

A MULTI-SPHERE PARTICLE NUMERICAL MODEL FOR NON-INVASIVE INVESTIGATIONS OF NEURONAL HUMAN BRAIN ACTIVITY

Guido Ala^{1, *} and Elisa Francomano²

¹Dipartimento di Ingegneria Elettrica, Elettronica e delle Telecomunicazioni, di Tecnologie Chimiche, Automatica e Modelli Matematici, Università degli Studi di Palermo, Viale delle Scienze, ed. 9, Palermo I-90128, Italy

²Dipartimento di Ingegneria Chimica, Gestionale, Informatica, Meccanica, Università degli Studi di Palermo, Viale delle Scienze, ed. 6, Palermo I-90128, Italy

Abstract—In this paper, a multi-sphere particle method is built-up in order to estimate the solution of the Poisson's equation with Neumann boundary conditions describing the neuronal human brain activity. The partial differential equations governing the relationships between neural current sources and the data produced by neuroimaging technique, are able to compute the scalp potential and magnetic field distributions generated by the neural activity. A numerical approach is proposed with current dipoles as current sources and going on in the computation by avoiding the mesh construction. The current dipoles are into an homogeneous spherical domain modeling the head and the computational approach is extended to multilayered configuration with different conductivities. A good agreement of the numerical results is shown and, for the first time compared with the analytical ones.

1. INTRODUCTION

Bio-magnetic fields are caused by electric currents in conducting body tissues like the brain. Neural current sources in the brain produce external magnetic fields and scalp surface potentials that can be recorded in the neighborhood of the human head. The potentials recorded across many electrodes on the scalp form an evoked potential map containing important information about the brain generators;

Received 9 November 2012, Accepted 29 November 2012, Scheduled 12 December 2012

* Corresponding author: Guido Ala (guido.ala@unipa.it).

these potentials can be measured using brain imaging techniques. Magnetoencephalography (MEG) is a non-invasive brain imaging technique for monitoring brain function. With MEG it is possible to study neurophysiologic processes underlying mental acts in healthy and awake humans in a totally non-invasive way. MEG has already proven itself a useful tool in studies of human neurophysiology and information processing [1,2]. In searching the unknown sources, by means of the analysis of the measured data, an inverse problem has to be solved. In order to handle this task, the forward problem has to be firstly approached (i.e., how to evaluate the magnetic field and the electric potential arising from a known source). Since it is involved more and more times, the computational efforts have to be reduced as much as possible. To this aim, appropriate models have to be chosen both for the biological conductor and from the computational point of view. In this paper, the forward problem has been investigated by considering the quasi-static approximation equations governing the relationships among neural current sources and the data produced by means of neuroimaging technique [3]. The electric potential is obtained by solving Poisson's equation with proper boundary conditions, and the magnetic induction is obtained by means of the Biot-Savart law. The head models determining these solutions, generally assume the head as a piecewise homogeneous conductor. The usually adopted single spherical shell is too unrealistic as a model for the head, due to the large difference between the conductivities of brain and skull. The typical multilayered model includes three layers for the brain, skull, and scalp. For the case where the head is assumed to comprise a set of nested isotropic concentric spheres, each of constant conductivity, the analytic solutions exist [4]. Therefore, the numerical approach proposed in this paper refers to the multilayered model. In scientific literature, it has been argued that brain generators can be reasonably described by current dipoles within the brain [5–12]. It is a widely used concept in neuromagnetism and it is considered as a good approximation for a small source viewed from a remote field point. Therefore, current dipoles approximating localized sources are taken into account as the start points to validate the computational model.

