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Network Structure and Optimal Technological Innovation

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The role of networks in the development of technological innovation has attracted much theoretical and empirical attention. Yet much of the work has explored the role of undirected and homogeneous networks. In real cases many networks are directed. The flow of information, benefits or observations is directed from one node towards another node. Real networks are also heterogeneous: few nodes have high degree while many others have small degree. In this paper, we report on the results of an evolutionary agent based model in which a group of agents, in our case firms, collectively search a complex (rugged) technological landscape and observe each others solutions with different frequencies through different observation networks. Two groups of networks are considered in the analysis, the first group comprises undirected networks that vary in terms of efficiency while the second group comprises directed networks that vary in terms of homogeneity of node degree. We find that collective innovation (exploration of a technology landscape) improve average fitness over independent search because information about good innovations can diffuse faster through the network at an early stage. Moreover, we find that efficient networks outperform inefficient ones in the first stages of search, but some of the inefficient ones do marginally better in later stages. Finally, we find that degree-homogeneous and undirected networks achieve better fitness on average than heterogeneous and directed ones. We explain these results from the perspective of system-level technological innovation and in light of the theory of technological lock-in. We conclude by discussing implications for technological innovation and possible extensions.

Keywords: Fitness landscape, network efficiency, observation probability, degree-heterogeneity, technological lock-in

1. Introduction

Which network structures are best for promoting system-level technological innovation ? The importance of this question stems from two facts: first, technological innovation is a main driver of productivity and economic growth [51], and second the process of technological innovation is increasingly stylized as a collective phenomena of networked agents [14, 45, 54]. Similar to other problems that arise in economic contexts, technological innovation is rather a complex problem in the sense that it typically requires constrained optimization along multiple dimensions [18, 22, 33, 40]. According to [22], complexity theory is an important and promising field to the study of technological innovation.

In this regard, two approaches to modeling, namely network science and evolutionary search on fitness landscapes, have captured particular attention.

Fitness landscapes are used to model the space of solutions in a complex problem. Generally speaking, a fitness landscape can be defined as a mapping from a multidimensional space, which is usually referred to as the problem space, solution space, or technological graph, to some measure of goodness or fitness. When faced with a complex problem, agents are assumed to be located somewhere on the fitness landscape of that problem. Finding the optimal solution on the fitness landscape requires a balance between local exploitation, which consists in small incremental moves on the landscape, and extended exploration, where long distance search is required. Achieving balance between exploration and exploitation is challenging, because too much exploitation can result in sub-optimal performance in the long run and too much exploration is costly and forgoes the benefits of exploitation in the short run [37]. In many cases, the exploration-exploitation trade-off is further complicated by the presence of multiple problem solvers or innovators on the fitness landscape. The collective aspect of technological innovation justifies the use of network science. The presence of multiple searchers can be beneficial because innovators might benefit from the experience of others, but on the other hand it can be harmful if learning leads the collective to converge to a sub-optimal solution. In this paper, optimal technological innovation will be defined as a process which avoids the trap of technological lock-in. A lock-in is a form of path-dependency in which a system of innovating agents adopt or develop a certain technology not because of its superior performance, but rather because of the structure of interaction in the system that leads to an early convergence to such sup-optimal solution [1].

By adopting a collective view on technological innovation a question arises : why would firms search a fitness landscape collectively rather than doing it singly and seize all the related benefits ? In the extant literature, there have been several interpretations of why and how agents may decide to search or innovate collectively in certain economic contexts. In one approach, it has been recognized that innovating firms may not be able to appropriate all the benefits arising from their innovations, so that firms do not have incentives to carry out the scientific research needed to innovate singly [42, 55, 60]. One reason for such a scenario could be found in the presence of a weak patent system that does not protect technological innovations. As a consequence of this, one possible solution is cooperative R&D where a multitude of firms formally or informally share knowledge concerning the development of a certain technological innovation [30]. Similarly, when the knowledge base of an industry is complex, then it may become more convenient to collaborate and share the costs and benefits of complex R&D projects [44, 56]. Several empirical studies have shown that there are advantages for market participants to share information in many situations. In this cases, those who refuse to share any knowledge might end up excluded from the settings in which these exchanges of knowledge take place [29, 46, 47, 49, 60]. Reference [53] found that firms that share knowledge with competitors had better performance than those who do not. Creating shared technical standards can be another reason that is motivating firms to innovate collectively [14]. Another study [62] showed that knowledge sharing has positive effect on innovation capabilities. Finally, an important reason that can justify information sharing is the situation in which a set of roughly equivalent market agents are all trying to solve the same problem. In this case there is an increase of the incentive to know how the others are approaching the same problem. This situation can be encountered in real life like when doctors and professors are working on the same problem or state governments are formulating their policies [36].

From the modeling perspective, there exists a wide range of theoretical models of collective tech-

nological innovation. Studies modeling technological innovation as a collective phenomena rely on theories and concepts like imitative innovation [41], organizational learning [37], knowledge barter models [10], collective invention [9], private-collective invention [43], alliance networks [27], networks of innovators [13], and open innovation [12].

One particular line of research on complex problem solving by collectives has concluded that the efficiency of networks, defined as the speed at which networks disseminate information, can have a relevant effect on the performance of the collective of agents for the solution of those problems that require extended exploration [2, 36, 39]. Empirical evidence has shown that shorter path lengths is positively correlated with system-level technological innovation [17]. Although these studies have spurred an interesting debate on whether network efficiency can improve collective performance, little has been done regarding the role of other network features like degree-heterogeneity and edge direction. If a network of innovators has a heterogeneous degree distribution, this could mean that high-degree nodes are more influential, therefore increasing the chance that the system will be driven to a sub-optimal solutions if influencers perform poorly. In fact, in the presence of influential agents, a particular form of communication or observation may emerge according to which the majority of agents acquire their information from a small subset of the group. Work on this phenomena started with the study of [32, 35] who investigated the effect of personal contacts and media on consumer and voting decisions with regard to products, movies and fashion. Their main finding was that personal contacts play a crucial role in disseminating information which in turn influence individual's decisions and choices. The study, which relied on a sample of 4000 individuals, revealed that 20 percent of the whole sample was the source of information for the rest. In a similar study, [16] identified 20 percent of their sample of 1400 individuals as the main source of information about food items, household products, and drugs. Although the role of opinion leaders in the adoption of technological innovation has been to some extent investigated in literature [7], little has been done to explore the role of heterogeneous and directed networks in the process of searching a technological landscape.

Building on two of the above lines of research, namely the role of network efficiency and heterogeneity in the process of technological innovation, this paper reports on the results of an evolutionary agent based model in which a group of firms collectively search a complex (rugged) technological landscape and observe each others solutions with different frequencies through different relationship networks. Two families of networks are used in the simulations, the first family includes undirected networks which vary in terms of efficiency, i.e. average path length. The second family includes directed networks which vary in terms of degree heterogeneity. We show that network efficiency and heterogeneity of agent's degree are important factors influencing the average performance of the system.

The remainder of this paper is structured as follows. In Section 2, we provide an overview of the model, the technological fitness landscape and the behavioral rules of innovating firms. Section 3 offers a detailed discussion of the results and their implications. Finally, conclusions and a research agenda are discussed in the last Section.

