Dear Author,

Below are the queries associated with your article; please answer all of these queries before sending the proof back to AIP. Author please indicate the correct color processing option from the list below:

1. Author, please confirm Figure number(s) that should appear as color in print. Please know that any associated mandatory fees will apply for figures printed in color.
2. Author, please confirm Figure number(s) that should appear as color online only, there will be no fees applied.
3. Author, your paper currently does not include any color figures for online or print. If color is needed please indicate which figures it should be applied to and whether it is color in print or online.

<table>
<thead>
<tr>
<th>Location in article</th>
<th>Query / Remark: click on the Q link to navigate to the appropriate spot in the proof. There, insert your comments as a PDF annotation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AQ1</td>
<td>Please check electronic mail address as it differs from that provided on the cover sheet.</td>
</tr>
<tr>
<td>AQ2</td>
<td>For the citations in the abstract, if there are fewer than three authors in a reference, please list the names of all authors in the abstract.</td>
</tr>
<tr>
<td>AQ3</td>
<td>Please re-word the last sentence of the abstract, if possible.</td>
</tr>
<tr>
<td>AQ4</td>
<td>References 7, 8, and 10 were not cited in the original paper, but were added here. Please check their insertion here.</td>
</tr>
<tr>
<td>AQ5</td>
<td>In the Reference List, please supply the names of all authors in each reference where the term “et al.” is used. Use of et al. is not permitted.</td>
</tr>
</tbody>
</table>

Thank you for your assistance.
Magnetostochastic resonance under colored noise condition

Marco Trapanese
Dipartimento di Ingegneria Elettrica, Elettronica e delle Telecomunicazioni, Palermo University, Palermo, 1-I90128 Palermo, Italy

(Presented 1 November 2011; received 23 September 2011; accepted 21 December 2011; published online xx xx xxxx)

Stochastic resonance (SR) is an amplification of the system output in correspondence of well-defined finite values of the noise strength that is injected into the system [Gammatoni et al., Rev. Mod. Phys. 70, 223 (1998), Grigorenko et al., IEEE Trans. Magn. 31, 2491 (1995), Mantegna et al., J. Appl. Phys. 97, 10E519 (2005)]. In order to clarify the influence of a colored noise, in this paper magnetostochastic resonance (MSR) in magnetic systems described by the dynamic Preisach model is numerically investigated in the presence of colored noise. It is shown that noise spectrum affects MSR, white noise, 1/f and 1/f² noise induce in magnetic systems described by the dynamic Preisach model MSR, the maximum level of signal-to-noise ratio (SNR) obtained by using white noise but 1/f noise presented a range where SNR value is higher than the case of white noise; maximum signal amplification is obtained for white noise. © 2012 American Institute of Physics. [doi:10.1063/1.3680083]

INTRODUCTION

Stochastic resonance (SR) is a well-known phenomenon characterized by an amplification of the system response for certain finite values of the noise strength injected into the system. In particular, the signal-to-noise ratio (SNR) and the signal amplification show nonmonotonic behaviors with a maximum as a function of the noise intensity. SR has been experimentally observed in many physical systems and also in magnetic systems. Some theoretical approaches have been developed to describe SR (for a theory of SR in magnetic systems see Ref. 6 and for a review see Ref. 3) for bistable systems, but no theoretical approach has been developed to describe SR in systems that present a magnetic-like hysteretic behavior. In all the above-recalled approaches, MSR has been numerically investigated in the presence of colored noise. In DPM, the magnetization M(t) at the generic time t is given by

\[ M(t) = M_s \int_0^\infty d\mu \int_{-\infty}^\infty p(h_c, h_a) \cdot \varphi(h_c, h_a, t) dh_a, \]

where \( M_s \) is the saturation magnetization, \( p(h_c, h_a) \) is the Preisach model density function, and \( \varphi(h_c, h_a, t) \) describes the state of each elementary Preisach model loop at the time t. \( \varphi(h_c, h_a, t) \) varies according to

\[ \frac{\partial \varphi(h_c, h_a, t)}{\partial t} = \begin{cases} k[H(t) - (h_a + h_c)] & \text{if } H(t)(h_a + h_c) \\ k[H(t) - (h_a - h_c)] & \text{if } H(t)(h_a - h_c) \end{cases}, \]

where k is an unknown parameter. The dynamic model becomes equivalent to CPM if the parameter k becomes infinite, because, in this case, the function \( \varphi(h_c, h_a, t) \) can assume only the values -1 and +1. The parameter k quantifies the finite rate of the switching of the hysterons of DPM.

THE NUMERICAL APPROACH

In this paper, the external magnetic field \( (h_{\text{ext}}) \) applied to a magnetic material has two components, one small sinusoidal component added to a colored noise component.
FIG. 1. (Color online) FFT of the output of the system. Diamonds represent white noise, squares 1/f noise, and triangles 1/f² noise.

FIG. 2. (Color online) SNR vs noise intensity. Diamonds represent white noise, squares 1/f noise, and triangles 1/f² noise. The maximum SNR is reached in the case of white noise but 1/f noise presents a broader maximum, and an area where SNR values are higher than white noise case.