Several grid based numerical approaches [13–18] have been proposed in technical literature, but they may become computationally inefficient for complex domain related to the human head and brain. The meshless methods have recently known a great success in the simulation of a wide variety of problems as a valid computational alternative to grid ones [19–23]. They share common features such as the avoidance of the use of grids, but are different in functions approximation and computational processes. In the past decades,

many numerical methods, have been successfully developed to solve partial differential equations (PDEs) all depending on a suitable generation of a mesh: this is usually a difficult task for problems involving complicated and irregular geometries. The meshless methods are increasing their influence in physical applications with the aim to eliminate the generation of meshes and by constructing the approximation only in terms of particles. Therefore, a set of particles are used instead of meshing the domain of the problem [24, 25]. In this paper, the Poisson's equation with suitable boundary conditions for multilayered homogeneous, isotropic spherical domain is studied by using the Smoothed Particle Hydrodynamics (SPH) method [19–23, 26–32]. The authors have reformulated the SPH for the Poisson's equation, by modelling the electric field behaviour in a conducting medium, in order to satisfy the second order of consistency. The stationary hypothesis has been adopted by following a consolidated approach in technical literature [1–16, 24, 25]. Validation of the proposed approach has been performed with different simulation results and, for the first time, by comparing multilayered models with different conductivities with the analytical ones, for a typical case study. The paper is organized as follows. In Section 2, the forward problem is briefly described. In Section 3, the mathematical model and the numerical approach are reported. In Section 4, simulation results are discussed and compared with the analytic ones referring to multilayered models.

2. FUNDAMENTALS OF THE FORWARD PROBLEM

Solving the forward model represents the first step towards the reconstruction of the spatial-temporal activity of the neural sources. Neural current sources in the brain can be separated into two components, the *primary currents* term representing the impressed neural currents, and the *secondary currents* that are a result of the macroscopic electric field. The primary currents are considered to be the sources of interest in neuroimaging technique, such as MEG, since they represent the areas of neural activity associated with a given sensorial, motor or cognitive processes. The recent development of dedicated measurement systems (i.e., the SQUID [33, 34]), offers the potentialities for MEG to produce accurate estimates of the location and temporal evolution of these underlying primary sources. Localization of the cortical regions responsible for this activity, is of importance in the correct diagnosis and treatment of several brain pathologies. In the context of the localization of neural sources, the *forward problem* is to determine the electric field E and the magnetic

field B that result from primary current sources J^i . The forward problem begins with known distributions of the conductivity σ and the brain's total electrical current sources with magnetic permeability assumed to be equal to μ_0 everywhere in the conductor domain Ω , by using the following assumptions:

$$\begin{aligned} E &= -\nabla\Phi \\ \nabla \times B &= \mu_0 J \\ \nabla \cdot B &= 0 \\ J &= J^i + J^\Omega \end{aligned} \quad (1)$$

where Φ is the electric potential, J the total current density, and $J^\Omega = \sigma E$ the volumetric current. In order to obtain E and J , the electric potential must be computed and $\nabla \cdot J = 0$ holds. The scalar electric potential is obtained by solving the following boundary value problem by considering that no current can flow out of the skull:

$$\begin{cases} \nabla \cdot J^i(p) = \nabla \cdot (\sigma(p)\nabla\Phi(p)) & p \in \Omega \\ n \cdot (\sigma(p)\nabla\Phi(p)) = 0 & p \in \partial\Omega \end{cases} \quad (2)$$

n as the unit vector normal to the domain contour. The magnetic field is obtained by the well-known following integral:

$$B(p) = \frac{\mu}{4\pi} \int_{\Omega} J(q) \times \frac{p - q}{\|p - q\|^3} d\Omega \quad (3)$$

In the forward problem, the current source is typically approximated by a current dipole $J^i(p) = Q\delta(p - p_0)$ [9–11] located at position p_0 with moment Q . A current dipole with a moment Q is a concentration of the impressed current to a single point where δ is the Dirac delta.