2. The Model

First we give a schematic description of our model, network structure, details of the search dynamics and observation behavior of agents. Agents in our model are firms and all firms are assumed to face the same initial problem : finding the optimal technological innovation with the highest fitness in a given techno-

logical space (or fitness landscape) without knowing the structure of this technological space. Solutions are called technological innovations (or simply innovations) and the fitness landscape represents the space of all possible technological innovations.In our setting, the fitness landscape has one global peak and the optimal technological innovation would be to find that global peak. We assume that there is a network of relationships between firms such that neighboring firms (two linked nodes) can observe the innovations according to their position in the network. Specifically, for undirected networks the observation can be bidirectional whereas for directed networks the observation will be possible according to the direction of the arc (or arcs) connecting the two nodes. The fitness of a technological innovation is assumed to be related to the profitability of an innovation such that the higher the fitness the higher the expected profits.

The assumption of observing neighboring firms is justified by the fact that all firms face the same initial problem which require them to search for an innovation (a solution) in the fitness landscape. In settings where agents can observe one another's choices, then it would be a rational decision to learn from one another [3, 23, 59]. It is worth noting that in this model we are not concerned with the costs of observation or absorptive capacity of firms (see [9]), rather we focus on the role of firm network structure on performance. Performance will be measured as the average fitness of the innovations of all firms at one time step and by the number of firms who find the global peak. Next we turn to illustrate our model in more detail.

2.1 *The Fitness Landscape*

Technological innovation is a complex problem. A problem is said to be complex if at least two conditions are satisfied: first, the solution of the problem is made up of several components where each component can assume one of different values or attributes. For example, in constructing a jet fighter, several elements like speed, weight, maneuverability, firing power, and protective armoring need to be considered. The second condition is the existence of non-linear relationships between the components of a solution such that the total evaluation of the goodness (fitness) of a solution is not simply the sum of the goodness of the individual parts. Non-linear dependencies can act as a constraint on achievable fitness. For example, increasing the weight of the jet fighter will decrease its maximum speed. The presence of multiple dimensions that need to be optimized implies that there are different solutions to choose from, and the presence of complex dependencies between problem components imply that the space of solutions admits many local peaks with different fitness values. Because of these two features, complex problems are usually modeled using the theory of fitness landscapes [36].

A variety of fitness landscape models have been used to model complex problems. One popular model is the NK model proposed by Stuart Kauffman to describe the effect of epistasis on the structure of gene fitness landscapes [36]. Epistasis refers to the effect that one gene has on another. In the NK model, N represents the dimension of the problem and K controls the level of interdependence among the components the N components. If k=0, then interaction between components is absent and, therefore, each component contributes independently to the fitness of a given solution. On the other extreme, if k=N, then every component interact with all other components and the fitness contribution of one component influences the fitness contribution of all the other components. By varying K, one can control for the ruggedness (complexity) of the fitness landscape. Other popular models used in literature include the random field models, which are random functions of multidimensional parameters which can be simulated using different correlation lengths and probability distributions. [33]. Although these

models have been widely used in the description of complex systems, we follow similar steps along the line suggested by [39]. In our approach, the goal is not to replicate standard fitness landscapes like the NK or random field models which are mostly artificial, rather we aim at capturing more qualitative features of real world problems. The feature which we want to capture is the presence of multiple local peaks and one global peak, where the system will be said to experience a technological lock-in if it converges to one of the local peaks. We note that the focus is not on the fact that there is a unique global optima but rather our aim is to create a context where there is at least one solution that is significantly better than every other solution. To this end, the landscape is constructed in such a way to have many peaks with values between 100 and 5000 and a single peak with fitness value of 10000 which corresponds to the global optima. In this way, the resulting landscape, which has a global peak, is complex to navigate since it requires extended exploration in order to find the optimal (global) innovation. We note that firms are assumed to lack information about the structure of the landscape and the existence of a global optimum.

The space of technological innovations (fitness landscape) is modeled as a 2-dimensional regular grid of size 1000 x 1000. The assignment of fitness values to the grid points is done in four stages:

1- The grid is divided into 100 x 100 smaller and equally sized blocks, resulting in a total of 100 blocks.

2- Evenly spaced values within the interval [100,5000] are generated with spacing between values of 50. The result is a list of 99 numbers, to which we add the value 10000 that will be used as a global optima, i.e. [100,150,200,250,...5000,10000].

3 -To each of the blocks obtained in step 1, we assign randomly and without replacement a number from the generated values in step 2 to the point at the center of each block.

4 - We assign values to all other coordinates inside each block by following an iterative process where nodes of distance 1 from the central point of a block are assigned fitness values equal to a fraction of the fitness assigned to the center (see step 3) , nodes at distance 2 are assigned a smaller fraction of the fitness of the central node and so on , resulting in a fitness landscapes which has many peaks and valleys and high variance in the fitness values of local peaks.

A schematic illustration of the fitness landscape is shown in figure 1. We point out that in this paper the fitness landscape is fixed, i.e. does not expand, and static, i.e. the fitness values of innovations does not change over time.

2.2 *Network Configuration*

The key question of this paper is the impact of network efficiency, degree heterogeneity edge direction, and observation probability on the average performance of the innovation system. For this reason, in our model, firms are located on a network which defines the interaction structure among firms. We consider only networks that have a single component (i.e. there is at least one path connecting each pair of nodes in the network). In the simulations presented, we examine two basic types of network: a family of undirected networks which vary in terms of efficiency, and a family of directed networks which vary in terms of heterogeneity of degree. In the undirected networks, observation can be done in two ways

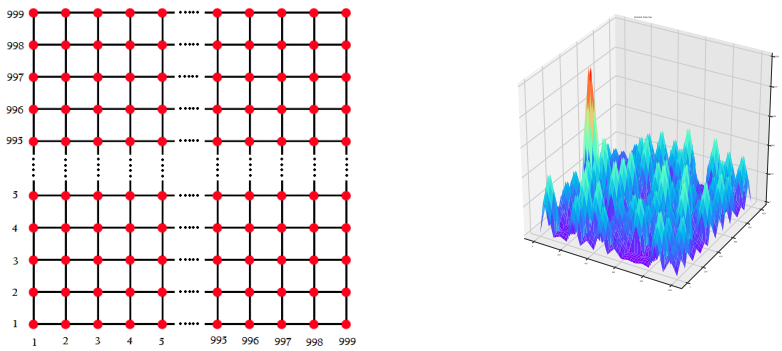

FIG. 1: The Fitness landscape. The fitness landscape is generated by dividing a 1000 x 1000 grid (left panel) into 100 blocks of size 100 x 100 and assigning them different fitness values. An example of resulting landscape is shown in the right panel of the figure.

meaning that node A can observe neighboring node B and vice versa. In directed networks observation is one way, meaning that if A has a directed link towards B and B doesn't have a directed link to A then only A can observe B but not vice versa. [24].

In the investigation of the role of network efficiency, we consider eight networks that are analog to those proposed in [39]. Specifically, we generate eight networks having 64 nodes and a fixed degree of 3 for each node, but with different structural properties related to (a) closeness centrality [20], (b) betweenness centrality [19], (c) clustering coefficient [48] and (d) network constraint [6]. It has been shown that such network attributes have an influence on the average path length of the network, which has been used as an indicator of network efficiency [36, 39]. Therefore, by manipulating these network attributes, we are able to obtain networks of the same size, average degree, and number of edges but with different levels of efficiency.

