Finite-Difference Time-Domain simulation of towers cascade under lightning surge conditions

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***Abstract* – In this article is presented the simulation, using FDTD technique, of the behavior of a transmission line consisting of towers connected by rope protection, in the presence of lightning current and a non-linear response of the grounding system, subject to ionization. FDTD method is used to solve both the Maxwell's equation and Telegraph equations.**

***Index Terms*—Grounding, lightning, ionization, Maxwell equations, FDTD**

I. Introduction

Simulate the behavior of a section of the high voltage line subjected to lightning strike may require to get more information on propagation of overvoltage waves in different materials. Such simulations are useful to evaluate following significant electromagnetic interference (EMI) arising in electric and electronic apparatus placed inside structures to be protected and also to evaluate dangerous conditions for persons. The guard cable, playing the role of antenna for the capture of the lightning strike, connects different ground electrodes, by means of the tower structure. The voltage reached by the guard cable, depends on the behavior of the ground system, whose behavior is nonlinear.

Over the years, different studies have been focused on the behavior of the electrodes buried, the assumptions made were different, but experimental and theoretical studies have shown that, when the surge current leaking into the soil by the different parts of the earth electrode increases, the electric field on the lateral surface of the electrodes can overcome the electrical strength. Thus the ionization phenomenon takes place [1-6, 21]. This phenomenon has a great influence on the performance of concentrated electrodes fed by high magnitude currents as the vertical rod used for grounding towers. A deionization process follows the local soil breakdown; it starts when the surge current begin to decrease and stops when the stationary values of soil parameters are restored.

The ionization process and the following de-ionization modifies the electromagnetic transient behavior of the system with respect to the case of absence of soil breakdown, but also the connection between different towers and respective ground system also affect the global behavior [7].

To deal with the nonlinear behavior of the system is necessary to make use of a time-domain solver, which can consider the transient resistance of the ground system.

In order to correctly take into account the non linear behavior of the earth electrodes during the ionization and the subsequent deionization processes, a finite difference time domain (FDTD) scheme [22, 23], applied to Maxwell's equations [24] and time-varying constitutive relations of soil [2, 9], can be used.

The FDTD method, applied to Maxwell's equations, describes the electromagnetic environment in which the electromagnetic phenomena are developing with dense lattice, this is both an advantage, since there is not the need of particular assumptions about the geometry of the part of the ground subject to the phenomenon of ionization, but at the same time is a limitation in describing environments extended as a cascade of towers.

To describe the connection between different earthing systems simultaneously subject to the phenomenon of ionization, the FDTD algorithm is applicable to the Telegrapher equations [25]. With the approach of the transmission lines it is possible to describe the behavior of both the rope guard that of the tower, assuming the lumped inductance, capacitance and resistance do not vary to the passage of the lightning current.

The article is divided into three parts: the first describes the phenomenon of ionization and also the approaches that have been used over time to take it into account in the behavior ground systems; in the second part the model to describe the systems in the air (tower and guard rope); finally in the third part simulations of air and ground system are presented.

II. Ground electrode

The ionization models available in technical literature, are based on the simplified assumption that the ionized zones of the soil around the earth electrodes are well-shaped and predefined. Since it is difficult to guess the shape of the ionized zone around the earth electrodes, in technical literature different conjectures have been adopted based on different models of the physical phenomenon describing the soil ionization and deionization processes. One commonly adopted approach models a given concentrated electrode embedded in an ionized soil as an electrode of modified transversal dimensions into a non ionized soil. This assumption is acceptable when only the transient resistance is needed, resistance seen as a parameter that characterizes all the ground system, but is not useful for assessing the induced potential, since the common formulas [21] used provide that the longitudinal dimensions are substantially larger than those transversal. A time variable radius of the electrode is forced by assuming that the electric field may not exceed the critical value [3,4,7]. However, this model is far from the real ionization mechanism, since the ionized zones of the soil around the electrodes are considered well-shaped and substituted by an electrode of larger dimension. In Fig. 1 the shape of the ionized zone around a vertical rod, adopted as hypothesis in technical literature, are sketched.

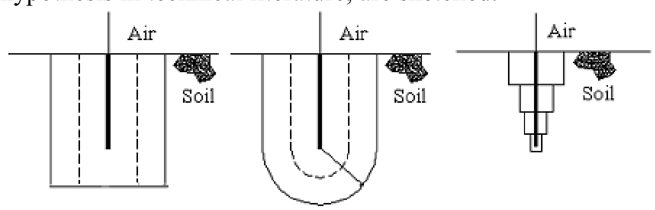

Fig. 1 – Shapes of the supposed ionized zones by Bellaschi, Darveniza and Geri, for a vertical rod. The existence of different conjectures is symptomatic of the difficulty in guessing the shape of the ionized zone.

A more useful approach, proposed by Darweniza and Liew for concentrated earths [2], considers a space-time variable soil resistivity function in the region surrounding the electrode. This time variable resistivity is a non-linear function of the electric field. In spite this approach is used in conjunction with an initial hypothesis on the shape of the ionized zones of the soil surrounding the concentrated electrode, the use of a space-time variable soil resistivity function seems to be more closer to the physical phenomenon than the variable geometry approach previously synthesized. In order to overcome the difficulty of a correct guess of the ionized zone, a space-time variable resistivity approach can be usefully employed together with a FDTD scheme based on the numerical solution of the Maxwell's equations.