The scalar potential $\Phi(p)$ is expressed as the sum of two terms, $\Phi(p) = \phi_F(p) + \phi(p)$, where $\phi_F(p)$ is the electric potential generated by a dipole source in an infinite homogeneous medium which is known as:

$$\phi_F(p) = \frac{1}{4\pi\sigma} \frac{p - p_0}{\|p - p_0\|^3} q \quad (4)$$

The single spherical shell is too unrealistic as a model for the head due to the large difference between the conductivities of brain and skull. The typical multi-sphere model includes three layers for the brain, skull, and scalp; some also include a cerebrospinal fluid layer. For the case where the head is assumed to comprise a set of concentric spheres, each of constant conductivity, analytic solutions exist for MEG. By considering a set of contiguous M regions, each of constant conductivity σ_k , $k = 1, \dots, M$, a closed form potential expression for a dipole within a homogeneous sphere is available in [4], where the

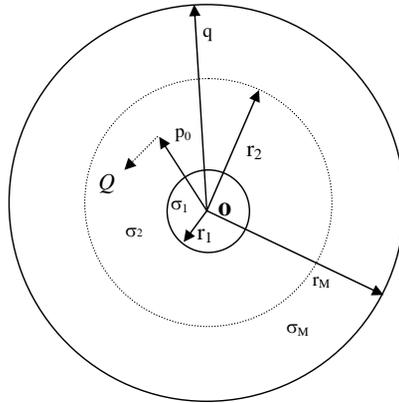


Figure 1. Multi-sphere head model.

conductivities $\sigma_1, \sigma_2, \dots, \sigma_M$ are arranged from the innermost sphere to the outer most (Fig. 1). A dipole is located within the innermost layer at p_0 and the observed potential on the M -th layer sphere surface at q is obtained as follows:

$$\Phi(p_0, q, Q) = \frac{Q}{4\pi\sigma_M q^2} \sum_{n=1}^{\infty} \frac{2n+1}{n} \left(\frac{p_0}{q}\right)^{n-1} \times f_n (n \cos \alpha P_n(\cos \gamma) + \cos \beta \sin \alpha P_n^1(\cos \gamma)) \quad (5)$$

where α is the angle between dipole location and dipole moment Q , γ is the angle between the p_0 and q , β is the angle between the planes defined by p_0 and Q and by p_0 and q , P_n and P_n^1 are the Legendre and associated Legendre polynomials respectively, and:

$$f_n = \frac{n}{nm_{22} + (1+n)m_{21}} \quad (6)$$

The coefficients m_{21} and m_{22} are found by:

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \frac{1}{(2n+1)^{M-1}} \times \prod_{k=1}^{M-1} \begin{bmatrix} n + \frac{(n+1)\sigma_k}{\sigma_{k+1}} & (n+1) \left(\frac{\sigma_k}{\sigma_{k+1}} - 1\right) \left(\frac{r_q}{r_k}\right)^{2n+1} \\ n \left(\frac{\sigma_k}{\sigma_{k+1}} - 1\right) \left(\frac{r_k}{r_q}\right)^{2n+1} & (n+1) + \frac{n\sigma_k}{\sigma_{k+1}} \end{bmatrix} \quad (7)$$

3. NUMERICAL FORMULATION

The volume domain Ω is modelled by different concentric layers Ω_i with different constant conductivities σ_i , the boundary $\partial\Omega_i$ between

the domains Ω_i and Ω_{i+1} is considered to be regular. By considering the above assumptions, the Equation (2) can be rewritten as follows:

$$\begin{cases} \sigma(p)\nabla^2\phi(p) = 0 & p \in \Omega \\ \phi_i(p) = \phi_{i+1}(p) & p \in \partial\Omega_i \\ n \cdot (\sigma_i(p)\nabla\phi(p)) - n \cdot (\sigma_{i+1}(p)\nabla\phi(p)) & \\ = n \cdot (\sigma_{i+1}(p) - \sigma_i(p))\nabla\phi_F(p) & p \in \partial\Omega_i \end{cases} \quad (8)$$

