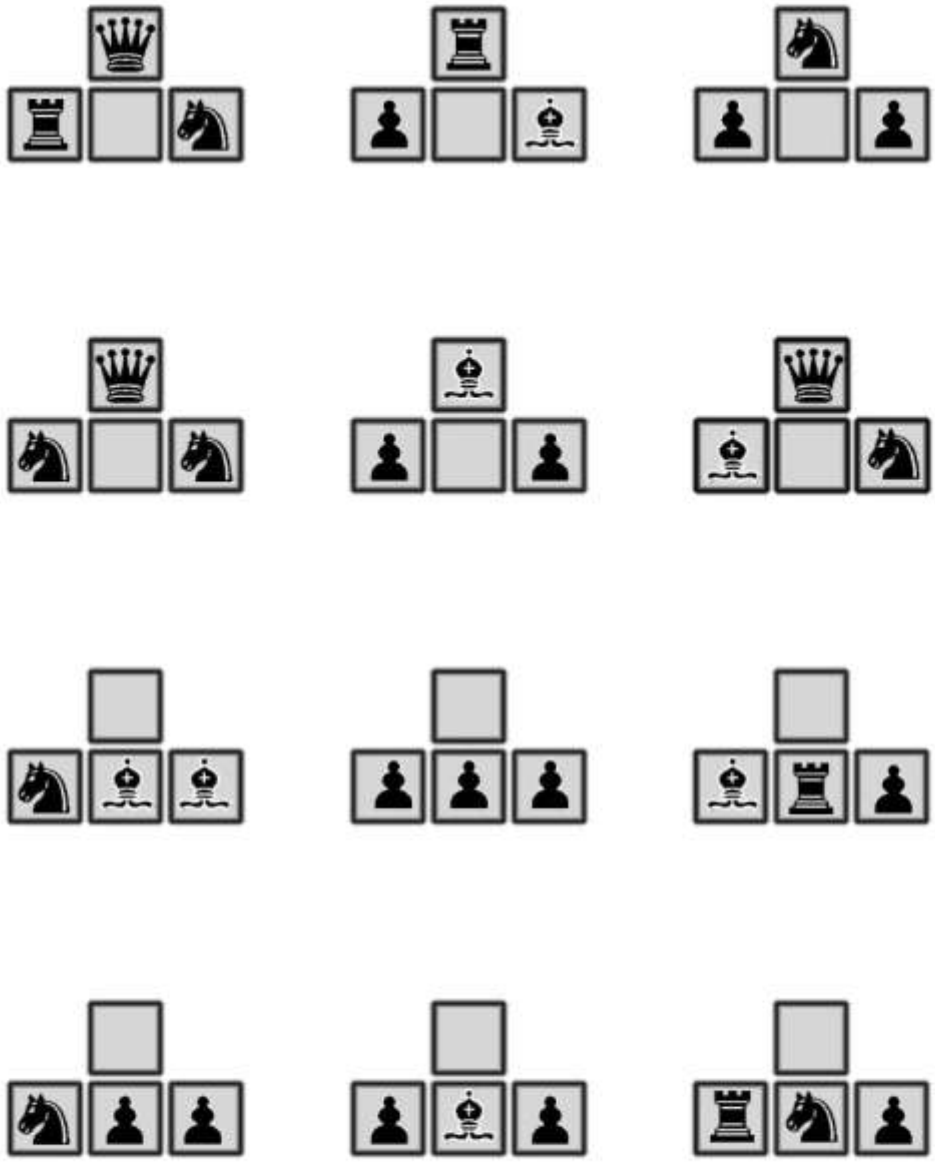
Difendere il pezzo attaccato con unica possibilità di spostarlo

10) Difesa con copertura (hai un pezzo che sta per essere catturato, con che mossa puoi difenderlo?)

In queste posizioni non è possibile salvare il pezzo spostandolo perchè verrebbe ugualmente catturato, ma si può difenderlo.

Lo scambio favorevole o sfavorevole si affronta con un semplice conteggio matematico.
Sappiamo che questo è un approccio puramente scolastico, quando andranno al circolo scacchistico impareranno che non è sempre così.
Per adesso questa è una certezza, che serve come punto di riferimento per permettere ai ragazzi di operare delle scelte, grazie alla presenza di punti fermi, stabiliti, senza i quali nessun processo di calcolo può essere iniziato.
Ricordiamoci che è tutto finalizzato allo scacco matto, e che vincere non è uguale a perdere...non è la stessa cosa.
La riflessione che dovete portare in classe è che la loro concentrazione deve essere utilizzata proprio al controllo del materiale, perché non si facciano mangiare i pezzi senza fare niente per evitare di andare in svantaggio. Dovranno fare esperienza del fatto che effettivamente un cambio tra pezzi di pari valore è paritario, se i pezzi sono di valore differente può essere vantaggioso o svantaggioso. Non devono evitare sempre il cambio dei pezzi e nemmeno cercare di evitare la cattura con la sola opzione "fuggire", che è sempre la più usata.
Prima di fornire loro il lavoro con i diagrammi, fate sempre degli esempi chiari alla murale perché possano capire cosa state dicendo e che cosa state cercando di ottenere da loro.

6) Scrivi il simbolo di un pezzo, nel quadratino vuoto, per equlibrare i valori tra le tre caselle sotto e la casella sopra.
D=9 - T=5 - A=3 - C=3 - P=1



7) Che pezzo indifeso puoi catturare?

1)

2)

3)

4)

5)

6)

7)

8)

9)

10)

11)

12)

8) Quale pezzo indifeso è meglio catturare?

1)

2)

3)

4)

5)

6)

7)

8)

9)

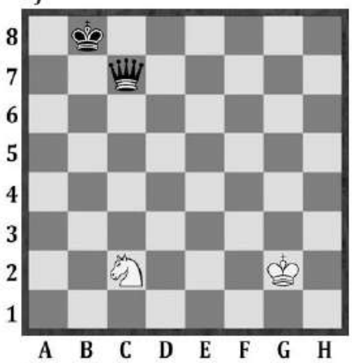
10)

11)

12)

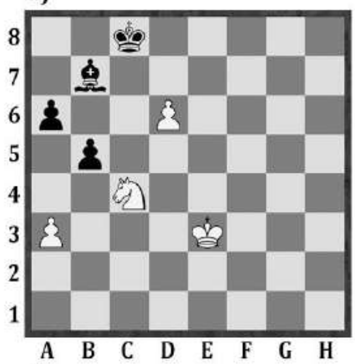
9) **Hai un pezzo che sta per essere catturato, dove lo sposti per non fartelo mangiare?**

1)



2)

3)



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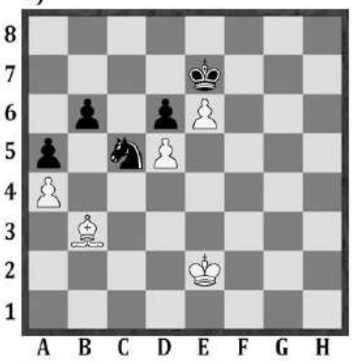

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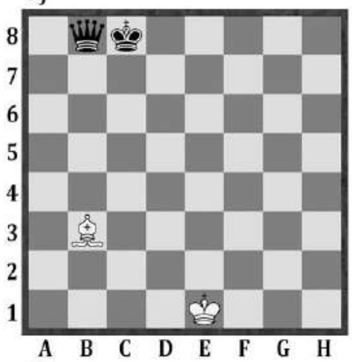

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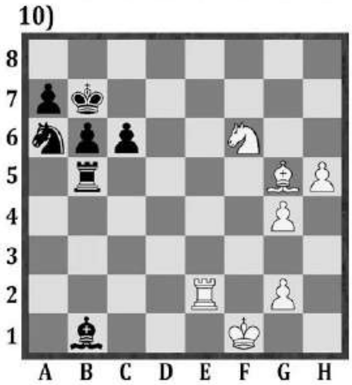

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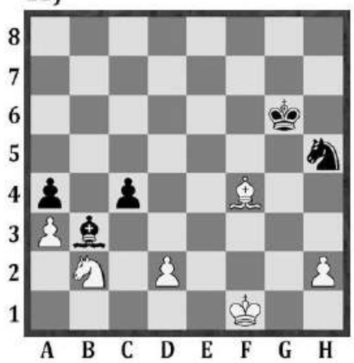
12)



10) Hai un pezzo che sta per essere catturato, con che mossa puoi difenderlo?