In constructing the networks, we use the following algorithm. Firstly we create random networks with 64 nodes all of them with fixed degree 3, therefore with a number of edges of 96. Secondly, for a sufficiently large number of iterations, we choose randomly two edges per iteration and perform a degree preserving double edge swap. A double edge swap deletes two randomly chosen edges x-y and z-w and creates the new edges x-z and y-w. With this rewiring approach besides keeping the node degree fixed, we ensure that the graph remains connected. During our search for a network characterized by maximum value of a given network indicator only rewiring procedures that increased or decreased one of the properties of the four network metrics mentioned above are accepted. The properties examined are the average value, the variance, the maximum value and the minimum value. For example, the network with "Maximized Maximum Closeness" is the one where the most central node (in closeness terms) is as close to other nodes as possible. With this procedure, we control the degree of efficiency of the network, obtaining eight networks analog to those investigated in [39]. Figure 2 shows the undirected networks used in the simulations of our agent based model. The parameters of these networks are summarized in

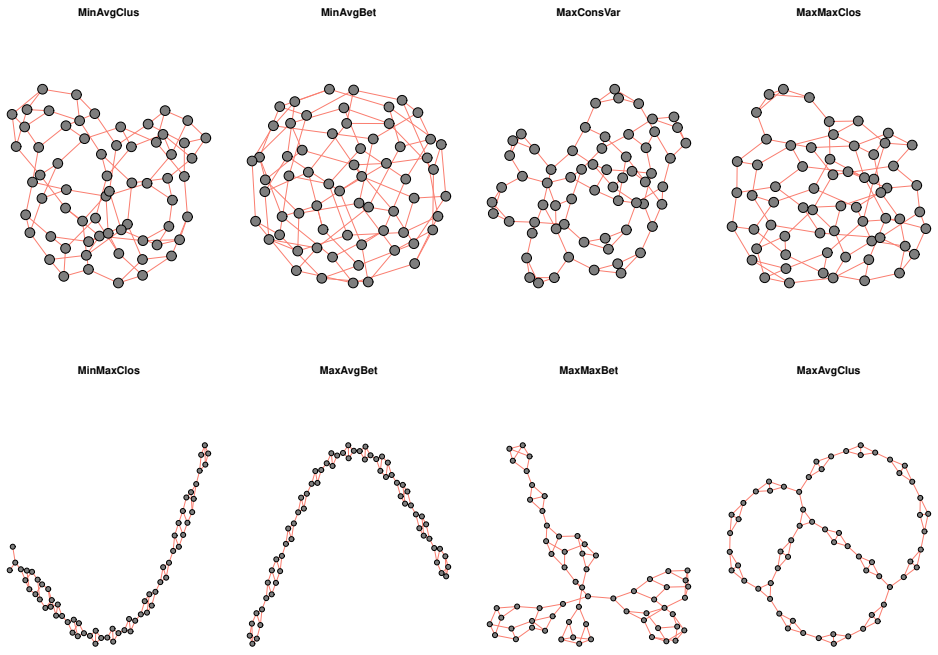

FIG. 2: The networks used in simulations investigating the role of network efficiency. The panels of the upper row shows networks which have high efficiency in terms of path length. From left to right they are characterized by minimized average clustering, minimized average betweenness, maximized variance in constraint and maximized maximum closeness. The four networks in the panels of the lower row have low efficiency in terms of path length. From left to right they are characterized by minimized maximum closeness, maximized average betweenness, maximized maximum betweenness and maximized average clustering.

the Table provided in the Appendix.

To examine the role of degree heterogeneity and one way observation, we also generate four networks with 64 nodes and a total number of directed edges equals to 96. The procedure used in the generation of these networks is the following: firstly we fix the number of hubs (H) we want to have in the network and we connect them with each other by directed links with probability 0.5. Secondly, with probability 0.95, we assign a directed link from any of leaf nodes to a randomly chosen hub, otherwise with probability 0.05 a link is assigned to a randomly chosen node of the periphery of the network. During the generation of the network, we make sure that the network is weakly connected and nodes with degree k can have an outgoing link only towards nodes with degree greater than or equal k. For every value of H, we generate many realizations of these networks. The final choice is based on the value of the degree variance. As shown in figure 3, the four selected networks present different levels of degree homogeneity. It could be noticed that by increasing degree heterogeneity, the resulting networks resemble networks with a core-periphery structure where the core is densely connected and its nodes have high degree while the periphery of the network is made of sparsely connected nodes which are mostly linked to the core [11]. Figure 3 presents the four directed networks used in the paper.

FIG. 3: The networks used in the simulations of firms located in directed and heterogeneous networks. By increasing the variance of degree the network assumes a structure progressively closer to a core periphery network.

2.3 *Simulation procedure*

The aim of this paper is to explore the role of the network structure on the overall performance of searching firms. In particular, we investigate whether collective search, conducted by using observational learning with some probability, improves the performance of the system with respect to independent search. We also investigate whether efficiency, degree heterogeneity, and edge symmetry of networks have an effect on the collective performance of firms. In addition to network structure, we also examine the role of the amount of observational learning, quantified through the probability p of observing a neighbor performance.

In studying the behavior of complex systems, agent based simulations and computational modeling are powerful tools, allowing the modeler to examine the behavior of the system for different settings of its features. Our aim is to develop an understanding of the role of network efficiency, degree heterogeneity, edge direction, and observation frequency on the optimization performance of the system of networked economic actors.

Each simulation is structured as follows: We assign random initial solutions to the 64 firms in the fitness landscape. This guarantee that the system has a high level of initial heterogeneity in the adopted solutions. Solutions are labeled in the form of 2-tuples, for example, one solution could be (10,20), another (100,500), and so on. We then systematically vary the observation probability p. Specifically, we consider the following values linearly spaced [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]. At each round of the simulation, we assumed that firms choose randomly between two strategies: (a) with probability p the firm observes the innovation of a randomly chosen neighboring firm, and if the fitness value of the innovation of the observed firm is higher, the observing firm will move on the position of the fitness landscape of the neighboring firm, otherwise it will stay in its position [1]. (b) with probability $1-p$ (i.e. probability of autonomous search), the firm conducts independent search by visiting a random neighboring innovation on the fitness landscape. For example, if the actual position in the fitness landscape of a firm is (50,50), then the possible neighboring innovations are (51,50),(49,50),(50,49),(50,51) [2]. If the fitness value of the newly visited innovation (F_{New}) is greater than the fitness of the previous innovation F_{Old}, then the firm will move on the fitness landscape to the new innovation with probability $1-\exp^{\frac{-\Delta E}{T}}$, where ΔE is simply F_{New} - F_{Old} and T is a temperature parameter that controls for randomness in accepting the new innovations. In this paper we report results obtained for $T = 300$, but we also run simulations with $T = 100$ and $T = 10$. We note that when $p = 0$, then firms are searching independently and don't observe each other, which is the equivalent of having no network in which observation can take place. On the other hand, when $p = 1$, then only observation takes place and search on the fitness landscape happens by observing neighboring firms and moving to their location if they have fitter solutions. Each simulation is run by performing 1000 rounds with asynchronous update. Results are reported by averaging the dynamics across 200 simulations. For each simulation, we use a different set of initial conditions of firms (positions in the fitness landscape) and these 200 sets of initial conditions are the same for all networks.