In solving this boundary value problem the SPH meshless kernel method has been considered. Namely, the domain Ω is described by introducing a number of particles arbitrarily distributed to cover the problem domain, and by approximating the function ϕ_p in a point $p \in \Omega$ with a kernel function $W_{qp} = W(p - q, h)$ [21] involving all nearest neighboring particles (NNP) q of particle p :

$$\phi_p^h = \sum_{q \in \Omega} \phi_q W_{qp} V_q \delta_{qp} \quad (9)$$

where V_q is the measure of the support domain surrounding the particle q . The problem is discretized and the unknowns are computed solving the following linear system:

$$\begin{cases} \sum_{q \in \Omega} K_{pq} \phi_q = 0 & p \in \Omega \\ \sum_{q \in \Omega} Z_{pq} \phi_q = 0 & p \in \partial\Omega_i \\ \sum_{q \in \Omega} H_{pq} \phi_q = n \cdot (\sigma_{i+1} - \sigma_i)\nabla\phi_F(p) & p \in \partial\Omega_i \end{cases} \quad (10)$$

The matrix K_{pq} is obtained as:

$$K_{pq} = S_p G_{qp} V_q \delta_{qp} \quad (11)$$

where:

$$\begin{aligned} S_p &= (0, 0, 0, 0, \sigma_p, \sigma_p, \sigma_p, 0, 0, 0), \\ G_{qp} &= F_p^{-1} (W_{qp}, \partial_r W_{qp}, \partial_{rr} W_{qp}, \partial_{rs} W_{qp})^T \end{aligned} \quad (12)$$

$r = x, y, z$, $(r, s) \in \{2 - \text{combinations from } \{x, y, z\}\}$, $\partial_r = \frac{\partial}{\partial q_r}$, $\partial_{rr} = \frac{\partial^2}{\partial q_r \partial q_r}$, $\partial_{rs} = \frac{\partial^2}{\partial q_r \partial q_s}$ and

$$F_p = \begin{pmatrix} \sum_{q \in \Omega} W_{qp} V_q \delta_{qp} \\ \sum_{q \in \Omega} \partial_r W_{qp} V_q \delta_{qp} \\ \sum_{q \in \Omega} \partial_{rr} W_{qp} V_q \delta_{qp} \\ \sum_{q \in \Omega} \partial_{rs} W_{qp} V_q \delta_{qp} \end{pmatrix} \left(1, (q_r - p_r), \frac{(q_r - p_r)^2}{2!}, (q_r - p_r)(q_s - p_s) \right) \quad (13)$$

Z_{pq} is a sparse matrix whose values for each row are zero except for two entries equal to 1 and -1 , respectively and the H_{pq} is the following matrix:

$$H_{pq} = (\sigma_i - \sigma_{i+1})N_p G_{qp} V_q \delta_{qp} \tag{14}$$

by assuming:

$$N_p = (0, n_{x_p}, n_{y_p}, n_{z_p}, 0, 0, 0, 0, 0) \tag{15}$$

Therefore, the volume current $J^\Omega(p)$ is computed for all $p \in \Omega$ as follows:

$$J^\Omega(p) \cong -(0, \sigma_P, \sigma_P, \sigma_P, 0, 0, 0, 0, 0) \sum_{q \in \Omega} G_{qp} \Phi_q V_q \delta_{qp} \tag{16}$$

The magnetic field is subsequently obtained as follows:

$$B(p) \cong \frac{\mu}{4\pi} \sum_{q \in \Omega} \frac{J^\Omega(q) \times (p - q)}{\|p - q\|^3} V_q \tag{17}$$

4. NUMERICAL RESULTS

In this section, numerical simulations are reported and compared with the analytical results. A set of contiguous $M = 2, 3$ regions have been considered and a comparison between the computed results and the analytical solutions is performed.

