AUTHOR QUERY FORM

AIP **American Institute** **of Physics**	**Journal: J. Appl. Phys.** **Article Number: 522291JAP**	**Please provide your responses and any corrections by** **annotating this PDF and uploading it to AIP's eProof** **website as detailed in the Welcome email.**

Dear Author,

Below are the queries associated with your article; please answer all of these queries before sending the proof back to AIP. Author please indicate the correct color processing option from the list below:

1. Author, please confirm Figure number(s) that should appear as color in print. Please know that any associated mandatory fees will apply for figures printed in color.

2. Author, please confirm Figure number(s) that should appear as color online only, there will be no fees applied.

3. Author, your paper currently does not include any color figures for online or print. If color is needed please indicate which figures it should be applied to and whether it is color in print or online.

Location in article	Query / Remark: click on the Q link to navigate to the appropriate spot in the proof. There, insert your comments as a PDF annotation.
AQ1	Please shorten your paper so that it fits within the 3 page length limit for MMM contributed papers.
AQ2	Please clarify if Transrapid is the specific name of the system and should be capitalized, or if it should be lowercased here: "used, for example, in Transrapid transportation systems"
AQ3	Later in this paper the acronym used is EMALS for this phrase; which is correct: "proposed for an electromagnetic aircraft launch system (EALS)"
AQ4	Please clarify your meaning here: "One between the inductor and the induct is always longer than the other." Do you mean that one of the topologies has an induct that is always longer than the inductor, or something else?
AQ5	Please spell out the acronym PM on first use here: "identical for superconducting and PM linear synchronous motor". Should this be permanent magnet?
AQ6	Please confirm that this edit is OK, to change permeance to permanence here: "where Λ is the position-dependent permanence"
AQ7	Please confirm that this sentence is correct as written: "first one contains a half pole per phase (HPP), the second one contains one phase per pole (OPP)
AQ8	This text in Sec. III B is an exact duplicate of the last paragraph in Sec. II. Please try to reword it or possibly delete some text in one or the other location: "Deviations from the ideal design can be caused by errors in the induct coils placement, error in the inductor coils placement, a non uniform air gap dimension, deformation in the structure of either the moving or the stationary part of the machine, angular displacement in the multi sided machine, and so on. These kinds of errors can naturally be present also in a rotating machine but the main difference between a rotating and a linear machine is that in the former case the structure of the machine is periodic (i.e., the rotating part faces the same geometrical structure for each performed turn) and in the latter case the structure of the machine is not periodic. As a result, deviations from idealities in a rotating machine can be easily included in the mathematical analysis of the machine by using Fourier analysis and introducing some ad hoc coefficients; in linear machines this approach cannot be followed straightforwardly."
AQ9	Please ensure that the publisher's name and location are correct for Refs. 2 and 4.
AQ10	Please check the journal title in Ref. 5. Should this be IEEE Trans. Appl. Supercond.?

Thank you for your assistance.

AQ1

A model of a linear synchronous motor based on distribution theory

Marco Trapanese[a)]
Dipartimento di Ingegneria Elettrica, Elettronica e delle Telecomunicazioni, Palermo University, Palermo, I-90128 Palermo

(Presented 31 October 2011; received 23 September 2011; accepted 1 December 2011; published online xx xx xxxx)

The fundamental idea of this paper is to use the distribution theory to analyze linear machines in order to include in the mathematical model both ideal and non ideal features. This paper shows how distribution theory can be used to establish a mathematical model able to describe both the ordinary working condition of a Linear Synchronous Motor (LSM) as well the role of the unavoidable irregularities and non ideal features. © *2012 American Institute of Physics*. [doi:10.1063/1.3679046]

I. INTRODUCTION

The fundamental idea of this paper is to use the distribution theory to analyze linear machines in order to include in the mathematical model both ideal and non ideal features.