1)

2)

3)

4)

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11)

12)

11) Difesa mista (hai un pezzo che sta per essere catturato, lo sposti o lo difendi?) - diagrammi

Una scelta che va determinata in base alla posizione.

12) Cattura il pezzo che non è difeso (individua e cattura il pezzo indifeso) - diagrammi

In queste posizioni dovranno catturare, distinguendoli da quelli difesi, i pezzi indifesi.

13) Scambio favorevole (esegui la cattura che ti dà maggior vantaggio) - diagrammi

Il vantaggio maggiore si può ottenere solo grazie ad uno scambio, è un livello più alto rispetto al vantaggio dato da una cattura che non porta conseguenze. In alcune posizioni c'è anche un pezzo indifeso.

14) Cattura o scambio favorevole (che mossa ti fa guadagnare maggiore materiale?) - diagrammi

In questi diagrammi bisogna operare una scelta, osservando la posizione, che può essere di tipo 12 o 13.

15) Difesa e scambio (hai un pezzo minacciato) - diagrammi

Possono difendere in modi differenti. In alcuni casi potranno anche non fare una mossa di difesa.

16) La leggenda di Sissa e i grandi numeri

La leggenda di Sissa è un ottimo metodo per introdurre i numeri grandi oltre il miliardo e il concetto di elevamento a potenza. Partendo dalla storiella dei chicchi di grano chiesti in premio da Sissa per aver fatto apprezzare ad un sovrano annoiato le regole del gioco degli scacchi, è possibile dimostrare come la semplice operazione di raddoppio (moltiplicazione per 2), a partire da 1 chicco di grano posto sulla prima casa fino alla 64^, permetta di raddoppiare l'ordine di grandezza ad ogni traversa.
Per questa lezione è necessario che l'istruttore racconti la storiella di Sissa, facendo leva sulle sue doti di narratore, per poi passare alla seguente spiegazione di tipo matematico che può senz'altro essere interessante e utile per gli alunni. Per comodità consideriamo soltanto i numeri che si ottengono ad ogni fine traversa per facilitare la comprensione di quanto esposto.

Traversa	Numero finale	Ordine di grandezza		
1^	128	Centinaia	10^2	100
2^	32.768	Decine di	10×10^3	10.000
3^	8.388.608	migliaia	10^6	1.000.000
4^	2.147.483.648	Milioni	10^9	1.000.000.000
5^	549.755.813.888	Miliardi	$10^2 \times 10^9$	100.000.000.000
6^	140.737.488.355.328	Centinaia di	$10^2 \times 10^{12}$	100.000.000.000.000
7^	36.028.797.018.963.968	Miliardi	10×10^{15}	10.000.000.000.000.000
8^	9.223.372.036.854.775.808	Centinaia di Bilioni[1] Decine di Biliardi[2] Trilioni[3]	10^{18}	1.000.000.000.000.000.000

nota 1: Centinaia di Migliaia di Miliardi
nota 2: Decine di Milioni di Miliardi
nota 3: Miliardi di Miliardi

Sommando tutti i numeri a partire da 1 sulla prima casa e raddoppiando di volta in volta fino all'ultima casa della scacchiera, Il numero finale che si ottiene è: **18.446.744.073.709.551.615** = 264 - 1.
Il metodo è utile per introdurre la seguente scala per ordini di grandezze:

mille	1.000
milione	1.000 migliaia
miliardo	1.000 milioni
bilione	1.000 miliardi
biliardo	1.000 bilioni
trilione	1.000 biliardi
triliardo	1.000 trilioni
quadrilione	1.000 triliardi
quadriliardo	1.000 quadrilioni

Il numero totale ottenuto si legge:
18 Trilioni 446 Biliardi 744 Bilioni 73 Miliardi 709 Milioni 551 Mila 615.

11) Hai un pezzo che sta per essere catturato, lo sposti o lo difendi?

1)

2)

3)

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12)

12) Individua e cattura il pezzo indifeso.

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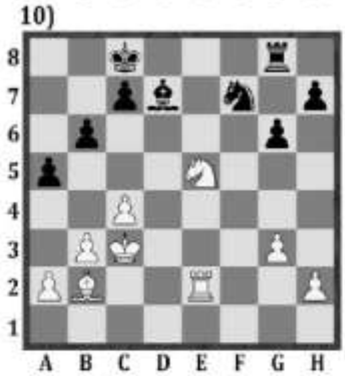

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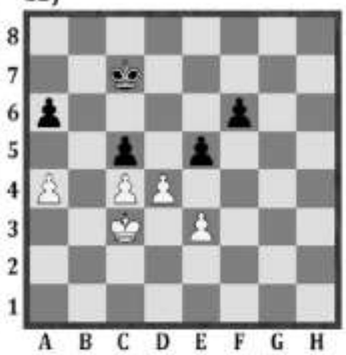

13) Esegui la cattura che ti dà maggior vantaggio.

1)

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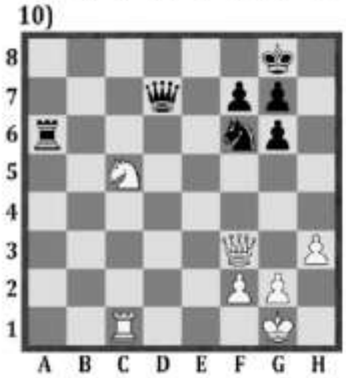

11)

12)

14) Che mossa ti fa guadagnare maggiore materiale?

1)

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11)

12)

15) Hai un pezzo minacciato

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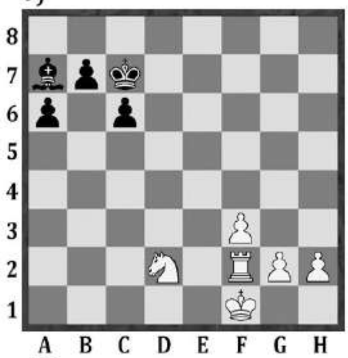

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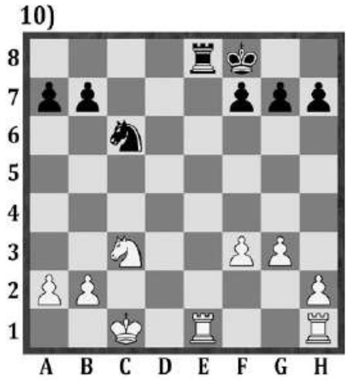

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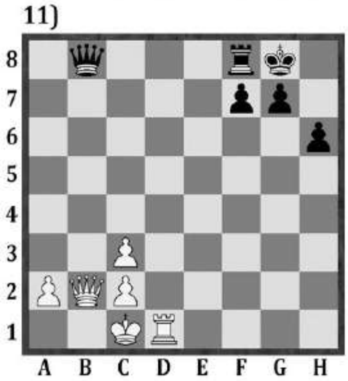


12)

Consigli in quattro mosse per un corso di Scacchi rivolto agli alunni della scuola primaria.
(di Carlo Alberto Cavazzoni)

"La prima mossa: appassionare"