In Fig. 2, the analytic and the simulated electric potential are considered by taking into account two concentric spheres centred in $(0, 0, 0)$ with radius $r_1 = 0.08$ m, $r_2 = 0.1$ m, and layers conductivities $\sigma_1 = 0.2$ (Ωm)⁻¹, $\sigma_2 = 0.01$ (Ωm)⁻¹, respectively. The dipole is in

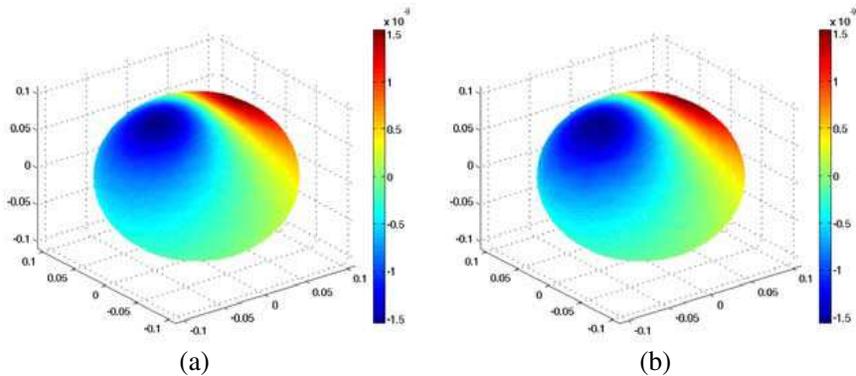


Figure 2. (a) Analytic electric potential [V]. (b) Simulated electric potential [V] on the external sphere surface.

$(0, 0, 0.05)$ with moment $J * (1, 0, 0)$ where $J = 4.5 * 10^{-12} \text{ A/m}^2$. A random particles distribution has been chosen. A good agreement has been reached.

In Fig. 3, the analytic and the simulated electric potential are considered by taking into account three concentric spheres centred in $(0, 0, 0)$ with radius $r_1 = 0.07 \text{ m}$, $r_2 = 0.08 \text{ m}$, $r_3 = 0.1 \text{ m}$. The conductivities are set to $\sigma_1 = 0.2 (\Omega\text{m})^{-1}$, $\sigma_2 = 0.01 (\Omega\text{m})^{-1}$ and $\sigma_3 = 0.001 (\Omega\text{m})^{-1}$ respectively and the dipole is in $(0, 0, 0.05)$ with moment $J * (1, 0, 0)$ where $J = 4.5 * 10^{-12} \text{ A/m}^2$. The results are, also in this case, referred to a random particles distribution, as sketched in Fig. 4. As it can be observed in Fig. 3, a satisfactory agreement has been also obtained in this case.

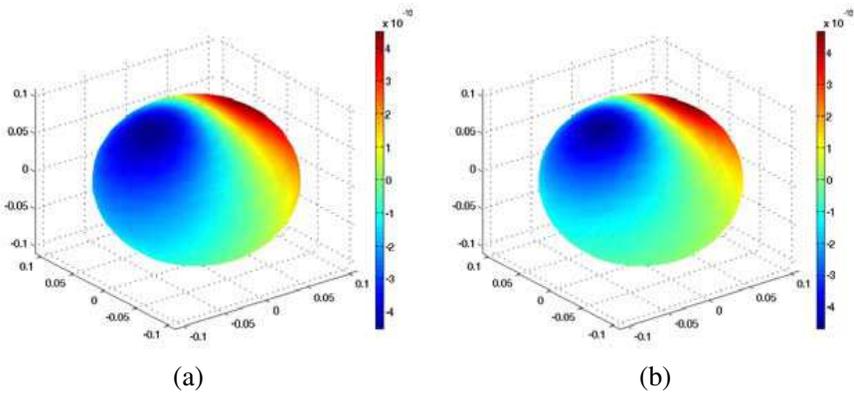


Figure 3. (a) Analytic electric potential [V]. (b) Simulated electric potential [V] on the external sphere surface.

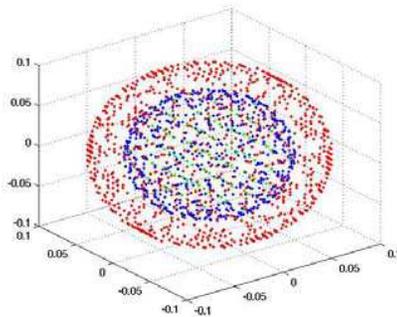


Figure 4. Random particles distribution for the three layers spherical model.

