Concrete arch bridges erected by suspended cantilever method vs cable–stayed bridges

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Summary
The suspended cantilever construction is nowadays one of the most widely used method for the erection of medium and large span concrete bridges. This technique is the only choice when use of temporary supports is not possible. In arch bridges construction, it allows to eliminate conventional centerings that often caused the abandonment of this structural solution for economical reasons. One of the main problems to solve when the construction of an arch or a cable–stayed bridge is made by cantilevering, concerns the tensioning sequence and the adjustment of cables at each construction stage. When all permanent loads are applied, the main aim is to reach the final design geometry (dead load configuration). Moreover in concrete bridges the influence of creep on stresses and deformations must be taken into account. A study is presented in which a comparison between the construction stages of a concrete arch bridge and a concrete cable–stayed bridge, both erected by suspended cantilever method, is done. Results of analysis methods are examined and discussed. Staged construction analyses are performed on both typologies to show differences and similarities taking also into account the effects of time-dependent phenomena.

Keywords
bridge, arch, cantilever construction, cables

Theme
bridges – design – structural analysis

1. Introduction
A study about construction stages of arch bridges built by the suspended cantilever system is developed here. The target of the study is to evaluate an optimal stay stressing sequence which allows, at the end of construction stages, to reach the optimal structural configuration in terms of stress and strain patterns. By looking at the construction stages of arch bridges with upper deck built by suspended cantilever method, it is evident the strong analogies with construction phases of concrete cable-stayed bridges, the main difference being the geometric axis line, which is curvilinear in the arch bridge. All others implications are extremely similar: a structure composed of different partial cantilevered structures suspended to stays from a pylon, in order to avoid too large deflections and stresses due to long cantilevers. The main topic in using stays for arch bridge construction is to reach the final geometric configuration, but it is also one of the most important items for cable-stayed bridges. At the same time, stay adjustments allows to modify bending moment diagram for cable-stayed decks or arches. So, flexural implications must be evaluated also for arch bridges. Naturally, besides clear analogies there are some important differences between these two kind of bridges. First the final static scheme: cable-stayed bridge has a behaviour near to that of a continuous beam on elastic supports. The final configuration is mainly governed by flexural effects and stays remain as a fundamental part of the structural behaviour for all bridge life. Moreover bending moments have to be controlled in each construction stage and the general structural behaviour does not modify from partial intermediate

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configurations to the final one. Arch bridges instead has a mainly final axial behaviour and stays are removed after the completion of the arch, being only temporary elements, while during intermediate stages, the suspended cantilevers have a main flexural behaviour. So, there is a strong difference between construction static schemes and the final one.

Figure 1: Partial construction schemes of arch and cable-stayed bridges.

When concrete bridges are designed, time-dependent phenomena as shrinkage and creep have to be evaluated. They have different effects on the two structures, being them related to the structural behaviour and the sequence of load applying.

Generally, two main items related to time-dependent phenomena have to be considered in cable-stayed bridges: the application of sustained loads in a structure with elastic restraints (which implies relaxation of stress in concrete) and the redistribution of internal forces due to delayed restraints added during construction stages. In order to minimize the change of stresses due to creep, it is possible to exploit an important consequence of the first theorem of linear viscoelasticity, regarding homogeneous concrete structures statically determined for rigid external restraints in which there are also additional elastic (e.g. steel) ones. If elastic restraints are introduced at the same time of sustained loads, or immediately after, and they are forced up to the values of the reactions for equivalent rigid restraints, the initial stress distribution is not affected by creep and remains unchanged. In this case in fact all the points restrained remain fixed at successive times for that sustained load; as a consequence of the first theorem of linear viscoelasticity the initial stress distribution is not modified by creep [1,2]. This result is useful to reduce creep effects in cable-stayed bridges, by imposing the vertical component of the stay forces equal to the value of the vertical reactions of rigid supports. So, the final value of reactions is introduced from the beginning and it is not modified by creep [3].

Then, in order to minimize creep effects, it is possible to follow this strategy by giving the desired stay stresses for achieving the final dead load configuration as that of a beam on rigid supports, in which no changes due to creep are to be expected in time. It implies initial cable forces that allow null displacements of stay anchorages on the deck during construction stages. Moreover a final adjustment need at the last stage with superimposed dead loads due to finishing works and final deck prestressing applied [4].

After that, live loads will be applied but they do not cause time-dependent stress and strain. By this way, the scheme is forced to the final elastic one by minimizing the effects of creep, but it implies a forward analysis to take into account staged construction [5].