E' difficile parlar facile del nobile gioco degli Scacchi , che per la sua struttura ed architettura è ricco di complessità matematiche e psicologiche di alto spessore.
Soprattutto se l'insegnamento è rivolto agli alunni delle scuole primarie, sono fondamentali la chiarezza e l'apertura ad una dimensione inusuale della didattica tradizionale .
Già il verbo insegnare non mi pare totalmente opportuno ed efficace: gli Scacchi sono una passione e le passioni non si possono insegnare ; al piu' si possono trasmettere.
Di conseguenza la prima mossa di un corso di Scacchi , a mio parere, deve avere per obiettivo il raggiungimento di un atmosfera favorevole alla germinazione della passione scacchistica .
Deve essere il simbolo di quello che verrà dopo.
E' importante coinvolgere i bambini nei nostri sogni. Essere avvincenti per suscitare un vivace interesse. Parlare con calore ed un linguaggio consono ai giovanissimi. Come? Con fiabe, leggende o filastrocche a soggetto scacchistico .
Anche con ricerca storica che comprenda soggetti con particolari del nostro gioco.
Utilizzare in abbondanza lo stupore , la gestualità e scegliere un abbigliamento ad hoc; perchè il linguaggio non verbale può rappresentare il 70 % della comunicazione .
Essere quindi degli istruttori-attori e persone che hanno conservato la curiosità e la magia dell'infanzia.
Trasmettere una passione è un'arte che va imparata perchè altrimenti il corso di Scacchi non solo rischia l'insuccesso ma può decretare l'allontanamento perenne dal gioco piu' affascinante che l'uomo abbia inventato.
Date il meglio nelle prime ore e ricordatevi cosa affermava Jean Jéareus "Non si insegna quello che si sa o si crede di sapere ; si insegna o si può insegnare quello che si è ".

"La seconda mossa: Istruire ed educare"

Da bravi educatori abbiamo il dovere prioritario di aiutare i nostri allievi a crescere in tutti i sensi, con lo scopo di farli diventare prima campioni nella vita ed eventualmente poi campioni negli Scacchi.
E' necessario in questa fase far comprendere i valori etici di cui gli Scacchi sono depositari
come :il valore della non violenza , della lealtà, della convivenza civile, del coraggio, della ragione, dell'amicizia e del sacrificio. Ricordare che il gioco dei Re è un linguaggio universale e che è l'unica guerra che unisce i popoli.
Poi suggerisco in collaborazione con le maestre di allargare l' educazione scacchistica favorendo momenti di interdisciplinarità, creando collegamenti con le materie curricolari come l'inglese, l'educazione musicale, la storia, la geografia, la matematica e l'educazione all'immagine. Come? Utilizzando un pò di fantasia !

Alcuni esercizi da esempio :

Italiano
1) Trovare cinque parole che facciano rima con :
pedone --
regina --
alfiere---
cavallo--
torre ---
re ---

2) Scrivere una poesia sugli Scacchi

Alla finestra

Un giorno mi svegliai
guardai alla finestra e cosa vidi?
Un 'immensa scacchiera bianca e nera
con gli eserciti di tutto il mondo
che insieme facevano un allegro girotondo
Quarta A

3) Scrivi un pensiero su questa domanda
"Cosa pensi del gioco degli Scacchi ?

Matematica

1) La Regina Margherita ha messo in fila tre pedoni numerandoli secondo un segreto aritmetico , riesci a trovare quale numero dovra' scrivere sul quarto pedone ?
10 14 18 ?

2) Nella classe 3B della scuola di Valle delle Rose ci sono 8 scacchiere e 8 scatole di pezzi.
I bambini giocando hanno perso alcuni pezzi e la situazione è la seguente :

Prima scatola mancano la regina bianca e due pedoni neri.
Seconda scatola completa
Terza scatola ; mancano il cavallo bianco e un pedone nero.
Quarta scatola :manca il re nero e l'alfiere bianco.
Quinta scatola completa
Sesta scatola: mancano tre pedoni neri.
Settima scatola manca la torre nera e l'alfiere bianco.
Ottava scatola completa.
Riesci a capire quanti ragazzi potranno giocare contemporaneamente con scacchiere complete il torneo della classe ?
Il Grande Maestro Gatto Matto ha tenuto una simultanea contro una comunità di topi ottenendo i seguenti risultati:
sette topi non hanno perso, 15 topi non hanno vinto, tre topi hanno pareggiato.
Quanti topi hanno vinto? Quanti topi hanno giocato in totale?

Educazione all'immagine

Esegui un disegno a soggetto scacchistico con il seguente tema :
"Tutti possono giocare a Scacchi ! "

Questi sono solo alcune idee e sarà sorprendente vedere cosa i bambini riescono a escogitare .

"Terza mossa ; l'elogio della sconfitta."

Spesso un ostacolo alla diffusione degli scacchi, in ambito scolastico è il timore, che hanno alcuni insegnanti, dell'agonismo scacchistico. Il confronto tra intelligenze potrebbe arrecare delusioni per gli sconfitti? Io non condivido questa paura, perchè viaggiando la vita i giovani incontreranno comunque un susseguirsi di vittorie e sconfitte su vari campi. Occorre non renderli fragili ed insegnar loro a perdere. Spiegargli bene che l'avversario non e' un nemico, che la sconfitta va accettata con il sorriso e che da essa si possano trarre degli insegnamenti che consentiranno di migliorarsi.

Di fronte ad una sconfitta il vinto dovrà analizzare gli errori commessi e rafforzare i propri punti deboli. In tal modo trasformerà la sconfitta in una vittoria.

"Quarta mossa: la vittoria piu' bella"

Se avrete fatto le prime tre mosse con precisione la vittoria verrà automaticamente.
I bambini non solo avranno appreso le regole, le tattiche e le strategie del gioco ma ne saranno anche innamorati. Gli Scacchi li accompagneranno per tutta la vita regalandogli momenti di autentica felicità e arricchendoli in saggezza ed entusiasmo.
Diventeranno anche dei vostri complici. Appassioneranno i propri genitori, i nonni, i fratelli e gli amici. Iniziando una positiva e divulgativa catena di Sant'Antonio.
Ma quello che piu' conta avrete insegnato a tanti giovani a dare scacco matto alle difficoltà di quel gioco ancora piu' complesso che si chiama La Vita !

Buon lavoro , in bocca al lupo!

Considerazioni metodologiche sulla didattica scacchistica nelle scuole.
(di Sebastiano Paulesu)

Più volte si è dibattuto su quale debba essere la metodologia da proporre nelle scuole, specie quelle primarie, per insegnare il gioco degli scacchi. Il progetto SAM è un'occasione utilissima per fornire agli Istruttori di scacchi qualche consiglio in proposito.
Il grande privilegio di poter proporre ai bambini un gioco ci fornisce subito un grande "appeal" nei loro confronti, e sarebbe davvero un peccato sprecarlo in poche ore proponendo loro delle noiose lezioni "frontali" poco coinvolgenti dal punto di visto motivazionale.
Per evitare questo è necessaria una comunicazione empatica, ed un rapporto Istruttore-Allievo pressoché paritario per poter instaurare un tipo di lezione interattiva (cioè con grande partecipazione attiva dei bambini) che crei l'atmosfera giusta sia per il divertimento che per l'apprendimento dei contenuti che grazie al gioco si possono veicolare.
Il primo passo – come giustamente notato da più parti – è la fondamentale accortezza di poter chiamare i bambini col loro nome e far sì che essi si sentano direttamente chiamati in causa ogni volta che abbiamo bisogno di catturare la loro attenzione. Infatti una cosa è dire "Ragazzi c'è troppa confusione, fate un po' di silenzio per cortesia..." Oppure :" Stefano e Mario, non chiacchierate tra voi, altrimenti perderete l'argomento della lezione e poi gli altri diventeranno più bravi di voi..."
Proprio questo vuol essere il mio contributo alla discussione attuale: mettere in pratica un gioco – da proporre ai bambini già alla prima lezione – per iniziare ad imparare i loro nomi a memoria, e consolidarli di lezione in lezione.