5. CONCLUSIONS

In this paper, a multi-sphere particle numerical method is proposed to estimate the neuronal human brain activity. The Poisson's equation with Neumann conditions, governing the relationships between neural current sources and the data produced by means of the neuroimaging technique, is taken into account to compute the scalp potential and magnetic field distributions generated by the brain generators. The localized current sources are simulated as current dipoles into a spherical domain modeling the head. The meshless computational approach is extended to multilayered model with different conductivities arranged from the innermost layer to the outer most. A satisfactory agreement of the simulated electric potential is shown compared with the analytical ones.

REFERENCES

1. Hämäläinen, M., R. Hari, R. J. Ilmoniemi, J. Knuutila, and O. V. Lounasmaa, "Magnetoencephalography — Theory, instrumentation and applications to noninvasive studies of the working human brain," *Rev. Mod. Phys.*, Vol. 65, 413–497, 1993.
2. Darvas, F. and D. Pantazis, "Mapping human brain function with MEG and EEG: Methods and validation," *Neuroimage*, Vol. 23, S289–S299, 2004.
3. Plonsey, R., *Biomagnetic Phenomena*, McGraw-Hill, 1969.
4. Zhang, Z., "A fast method to compute surface potentials generated by dipoles within multilayer anisotropic spheres," *Phys. Med. Biol.*, Vol. 40, 335–349, 1995.
5. Bishop, G. H., "Potential phenomena in thalamus and cortex," *Electroencephalography and Clinical Neurophysiology*, Vol. 1, Nos. 1–4, 421–436, 1949.
6. Brazier, M. A. B., "A study of the electrical field at the surface of the head," *Electroencephalography and Clinical Neurophysiology Supplement*, Vol. 2, 38–52, 1949.
7. Sholl, D. A., *The Organization of the Cerebral Cortex*, Wiley, 1956.
8. Landau, W. M. and E. Potentials, *The Neurosciences — A Study Program*, G. C. Quarton, T. Melnechuk, and F. O. Schmitt, Rockefeller University Press, 1967.
9. De Munck, J. C., B. W. van Dijk, and H. Spekreijse, "Mathematical dipoles are adequate to describe realistic generators of human brain activity," *IEEE Trans. Biomed. Eng.*, Vol. 35, 960–966, 1988.

10. Sarvas, J., "Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem," *Phys. Med. Biol.*, Vol. 32, 11–22, 1987.
11. Schimpf, P. H., C. Ramon, and J. Haueisen, "Dipole models for the EEG and MEG," *IEEE Trans. Biomed. Eng.*, Vol. 49, 409–418, 2002.
12. Yao, D., "Electric potential produced by a dipole in a homogeneous conducting sphere," *IEEE Trans. Biomed. Eng.*, Vol. 47, 964–966, 2000.
13. Gençer, N. G. and I. O. Tanzer, "Forward problem solution of electromagnetic source imaging using a new BEM formulation with high-order elements," *Phys. Med. Biol.*, Vol. 44, 2275–2287, 1999.
14. Tanzer, O., S. Järvenpää, J. Nenonen, and E. Somersalo, "Representation of bioelectric current sources using whitney elements in the finite element method," *Phys. Med. Biol.*, Vol. 50, 3023–3039, 2005.
15. Haueisen, J., C. Hafner, H. Nowak, and H. Brauer, "Neuromagnetic field computation using the multiple multipole method," *Int. J. Numer. Model.*, Vol. 9, 145–158, 1996.
16. Pohl-Alfaro, M., O. Yáñez-Suárez, J. R. Jiménez-Alaniz, and V. Medina-Bañuelos, "Realistic meshless conductor model for EEG inverse problems," *International Journal of Bioelectromagnetism*, Vol. 10, 176–189, 2008.
17. Ala, G., E. Francomano, and F. Viola, "A wavelet operator on the interval in solving Maxwell's equations," *Progress In Electromagnetic Research Letters*, Vol. 27, 133–140, 2011.
18. Ala, G., M. L. Di Silvestre, F. Viola, and E. Francomano, "Soil ionization due to high pulse transient currents leaked by earth electrodes," *Progress In Electromagnetics Research B*, Vol. 14, 1–21, 2009.
19. Liu, G. R., *Mesh Free Methods — Moving beyond the Finite Element Method*, CRC Press, Boca Raton, 2003.
20. Gingold, R. A. and J. J. Monaghan, "Smoothed particle hydrodynamics: Theory and application to non-spherical stars," *Mon. Not. Roy. Astron. Soc.*, Vol. 181, 375–389, 1977.
21. Liu, M. B. and G. R. Liu, "Smoothed particle hydrodynamics (SPH): An overview and recent developments," *Archives of Computational Methods in Engineering*, Vol. 17, 25–76, 2010.
22. Monaghan, J. J., "An introduction to SPH," *Comput. Phys. Commun.*, Vol. 48, 89–96, 1988.
23. Monaghan, J. J., "Smoothed particle hydrodynamics," *Annu.*

