

LIBER AMICORUM

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**CONFERENCE IN HONOR OF GIULIANELLA COLETTI'S 70TH BIRTHDAY**

*December 14-15, 2018*

Department of Mathematics and Computer Science (University of Perugia, Perugia)  
and Palazzo Trinci (Foligno)

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## Chapter 11

# Algebraic aspects and coherence conditions for conjunctions among conditional events by Angelo Gilio and Giuseppe Sanfilippo

We deepen the study of a notion of conjunction among conditional events, introduced in previous papers in the framework of coherence. This notion of conjunction, differently from other approaches, is given in the setting of conditional random quantities. We show that some well known properties which are satisfied by conjunctions of unconditional events are also satisfied by conjunctions of conditional events. In particular we examine an additive property and a decomposition formula, by also obtaining a generalized inclusion-exclusion formula. Then, by exploiting the notion of conjunction, we introduce the set of constituents generated by  $n$  conditional events. Moreover, under logical independence, we give a necessary and sufficient condition of coherence for the prevision assessments on a family  $\mathcal{F}$  constituted by  $n$  conditional events and all possible conjunctions among some of them. This condition of coherence has a simple geometrical characterization in terms of a suitable convex hull. Such a characterization amounts to the solvability of a linear system as in the case of unconditional events. Then, we illustrate the set of all coherent assessments on the family  $\mathcal{F}$  by a list of linear inequalities on the components of the prevision assessment. Finally, given a coherent assessment  $\mathcal{M}$  on  $\mathcal{F}$ , we show that every possible value of the random vector associated with  $\mathcal{F}$  is itself a particular coherent assessment on  $\mathcal{F}$ .

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