Personalmente, già da diversi anni, ho sperimentato un sistema semplice che consiste nel presentarmi e chiedere – in ordine di banco – al primo bambino di ripetere il mio nome più il suo; al secondo bambino chiedo di ripetere il mio nome, il nome del suo compagno più il suo; al terzo chiedo di ripetere da capo, aggiungendo al mio nome, il nome del 1° bambino, il nome del 2° bambino ed il suo nome... Così, ad ogni ripetizione inizia per l'istruttore una memorizzazione dei nomi dei bambini (immaginate di dover sentire dalle 15 alle 25 ripetizioni a seconda delle classi), che continuerà durante tutta la lezione, quando continueremo a chiamarli per nome chiedendo di rispondere a piccoli quesiti alla scacchiera murale ("Michele, cosa può catturare il pedone bianco, la Torre o l'Alfiere?").
Secondo la mia esperienza il coinvolgimento dei bambini grazie a questo piccolo accorgimento è davvero strabiliante: sia i bambini che gli insegnanti rimangono colpiti della confidenza che si instaura già dopo poche ore di lezione. A questo punto qualcuno si chiederà: questo può andare bene quando si ha a che fare con una sola classe, ma come può fare un istruttore che ha decine di classi a ricordare il nome di tutti i suoi allievi? Rimarrete allora stupiti quando vi dirò che io – pur non avendo una memoria prodigiosa – tengo a mente il nome di centinaia di bambini, perché utilizzando costantemente questo metodo dopo 3 o 4 lezioni non ho più bisogno di chiedere loro di ricordarmi i loro nomi.

Però le prime volte mi aiutavo con alcuni trucchi: tipo trascrivere al termine della lezione tutti i nomi dei bambini incontrati nell'ordine di banco (anche se spesso poi ce li cambiano...); oppure proporre negli ultimi 5 minuti dei giochi in cui devono dire parole che iniziano con l'iniziale del loro nome...
Questi giochi nel tempo sono entrati a far parte della mia personale metodologia, acquisendo una dignità di esercizio di "relax" dopo le partite giocate, ma originariamente mi servivano solo a memorizzare i nomi dei bambini. Così, ho imparato ben presto che c'è un tempo tecnico, di solito 15 o 20 minuti, in cui i bambini possono essere intrattenuti nella lezione teorica (prima che inizino esplicitamente a chiedere "Ma quando giochiamo?"): e allora io utilizzo il sistema della domanda a ciascuno (un giro in classi con 20 bambini oppure due giri con classi meno numerose) che mi consente di non sforare oltre la loro curva di attenzione.
A costo di sembrare noioso ribadisco che ogni domanda – utile per avere un feedback delle informazione proposte – deve essere sempre formulata chiamando il bambino intervistato con il proprio nome: "Piero, chi muove in diagonale l'Alfiere o la Torre?". Il vantaggio è che quando un bambino vuole parlare quando non è il suo turno è efficacissimo dirgli "Gabriele, ho chiesto a Barbara, tu ti chiami Barbara??" e questo, come sapranno gli istruttori è molto importante per tenere una buona disciplina della classe: a volte è sufficiente prospettare lo "spauracchio" che perderanno il loro turno di risposta!

Queste dinamiche sono molto importanti, perché in ogni classe ci sono sempre bambini che:

A) Intervengono con domande (Lo Sbandieratore e il Savio)
B) Ascoltano con attenzione (Il Pedante e il Curioso)
C) Si distraggono (Palinfrasca e il Filosofo)
D) Disturbano i compagni (Attaccabottone)
E) Fanno rumori molesti (L'Eclatante o Roboante)
F) Fanno battute spiritose (Il Giullare)
G) Pongono domande non pertinenti ((Il Petulante e Lo Sbandieratore))
H) Pongono domande pertinenti (Il Savio)
I) Aiutano gli altri compagni ad eseguire il lavoro (Il Samaritano)
J) Eseguono il lavoro senza guardare il lavoro degli altri (Il Solipsista e Lo Scacchista)
K) Spiegano ai compagni come deve essere svolto il lavoro (L'Erudito)
L) Si mostrano infastiditi se un compagno chiede loro spiegazioni (L'Intransigente)
M) Si mostrano interessati (Il Curioso e il Savio)
N) Si mostrano disinteressati (Cacciafarfalle)
O) Eseguono il compito assegnato (Il Vigile)
P) Non eseguono il compito assegnato (Il Disertore)
Q) Si organizzano autonomamente (Il Demiurgo)
R) Non si organizzano affatto (Il Fatalista)
S) Aspettano che gli vengano impartiti ordini precisi (Il Soldato)
T) Fanno proposte (L'Artista)
U) Eseguono passivamente (Il Cameriere)
V) Chiedono ripetutamente ulteriori delucidazioni (Il Professore)
W) Non chiedono delucidazioni nonostante sia evidente qualche difficoltà (Il Facilone)
X) Si isolano dal gruppo (L'Eremita)
Y) Non partecipano (L'Invisibile)
Z) Stanno nel gruppo a guardare gli altri (Il Passeggiatore)
AA) Disturbano il lavoro degli altri (Attila)
BB) Parlano tra loro di altri argomenti ma eseguono il lavoro assegnato (Multimedia)
CC) Esprimono la propria opinione (L'Opinionista)
DD) Esprimono valutazioni su se stessi a voce alta (Il Plateale)
EE) Esprimono giudizi negativi sul lavoro degli altri (Criticone)
FF) Esprimono giudizi positivi sul lavoro degli altri (L'Esteta o il Fan)
GG) Polemizzano (Tiraemolla)
HH) Lavorano isolandosi dal gruppo (L'Eremita)

Considerazioni finali

In questo protocollo si vuole solo dare una traccia fondamentale da rispettarsi nei tempi, nelle finalità e negli argomenti proposti.
Con questo non si vuole negare i punti di forza che ogni Istruttore capace ha saputo sviluppare con l'esperienza. Gli esempi necessari per presentare il lavoro con i diagrammi possono senz'altro essere molteplici, così come possono essere introdotte altre forme di interazione con gli allievi basate su giochi differenti, anche se qui non sono stati menzionate.
La principale forza di apprendimento che gli Istruttori possono far scaturire dai loro allievi è la MOTIVAZIONE, che può solo derivare dall' ENTUSIASMO che hanno saputo suscitare con la loro creatività.
Le prime 10 ore tendono velocemente verso la partita completa e corretta con tutti i pezzi, perché gli alunni possano quanto prima appassionarsi e giocare tra loro anche al di fuori delle ore del corso. La pratica spontanea è un valore aggiunto a quello dell'Istruttore, che può fornire motivazione ed esperienza ai ragazzi anche senza la sua presenza.
Questa è la risorsa principale che può avere un istruttore, deve essere ricercata e favorita in ogni modo per tutta la durata del corso.
E' quindi fortemente consigliato per tutti i ragazzi che parteciperanno alla ricerca il gioco libero durante l'intervallo, a casa, e se voglio anche al circolo.
Se le scuole non posseggono giochi completi imprestatene un paio alle classi dove operate, perché con questo potreste stimolare la nascita di una dinamica di gruppo che, come ha testimoniato la ricerca "Gli scacchi: un gioco per crescere", è assolutamente necessaria per esprimere il vero potenziale del gioco degli scacchi in ambito scolastico.

Il protocollo è stato scritto da:

Alessandro Dominici
Alexander Wild
Giuliano D'Eredità
Marcello Perrone

Appendix 6
SAM Pre test

Ti chiediamo qualche notizia su di te:

Tu sei:

- ○ Maschio
- ○ Femmina

All'inizio della terza sapevi già giocare a scacchi?

- ○ Si, bene
- ○ Si, ma non molto bene
- ○ No

Chi ti ha insegnato?

- ○ mio padre
- ○ mia madre
- ○ mio fratello/mia sorella
- ○ altri parenti
- ○ un mio amico/una mia amica

Come ti chiami di nome?

Per favore scrivi la lettera con cui inizia il tuo cognome

In che mese dell'anno sei nato?

