Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 199 (2017) 1423–1428

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Dynamic response of equivalent orthotropic plate model for stiffened plate: numerical-experimental assessment

Giuseppe Battaglia[a], Alberto Di Matteo[a], Antonina Pirrotta[a,b,]*, Giorgio Micale[c]

[a]*Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale e dei Materiali, Università degli Studi di Palermo, Viale delle Scienze, 90128 Palermo, Italy*

[b]*Department of Mathematical Sciences, University of Liverpool, Liverpool, UK*

[c]*Dipartimento della Innovazione Digitale ed Industriale, Università degli Studi di Palermo, Viale delle Scienze, 90128 Palermo, Italy*

Abstract

Over the last two decades, homogenization-based modeling techniques have attracted considerable attention. In fact, through these methods, structures such as corrugated or stiffened plates, commonly referred to as structurally orthotropic plates, can be approximately studied as equivalent flat plates with orthotropic behavior. Specifically, these homogenization techniques allow for the direct determination of the equivalent flexural and torsional rigidities which appear in the governing equation for the deflection of the equivalent orthotropic plate. It is worth noting that, the determined equivalent material properties retain the dependence on the geometric parameters of the original corrugated or stiffened plates. This is rather convenient since, in this way, optimal design of the structural geometry of such systems may be easily investigated. In this regard, here an experimental and numerical analysis is developed on the reliability of the equivalent orthotropic plate model for plates reinforced with set of equidistant stiffeners in one direction. Numerical results, obtained considering the common case of rectangular plates, are validated via pertinent finite element modeling and experimental data

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: Stiffened plates; Experimental analysis; Equivalent orthotropic model;

* Corresponding author. Tel.: +39-09123896756.
E-mail address: antonina.pirrotta@unipa.it

1877-7058 © 2017 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.387

1. Introduction

Thin plates and membranes are widely used in civil engineering, mechanical and chemical engineering. In many practical cases, structural properties of the plate differ in two mutually perpendicular directions, so that the plate is described as orthogonally anisotropic or, in short, orthotropic. Such anisotropy can be due to the inherent orthotropic characteristic of the material, or it can be introduced by ribs, corrugation or stiffeners, which generally referred to as structural orthotropy. Examples include plates reinforced with set of equidistant stiffeners in one or two directions, open gridworks and corrugated plates. A rather special case is that encountered in the field of Salinity Gradient Power (SGP) where Reverse Electro-Dialysis (RED) units are employed to directly convert the salinity gradient into electric energy. In this case the above mentioned plates are in fact ion-exchange membranes suitably arranged within a stack which may contain up to several hundreds of such plates.

On this base, corrugated and stiffened plates have been deeply investigated. Although stiffened plates show economical and mechanical advantages, their analysis and optimization are arduous tasks. For this reason, they are usually analyzed by finite element (FE) methods, which clearly require high computational effort especially for wide and complex geometries. On the other hand, in some cases mechanical behavior of a stiffened or corrugated plate can be determined studying an equivalent orthotropic thin plate. It is worth noting that, although the actual structural behavior of plate stiffener assemblies cannot be completely replaced by that of orthotropic plates, experimental data indicate good agreement with such idealization, provided that the relatively small stiffeners are uniform and closely spaced. Clearly, this approach yields several advantages, such as reducing the computational effort that finite element method would have demanded, especially in preliminary design.

Therefore, several research efforts have been devoted to determining the required equivalent orthotropic plate parameters. In this regard, many studies have dealt with the cases of corrugated plates, introducing several different formulations. A rather detailed account of these models is presented in [1], where also results of an experimental investigation has been presented to estimate the reliability of the equivalent models. Note that, in this case results show that the equivalent model is in good agreement with the first few mode shapes of the plates, while it is less accurate for higher frequencies.

As far as orthogonally stiffened plates are concerned, first studies date back to the 50th [2, 3], where a procedure has been developed for determining the stiffness rigidity properties of a reinforced plate by equidistance grooves stiffeners. Few other studies are presented in literature on this topic, and, to the best of the authors' knowledge, no experimental comparison in terms of natural frequencies (different from the fundamental one) and mode shapes have been presented so far.