In arch bridges built by the suspended cantilever method the effects of time-dependent phenomena on stresses in transient construction phases is not significant as it occurs in cable-stayed bridges. So the main item to be considered is the achieving of dead load configuration in terms of construction strains with a low
importance of creep stress redistribution.
The aim of reaching the desired final geometry is very important for service life of cable-stayed bridges but much more for arch bridges, in which the geometric differences between arch axis line and the anti-funicular line of dead loads causes the birth of bending moments into the arch. Moreover, this is fundamental for slender arches in which the shape has to be maintained with a great degree of precision in order to avoid instability and second order effects.

A reach literature can be found about stay stressing optimization of cable-stayed bridges but much less about arch bridges with suspended cantilevers. Construction sequences of these bridges have been analyzed by Au et al. [6] through the stress balance method in order to establish a tensioning procedure. They presented a study in which a two-phase tensioning method is considered and stay stresses are minimized. Li et al. [7] presented a study of cable forces optimization for cantilever construction of arch bridges by evaluating an objective function in terms of moments and stresses in the arch. A two-phase stressing procedure has been implemented by these authors but no evaluations on intermediate stages are given. Many other examples can be found in literature of bridges built by this methodology in which a multiple-stage stressing procedure have been used [8,9] but no optimization studies have been presented.

Janjic et al. applied the unit load method for construction stages of cable-stayed bridges and concrete arch bridges, by an optimization of stay stressing sequence in which the target is a given distribution of bending moments at the final construction stage [10].

In the present study a procedure of stay stressing during construction stages, optimized and used by authors for concrete cable-stayed bridges [5] has been extended to construction of concrete arches, in order to reach the above mentioned target. It is based on a two-stage stressing method which aims to maintain construction configuration near to the final desired profile of the arch. The procedure is discussed and numerical results are given on practical applications in order to compare the behaviour of arch and cable-stayed bridges and to supply useful information to designers for construction stages of these structures.

2. Stay stressing in concrete cable-stayed bridge construction
Let us consider a cable-stayed bridge built by the cantilever method. Bridge general configuration is self-anchored until the back span has reached its final length, then it could be partially earth anchored or not. It has a general 2-D symmetric scheme, with a mixed fan-harp stays arrangement. Geometric non-linearity due to cable sag can be taken into account by performing an iterative procedure in which the modified Dischinger elastic modulus (by the Ernst hypothesis) has to be computed. In case of short stays, geometric non-linearity can be neglected and the linear elastic solution is very close to the non-linear one.

![Figure 3: Proposed stay stressing sequence for cable-stayed bridges](image-url)

The proposed strategy for taking into account time-dependent phenomena during erection consists of a forward staged construction analysis of the bridge in which at each stage the main aim is to reach the
configuration of a partial beam on rigid supports under dead loads, construction equipment loads and temporary prestressing.

To achieve the exact theoretical configuration, a re-stressing of all stays in all stages would be done, but it is not possible to do that, because too many adjustments of stay forces make complicated the erection procedure. On the other hand, every time the stay is adjusted by stressing it to a different value, the wedge into the anchorage engraves the wires of the strand; too many adjustments lead to damaged wires and reduce the fatigue strength of stays. The two-step procedure is the following one:

- at each stage two stays are stressed: the new one and the previous one, reaching the stay force values, close to that of the continuous beam on rigid supports;
- in the final configuration, when the structure is completed, all stays are re-stressed and adjusted to achieve the final “dead load configuration”, by considering all permanent loads, definitive prestressing, creep and shrinkage effects on axial shortening.

At the end of this procedure deformed shape and bending moments into the deck are very close to those of the continuous beam on rigid supports and consequently the final values of stay forces are determined.

![Bending moment diagrams](image)

**Figure 4: Bending moment diagrams in deck and pylon for construction stages.**

In each phase of the procedure, the first unknown is the value of pre-stress to be given to the new stay introduced into the structure and the adjustment value of the previous one. In order to find these values, the influence matrix method has been applied. By considering as direct unknown variables the specific imposed strains of stays due to stressing, cable elongations and prestress forces will be indirect unknowns. Therefore maximum number of unknowns will be the same as stays number to be stressed. It is important to choose the smallest number for a good conditioning of the mathematical problem, by considering the number of control points in which an assigned displacement is imposed. These are the vertical displacements of the deck stay anchorage points and the horizontal ones of the pylon, for earth-anchored stays. With \( n \) stays to be stressed and \( n \) displacements to be controlled, the influence matrix \( D \) is symmetric and its coefficients can be obtained by evaluating the displacement \( \delta_j \) of the \( j \)-th control point (\( j \leq n \)) due to the unit imposed strain of the \( i \)-th stay (\( i \leq n \)). Considering the \( n \)-dimensional vector \( \mathbf{d} \) of displacements of profile control points and the \( n \)-dimensional vector \( \mathbf{e} \) of imposed strains (stay pre-stresses), it can be written \([11]\):

\[
\mathbf{d} = D\mathbf{e} + \mathbf{d}^* \tag{1}
\]

where \( \mathbf{d}^* \) is the vector of displacements induced by loads applied at that stage in the control points. By making null the displacement of control points (zero-displacement method), the solution is given by the relation:

\[
\mathbf{e} = D^{-1} \mathbf{d}^* \tag{2}
\]