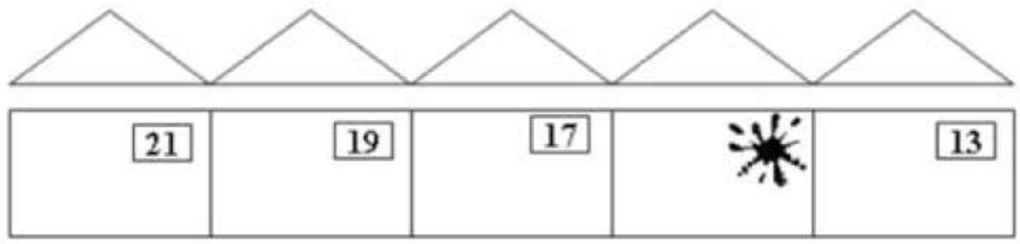
- ○ Gennaio
- ○ Febbraio
- ○ Marzo
- ○ Aprile
- ○ Maggio
- ○ Giugno
- ○ Luglio
- ○ Agosto
- ○ Settembre
- ○ Ottobre
- ○ Novembre
- ○ Dicembre

Quanti fratelli/sorelle hai?

- ○ Nessuno
- ○ Uno
- ○ Due
- ○ Tre
- ○ Quattro
- ○ Cinque o più

Troverai nelle prossime pagine 22 domande di matematica.

Una sola risposta è quella giusta.
Fai una X sulla risposta che ritieni giusta.
Se vuoi correggere una risposta già data,
scrivi NO sulla crocetta che hai già messo
e metti un’altra crocetta sulla nuova risposta scelta.



1. Anna abita nella casetta con il numero coperto dalla macchia.

Scopri qual è

- 14
- 15
- 18

2. Elena ha raccolto 18 conchiglie al mattino e 13 il pomeriggio. Quante conchiglie ha ora Elena?

- 33
- 31
- 38

Osserva la seguente tabella:

	A	B	C
1			
2			
3			

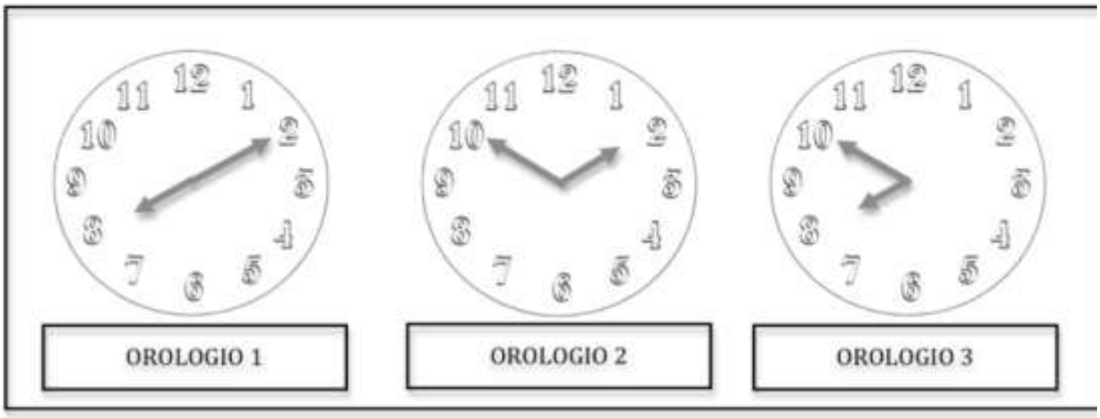
3. Il sole si trova nella casella:

- ◯ A 1
- ◯ B 3
- ◯ C 1

4. Il maestro ha 3 scatole da 8 matite ciascuna e regala 1 matita ad ognuno dei suoi 22 alunni.

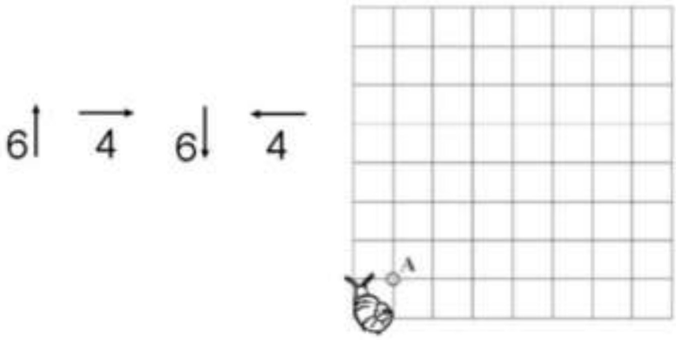
Al maestro:

- ◯ non restano matite
- ◯ resta una matita
- ◯ restano due matite



5. Quale orologio indica le 8 e 10?

- Orologio 1
- Orologio 2
- Orologio 3



6. Immagina che la chiocciola parta dal punto A e segua il percorso indicato dai numeri e dalle frecce, quindi si sposti in su di sei caselle, poi vada a destra di quattro, poi scenda di sei e torni al punto di partenza.

Quale forma ha il percorso della chiocciola?

- Triangolo
- Rettangolo
- Quadrato

7. Quale delle seguenti scritture corrisponde a "cento è maggiore di cinquanta"?

- ○ 100 < 50
- ○ 100 = 50
- ○ 100 > 50

8. Sandro ha 28 euro. Il suo amico Stefano ne ha la metà. Quanti euro ha Stefano?

- ○ 14
- ○ 18
- ○ 24

9. Al luna park vince chi colpisce la mano destra di un pagliaccio.
Anna prende la mira e lancia la pallina nera quattro volte.
Indica per ogni figura se Anna ha colpito la mano destra del pagliaccio

a

- ○ SI
- ○ NO

c

- ○ SI
- ○ NO

b

- ○ SI
- ○ NO

d

- ○ SI
- ○ NO

Nella mensa della scuola la cuoca registra
su un cartellone le mele mangiate in tre giorni.

Osserva il cartellone:

10. Quante sono le mele mangiate nei tre giorni?

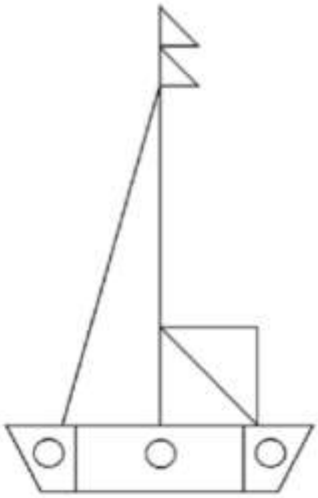
- ○ 9
- ○ 30
- ○ 90

11. Osserva il riquadro

17 + 46 = 60 + 3

Perchè quello che è scritto nel riquadro è corretto?

- ○ Perchè ci sono due numeri a destra e due a sinistra del segno di uguale
- ○ Perchè il risultato della prima addizione è uguale al risultato della seconda addizione
- ○ Perchè 60 è il risultato di 17 + 46



12. Osserva la barchetta

In tutto i triangoli sono:

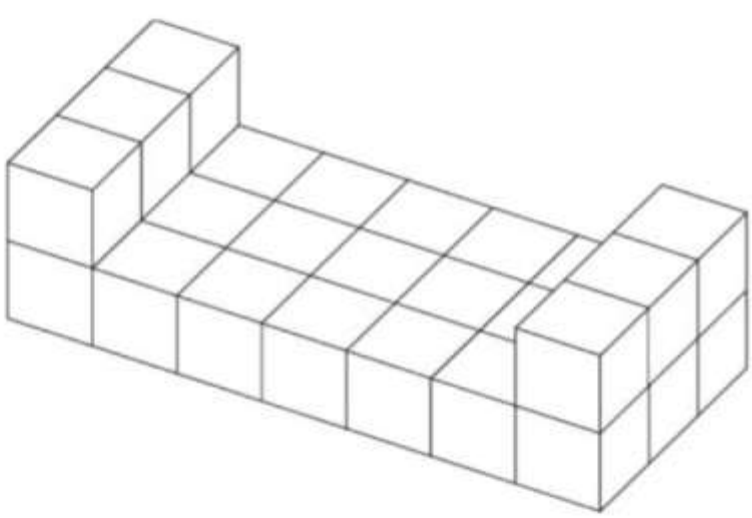
- 3
- 5
- 7

13. La maestra ha preparato un cartellone e ha chiesto a ogni bambino della sua classe di scrivere il proprio nome al posto giusto. Alcuni hanno scritto il loro nome e il cartellone si presenta così:

	Hanno gli occhiali	Non hanno gli occhiali
Maschi	Luca	Carlo
Femmine	Silvia, Jasmine	Teresa

Paola non ha gli occhiali, dove scriverà il proprio nome?

