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An extrinsic interface developed in an equilibrium based finite element formulation

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Abstract

The phenomenon of delamination in composite material is studied in the framework of hybrid equilibrium based formulation with extrinsic cohesive zone model. The hybrid equilibrium formulation is a stress based approaches defined in the class of statically admissible solutions. The formulation is based on the nine-node triangular element with quadratic stress field which implicitly satisfy the homogeneous equilibrium equations. The inter-element equilibrium condition and the boundary equilibrium condition are imposed by considering independent side displacement fields as interfacial Lagrangian variable, in a classical hybrid formulation.
The hybrid equilibrium element formulation is coupled with an extrinsic interface, for which the interfacial separation is zero for a sound interface. The extrinsic interface is defined as a rigid-damage cohesive zone model (CZM) in the rigorous thermodynamic framework of damage mechanics and is defined as embedded interface at the hybrid equilibrium element sides.

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Keywords: CZM, Extrinsic interface, Hybrid equilibrium element, Delamination

1. Introduction

The intrinsic relation between traction and displacement jump in a classic displacement based finite element formulation are well known and several contributions have been proposed by the author: with continuous transition from cohesive to frictional behaviour in a consistent thermodynamic framework (Parrinello et al. (2009), Parrinello et al. (2013)), with independent mode I and mode II fracture energies in Parrinello et al. (2016) and in Parrinello et al.

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(2019), under large displacement conditions in Parrinello and Borino (2018), in the extended finite element method in Parrinello and Marannano (2018) and for the low cycles fatigue in Parrinello, Benedetti, Borino (2018) and in Marannano et al. (2015). The development and use of CZMs in the class of extrinsic formulations, that is with initial rigid behaviour to avoid the unphysical use of penalty terms, has been approached by the discontinuous Galerkin method by Lorenz (2008), Gulizzi et al. (2018), Mergheim et al. (2004) and Nguyen (2014).

In the present paper, an extrinsic CZM is defined by a rigid-damaging interface embedded at the element sides of Hybrid Equilibrium Elements (HEE). The use of finite element formulation based on stress fields which satisfy homogeneous equilibrium equations are known in literature and have been proposed by De Almeida (2006) and Kempeneers (2010) as numerical tool for error estimation compared to classical displacement based analyses. The equilibrium elements formulations has been proposed in hybrid formulation, with independent stress fields on each element by de Almeida (1991), de Almeida (1996) and by Parrinello (2013), and the solution satisfy the equilibrium condition throughout the domain with codiffusive traction at the element sides. Higher order hybrid equilibrium formulation has been proposed in Olesen (2017).

In the present paper the hybrid equilibrium formulation is defined by the element stress fields, defined as quadratic polynomial function, which implicitly satisfying homogeneous equilibrium equation, and displacement polynomial laws at the element sides. The displacement are independently defined for each side as Lagrangian variables enforcing the inter-element equilibrium condition and the boundary equilibrium condition

2. The hybrid equilibrium element

In the present paper, the nine-node triangular hybrid equilibrium element proposed by Parrinello (2013) for two dimensional membrane problems, with quadratic stress field, is adopted. The finite element is represented in Fig. 1, with a local Cartesian reference (x_1, x_2) centred at vertex 1 and the membrane stress fields, which satisfies equilibrium equations and for null body force, are defined by the following quadratic polynomial functions

$$\sigma_1 = a_1 + a_2 x_1 + a_3 x_2 + 2a_4 x_1 x_2 + a_5 x_1^2 + a_6 x_2^2 \tag{1}$$

$$\sigma_2 = a_7 + a_8 x_1 + a_9 x_2 + 2a_{10} x_1 x_2 + a_{11} x_1^2 + a_5 x_2^2 \tag{2}$$

$$\tau_{12} = a_{12} - a_9 x_1 - a_2 x_2 - 2a_5 x_1 x_2 - a_{10} x_1^2 - a_4 x_2^2 \tag{3}$$

which can be represented in compact notation as $\boldsymbol{\sigma} = \mathbf{S} \cdot \mathbf{a}$, where $\boldsymbol{\sigma} = \left[\sigma_1, \sigma_2, \tau_{12}\right]^T$ and $\mathbf{a} = \left[a_1, \ldots, a_{12}\right]^T$ collects the generalized stresses variables.

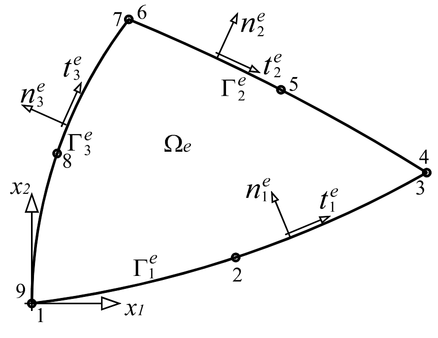

Figure 1: Nine node hybrid triangular finite element

Independent displacement fields are considered as lagrangian variable at each side in order to impose the inter-element equilibrium condition for internal sides between the two adjacent elements or in order to impose the boundary equilibrium condition at the boundary element sides. The quadratic displacement field is considered with three independent nodes for every element side by a classic isoparametric formulation.