In this way a configuration close to that on rigid supports, can be achieved at each construction stage and for the 1st theorem of linear viscoelasticity, no stress redistribution due to creep is induced in every partial structure. If the same procedure is followed for the final configuration after the end of erection, by considering all dead loads, definitive prestressing and axial shortening due to creep and shrinkage, the configuration of a beam on rigid supports can be definitely achieved, before the live loads are applied.
3. Stay stressing in suspended cantilever construction of arch bridges

The procedure described above and applied by the authors to cable-stayed bridges has been extended also to arch bridges in order to achieve the desired geometrical configuration of the arch at key closure. It is necessary, as in the previous case, to establish the initial cable forces and stay stressing sequence for all construction stages. As previously seen elastic schemes of partial structures have to be considered by applying the zero-displacement method in order to evaluate the initial cable forces of new stays and the adjustments of the previous stays attached (figure 5). In each scheme the control points at which a null displacement has to be imposed are the anchorage points of stays on the arch segments and vector \( d^* \) of eq. (1) is given by vertical displacements due to dead and equipment loads. Control points of backstays are that of pylon anchorages. Vector \( e \) contains the values of imposed strains to stays, i.e. the values of prestressing forces.

![Figure 5: Elastic partial construction schemes of an arch bridge.](image)

By evaluating the vector \( d^* \) of control points displacements and the influence matrix \( D \) due to unitary prestressing forces into stays, eq. (2) gives the initial cable forces to be applied to new stays attached and the values of adjustments for stays previously attached. Prestress forces have to be applied to a forward staged construction analysis in order to obtain final stress and strain patterns. Even if the desired geometry is achieved, a careful evaluation of maximum bending moments of the cantilever arch has to be performed at each stage.

For cable-stayed bridge the same evaluation can be done by a classical backward analysis, which is a staged construction analysis that, starting from the final scheme completed, deconstructs it by following the inverse order of construction. From this kind of analysis the initial cable forces of stays can be found, but a one-stage stressing procedure is performed. In the case of arch bridges built by suspended cantilever method, backward analysis often cannot be applied. In fact, this methodology gives negative (compressive) values of initial cable forces for some stays, due to the curvilinear axis line, and it cannot be accepted. A negative value of cable forces can be accepted only as adjustment with respect to the previous cable tensile force because it could be necessary a partial release of stay stress in some phases (naturally the total cable stress must be a tensile one in all stages).

Attention has to be put also to the final stress adjustment. It is very important for cable-stayed bridges in order to obtain the final desired profiles of deck and pylon and the desired diagram of bending moments in the deck after superimposed dead loads due to finishing works are applied on the bridge. It can coincide with that of a continuous beam on rigid supports when redistribution of stresses due to creep has to be minimized in concrete cable-stayed bridges, as seen in the previous section.

In the case of the arch bridge, the last adjustment has to be done only to compensate construction errors or unexpected deformations and final bending moment diagram has to be considered in the following stages. The value and distribution of these residual bending moments on the arch are important for service life considerations. Bending moment diagram at the end of construction remains into the arch and adds to the effect of thrust loss due to axial shortening of redundant structures.

During service life effects of moving loads have to be superimposed to those of construction. So, it needs to verify that total bending stresses are always acceptable for the arch. Moreover during and after construction, the influence of temperature has to be evaluated.

Finally, the proposed procedure aims to reduce the number of stay stress adjustments at each stage. In fact, for arch bridges, it is common the re-stressing of all stays in all stages [8], that is a complicated inconvenient, in order to achieve the desired geometry and control the maximum values of bending moments. The proposed procedure avoids it, obtaining the same final result.
4. Numerical application

A numerical application developed on a case-study of a concrete arch bridge with an upper deck is presented. The arch is built by the suspended cantilever method and the proposed procedure of stay stressing has been implemented. Geometric characteristics of the bridge are shown in figure 6. The bridge is composed of two twin arches with parabolic profile, rise $f = 19.20$ m and span length $L = 90$ m. Arches lie on two vertical planes 3.20 m away from each other. They have rectangular cross section 1.0 m wide, 1.80 m high at arch feet, 1.30 m high at key segment.

![Figure 6: Geometric characteristics of the bridge.](image)

The concrete bridge deck is 7.0 m wide and it is subdivided into ten spans with variable length from 13.5 m to 17.5 m. The construction of the two symmetrical half-arches by cantilevering starts after the access spans have been completed. Every arch is built by nine stages, from casting of the first arch segment to provisional stays dismantling (fig. 7).