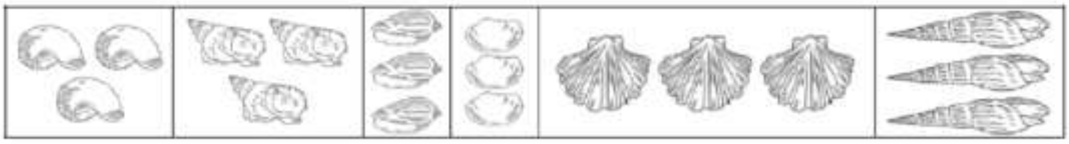
- Dove l'ha scritto Teresa
- Dove l'ha scritto Carlo
- Dove l'ha scritto Silvia



14. Di quanti cubetti è fatta questa costruzione?

- ◯ 23
- ◯ 25
- ◯ 27

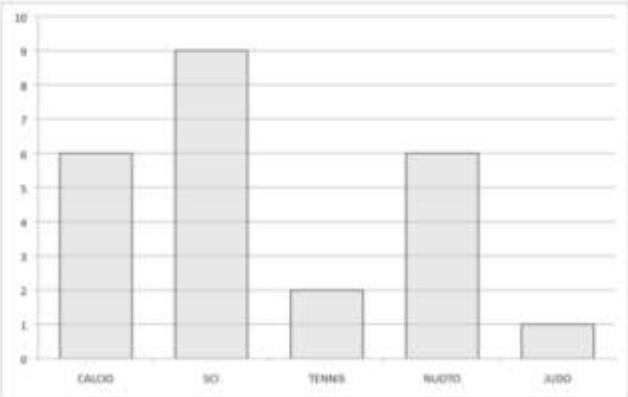
15. Marco ha sistemato le sue conchiglie in una scatola con 6 scomparti.
In ogni scomparto ha messo 3 conchiglie.



Quale dei seguenti calcoli per trovare il numero delle conchiglie è <u>sbagliato</u>?

- ◯ 6 + 3
- ◯ 6 x 3
- ◯ 3+3+3+3+3+3

16. Nella classe di Michele la maestra ha chiesto qual è lo sport preferito; ogni alunno ha scelto un solo spor
Alla fine hanno costruito questo grafico:



Che cosa rappresenta ogni rettangolino come questo --------> 

- La scelta di un bambino
- Un gruppo di bambini
- Uno sport

17. Andrea, Marco e Luca giocano a figurine. Luca ha 13 figurine più di Andrea. Marco ha 12 figurine più di Luca.

Chi ha più figurine?

- Andrea
- Marco
- Luca

18. **Mara raccoglie 24 fiori e ne regala 4 a ognuna delle sue cugine.**
Quante sono le cugine di Mara, sapendo che non le rimane nessun fiore?

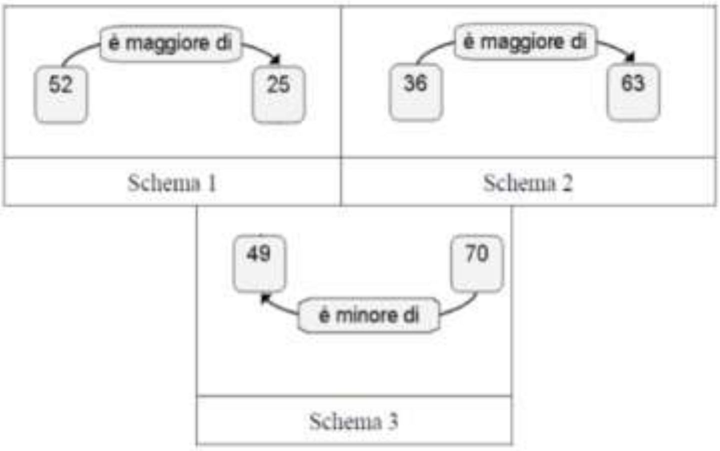
- ◯ 4
- ◯ 6
- ◯ 8

19. **Osserva la seguente operazione:**

23 - 11 = 12

Qual è il problema che si può risolvere con questa operazione?

- ◯ Paolo possiede 23 macchinine rosse e 11 macchinine blu. Quante macchinine rosse ci sono in più di quelle blu?
- ◯ Paolo possiede 23 macchinine rosse e 11 macchinine blu. Quante sono in tutto le macchinine?
- ◯ Paolo possiede 23 macchinine rosse e 11 macchinine blu. Quante sono le macchinine blu?



20. **In quale dei seguenti schemi la relazione indicata dalle frecce è corretta?**

- ◯ Schema 1
- ◯ Schema 2
- ◯ Schema 3

21. Quale numero corrisponde a "2 unità e 1 centinaio"?

- 100
- 102
- 200

22a. Quale tra i seguenti oggetti è il più leggero?

- Libro
- Foglio da disegno
- Quaderno

22b. Quale tra i seguenti oggetti è il più lungo?

- Bastone
- Stuzzicadenti
- Matita

22c. Quale tra i seguenti recipienti può contenere più acqua?

- Cucchiaio
- Bicchiere
- Bottiglia

Appendix 7
SAM Post test

Ti chiediamo qualche notizia su di te:

Come ti chiami di nome?

Per favore scrivi le prime due lettere con cui inizia il tuo cognome

Tu sei:

- Maschio
- Femmina

Quanto ti piace la matematica?

- Molto
- Abbastanza
- Poco
- Per niente

A scuola sei bravo in matematica?

- Sì, molto
- Sì, abbastanza
- No, poco
- No, per niente

Tu sai giocare a scacchi?

- Si, bene
- Si, ma non molto bene
- No

In che mese dell'anno sei nato?

- Gennaio
- Febbraio
- Marzo
- Aprile
- Maggio
- Giugno
- Luglio
- Agosto
- Settembre
- Ottobre
- Novembre
- Dicembre

Se hai risposto che sai giocare a scacchi, bene o non molto bene, chi ti ha insegnato?

- Mio padre
- Mia madre
- Mio fratello/sorella
- Altri parenti
- Un mio amico/amica
- Il maestro/La maestra di scacchi a scuola

In che anno sei nato/a?

- 2000 (o prima)
- 2001
- 2002
- 2003
- 2004 (o dopo)

Tua madre è nata in Italia?

- Si
- No
- Non so

In casa hai una cameretta tutta tua, che non dividi con fratelli o sorelle?

- Si
- No

In casa tua c'è almeno un computer?

- Si
- No

Tuo padre è nato in Italia?

- Si
- No
- Non so

Tu sei nato in Italia?

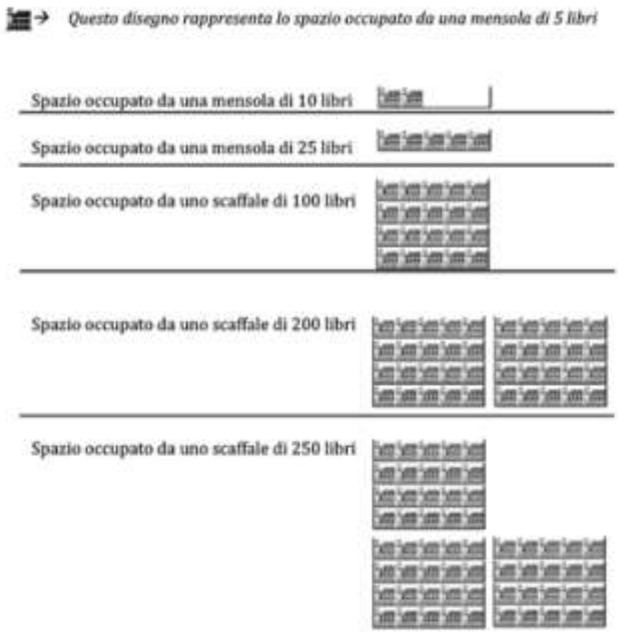
- Si
- No
- Non so

Se hai risposto che non sei nato/a in Italia, quanti anni avevi quando sei venuto/a in Italia?