In this work, the validity of the equivalent orthotropic model developed in [2] for a reinforced plate by equidistance grooves stiffeners have been investigated. Experimental measurements of frequency and modes of vibration have been evaluated using both laser scanning vibrometer and impulsive tests. In addition, a FE model for the real stiffened plate has been developed. The vibration frequencies of the theoretical equivalent orthotropic plate have been compared to both the experimental measurements and FE results in order to evaluate their reliability.

2. Problem Formulation

Consider a thin orthotropic homogeneous plate with density ρ and thickness h. Assuming the material to be linearly elastic and that the orthotropy axes are aligned with the x- and y-axes of a Cartesian system in the plane of the middle surface, the differential equation for the free vibration of the plate can be written as

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = \rho h \frac{\partial^2 w}{\partial t^2} \tag{1}$$

where w is the plate displacement in the vertical direction, and the parameters D_x, D_y, and H are the so-called flexural and torsional rigidities which are given in terms of the material properties as

$$D_x = \frac{E_x h^3}{12\left(1-\nu_{xy}\nu_{yx}\right)};\ \ D_y = \frac{E_y h^3}{12\left(1-\nu_{xy}\nu_{yx}\right)};\ H = D_1 + 2D_t;$$
$$D_1 = \nu_{xy}D_y = \nu_{yx}D_x;\ \ D_t = \frac{G_{xy}h^3}{12} \tag{2}$$

in which E_x and E_y are the Young's moduli, v_{xy} is the Poisson's ratio, G_{xy} is the shear modulus and $v_{yx}=v_{xy}E_y/E_x$.

Note that, the shear modulus of the orthotropic material can be approximately expressed in terms of Young's moduli as follows [4]

$$G_{xy} \approx \frac{\sqrt{E_x E_y}}{2\left(1+\sqrt{\nu_{xy}\nu_{yx}}\right)} \tag{3}$$

It is worth stressing that, for generic boundary conditions, all four parameters in Eq. (1) must be found from Eq.(2), or else appropriately defined in case of an equivalent thin plate model applicable to structural orthotropic plates. Clearly, for plate made of isotropic material, that is $E_x= E_y=E$ and $v_{yx}=v_{xy}=v$, the flexural rigidity is given as $D_x = D_y = H = Eh^3/12\left(1-\nu^2\right)$. Thus, in this case, the number of independent parameters reduces from four in Eq. (1) to two (Young's modulus and Poisson ratio).

As well-known, solution of Eq. (1) strongly depends on the boundary conditions and plates shape. Considering the simpler case of rectangular shaped plate, exact solutions in terms of mode shapes and natural frequencies are available for very few cases [5] of boundary conditions, while for generic boundary conditions Rayleigh-Ritz procedure is commonly adopted [6].

2.1. Equivalent orthotropic properties of stiffened plates

As mentioned previously, stiffened plates, as well as corrugated plates, can be approximated as equivalent orthotropic flat plates. Clearly, a proper selection of the equivalent rigidities plays a significant role in the accuracy of the model. Note that, for corrugated plates various different equivalent rigidities have been defined in literature [1]. On the other hand, as far as orthogonally stiffened plates are concerned, the equivalent rigidity properties given in [3], based on the work in [2], are commonly adopted. In this regard, for a rectangular plate reinforced by equidistant stiffeners in one direction of thickness $\tilde{h}$, disposed symmetrically with respect to the middle plane of the plate (Fig. 1(a)), and assuming that both the plate and the stiffeners are made of isotropic material, the equivalent rigidities are given in [3] as

$$D_x = H = \frac{E\,h^3}{12\left(1-\nu^2\right)};\ \ D_y = \frac{E\,h^3}{12\left(1-\nu^2\right)} + \frac{\tilde{E}\,I}{\left(s+L_s\right)} \tag{4 a,b}$$

in which E and v are the elastic constant of the plate material, $\tilde{E}$ is the Young modulus of the stiffeners, I the moment of inertia of the stiffener taken with respect to the middle axis of the cross section of the plate, s is the distance between two stiffeners and L_s is the width of the stiffeners.

It is worth mentioning that, equivalent rigidities in Eq. (4a) are lower bound of more exact equivalent rigidity parameters given in [2] as

$$D_x = \frac{a^5}{30\int_0^b \frac{\left(ax-x^2\right)^2}{D(x)}dx};\quad D(x) = \frac{E\left[h(x)\right]^3}{12\left(1-\nu^2\right)} \tag{5 a,b}$$

where $h(x)$ is the total thickness of the plate-stiffener combination.