[1] We note our model assumes implicitly that knowledge about the fitness of observed neighbors has moderate level of complexity such that it can flow easily from one firm to another [52]

[2] In technological innovation studies, the behavior of local search for new innovations, i.e., trying combinations of existing innovations which are to somehow similar to what an agent already use, is treated as a stylized fact. see [50] for a discussion of stylized facts about technological innovation

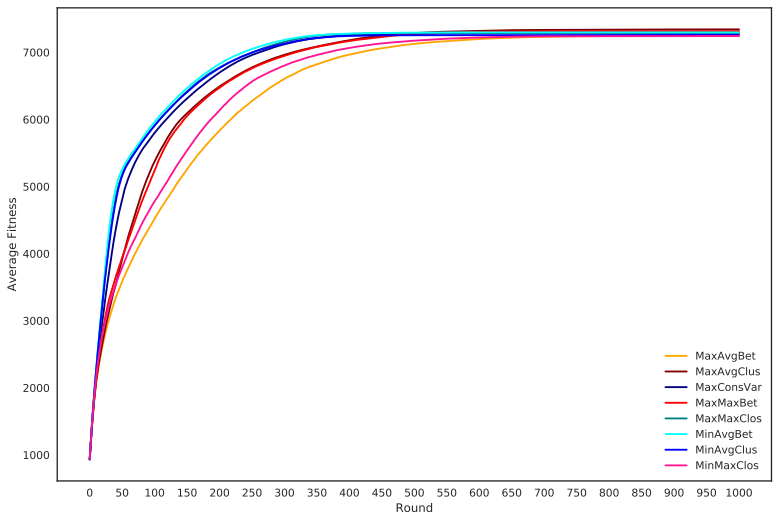

FIG. 4: Average fitness for all eight undirected observation networks. The observation probability is $p = 0.3$.Efficient networks are blue lines and inefficient ones are red lines.

3. Results

3.1 *Efficient vs. Inefficient Networks*

3.1.1 *Efficient Networks Perform Better than Inefficient Networks* As a first result, our simulations show that efficient networks achieve higher average fitness than inefficient ones in the short-medium term, but inefficient networks do marginally outperform in the long run for all observation probabilities p greater than 0. Figure 4 plots the average performance over 200 simulations of the efficient and inefficient networks over time for one value of the observation probability ($p = 0.3$). Figure 4 shows a clear pattern. When information circulates more efficiently, firms are able to get information about better local optima faster and at an earlier stage than when they are located in inefficient networks.

By circulating information about the best innovation more rapidly, efficient networks allow firms to position themselves close to high fitness solutions quicker than in the case of inefficient networks. However, given that the landscape which we use requires extended search to find the global peak, efficient networks are more likely to experience a convergence to sub-optimal solutions. In other words, efficient networks might induce firms to experience a technological lock-in [1]. On the other hand, inefficient networks do not circulate information quickly given the higher average path length and therefore they have more time to conduct extended search which would increase the chance of finding the global peak more often but at a later stage. Figure 5 shows this behavior clearly. In the left panel, we report the average over 200 simulations of the number of agents who are located at the local peak at each round, and on the right panel we show the average of the maximum fitness value attained by a firm (the best

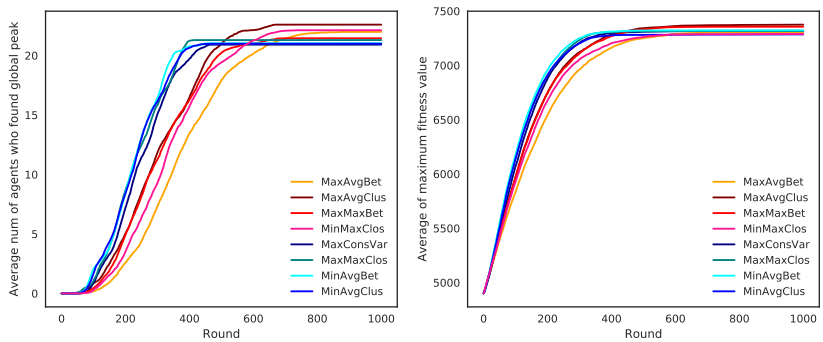

FIG. 5: Simulations of firms' search located in all eight undirected observation networks. The observation probability is $p = 0.3$. Efficient networks have colors in the blue region of the color spectrum whereas inefficient ones have colors in the red region of the color spectrum. Left panel: average number of agents finding the global peak. Right panel: average value of the maximum fitness value discovered by firms.

performer) at each round. From figure 5, it could be seen that efficient networks allow firms to find the global peak more often in the short-medium term, but in the long run more agents on average find the global peak in inefficient networks. We note that after checking the fitness values attained by firms in the last round (round 1000), it was found that approximately half the time (100 simulations) most firms were at the global peak, while in the other half of simulations no firm could find the global peak. This can be attributed to the fact that some initial conditions are more favourable for finding the global peak. Similar behavior was observed for the maximum fitness per round as shown in the right panel of figure 5.

To statistically validate these results, we conduct a multiple hypothesis testing to examine the difference between the average performance of the firms located in the different eight networks at rounds 50, 75, 100 and 200. The total number of t-tests we perform is equal to 112 (= 8x7/2 * 4). In our multiple hypothesis test procedure, we adopt the customary statistical threshold of $\alpha = 0.05$. To correct for familywise error, we apply the Bonferroni correction [58] by dividing α by 122, resulting in a corrected threshold equal to 0.0004098. Figure 9 in the appendix shows the results of the statistical tests. These results show that the average performance of efficient and inefficient networks are in general different, however with some exceptions. For rounds 50 and 75, the t-tests comparing efficient and inefficient networks detect several statistical differences. However, when considering round 100 and 200, we gradually start to observe that the t-tests do not reject the hypothesis that the average values are the same. However, it is worth noting that this might be due to the fact that the Bonferroni correction is the most stringent methods in multiple test corrections. In fact, it is well known that this correction guarantees a very low number of false positives but this high precision is often achieved at the cost of having a large number of false negatives.

Finally, our results show that the performance at the system level improve on average when we decrease the value of the parameter T. For T=100 and T=10, all networks achieve better average fitness and the global peak is found more often. This can be interpreted by considering that lower values of T allow autonomous searchers (which happens with probability 1-p as we showed in section 2.2)

to accept new solutions more often than in the presence of higher values of T. When T is high then the average performance is mainly driven by firms observing each other and therefore the role of the network structure is more important. On the other hand, when T is low, then autonomous search contributes more to the average performance and the role of the network becomes less pronounced. One would expect that in real life firms have limited resources and therefore it is unlikely that a firm invests heavily in both autonomous search as well as observing other innovating firms. To achieve a balance between autonomous vs. observational search, firms may be constrained by several factors like (i) existence of technological paradigms [15] which compels firms to check the prevailing innovative trend, (ii) absorptive capacity which limits firm's understanding of other firms' solutions [8], and (iii) dynamic capabilities [57] which limit firm's ability to adapt to new solutions.

3.1.2 *Collective Search Performs Better than Independent Search* Regardless of network efficiency, our results demonstrate that search with observation probability greater than zero produces higher average fitness in the medium and long run than autonomous search. Figure 6 plots the average performance of searches performed by firms when they are located in a network characterized by a maximized value of the maximum closeness for several different values of the observation probability. What figure 6 shows is that for moderately high values of the observation probability (as, for example, 0.5,0.6, and 0.7), the system achieves higher performance in the short and medium run. On the contrary, for lower values (0.1,0.2, and 0.3) the system achieves higher performance in the long run. When observation probability is moderately high, then firms will be copying each others' solutions more frequently, resulting in a quick improvement in the short and medium run, but increasing the likelihood that the system will be unable to find valuable solutions that require more exploration. This is the reason why simulations with lower observation probabilities do perform poorly at the early stage, but later they achieve higher performance which is due to the fact that they are given more time to explore the space of solutions.These simulations tell us that in order to achieve an optimal performance at system level, one needs to guarantee a balance between how often firms observe each other and how often they search in isolation.