The equation of the single hybrid equilibrium element is defined as

$$\begin{bmatrix} \mathbf{C}_e & \mathbf{H}_e \\ \mathbf{H}_e^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{a}_e \\ \mathbf{u}_e \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{q}_e \end{bmatrix}$$

(4)

where the compliance matrix $\mathbf{C}_e$ is symmetric, positive definite and not singular, so that it can be inverted and the relevant degrees of freedom $\mathbf{a}_e$ can be condensed out at the element level.

3. Extrinsic interface CZM embedded at the element side

The HEE formulation can be defined with an embedded interface at the element sides and it is particularly effective for the modelling of an extrinsic interface, which imposes null separation displacement between the positive and negative edges of the interface, up to the initial damage activation condition is attained. The rigid-damage CZM is developed in the rigorous thermodynamic framework of damage mechanics with the following linear relationship between displacement jump $[\![u_j]\!]$ and traction s_i

$$[\![u_i]\!] = \frac{1}{2}\frac{\omega}{1-\omega} A_{ij}^{el} s_j \quad (5)$$

where: $0 \le \omega \le 1$ is the damage variable; A_{ij}^{el} is the interface compliance diagonal matrix. The traction separation law produces perfect bonding with null separation displacement for the undamaged interface $(\omega = 0)$ and an elastic-damaging relation at the damaged condition, with linear elastic unloading.

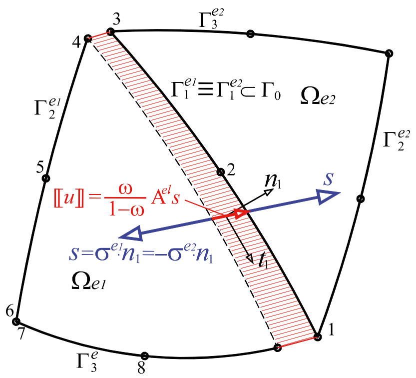

Figure 2: extrinsic interface embedded at the side Γ_1 of element e_1

For an interface embedded at the side Γ_1, as depicted in Fig. 2, the traction vector can be defined as function of the generalized stress variables $\mathbf{a}_e$, that is $\mathbf{s}(x) = \boldsymbol{\sigma}^T \mathbf{n}_1^e = \mathbf{a}_e^T \mathbf{S}^T(x)\mathbf{n}_1^e$, and the equation of the single HEE with an embedded interface is

$$\begin{bmatrix} \mathbf{C}_e + \mathbf{C}_e^{\Gamma}(\omega) & \mathbf{H}_e \\ \mathbf{H}_e^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{a}_e \\ \mathbf{u}_e \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{q}_e \end{bmatrix} \quad (6)$$

where

$$\mathbf{C}_e^{\Gamma}\left(\omega\right) = \int_{\Gamma_1^e} \frac{\omega}{1-\omega} \mathbf{S}_e^T \mathbf{n}_s^e \, \mathbf{A}^{el} \mathbf{n}_s^{eT} \mathbf{S}_e d\Gamma \tag{7}$$

is the additional interface compliance matrix due to the damaged embedded interface, which is identically null for a perfectly bonded interface ($\omega = 0$). In HEE the extrinsic interface can be embedded in the element side by including the additional compliance matrix in the element stiffness matrix, without any additional degrees of freedom.

3.1. Damage activation condition.

In the framework of damage mechanics, the damage activation and evolution is governed by the damage conjugated variable, that is the energy release rate *Y*, defined as function of the traction components in the following form

$$Y = \frac{1}{2} \frac{1}{\left(1-\omega\right)^2} s_i A_{ij}^{el} s_j \tag{8}$$

and the damage activation condition is defined as

$$\phi_d = Y - Y_0 - \chi\left(\xi\right) \tag{9}$$

where Y_0 is the initial damage threshold and $\chi\left(\xi\right)$ is the internal static variable, which governs the softening behaviour. The proposed model is isotropic with the following interface elastic compliance matrix $A_{ij} = \delta_{ij} / k_0$, with δ_{ij} the Kronecker delta and k_0 an isotropic stiffness parameter. The initial damage threshold is defined as the complementary elastic strain energy release rate $Y_0 = s_0^2 / 2k_0$ where s_0 is the interface strength. The internal static variable is defined in Parrinello et al. (2009) and produces bilinear response both in tensile test and in shear test. Logarithmic response can be obtained with the internal variable defined by Borino et al. (2009).

4. Numerical simulation

The mode I DCB delamination test has been numerically simulated by the propose HEE formulation with extrinsic embedded interface. The thickness of the two partially delaminated legs is *t* = 5mm and the initial delamination length is *a* = 50mm. The known delamination surface is modelled by the extrinsic interface embedded at the element sides. The whole specimen is discretized by a unique mesh and the delamination surface is not specifically discretized, but it is simply defined as geometric locus in the input file and it does not requires additional degrees of freedom.