Further, as far as the term $H=D_1+2D_t$ is concerned, it is suggested to find D_t from experimental data, while D_1 and the two Poisson's ratios for the equivalent orthotropic plate can be obtained as

$$D_1 = \nu D; \quad \nu_{xy} = \frac{D_1}{D_y}; \quad \nu_{yx} = \frac{D_1}{D_x} \tag{6}$$

Clearly, taking into account Eq. (6), an approximate value for the shear modulus G_{xy} can be retrieved from Eq. (3), therefore leading to the torsional rigidity D_t as in Eq. (2), where h should be assumed as the thickness of the equivalent orthotropic plate h_e.

In order to investigate on the reliability of this formulation, in terms of natural frequencies and mode shapes, in the following section results of a detailed experimental campaign will be presented.

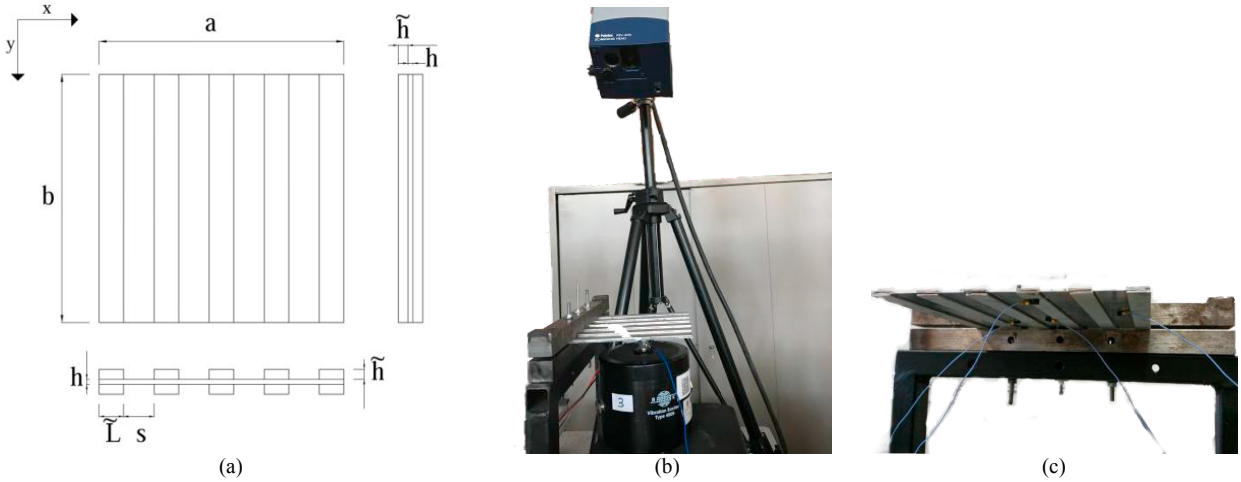


Fig. 1: Stiffened plate: a) Geometrical parameters (op, side and front views); b) Experimental set-up with the laser vibrometer; c) Experimental set-up for the impulsive tests.

3. Experimental investigation

3.1. Experimental set-up and data acquisition

Natural frequencies and mode shapes of a rectangular homogeneous and stiffened plate have been obtained by a series of experimental tests performed in the Laboratory of Experimental Dynamics at the University of Palermo. It was decided to employ a cantilever plate, thus having just one side clamped and the other free, since these boundary conditions are particularly feasible for experiments compared to others common cases. In this regard, firstly an homogeneous flat aluminum plate, whose characteristic are reported in Tab. 1, has been fixed on one side on a rigid frame, in order to determine Young's modulus and Poisson ratio of the homogeneous plate. Secondly, on the same plate six stiffeners, made of an Aluminum alloy, have been carefully attached, so as to obtain a reinforced plate with equidistant stiffeners in one direction.