3.2 *Homogeneous vs. Heterogeneous Networks*

3.2.1 *Degree heterogeneity has a negative effect on average performance* Simulations for directed networks of firms with different degree show that the increase of degree heterogeneity has a negative effect on the average performance of the system. As we mentioned in section 2.2, degree heterogeneity is measured by the variance in degree of nodes, where higher variance indicates more heterogeneous networks. Figure 7 illustrates the behavior of the system for four directed networks (simulations are performed for observation probability $p = 0.3$). As expected, the network with degree variance 4 has by far outperformed all other networks with variance 10, 18 and 32 respectively. This behavior shows that by introducing heterogeneous nodes and one way observation structure, we reduce the sources of information available to the imitating firms, thus resulting in worse performance. The inferior performance can be attributed to the fact that the location of hubs in the technological landscape constrains their search for technological improvements and as a consequence limits the improvements of the imitating firms. If the initial position of hubs is poor (in fitness terms), then heterogeneous structures of firms with one-way observation are very likely to drive the system to perform poorly and increase the chance of a technological lock-in. Concerning the finding of the global peak, results show that, contrary to the family of efficiency networks, the family of directed and heterogeneous networks are not able to find

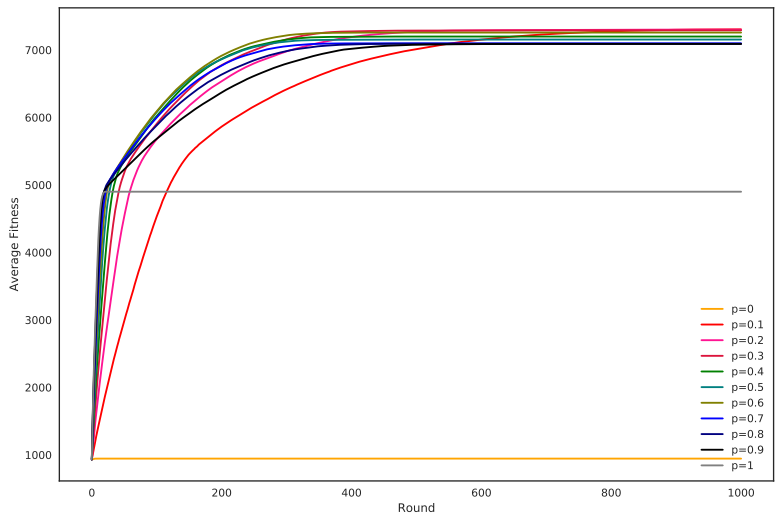

FIG. 6: Average fitness for a search performed by firms located in an undirected network with a maximized value of the maximum closeness. Different lines refer to different values of the observation probability p. These values are ranging from 0 to 1 in steps of 0.1

the global optima in all cases and for all observation probabilities when $T = 300$. This however is not the case for lower values of the parameter T. Our results show that by setting the value of T as $T = 100$, some firms searching in all four networks are able to find the global peak few times (in average 1 or 2 times). By setting $T = 10$, the system performs better and the global peak is found more frequently by firms located in all four networks.

To test the statistical validity of these results, we conduct a series of t-tests as we did in the previous section. The results of these tests are reported in figure 10 in the appendix. This time, it could be noticed that the mean value of the fitness obtained by firms located in different networks is statistically significant only when we compare the network having degree variance 4 with the other three networks.

It could be noticed in figure 7 that the average performance of the system is much lower for all four networks than it was with the degree regular and undirected networks examined in the previous section. To check the role of directed versus bidirectional observation, we repeat the same simulations on the four networks in figure 3 by ignoring edge direction and keeping the same network structure. Results for these simulations are shown in figure 8. Figure 8 shows that the average performance for all networks is higher than the case with directional observation (figure 7). When the structure of observation between firms is directional, then it is more likely that the system collectively misses improvement opportunities because of the lack of mutual observation. In the real world, factors pertaining to social and economic pressures can lead to the formation of heterogeneous networks [28]. In a recent study, authors of reference [25] called this phenomenon *the law of the few*. The name originates from the fact

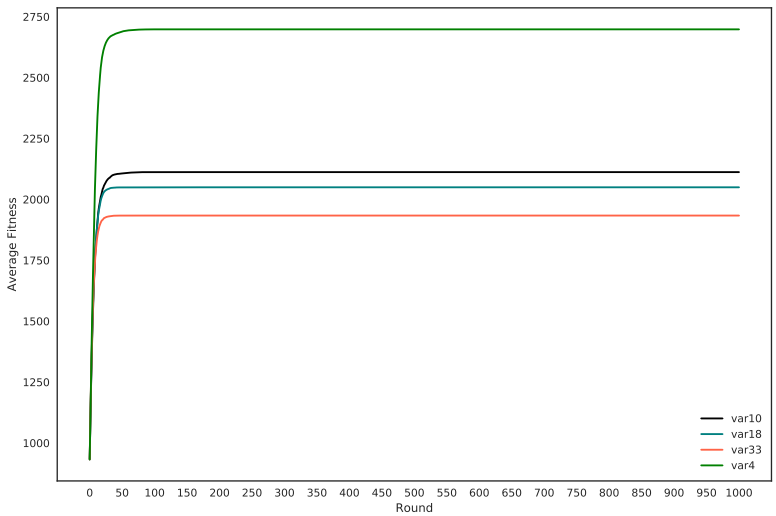

FIG. 7: Average fitness for a search performed by firms located in the four networks with heterogeneity in the degree discussed in the text. Edges of the network are directed and observations of the firms is directional. Each network is labeled by the value of the variance of the degree. Observation probability is $p = 0.3$

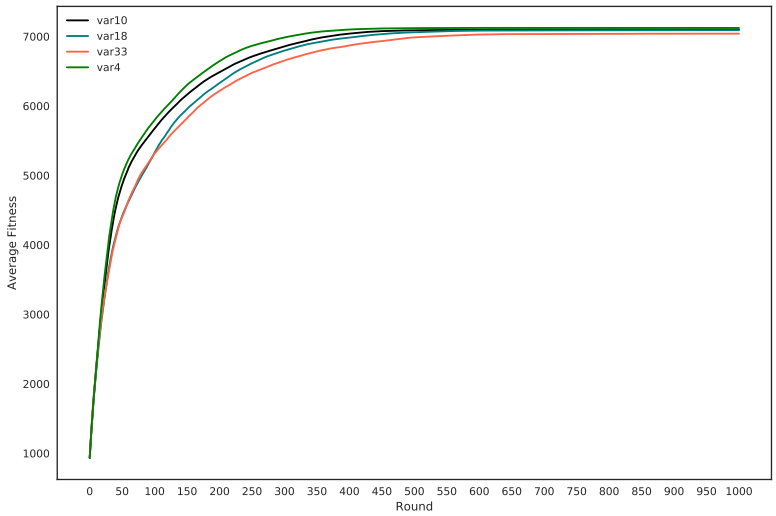

FIG. 8: Average fitness for a search performed by firms located in the four networks with heterogeneity in the degree discussed in the text but ignoring edge direction. Edges of the network are undirected and observations of the firms is bidirectional. Each network is labeled by the value of the variance of the degree. Observation probability is $p = 0.3$

that the majority of agents acquire most of their information from a small subset of other agents. In [25], the authors ask the question of whether the law of the few can emerge in settings where there are no differences between individuals. To answer this questions, they proposed a game where individuals are able to either acquire information personally or establish directed edges with other individuals to access their information. The main finding of the paper is that every equilibrium of the game exhibited the law of the few. The resulting network has a core-periphery structure. Core agents acquire information personally, while periphery agents acquire their information by using their links with the core. One economic reason of why the law of the few can emerge in firm networks relates to the role of past experience [5]. The knowledge base of firms is path dependent , therefore older firms can be in better position that new incumbent ones. The universality of such claim however cannot be confirmed, and because of the importance of such theory, an interesting debate in the technological innovation literature spurred around the idea of small vs. big innovators. This debate is usually referred to as the Schumpeter/Arrow debate [26]. For Joseph Schumpeter, concentrated markets with big companies have more resources to invest in RD and therefore are in better position to innovate. On the other hand, Arrow argued that big companies may have incentives not to innovate and therefore competition should be the main driver of innovation. Although in this paper we assumed that firms have the same search capability, our results highlight that in a market characterized by heterogeneous observation structure that favors the core over the periphery, the system-level performance is not optimal and more likely to experience technological lock-in.