The specimen has been discretized by a mesh of 400 HEEs and 3067 nodes. The restraining condition of spurious kinematic modes developed by the author in Parrinello (2013) has been imposed at four corners of the discretized domain. The bulk is modelled as isotropic and linear elastic with Young modulus $E = 111900 N / mm^2$ and Poisson ratio $\nu = 0.2$ (standard parameters for Carbon/epoxy composite material). The plane stress two-dimensional numerical simulation has been performed under displacement control condition and the corner nodes of upper and lower laminas are constrained, with the upper one subjected to imposed increasing vertical displacement. The full unloading is performed after delamination.

The fracture energy of the embedded interface is $G_I = 1N / mm$ with tensile strength $s_0 = 10N / mm^2$. The results of the numerical simulation are plotted in Fig. 3 in terms of imposed displacement *u* and reaction force *F*, and the results are compared to the analytical solution obtained in the beam and fracture mechanics (BFM) theory. The maps of the three stress components obtained by the numerical simulation, for the imposed displacement $u = 4.5mm$, are plotted in the Figs. 4a,c.

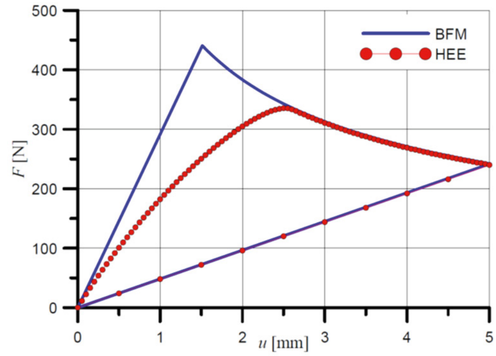

Figure 3: results of the numerical simulation in terms of imposed displacement u and reaction force F compared to the analytical solution obtained in the beam and fracture mechanics theory (BFM)

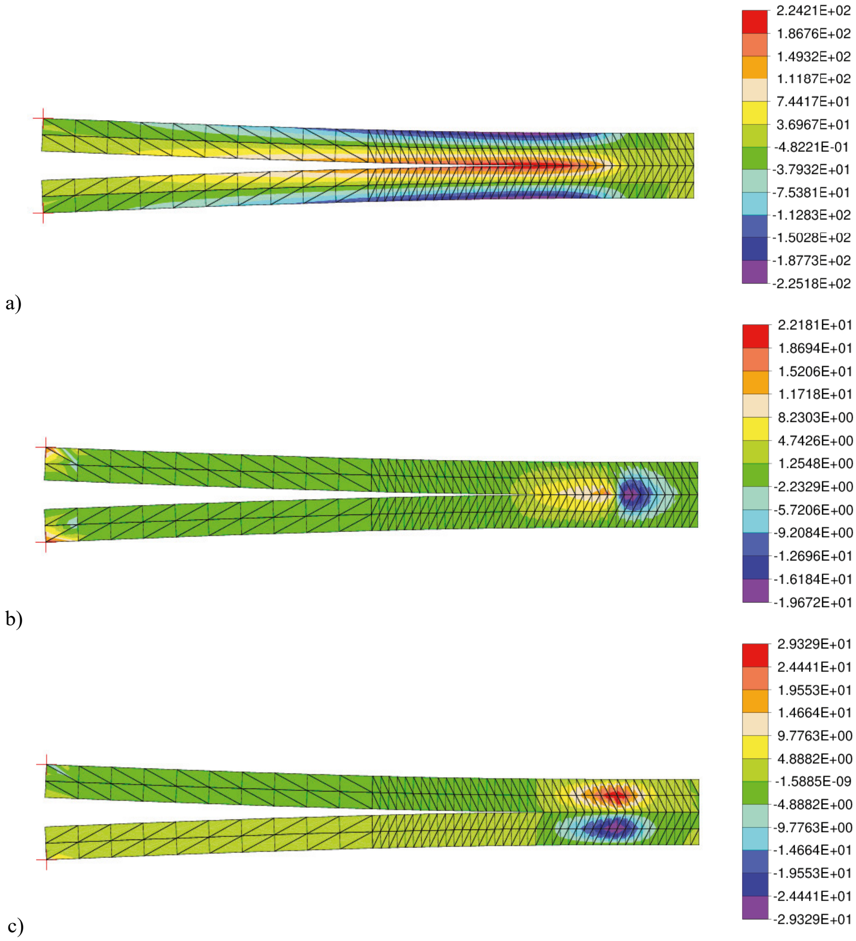

Figure 4: Maps of stress of the numerical simulation at loading condition $u = 4.5mm$: a) normal stress σ_x ; b) normal stress σ_y ; c) tangential stress τ_{xy}

5. Conclusion.

The hybrid equilibrium formulation satisfying the strong equilibrium condition throughout the element domain and with codiffusive tractions at the element sides has been developed with extrinsic embedded interface for the analysis of delamination problems. The extrinsic interface can be modelled on every element side, without additional degrees of freedom. Activation of the interface is driven by the a damage initiation condition, which depends on the traction components at the element sides. The proposed formulation has been implemented in an open source finite element code, the DCB delamination test has been numerically simulated and the results are compared to the analytical solution.

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