The main difference between rotating and linear machines is that the former have periodic structures and the latter do not. However, despite the fact that linear machines do not have a periodic structure, the mathematical tools generally used to describe and design them are based on Fourier analysis; see for example Refs. 1 and 2. This approach introduces a sort of periodic boundary condition in the description of the machine that limits in some way the analysis of this kind of machine. As an example, in this kind of analysis the role of the displacement of the armature coils from their ideal position as well as the error in the position of the field coils cannot be taken into account straightforwardly. On the contrary, an approach based on stochastic analysis can allow a deeper analysis.

This paper shows how distribution theory can be used to establish a mathematical model able to describe both the ordinary working condition of a linear synchronous motor (LSM) as well the role of the unavoidable irregularities and non ideal features.

II. THE LINEAR SYNCHRONOUS MOTOR

A linear synchronous motor (LSM) consists of two parts: the induct and the inductor.

The main magnetic field is generated by the inductor. The inductor may be assembled by using either traditional windings, permanent magnets, or superconducting magnets. The first solution is used, for example, in Transrapid transportation systems, the second has been proposed for an electromagnetic aircraft launch system (EALS), and the third solution is currently tested in the Japanese maglev system; this solution uses no iron in the magnetic circuit.

The induct is made of a polyphase system (usually a three-phase system) and generates a traveling field. Thrust is generated by the interaction between the inductor and the induct (Fig. 1).

A LSM inductor can be made of hundreds of poles distributed along hundreds of meters (e.g., transportation systems; Fig. 2) and quite often modular topologies are used.

Several motor topologies have been previously proposed and developed. One between the inductor and the induct is always longer than the other. The simplest linear motor topology is based on a single side inductor and a single side induct but most of the technologies developed use multiple inducts and inductors: in transportation systems a double sided induct is often used and in EALS a single inductor-multiple induct motor has been proposed.[3]

As a result, LSM are motors that tend to be quite large and probably are the largest motors technologically available. Such huge dimensions tend to present meaningful deviations from the ideal designed topologies. Such deviations can be caused by error in the induct coils placement, error in the inductor coils placement, a non uniform air gap dimension, deformation in the structure of either the moving or the stationary part of the machine, angular displacement in the multi sided machine, and so on. These kinds of errors can naturally be present also in a rotating machine but the main difference between a rotating and a linear machine is that in the former case the structure of the machine is periodic (i.e., the rotating part faces the same geometrical structure for each performed turn) and in the latter case the structure of the machine is not periodic. As a result deviations from idealities in a rotating machine can be easily included in the mathematical analysis of the machine by using Fourier analysis and introducing some ad hoc coefficients; in linear machines this approach cannot be followed straightforwardly.

III. MATHEMATICAL MODEL OF LINEAR SYNCHRONOUS MOTOR

The traditional approach to the mathematical model of a LSM describes a LSM as a multiple of a simple unit structure. This approach, as already stated, makes it difficult to include in the model the deviations from the ideal design that unavoidably are included in an actual machine. Moreover, a superconducting linear synchronous motor (SLSM)

AQ4

AQ2
AQ3

a)Electronic address: mtrap@diepa.unipa.it.

000000-2   Marco Trapanese
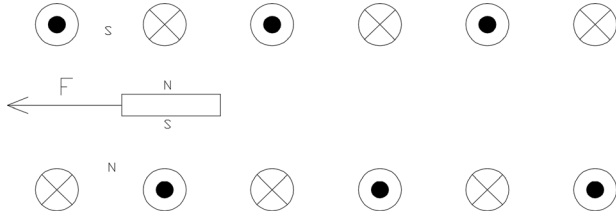
FIG. 1. Thrust generation in a LSM. Poles located in the inductor interact with the pole generated by the currents in the induct.

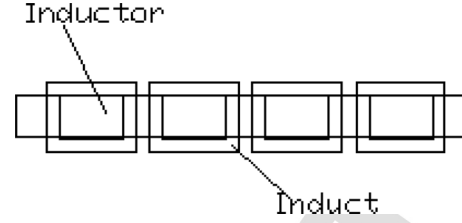89 differs from a LSM because of the use of superconductors to
90 obtain the inductor field. This permits an ironless induct, but
91 if iron losses, fringing, or skin effect are neglected the mag-
92 netic analysis of the motor is identical for superconducting
93 and PM linear synchronous motor.
94   In what follows, the traditional approach is preliminarily
95 presented and subsequently the new proposed approach is
96 illustrated. As an example, the analysis is carried out for the
97 geometry of a SLSM for EMALS proposed in Refs. 3 and 5.