Concerning the effect of observation probability, networks with degree heterogeneity showed less clear distinction between which observation probabilities are optimal for the system performance. The Results shows however that collective search, characterized by values of the observation probability greater than zero ($p > 0$), performs always better than independent search ($p = 0$). However, the difference in performance between different values of the observation probability is less pronounced than the case with the undirected and regular networks examined in the previous section.

4. Conclusion and Possible Extensions

Besides being a primary and outstanding example for a complex economic problem, technological innovation is nowadays increasingly stylized as a collective phenomenon of interactions between a network of innovating agents. To better understand the characteristics controlling the best network structure for promoting system-level innovation, this paper propose to analyze the role played by network efficiency, edge direction, and heterogeneity of degree on the average performance of a network of innovating firms. Our results show that the system achieves higher performance in the short-medium term when networks are efficient at circulating information about technological innovations. On the other hand, in the long run inefficient networks achieve marginally better performance in terms of average fitness and number of firms who found the global peak. We also find that the average performance of the system results optimal when the probability of observation is neither too small nor too large. This result shows that when innovating firms interact through an observation or imitation networks, then the optimal balance between collective exploration and individual exploitation is achieved by an intermediate value of the probability of observation. Finally, by introducing heterogeneity of the degree and directional observation, we find that the system performs much worse than in the case of undirected and homogeneous networks. By increasing the heterogeneity of the degree networks assume a more centralized structure primarily focused on the results obtained by the hubs, which constrains the search opportunities as a result.

As with any agent based model, our results are strongly dependent on the assumptions of our model. There are therefore important potential extensions in both empirical testing and modeling. On the empirical level, it would be useful to identify markets or market segments where technological innovation is being developed through observation and imitation of firms. It would be interesting also to investigate case studies of technological innovations collectively developed in settings where markets influencers are present. One potential candidate for empirical testing is the financial sector [34]. On the one hand, financial innovations are not usually protected by patents, thus making very likely that market participants observe and copy the innovations of others. On the other hand, the structure of firms active in financial markets is mostly of core periphery type, thus rendering it likely that the big financial institutions are more influential in the process of financial innovation that the small ones.

On the theoretical level, we purposely constructed the model with the minimum complexity necessary to understand the role of different network structures in supporting technological innovation. There is therefore space for extending the model. It would be useful to examine a wider array of network structures and their performance under different conditions. In [2], it was found that different network structures perform differently according to what learning strategy agents adopt. An interesting network structure for technological innovation is the modular structure, since innovating firms are known to form clusters [4, 45]. One might consider the performance of different networks to changes in system rules like competition or patents which might increase the incentives for more exploration over exploitation.Our paper has been concerned with the system-level (or network-level) performance of firm networks, but researchers can also examine the performance at the level of a single level [27]

An implicit assumption in our model is that search process is the same for all firms in that it is modeled as a random process of trial and error plus a constant probability of observation. The main advantage of this assumption is its simplicity. However, in a real world economic setting agents are likely to be more strategic than just following simple incremental optimizing paths and/or performing elementary observation of their neighbors. Agents can indeed use sophisticated heuristics and/or complex adaptive strategies [21, 38] . For example, economic agents might explore the landscape by means of hierarchical decomposition of the structure of the landscape [18].

Another implicit assumption in our model is that the fitness landscape is both static and fixed.There are valid reasons to believe that fitness landscapes are neither static nor fixed and this creates potential to add a dimension to the model that incorporate dynamic complexity. Fitness landscapes are static if environmental conditions that influence the fitness of solutions remains stable through time in the technological landscape. Phenomena whose context is changing rapidly will eventually result in the peaks and valleys moving up and down as time goes on. Furthermore, the fitness of solutions in the landscape can change as a result of the actions of an agent or a collection of agents acting and reacting to each other (endogenous drivers), or as a result of external shocks like a new regulation that reduces the profitability of an investment or product (exogenous impact). The assumption of fixed landscape can also be challenged, because the continuous emergence of novelty contributes to the expansion of fitness landscapes through time. Novelty creates new niches and opportunities that can be discovered. The original contribution to the idea of expanding spaces was introduced by Kauffman in his book *Investigation* where he coined the term *adjacent possible* to explain how and why novelty emerges. The idea of adjacent possible has also been applied to the study of technological innovation [31, 61].

In summary many possible extensions of our work are possible and the present work is proposing a basic framework that can be specialized and generalized to many dimensions.

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appendix

	AvgBet	MinBet	MaxBet	AvgClos	MaxClos	MinClos	AvgClus	VarConst
MinAvgBet	0.04	0.04	0.05	0.26	0.27	0.26	0.00	0.00
MinAvgClus	0.05	0.02	0.10	0.23	0.25	0.20	0.00	0.00
MaxMaxClos	0.05	0.01	0.08	0.24	0.27	0.19	0.03	0.00
MaxConsVar	0.06	0.00	0.17	0.20	0.25	0.16	0.22	0.02
MaxAvgClus	0.12	0.03	0.34	0.12	0.15	0.10	0.47	0.01
MaxMaxBet	0.11	0.00	0.67	0.13	0.19	0.09	0.12	0.01
MinMaxClos	0.22	0.00	0.51	0.07	0.09	0.05	0.19	0.02
MaxAvgBet	0.24	0.00	0.51	0.06	0.08	0.04	0.41	0.01