- Più di 6 anni
- Tra i 4 e i 6 anni
- Meno di 4 anni
- Non riesco a ricordarmelo

Quanti libri ci sono all'incirca a casa tua (esclusi i libri di scuola)?

- Nessuno o pochissimi (0-10 libri)
- Abbastanza da riempire una mensola (11-25 libri)
- Abbastanza da riempire uno scaffale (26-100 libri)
- Abbastanza da riempire due scaffali (101-200 libri)
- Abbastanza da riempire tre o più scaffali (più di 200 libri)



Fai la massima attenzione a queste istruzioni.

Troverai nelle prossime pagine 21 domande di matematica.
Ogni domanda ha quattro possibili risposte, ma una sola è quella giusta.
Per rispondere metti una crocetta nel cerchietto a sinistra della risposta che ritieni giusta, come nell'esempio seguente.

ESEMPIO 1

1. Quanti giorni ci sono in una settimana?

☒ Sette

○ Sei

○ Cinque

○ Quattro

È stata messa una crocetta nel cerchietto corrispondente al sette perché in una settimana ci sono sette giorni.
Se non sei sicura/o di una risposta, segna la risposta che ti sembra giusta e continua con la domanda successiva.
Se ti accorgi di aver sbagliato, puoi correggere scrivendo NO accanto alla risposta sbagliata e mettendo una crocetta nel cerchietto della risposta che ritieni giusta, come nell'esempio seguente.

ESEMPIO 2

2. Quanti minuti ci sono in 1 ora?

NO ☒ 30

○ 50

☒ 60

○ 100

In questo esempio la prima crocetta su 30 (sbagliata) è stata corretta con la crocetta su 60 (che è quella giusta).

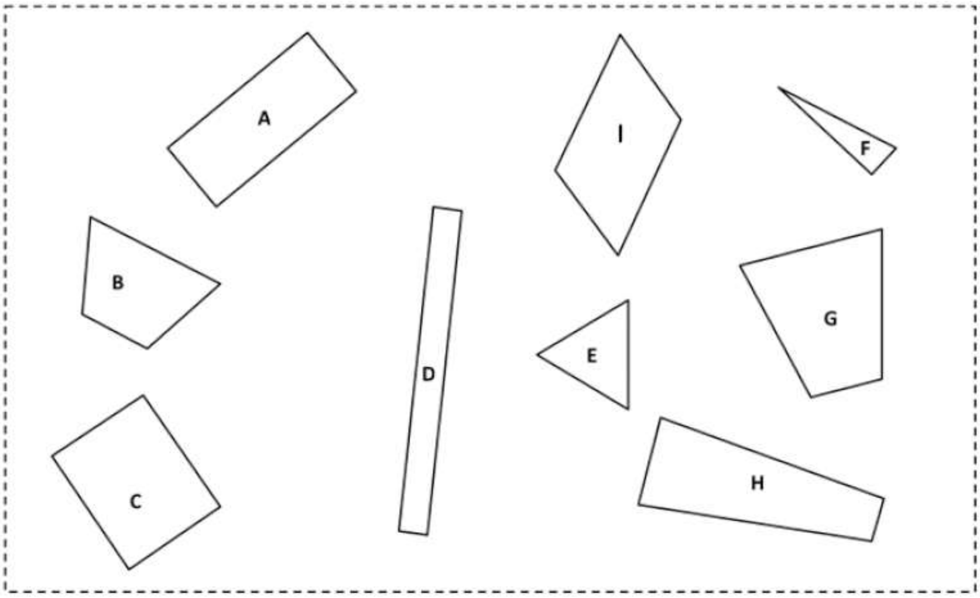
Per rispondere non puoi usare la calcolatrice. Deve comunque essere chiaro qual è la risposta che intendi dare. Non scrivere con la matita, usa soltanto una penna nera o blu.
Puoi usare le pagine bianche del fascicolo, un quaderno o gli spazi bianchi accanto alle domande per fare calcoli e/o disegni.
Hai a disposizione 1 ora per rispondere alle domande.
L'insegnante ti dirà quando cominciare a lavorare.
Quando l'insegnante ti comunicherà che il tempo è finito, posa la penna e chiudi il fascicolo.
Se finisci prima, puoi controllare le risposte che hai dato oppure puoi chiudere il fascicolo e consegnarlo all'istruttore di scacchi.

Non iniziare a lavorare finché l'insegnante non te lo dirà.

1. Quale delle seguenti scritture corrisponde a "centoventi è minore di centosessanta"?

- ◯ 120 = 160
- ◯ 120 > 160
- ◯ 120 < 160
- ◯ 120 <> 160

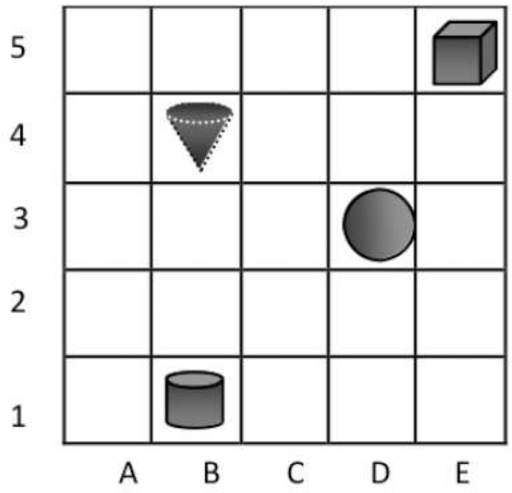
2. Osserva con attenzione le seguenti figure geometriche inserite nel riquadro:



Scrivi nelllo spazio qui sotto, una dopo l'altra, le lettere di tutti i rettangoli che hai trovato:

3. Carlo pensa un numero, ci aggiunge 79 e trova 102. Quale numero ha pensato?

- ◯ 23
- ◯ 181
- ◯ 33
- ◯ Non si può trovare il numero che ha pensato



4. In quale casella si trova il cono?

- ◯ B 1
- ◯ B 4
- ◯ E 4
- ◯ D 3

5. Nella mensa della scuola ci sono 15 tavoli da 6 posti ciascuno.
***Oggi* in mensa sono andati 85 alunni.**

***Oggi* in mensa:**

- ◯ non restano posti vuoti
- ◯ resta un posto vuoto
- ◯ restano cinque posti vuoti
- ◯ restano nove posti vuoti

Orologio 1 – Orario di inizio	Orologio 2 – Orario di fine
	

6. I due orologi indicano l'ora in cui Marco ha iniziato un gioco al computer (orologio 1) e l'ora in cui ha finito (orologio 2).

   Quanto tempo ha giocato Marco?

- 20 minuti
- 80 minuti
- 25 minuti
- 5 minuti

7. La chiocciola parte dal punto A e segue il percorso indicato dai numeri e dalle frecce: si sposta in su di 3 caselle, poi va a destra di 5, poi scende di 3 e poi torna al punto di partenza.

3↑ 5→ 3↓ 5←

Quale forma ha il percorso della chiocciola?

- Triangolo
- Quadrato
- Rettangolo
- Esagono

8. La mamma di Maria ha 56 euro nel portafogli. La mamma della sua amica Gianna ne ha la metà.
Quanti soldi ha nel portafogli la mamma di Gianna?

- 14 euro
- 28 euro
- 84 euro
- 102 euro

9. Ci sono persone che scrivono con la destra e ci sono persone che scrivono con la sinistra.
Questi ultimi si chiamano "mancini".
Guarda le figure seguenti e metti la crocetta su SI se i personaggisono mancini oppure NO, se non lo sono.