98 **A. The 1D traditional approach to the model of LSM**

99   The traditional approach to the model of a LSM consists
100 in modeling the LSM in terms of one pole and the total thrust
101 is calculated by adding the thrust provided by each pole.[3,4] If
102 one assumes that the general structure of a LSM is the one
103 reported in Fig. 3, the thrust generated by a pole pair can be
104 expressed as:

$$F(z_0,t) = \int_0^{2\tau_p} A(z,t)B_g(z,z_0,t)Ydz, \qquad (1)$$

107 where F is the thrust provided per pole pair, z_0 is the dis-
108 placement of the inductor with respect to the induct, t is
109 time, τ_p is the polar pitch, $A(z,t)$ is the armature current sheet,
110 $B_g(z,z_0,t)$ is the air gap flux density, and Y is the motor length
111 along the y direction.
112      $A(z,t)$ can be expressed as follows:

$$A(z,t) = \frac{dH_a(z,t)}{dt}, \qquad (2)$$

115 where H_a is the armature magneto motive force (MMF).

FIG. 2. (Color online) A photograph of a Transrapid, which shows clearly how the LSM is placed along the whole vehicle. As a result the length of the motor is equal to the length of the vehicle.

FIG. 3. General structure of a LSM.

116 $B_g(z, z_0, t)$ can be expressed as follows:

$$B_g(z,z_0,t) = \Lambda(z,z_0)\Big(H_a(z,t) + H_{PM}(z,z_0,t)\Big), \qquad (3)$$

119 where Λ is the position-dependent permanance (however, in
120 an ironless machine the position dependence can be
121 neglected) and where H_{PM} is the armature magneto motive
122 force (MMF).
123   The induct coils can be connected in several ways; how-
124 ever, the most studied are the following: the first one con-
125 tains a half pole per phase (HPP), the second one contains
126 one phase per pole (OPP). In Ref. 3 the corresponding
127 MMFs are reported. In Fig. 4 the thrust for a constant current
128 in the induct is shown. This figure can be interpreted as the
129 thrust provided by the motor when the displacement of the
130 rotor from the ideal position varies according to the coordi-
131 nate z and the synchronicity between the traveling wave and
132 the rotor is guaranteed.
133   As a result, in order to evaluate the thrust in Eq. (1), one
134 must be able to calculate H_{PM}, which can be evaluated in
135 several ways: if it is generated by a PM it can be evaluated
136 by using the physical parameters of the PM used, if it is gen-
137 erated by a bulk superconductor it can be found using the
138 "sand-pile model" along with the Biot-Savart law, and if it is
139 generated by a superconducting coil it can be evaluated by
140 modeling the coil through the Biot-Savart law.
141   The total thrust provided by the motor along the z direc-
142 tion is simply obtained by multiplying the thrust provide by
143 Eq. (1) by the number of pole pairs. The same simple
144 approach, which describes motors with multiple inductors
145 along the y direction, is already contained in Eq. (1) and is
146 represented by the multiplication by Y factor inside the inte-
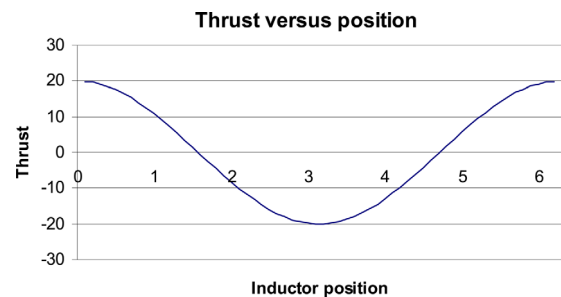147 gral. This approach simply assumes that each pole pair is

FIG. 4. (Color online) The thrust for a constant current in the induct is shown. This figure can be interpreted as the thrust provided by the motor when the displacement of the rotor from the ideal position varies according to the coordinate z and the synchronicity between the traveling wave and the rotor is guaranteed.