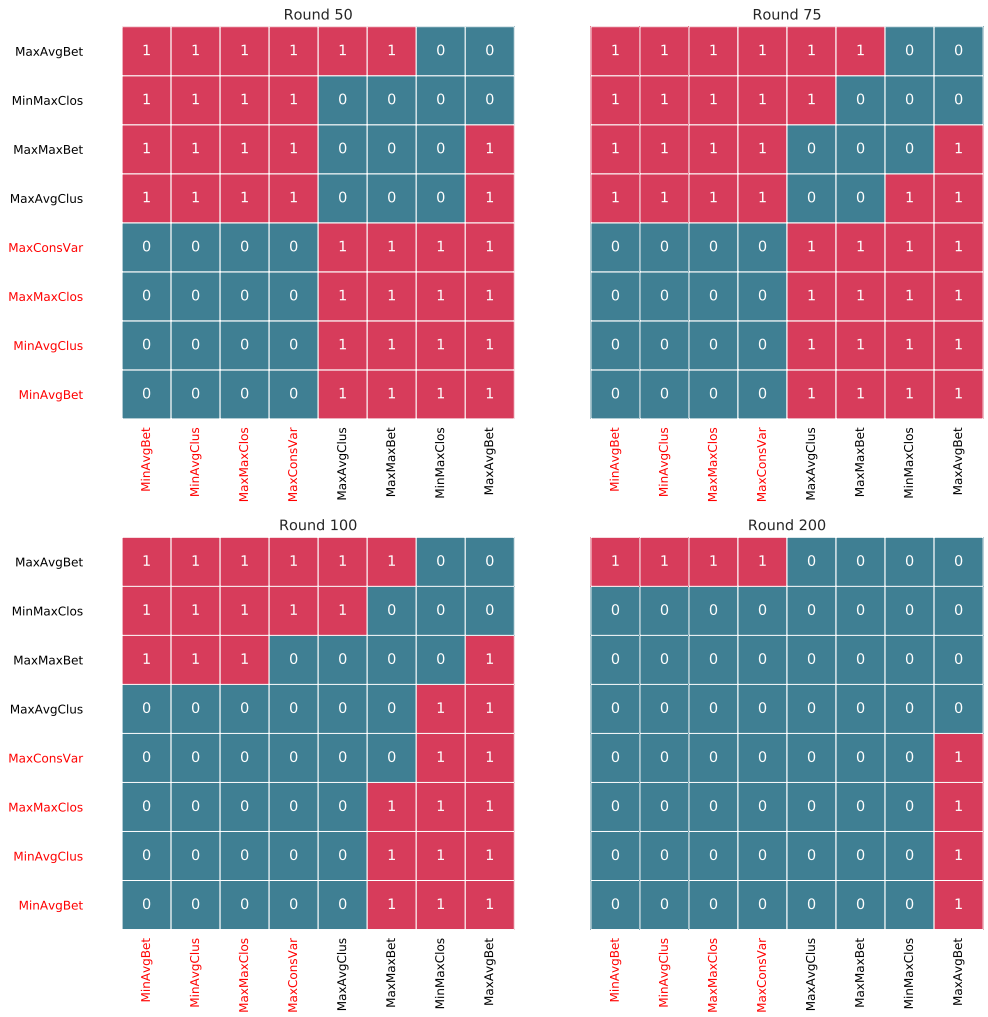

FIG. 9: Summary of the results obtained for a series of t-tests assessing the statistical significance of the difference in the average performance of firms searching in pairs of distinct undirected networks illustrated in figure 2. For all tests the observation probability is $p = 0.3$. The label 1 means that the statistical hypothesis that the average values are the same is rejected. The label 0 means that the statistical hypothesis that the average values are the same is not rejected. The statistical threshold used is 0.05. We apply Bonferroni multiple hypothesis test correction.

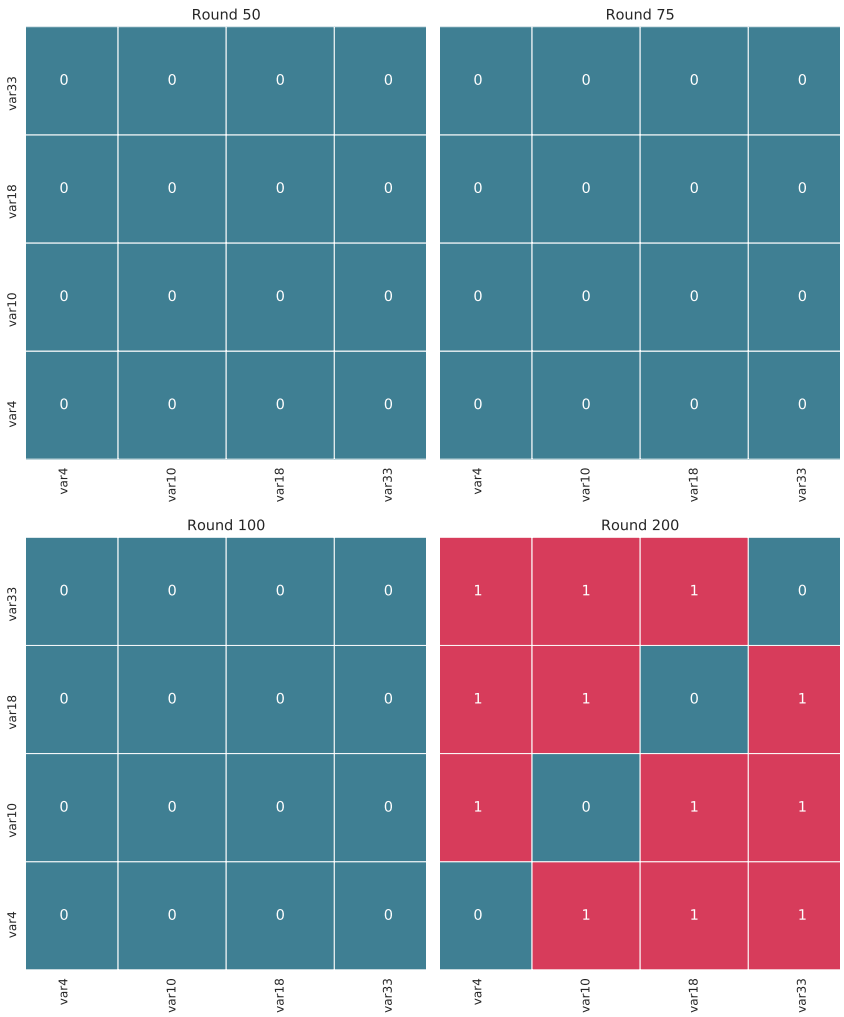

FIG. 10: Summary of the results obtained for a series of t-tests assessing the statistical significance of the difference in the average performance of firms searching in pairs of directed networks illustrated in figure 3. For all tests the observation probability is $p = 0.3$. The label 1 means that the statistical hypothesis that the average values are the same is rejected. The label 0 means that the statistical hypothesis that the average values are the same is not rejected. The statistical threshold used is 0.05. We apply Bonferroni multiple hypothesis test correction.

REFERENCES

1. Arthur, W. B. (1989) Competing technologies, increasing returns, and lock-in by historical events. *The economic journal*, **99**(394), 116–131.
2. Barkoczi, D. & Galesic, M. (2016) Social learning strategies modify the effect of network structure on group performance. *Nature communications*, **7**.
3. Bikhchandani, S., Hirshleifer, D. & Welch, I. (1998) Learning from the behavior of others: Conformity, fads, and informational cascades. *The Journal of Economic Perspectives*, **12**(3), 151–170.
4. Breschi, S. & Malerba, F. (2005) *Clusters, networks and innovation*. Oxford University Press.
5. Breschi, S., Malerba, F. & Orsenigo, L. (2000) Technological regimes and Schumpeterian patterns of innovation. *The economic journal*, **110**(463), 388–410.
6. Burt, R. S. (2009) *Structural holes: The social structure of competition*. Harvard university press.
7. Cho, Y., Hwang, J. & Lee, D. (2012) Identification of effective opinion leaders in the diffusion of technological innovation: A social network approach. *Technological Forecasting and Social Change*, **79**(1), 97–106.
8. Cohen, W. M. & Levinthal, D. A. (2000) Absorptive capacity: A new perspective on learning and innovation. In *Strategic Learning in a Knowledge economy*, pages 39–67. Elsevier.
9. Cowan, R. & Jonard, N. (2003) The dynamics of collective invention. *Journal of Economic Behavior & Organization*, **52**(4), 513–532.
10. Cowan, R. & Jonard, N. (2004) Network structure and the diffusion of knowledge. *Journal of economic Dynamics and Control*, **28**(8), 1557–1575.
11. Csermely, P., London, A., Wu, L.-Y. & Uzzi, B. (2013) Structure and dynamics of core/periphery networks. *Journal of Complex Networks*, **1**(2), 93–123.
12. Dahlander, L. & Gann, D. M. (2010) How open is innovation?. *Research policy*, **39**(6), 699–709.
13. DeBresson, C. & Amesse, F. (1991) Networks of innovators: A review and introduction to the issue. *Research policy*, **20**(5), 363–379.
14. Dodgson, M. (1994) *Technological collaboration in industry: strategy, policy and internationalization in innovation*. Routledge.
15. Dosi, G. (1982) Technological paradigms and technological trajectories: a suggested interpretation of the determinants and directions of technical change. *Research policy*, **11**(3), 147–162.
16. Feick, L. F. & Price, L. L. (1987) The market maven: A diffuser of marketplace information. *The Journal of Marketing*, pages 83–97.
17. Fleming, L., King III, C. & Juda, A. I. (2007) Small worlds and regional innovation. *Organization Science*, **18**(6), 938–954.
18. Fleming, L. & Sorenson, O. (2001) Technology as a complex adaptive system: evidence from patent data. *Research policy*, **30**(7), 1019–1039.
19. Freeman, L. C. (1977) A set of measures of centrality based on betweenness. *Sociometry*, pages 35–41.
20. Freeman, L. C. (1978) Centrality in social networks conceptual clarification. *Social networks*, **1**(3), 215–239.
21. Frenken, K. (2001) Fitness landscapes, heuristics and technological paradigms: a critique on random search models in evolutionary economics. In *AIP Conference Proceedings*, volume 573, pages 558–565. AIP.
22. Frenken, K. (2006) Technological innovation and complexity theory. *Economics of Innovation and New Technology*, **15**(2), 137–155.
23. Gale, D. & Kariv, S. (2003) Bayesian learning in social networks. *Games and Economic Behavior*, **45**(2), 329–346.
24. Galeotti, A. (2006) One-way flow networks: the role of heterogeneity. *Economic Theory*, **29**(1), 163–179.
25. Galeotti, A. & Goyal, S. (2010) The law of the few. *The American Economic Review*, **100**(4), 1468–1492.
26. Gilbert, R. (2006) Looking for Mr. Schumpeter: Where Are We in the Competition–Innovation Debate?. *Innovation policy and the economy*, **6**, 159–215.
27. Gilsing, V., Nooteboom, B., Vanhaverbeke, W., Duysters, G. & van den Oord, A. (2008) Network embeddedness and the exploration of novel technologies: Technological distance, betweenness centrality and density. *Research policy*, **37**(10), 1717–1731.