The frequency response function and the mode shapes of the plates have been measured with a Polytec laser scanning vibrometer model PSV-400, which is able of measuring displacements of a chosen grid of points of the plate without any contact through the laser signal (Fig. (1(b)). Excitation has been provided through a B&K shaker type 4809 connected with a stinger at the center of the plates, and the force signal has been acquired with a B&K force transducer type 8230 placed between the plates and the shaker. A sweep sine, of frequency range between 0.1 and 2000 Hz, has been used as forcing signal and repeated for each point of the grid, while the chosen sample rate was 2000 Hz. Finally, the modes and corresponding frequencies have been identified by means of the vibrometer

software version 9.2. Moreover, in order to obtain more precise data especially at the lowest frequency, impulsive tests have also been utilized for both the homogeneous and stiffened plates (Fig. 1(c)). In this regard, for each plate five tests have been performed and corresponding mean frequency response functions has been acquired. Specifically, a small impact hammer model PCB 086E80 has been used to infer the impulse, while four miniaturized PCB accelerometers model 352C23 have been employed to measure the acceleration responses on five points of the plates (Fig. 1(c)). Note that the mass of these accelerometers is almost negligible in comparison with the mass of the plates, so as to minimize their influence on the frequency responses. Further, signals have been then digitalized and acquired by means of a National Instruments NI 4497 PXI Acquisition Board provided inside the chassis of a National Instruments PXIe model 1082, and then processed in LabView and MATLAB environments.

3.2. Experimental results vis-à-vis numerical simulations

To verify the reliability of the equivalent orthotropic model for stiffened plates also in the frequency domain, comparison among experimental data and the theoretical models previously introduced has been performed.

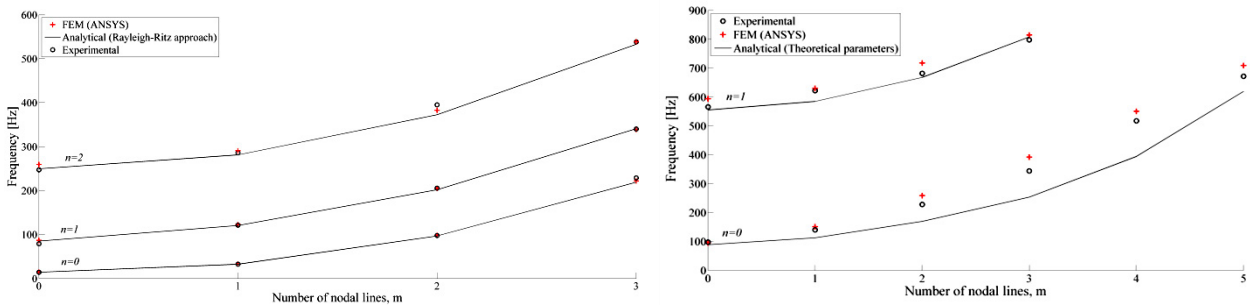
Firstly, material parameters (Young's modulus and Poisson ratio) of the homogeneous flat plate have been identified minimizing the mean squared error between the first two experimentally obtained natural frequencies, and the corresponding analytical one which are reported in [7]. In this regard, geometric information and the panel masses are summarized in Tab. 1, while values of the identified properties are given in Tab. 2. For further comparison, the natural frequencies of the plate has been determined also by FE analysis on ANSYS environment using these identified values. Specifically, for the FE analysis an hexahedral mesh with 395 nodes and 67.000 elements has been used. Pertinent results are shown in Fig. 2 (a). In this figure the modes are labeled by two mode indices, *m* and *n*, indicating the number of nodal lines approximately parallel to y- and x-axis, respectively. The frequencies are plotted over the mode index *m*. Lines connecting the symbols are guides to the eye only. Note that, a very good match between experimental and numerical natural frequencies can be seen in Fig. 2(a), thus proving the validity of the identified parameters.

Table 1. Geometrical properties of the tested plates.

	Flat plate	Stiffened plate
a	190 mm	190 mm
b	170 mm	170 mm
h	0.5 mm	0.5 mm
mass	0.044 kg	0.191 kg
$\tilde{h}$	-	2 mm
$\tilde{L}$	-	15 mm
s	-	20 mm

Table 2. Properties of the flat plate

E	68.10^9 GPa
ρ	2708 kg/m^3
ν	0.29



(a) (b)

Fig. 2: Comparison of experimental vis-à-vis numerical results in terms of natural frequencies: a) Flat plate; b) Stiffened plate. Continuous black line –Analytical results; Red crosses – FE results; Black circles – Experimental data.

On this base, equivalent orthotropic parameters have been retrieved using Eqs. (5-6), and corresponding numerical results in terms of mode shapes and natural frequencies for the stiffened plate have been obtained applying a Rayleigh-Ritz procedure, as detailed in [6].