148 from the magnetic point of view in exactly the same condi-
149 tion of all the other pole pairs. In a very large machine, under
150 extreme dynamic conditions this can be not true and a math-
151 ematical way to describe this situation is useful in order to
152 better model LSM.

B. Non idealities and the proposed approach

154 Deviations from the ideal design can be caused by errors
155 in the induct coils placement, error in the inductor coils
156 placement, a non uniform air gap dimension, deformation in
157 the structure of either the moving or the stationary part of the
158 machine, angular displacement in the multi sided machine,
159 and so on. These kinds of errors can naturally be present also
160 in a rotating machine but the main difference between a
161 rotating and a linear machine is that in the former case the
162 structure of the machine is periodic (i.e., the rotating part
163 faces the same geometrical structure for each performed
164 turn) and in the latter case the structure of the machine is not
165 periodic. As a result, deviations from idealities in a rotating
166 machine can be easily included in the mathematical analysis
167 of the machine by using Fourier analysis and introducing
168 some ad hoc coefficients; in linear machines this approach
AQ8 169 cannot be followed straightforwardly. Moreover, such devia-
170 tions are intrinsically stochastic so an attempt to describe
171 them by a deterministic analysis is not very fruitful.

172 The stochastic aspect of this issue can be modeled by
173 assuming that the motor consists of N pole pairs whose dis-
174 placement with respect the magnetic field generated by the
175 induct can be described by a variable z_{0i} (where i spans from
176 1 to N); as a result the total thrust F_{tot} provided by the motor
177 can be expressed as follows:

$$F_{tot} = \sum_{i=1}^{N} F(z_{0i}, t) = \sum_{i=1}^{N} \int_{0}^{2\tau_p} A(z,t) B_g(z, z_{0i}, t) Y dz. \quad (4)$$

179

180 If one assumes that z_{0i} is a stochastic variable then the thrust
181 also becomes a stochastic expression, which can be described
182 as $F_i = F(z_{0i},t)$. The total thrust becomes, therefore,

$$F_{tot} = \sum_{i=1}^{N} F_i. \quad (5)$$

183

185 If one can guess the stochastic distribution of the thrust and
186 assume that N is very large, Eq. (5) can be re-cast as follows:

$$F_{tot} = N \int_{-\infty}^{+\infty} F^*(z) P(z) dz, \quad (6)$$

188

189 where N is the normalization constant and is equal to the num-
190 ber of poles, F^* is the expression of the generated thrust for a
191 z displacement, and P is the probability distribution of z.

192 Equation (5) and Eq. (6) show clearly that the traditional
193 approach (i.e., the calculation of the total thrust as the multi-
194 plication of the total number of pole pairs by the thrust pro-
195 vided by a single pole pair) can be followed only if P is a
196 delta function, that is, if no deviations from the ideal design
197 are presented in the actual motor; in any other case the actual
198 thrust is less than the ideal one.

199 As a matter of fact, Eq. (6) provides more information:
200 It permits also the calculation of the reduction of the per-
201 formances if one assumes to know P.

IV. CALCULATION OF THE LOSS OF PERFORMANCES OF A LSM AFFECTED BY NON IDEALITIES

205 As already stated, looking at Fig. 4 one can see that the
206 thrust provided by each pole varies as a cosinusoid. As a
207 result, if one assumes a suitable probability function the total
208 provided thrust can be calculated using Eq. (6). In what fol-
209 lows, the deviations from idealities are assumed to be
210 described by a Gaussian law, whose average is zero and the
211 standard deviation is equal to σ. As a result, Eq. (6) can be
212 explicitly calculated and reads as follows:

$$F_{tot} = N \int_{-\infty}^{+\infty} F^*(z) P(z) dz$$
$$= \frac{N}{\sqrt{\pi \sigma^2}} \int_{-\infty}^{+\infty} T_{\max} \cos(zb) e^{-\frac{z^2}{\sigma^2}} dz = N e^{-\left(\frac{b^2 \sigma^2}{4}\right)} \quad (7)$$