28. Goyal, S. (2018) Heterogeneity and networks. In *Handbook of Computational Economics*, volume 4, pages 687–712. Elsevier.
29. Hicks, D. (1995) Published papers, tacit competencies and corporate management of the public/private character of knowledge. *Industrial and corporate change*, **4**(2), 401–424.
30. Johnson, P. S. (1973) *Co-operative research in industry: an economic study*. Halsted Press.
31. Johnson, S. (2011) *Where good ideas come from: the seven patterns of innovation*. Penguin UK.
32. Katz, E. & Lazarsfeld, P. F. (1966) *Personal Influence, The part played by people in the flow of mass communications*. Transaction Publishers.
33. Kauffman, S., Lobo, J. & Macready, W. G. (2000) Optimal search on a technology landscape. *Journal of Economic Behavior & Organization*, **43**(2), 141–166.
34. Khraisha, T. & Arthur, K. (2018) Can we have a general theory of financial innovation processes? A conceptual review. *Financial Innovation*, **4**(1), 4.
35. Lazarsfeld, P. F., Berelson, B. & Gaudet, H. (1965) *The people's choice: how the voter makes up his mind in a presidential campaign, by Paul F. Lazarsfeld [et al.]*. Columbia Univ. Press.
36. Lazer, D. & Friedman, A. (2007) The network structure of exploration and exploitation. *Administrative Science Quarterly*, **52**(4), 667–694.
37. March, J. G. (1991) Exploration and exploitation in organizational learning. *Organization science*, **2**(1), 71–87.
38. Marengo, L., Dosi, G., Legrenzi, P. & Pasquali, C. (2000) The structure of problem-solving knowledge and the structure of organizations. *Industrial and Corporate Change*, **9**(4), 757–788.
39. Mason, W. & Watts, D. J. (2012) Collaborative learning in networks. *Proceedings of the National Academy of Sciences*, **109**(3), 764–769.
40. McNerney, J., Farmer, J. D., Redner, S. & Trancik, J. E. (2011) Role of design complexity in technology improvement. *Proceedings of the National Academy of Sciences*, **108**(22), 9008–9013.
41. Mukoyama, T. (2003) Innovation, imitation, and growth with cumulative technology. *Journal of Monetary Economics*, **50**(2), 361–380.
42. Nelson, R. R. (1959) The simple economics of basic scientific research. *Journal of political economy*, **67**(3), 297–306.
43. Osterloh, M. & Rota, S. (2007) Open source software developmentJust another case of collective invention?. *Research Policy*, **36**(2), 157–171.
44. Powell, W. W., Koput, K. W. & Smith-Doerr, L. (1996) Interorganizational collaboration and the locus of innovation: Networks of learning in biotechnology. *Administrative science quarterly*, pages 116–145.
45. Preissl, B. & Solimene, L. (2003) *Dynamics of Clusters and Innovation*. Springer.
46. Rosenberg, N. (2010) Why do firms do basic research (with their own money)?. In *Studies On Science And The Innovation Process: Selected Works of Nathan Rosenberg*, pages 225–234. World Scientific.
47. Rosenberg, N. & Nelson, R. R. (1994) American universities and technical advance in industry. *Research policy*, **23**(3), 323–348.
48. Saramäki, J., Kivelä, M., Onnela, J.-P., Kaski, K. & Kertesz, J. (2007) Generalizations of the clustering coefficient to weighted complex networks. *Physical Review E*, **75**(2), 027105.
49. Schrader, S. (1991) Informal technology transfer between firms: Cooperation through information trading. *Research policy*, **20**(2), 153–170.
50. Silverberg, G. & Verspagen, B. (2005) A percolation model of innovation in complex technology spaces. *Journal of Economic Dynamics and Control*, **29**(1-2), 225–244.
51. Solow, R. M. (1957) Technical change and the aggregate production function. *The review of Economics and Statistics*, pages 312–320.
52. Sorenson, O., Rivkin, J. W. & Fleming, L. (2006) Complexity, networks and knowledge flow. *Research policy*, **35**(7), 994–1017.
53. Spencer, J. W. (2003) Firms' knowledge-sharing strategies in the global innovation system: empirical evidence from the flat panel display industry. *Strategic Management Journal*, **24**(3), 217–233.
54. Stuart, T. E. & Podolny, J. M. (1996) Local search and the evolution of technological capabilities. *Strategic*

Management Journal, **17**(S1), 21–38.

55. Teece, D. J. (1986) Profiting from technological innovation: Implications for integration, collaboration, licensing and public policy. *Research policy*, **15**(6), 285–305.

56. Teece, D. J. (1992) Competition, cooperation, and innovation: Organizational arrangements for regimes of rapid technological progress. *Journal of economic behavior & organization*, **18**(1), 1–25.

57. Teece, D. J., Pisano, G. & Shuen, A. (1997) Dynamic capabilities and strategic management. *Strategic management journal*, **18**(7), 509–533.

58. Tumminello, M., Micciche, S., Lillo, F., Piilo, J. & Mantegna, R. N. (2011) Statistically validated networks in bipartite complex systems. *PloS one*, **6**(3), e17994.

59. Venkatesh, B. & Goyal, S. (1998) Learning from neighbors. *Review of Economic Studies*, **65**, 595–621.

60. Von Hippel, E. (1987) Cooperation between rivals: informal know-how trading. *Research policy*, **16**(6), 291–302.

61. Wagner, A. & Rosen, W. (2014) Spaces of the possible: universal Darwinism and the wall between technological and biological innovation. *Journal of The Royal Society Interface*, **11**(97), 20131190.

62. Wang, Z. & Wang, N. (2012) Knowledge sharing, innovation and firm performance. *Expert systems with applications*, **39**(10), 8899–8908.