Finally, natural frequencies and mode shapes of the actual stiffened plate has been determined also by FE analysis on ANSYS environment using the real geometry of the plate, and parameters reported in Tab. 1 and 2.

In this regard Fig. 2(b) shows the experimental natural frequencies vis-à-vis FE results and pertinent numerical results using the Rayleigh-Ritz procedure with the equivalent orthotropic parameters.

As apparent, very good agreement is achieved .between experimental and FE results, thus proving the validity of the applied experimental procedure. Further, as shown in this figure, the equivalent orthotropic model leads to a satisfactory comparison with the experimental data, especially for lower values of m and n. However, as expected, for increasing values of m the accuracy of the equivalent orthotropic model decreases significantly.

Moreover, as shown in Fig. 3, analogous analysis has also been performed in terms of mode shapes. Specifically, in this figure, the first two mode shapes obtained experimentally are compared with those numerically determined with the Rayleigh-Ritz procedure with equivalent orthotropic model and FE analysis with ANSYS on the actual stiffened plate geometry. As it can be observed, these numerical methods lead to satisfactory agreement with the experimental data. Similar results have also been obtained for higher modes, here omitted for space constraints.

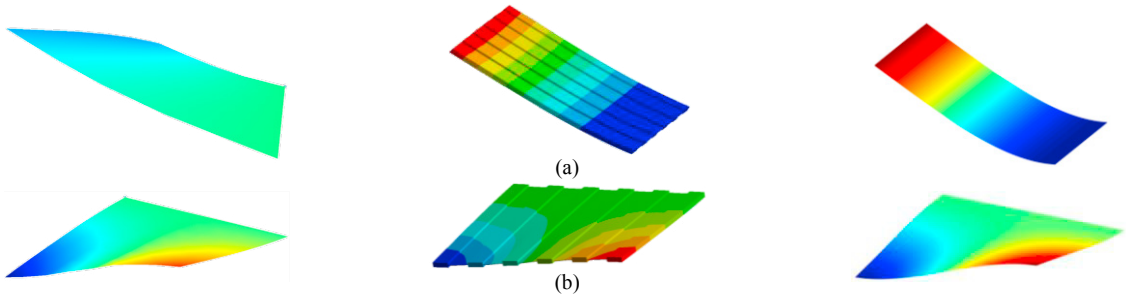


Fig. 3: Comparison of experimental vis-à-vis numerical results in terms of mode shapes: a) First mode; b) Second mode. Left column- Experimental data; Central column – FE results on ANSYS; Right column – Numerical results with equivalent orthotropic model.

4. Concluding Remarks

In this paper a numerical and experimental assessment on the reliability and accuracy of the equivalent orthotropic plate model for plates reinforced with set of equidistant stiffeners in one direction has been developed. Experimental measurements of frequency and mode shapes have been performed using both laser scanning vibrometer and impulsive tests. Further, a FE model of the real stiffened plate has been developed. Frequencies of the theoretical equivalent orthotropic plate have been compared to both the experimental measurements and FE results in order to evaluate their reliability. The fair agreement between measured natural frequencies and FE values indicates that the experimental setup was close to the intentions and the FE modeling was appropriate. Thus, these results can be considered as a reasonable reference to assess the accuracy of the equivalent orthotropic model. Results shows that the equivalent model generally underestimate the experimental data except for low-order modes, while good agreement has been obtained in terms of mode shapes.

References

[1] Y. Aoki, W. Maysenholder, Experimental and numerical assessment of the equivalent-orthotropic-thin-plate model for bending of corrugated panels, Int. J. Solids Struct., (2016) 1-13 DOI: 10.1016/j.ijsolstr.2016.07.042.
[2] N.J. Huffington, Theoretical Determination of Rigidity Properties of Orthogonally Stiffened Plates, J. Appl. Mech., (1956) 15-20.
[3] S. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill, New York, 1959.
[4] R. Szilard, Theory and Applications of Plate Analysis: Classical, numerical and engineering methods, John Wiley & Sons, New York, 2004.
[5] Y.F. Xing, B. Liu, New exact solutions for free vibrations of thin orthotropic rectangular plates, 89 (2009) 567-574.
[6] K.M. Liew, K.Y. Lam, S.T. Chow, Free vibration analysis of rectangular plates using orthogonal plate function, 34 (1990) 79-85.
[7] A.W. Leissa, Vibration of plates, NASA SP-160 (1969).