213

215 where b is the inverse of the polar pitch. Equation (7)
216 expresses the reduction of the provided thrust when devia-
217 tions from idealities are present. It can be seen how the pro-
218 vided thrust is affected by non idealities and the reduction
219 follows an exponential law. Equation (7) holds along the z
220 direction but an analogous result can be found also along the
221 Y direction

222 Equation (7) can be used to establish an upper limit on
223 the error that can be tolerated in order to achieve some
224 desired performances and is useful in order to give an esti-
225 mation of the actual thrust provided by a LSM.

V. VALIDATION

227 To verify the results expressed by Eq. (7), a numerical
228 model of a LSM has been constructed. In this model the
229 magnets have been supposed to be placed in ideal positions
230 but the armature winding positions have been supposed to be
231 affected by errors that can be described through a Gaussian
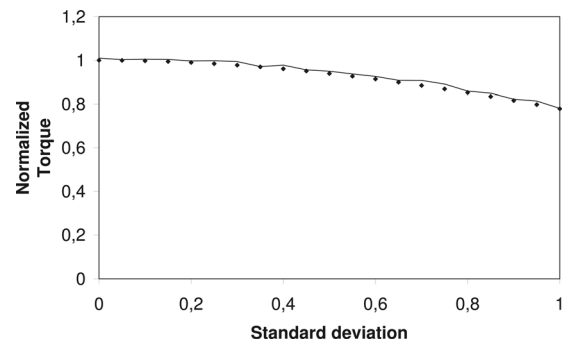232 distribution. The motor has been supposed to consist of 20

FIG. 5. A comparison between the normalized thrust obtained by the nu-
merical model of the LSM and the thrust obtained using Eq. (7). The contin-
uous line represents the normalized thrust calculated by the numerical
model, and the diamonds represent the thrust calculated according to Eq.
(7). Normalized thrust is expressed in adimensional units, σ in the unit of po-
lar pitch.

excitation magnets whose position was distributed according to a Gaussian law whose standard deviation can be varied numerically. The total thrust has been calculated by summing the thrust obtained from each magnet. Several runs with different standard deviation of this mathematical model have been performed. Figure 5 shows a comparison between the normalized thrust obtained by the numerical model of the LSM and the thrust obtained by using Eq. (7). The distribution of the results obtained with this mathematical approach followed that expressed by Eq. (7) with a deviation smaller than 2%.

VI. CONCLUSION

In this paper an approach to the calculation of the thrust provided by a LSM has been presented. This approach models the induct of a LSM as a series of coils interacting with inductor coils. In the calculation of the total thrust the deviation of the position of each coil from its ideal position is taken into account by using a probability distribution function. This explicitly gives the dependence of the total thrust from the standard deviation of this distribution function. It has been shown that the reduction of the provided thrust follows an exponential law, which decays as the standard deviation of the distribution function.

[1]Z. Q. Zhu and D. Howe, IEEE Trans. Magn., **29**(1), 124 (1993).

[2]S. Caldara, S. Nuccio, G. Ricco Galluzzo, and M. Trapanese, in *Proceedings of the 1997 IEEE Instrumentation & Measurement Technology Conference, IMTC*, Ottawa, Canada (IEEE, New York, 1997) Part 1, pp. 257–262.   AQ9

[3]G. Stumberger, D. Zarko, A. M. Timur, and T. A. Lipo, in *Proceedings of Electric Machines and Drives Conference IEMDC'03*, June 2003, (IEEE, New York, 2003) pp. 494–500.

[4]F. Profumo, A. Tenconi, and G. Gianolio, in *Conference Record of the IEEE Industry Applications Conference* (IEEE, New York, 2001) Vol. **3**, pp. 1984–1988.

[5]G. Stumberger, D. Zarko, A. M. Timur, and T. A. Lipo, IEEE Trans. Supercond., **14**(1), 54 (2004).   AQ10