- SI
- NO

- SI
- NO

- SI
- NO

- SI
- NO

10. Nella mensa della scuola, le maestre registrano su un tabellone quante bottiglie di acqua vengono consumate ogni giorno.
Le maestre indicano con il simbolo della goccia il consumo di 10 bottiglie di acqua.
Qui sotto vedi la registrazione dei primi tre giorni della settimana:

🌢=10 bottiglie consumate

Giorno	Bottiglie consumate
Lunedì	🌢🌢🌢
Martedì	🌢🌢🌢🌢
Mercoledì	🌢

Quante bottiglie di acqua sono state consumate nei tre giorni?

- 4
- 8
- 30
- 80

11. *Osserva il riquadro*

$$\boxed{6 \times 12 = 24 \times 3}$$

Perchè quello che è scritto nel riquadro è corretto?

- Perché ci sono due numeri a destra e due a sinistra del segno di uguale
- Perché 24 è il risultato di 6 x 12
- Perché il risultato della prima moltiplicazione è uguale al risultato della seconda moltiplicazione
- Perché 12 è il risultato di 3 x 6

12. La maestra ha preparato un cartellone e ha chiesto a ogni bambino della sua classe di scrivere il proprio nome al posto giusto.
Alcuni alunni hanno scritto il loro nome e il cartellone si presenta così:

	Gioca a basket	**Non gioca a basket**
Maschi	Sandro	Marcello, Nicola
Femmine	Lucia, Francesca, Chiara	Debora

Giorgio non gioca a basket. Dove scriverà il proprio nome?

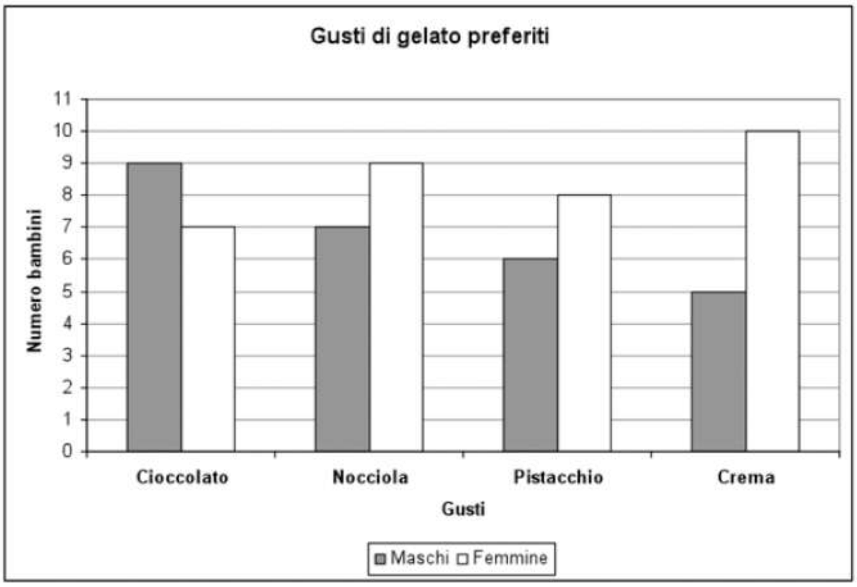
- ○ Dove l'ha scritto Marcello
- ○ Dove l'ha scritto Sandro
- ○ Dove l'ha scritto Francesca
- ○ Dove l'ha scritto Debora

13. Gianni ha sistemato i suoi giochi in 5 scatole. In ogni scatola ha messo 8 giochi.

Quale dei seguenti calcoli per trovare il numero totale dei giochi è sbagliato?

- ○ 5+5+5+5+5+5+5+5
- ○ 8+8+8+8+8
- ○ 5 x 8
- ○ 5 + 8

14. Un gruppo di bambini è stato intervistato all'uscita della scuola, chiedendo di dire il gusto di gelato preferito.
Il grafico seguente mostra i risultati delle interviste suddivisi per femmine (rappresentate dalle colonne bianche) e maschi (rappresentati dalle colonne grigie).



Quante sono le femmine che preferiscono il gusto nocciola?

- 7
- 8
- 9
- 10

Quanti sono in tutto i bambini (maschi e femmine) che preferiscono il gusto crema?

- 5
- 10
- 15
- 61

Quanti sono in totale i bambini (maschi e femmine) intervistati?

- 27
- 34
- 36
- 61

15. Elena, Martina, Alessandro e Gianni giocano con le figurine.
Martina ha 17 figurine più di Alessandro e di Elena. Gianni ha 15 figurine più di Martina.
Chi ha più figurine?

- ○ Martina
- ○ Alessandro
- ○ Gianni
- ○ Elena

16. Gli alunni di una scuola sono 96. Per giocare a pallavolo si devono formare squadre di 6 giocatori ciascuna.
Quale operazione devi fare per sapere quante sono le squadre di pallavolo che si possono formare in quella scuola?

- ○ 96 x 6
- ○ 96 : 6
- ○ 96 - 6
- ○ 96 + 6

17. Osserva la seguente operazione:

270 - 130 = 140

Qual è il problema che si può risolvere con questa operazione?

○ Gianni possiede 270 figurine blu e 130 figurine rosse.
Quante sono in tutto le figurine?

○ Gianni possiede 270 figurine blu e 130 figurine rosse.
Quante sono in più le figurine blu rispetto alle figurine rosse?

○ Stefania, per andare al mare, deve percorrere 140 kilometri.Luca deve percorrerne 130.
Quanti kilometri in meno deve percorrere Luca?

○ Stefania, per andare al mare, deve percorrere 140 kilometri.Luca deve percorrerne 130.
Quanti kilometri più di Luca deve percorrere Stefania?

18. Quale numero corrisponde a "6 unità e 3 centinaia"?

- 603
- 306
- 63
- 36

19. Quale tra i seguenti oggetti può pesare circa 50 grammi?

- Un vocabolario di italiano
- Un foglio da disegno
- Una poltrona
- Una merendina

Quale tra i seguenti oggetti può essere alto circa 2 metri?

- Una sedia
- Un armadio
- Una bicicletta
- Un palazzo di tre piani

20. Sulla porta di una libreria è esposto il seguente cartello.

LIBRERIA
ORARIO DI APERTURA:
Dalle 9.00 alle 13.00
Dalle 15.30 alle 19.30
APERTO TUTTI I GIORNI ESCLUSO
IL LUNEDÌ MATTINA
E LA DOMENICA

Per quante ore la libreria è aperta il Venerdì? (scrivi il numero delle ore nella casella)

Per quante ore complessive la libreria è aperta in una settimana?
(scrivi il numero delle ore nella casella)

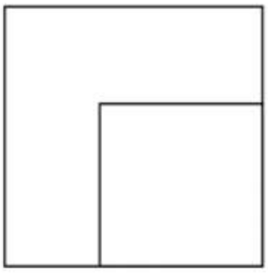


Figura A

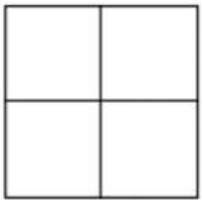


Figura B

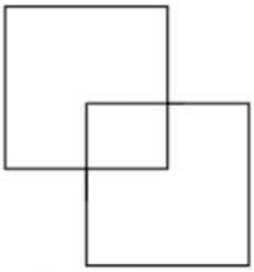


Figura C

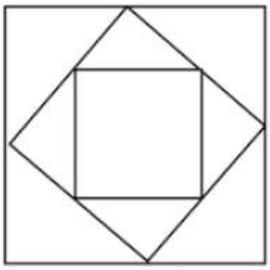


Figura D

21. In quale tra le figure vedi 5 quadrati? Indicala con la lettera.

- A
- B
- C
- D

Bravo! Hai finito!
Dillo al maestro di scacchi che è lì con te.

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