- Rev. Astron. Astrophys.*, Vol. 30, 543–574, 1992.
24. Von Ellenrieder, N., C. H. Muravchik, and A. Nehorai, “A meshless method for solving the EEG forward problem,” *IEEE Trans. Biomed. Eng.*, Vol. 52, 249–257, 2005.
 25. Demirel, O., B. Schrader, and I. F. Sbalzarini, “A parallel particle method for solving the EEG source localization forward problem,” *Proc. 6th Intl. Symp. Health Informatics and Bioinformatics — HIBIT*, 2011.
 26. Ala, G. and E. Francomano, “An improved smoothed particle electromagnetics method in 3D time domain simulations,” *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, Vol. 25, No. 4, 325–337, 2012.
 27. Ala, G. and E. Francomano, “Smoothed particle electromagnetics modelling on HPC-GRID environment,” *Applied Computational Electromagnetics Society Journal*, Vol. 27, No. 3, 229–237, 2012.
 28. Ala, G., G. Di Blasi, and E. Francomano, “A numerical meshless particle method in solving the magnetoencephalography forward problem,” *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, Vol. 25, 428–440, 2012.
 29. Ala, G. and E. Francomano, “A marching in time meshless kernel based solver for full-wave electromagnetic simulation,” *Numerical Algorithms*, Springer, 2012, ISSN: 1017-1398, DOI 10.1007/s11075-012-9635-1.
 30. Ala, G., E. Francomano, A. Tortorici, E. Toscano, and F. Viola, “Corrective meshless particle formulations for time domain Maxwell’s equations,” *Journal of Computational and Applied Mathematics*, Vol. 210, No. 1, 34–46, 2007.
 31. Ala, G., E. Francomano, A. Spagnuolo, and A. Tortorici, “A Meshless approach for electromagnetic simulation of metallic carbon nanotubes,” *Journal of Mathematical Chemistry*, Vol. 48, No. 1, 72–77, 2010.
 32. Di Blasi, G., E. Francomano, A. Tortorici, and E. Toscano, “A smoothed particle image reconstruction method,” *Calcolo*, Vol. 48, 61–74, 2011.
 33. Ahonen, A. I., M. S. Hämäläinen, M. J. Kajola, J. E. T. Knuutila, P. P. Laine, O. V. Lounasmaa, L. T. Parkkonen, J. T. Simola, and C. D. Tesche, “122-channel squid instrument for investigating the magnetic signals from the human brain,” *Physica Scripta*, Vol. T49A, 198–205, 1993.
 34. Zhang, Y. and L. Wu, “An MR brain images classifier via principal component analysis and kernel support vector machine,” *Progress In Electromagnetics Research*, Vol. 130, 369–388, 2012.