![Figure 7: Construction sequence of the arch bridge.](image)

Eight segments are suspended to seven cable pairs with equivalent steel diameter $\phi_1 = 0.0298$ m for the first three pairs and $\phi_2 = 0.0353$ m for the successive four pairs. Cables are anchored to a steel pylon and backstays have equivalent diameter $\phi_3 = 0.0462$ m. Segments are 6 m long and they are cast in situ through an auxiliary equipment which has been modelled as a uniform distributed load of $q_c = 20$ kN/m over the last segment built. Operations in each phase are the following ones:
- anchoring of stay at the tip of the last segment built;
- displacement of cast equipment for the new segment, reinforcements preparation and casting;
- stressing of the new stay, adjustment of the previous one and of the backstay, after the segment curing.

When the half-arches have been completed, the key segment, 2.0 m long, is cast in situ and the auxiliary stays are removed. After, the upper deck is built on vertical pillars. Concrete has $f_{ck} = 40$ MPa (C40/50).

Provisional stays are made of high tensile strength steel with $f_{ptk} = 1860$ MPa. Fib Model Code 90 has been implemented in order to evaluate shrinkage and creep effects.

<table>
<thead>
<tr>
<th>Construction stage</th>
<th>Proposed procedure</th>
<th>All stays re-stressed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Creep</td>
</tr>
<tr>
<td>08 (last cantilever)</td>
<td>-0.0052</td>
<td>-0.007</td>
</tr>
<tr>
<td>09 (key closure)</td>
<td>-0.0109</td>
<td>-0.0088</td>
</tr>
<tr>
<td>10 (stays removal)</td>
<td>-0.0095</td>
<td>-0.0041</td>
</tr>
</tbody>
</table>

*Table 1: Maximum vertical displacements of the arch [m].*

Figure 8 shows the deformed shape of the arch at an intermediate cantilever stage, soon before key segment
cast and after stays removal. The structure has been analyzed through a Finite Element model in which a forward staged construction has been implemented. Initial cable forces and adjustment values have been evaluated on partial elastic schemes in accordance with the procedure described above. Results obtained show a very little gap of deformed shape with respect to the desired geometric profile.

Another procedure in which all stays are adjusted at every stage has been implemented too, in order to compare results. Table 1 shows values of maximum vertical displacements registered by arch at the final stages of construction, underlining also the component due to creep. This component is very significant, being it about 50% of the total one. This phenomenon is important for arches in transient construction phases because of cast in situ segments are very sensitive to creep. Moreover in cable-stayed bridges, a final adjustment is done after superimposed dead loads are applied, so strain sensitivity to creep is strongly reduced. From table 1 values it can be seen that the same vertical displacements have been obtained by the two procedures implemented, but with the proposed approach, the target is achieved through less stay stressing operations.

Figure 8: Deformed arch shape during last construction stages.

Figure 9a shows instead the bending moment diagram soon before key segment casting. Maximum negative bending moment is registered at the last anchorage with a value of \( M_y = -1484 \, \text{kNm} \), that is acceptable for the arch section. Maximum bending moments obtained by the procedure with all stays re-stressed are very similar to those obtained with the proposed methodology. Figure 9b shows the diagram for the final stage after key closure and stays removal.

Figure 9: a) Bending moment diagram in the last cantilever stage. b) Bending moment diagram after key closure and stays removal

Figure 10: Axial force diagram in the last cantilever stage.

Maximum axial force during construction stages is instead \( N = -3800 \, \text{kN} \) and the diagram is shown in figure 10. Results are the same for the two sequences analyzed. After key closure and stays removal axial forces grow up and the final arch behaviour is established with maximum value \( N = -5248 \, \text{kN} \) at arch feet and minimum value \( N = -3769 \, \text{kN} \) at key section.

Stay forces show the necessity of partial but significant releasing of stays already attached when the new one is stressed. The same thing has been found by other authors [12]. Graphs of cable forces during construction stages are reported in figure 11. Maximum values of tensile stresses are located to backstays (marked with A
and B). Stay axial forces are all below the limits related to cable section and steel strength. From graphs it is evident that stay forces remain constant after the second adjustment; it means that an adjustment of all stays in all stages could be not convenient.

Figure 11: Stay forces developing during construction stages.

5. Conclusions
A study about stay stressing procedures in construction stages of concrete arch bridges built by the suspended cantilever method has been presented. Comparison with cable-stayed bridge construction has been reported by underlining analogies and differences between the two kind of structures. A methodology which takes into account time-dependent phenomena and optimizes cable forces in order to achieve the desired geometric profile of the arch has been presented and implemented. Numerical results of a case-study have been shown in order to explain and evaluate the effectiveness of the proposed procedure.

References