FIG. 3. (Color online) Signal amplification vs noise intensity. Diamonds represent white noise, squares 1/f noise, and triangles 1/f² noise. The maximum signal amplification is reached in the case of white noise.
\[ h_{\text{ext}} = H_s \sin t + D(t), \]  

(3)

where \( t \) is the time and \( D \) is the colored noise. \( D \) was generated by a suitable generator in which the type of noise and its root mean square was controllable. The frequency of the sinusoidal component was kept constant at the value of 1 in all the numerical simulations here presented and the dynamic features of the system were changed by letting \( k \) vary in the DPM and the correlation time in the noise generator. The value of \( h_{\text{ext}} \) was computed at several time steps. As a result, the time evolution of the magnetization of the system could be computed by inserting Eq. (3) in DPM [Eq. (1) and (2)]. A Lorentzian

Preisach distribution function was used in Eq. (1). Its expression is given in Ref. 5. The two parameters \( \sigma_c \) and \( H_0 \), which define the Lorentzian, were set equal to 0.1 and 1 respectively. This distribution generates a major loop of the static hysteresis that has a coercive field equal to 1 (see Ref. 5).

The magnetization was computed by discretizing the integral in Eq. (1) on a suitable grid. The grid on the Preisach plane is rectangular with \( 0 \leq H_r \leq 4 \) and \(-3 \leq H_u \leq 3 \) and it is made by at maximum 1000 \( \times \) 1000 points and the set of differential equations in Eq. (2) were solved by standard numerical techniques.

To compute the SNR and the power amplification, the fast Fourier transforms (FFT) of the magnetization were computed and the value of the component of the FFT for the frequency of the signal was used.

The SNR was calculated as

\[ \text{SNR} = 10 \log_{10} \left( \frac{P_1}{N_1(1)} \right) \]  

(4)

and the power amplification as

\[ \eta = 2 \left( \frac{M_1}{M_t} \right)^2. \]  

(5)

where \( P_1 \) is the output signal power level obtained from the FFT of the resulting magnetization at the frequency of the sinusoidal component, \( N_1 \) is the noise level obtained from the same FFT at the frequency of the sinusoidal component, \( M_1 \) is the component of the FFT at the frequency of the sinusoidal component, and \( M_t \) is the amplitude of the magnetization obtained with no noise pumped in the system.

The SNR, the power amplification and the behavior of the magnetization for several \( H_r \) and \( D \) and in correspondence of white, \( 1/f \) and \( 1/f^2 \) as a function of the parameter \( k \) have been computed.

In Fig. 1 the FFT of the time varying magnetization for an amplitude of \( H_s = 0.5 \) in the case of presence of noise with a value of \( H_{\text{rms}} = 0.8 \) and for \( k = 1000 \) is shown for the three types of noise. The amplitude of the harmonic of the FFT of the time varying magnetization at the frequency of the applied signal for a signal amplitude of \( H_s = 0.5 \) in the case of absence of noise is much smaller (1/1000)\(^5\) than the amplitude of the same harmonic when an external noise is applied. That means that the addiction of noise amplifies the harmonic value at the frequency of the signal. This, together with the nonmonotonic behavior of both SNR and \( \eta \), is the fingerprint of SR. Figure 1 shows the amplification of the harmonic value at the frequency of the signal and how white noise guarantees the maximum signal amplification; \( 1/f \) noise amplification is 20% less than white noise and \( 1/f^2 \) amplification is 40% less.

In Fig. 2 SNR is shown as a function of \( H_{\text{rms}} \) for the three types of noise at \( H_s = 0.5 \).

The maximum SNR is obtained for white noise, but \( 1/f \) noise seems to have a broader range where it is larger than white noise. This is due to the fact that in \( 1/f \) noise the noise reduction plays a role in the SNR by enhancing its value for a broader range, this tendency is confirmed in the \( 1/f^2 \) case where noise reduction plays a role in a broader range than the \( 1/f \) case. In the \( 1/f^2 \) case the maximum in SNR is reached at a noise rms (root mean square) value larger than the other cases (a noise RMS equal to 1.2). In Fig. 3 \( \eta \) [dB] is shown as a function of \( H_{\text{rms}} \) for the three types of noise at \( H_s = 0.5 \). Also in this case, white noise guarantees a higher signal amplification. However, in this case there is no range where \( 1/f \) noise presents a higher level of amplification. This is due to the fact that in signal amplification the level of noise is not included in the calculation.

**CONCLUSIONS**

In this paper, magnetostochastic resonance in the presence of colored noise has been investigated. It has been shown that:

1. Noise spectrum affects MSR.
2. White noise, \( 1/f \) and \( 1/f^2 \) noise induce in magnetic systems described by the dynamic Preisach model MSR.
3. Maximum level of SNR has been obtained by using white noise but \( 1/f \) noise presents a range where SNR value is higher than the case of white noise.
4. Maximum signal amplification is obtained for white noise.

**ACKNOWLEDGMENTS**

This work was supported by MIUR (project “Finanziamenti Ricerca Scientifica di Ateneo 2005” ex quota 60%).

2. R. Benzi et al., Tellus 34, 10 (1982).