Overcome the difficulty of managing the portion of soil that will vary its own characteristics, the parameter variable resistivity, connecting the electric field and the current density in the soil, has to be defined.

Two approaches can be used together with the FDTD technique, the first recalls the same assumptions Darveniza [10-12], the second those of Cooray [13]. In this paper the approach used in [2,10-12] has been followed.

When in the FDTD grid the local electric field E overcomes the electrical strength E_c, in the local cell the ionization begins, and the time variable resistivity law is expressed by the following:

$$\rho = \rho_0 \, e^{(-t/\tau_1)} \tag{1}$$

where ρ_0 is the stationary steady state resistivity, τ_1 is the ionization time constant. In the same manner, during the deionization process the following time variable resistivity law is used:

$$\rho = \rho_i + (\rho_0 - \rho_i) \cdot (1 - e^{t/\tau_2}) \cdot (1 - E/E_c)^2 \tag{2}$$

where ρ_i is the minimal values reached by the soil resistivity during the ionization process, τ_2 is the deionization time constant, E is the actual amplitude of the electric field.

The numerical implementation of the FDTD method combined with variable resistivity is described in [10-12] and for simplicity here is not reported.

The grounding electrodes commonly used for the connection to the earth of the towers are made of vertical rods. The FDTD method works with electric and magnetic field, which have to be converted into voltage and current: voltage to remote ground can be evaluated by summing elementary field from head of the rod until a distance ten time the length of the rod; current is forced by employing the ampere law around the head of the rod:

$$V = \sum_{i=1}^{\infty} E_x^i \Delta x \, , \quad I = \sum_{i=1}^{4} H_{x,y}^i \Delta x \, . \tag{3}$$

Fig.2 shows the use of previous formulas.

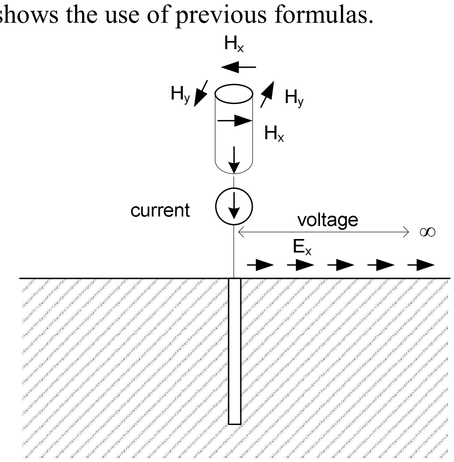

Fig. 2 – Scheme of a vertical rod in a FDTD simulation.

III. TRANSMISSION LINE MODEL

It is considered a section of the guard rope or a leg of the tower, called line in the following. The resistive and inductive effects along the line can be separated by the effects currents toward ground, which are conductive and capacitive. V (x, t) and I (x, t) are the instantaneous voltage and current in the line.

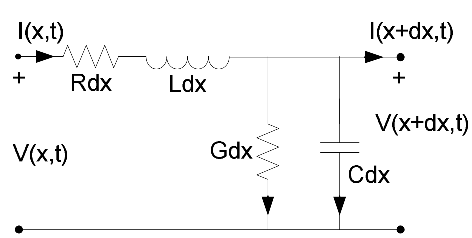

Fig. 3 – Section of the line.

Applying Kirchhoff's law for the voltages to determine correlation in voltage between the input section and output:

$$V(x,t) = Rdx \cdot I(x,t) + Ldx\frac{\partial}{\partial t}I(x,t) + V(x+dx,t),$$
$$I(x,t) = Gdx \cdot V(x+dx,t) + Cdx\frac{\partial}{\partial t}V(x+d,t) + I(x+dx,t), \quad (4)$$

in which appear the magnitudes R, L, G, C, respectively, longitudinal resistance, longitudinal inductance, conductance to ground, capacitance to ground, defined per unit length.

Applying elementary elaborations [25], the FDTD technique allows the writing of explicit formulas, in which the term later in time is obtained as a function of other quantities:

$$I_k^{n+1/2} = I_k^{n-1/2}\left(\frac{2L_k - R_k\Delta t}{2L_k + R_k\Delta t}\right) + -\frac{2\Delta t}{2L_k\Delta x + R_k\Delta t\Delta x}\left(V_{k+1/2}^n - V_{k-1/2}^n\right), \quad (5)$$

$$V_k^{n+1/2} = V_k^{n-1/2}\left(\frac{2C_k - G_k\Delta t}{2C_k + G_k\Delta t}\right) + -\frac{2\Delta t}{2C_k\Delta x + G_k\Delta t\Delta x}\left(I_{k+1/2}^n - I_{k-1/2}^n\right), \quad (6)$$

in which the points are on a lattice with a Δx and Δt spacing, and are identified by integer indices $x_k = k\Delta x$, $t_n = n\Delta t$.

As is usual the two equations (5) and (6) are staggered in time and space to be solved independently, and can therefore rewritten as:

$$I_k^{n+1/2} = A_k \cdot I_k^{n-1/2} + B_k\left(V_{k-1/2}^n - V_{k+1/2}^n\right),$$
$$V_{k+1/2}^{n+1} = E_{k+1/2} \cdot V_{k+1/2}^n + F_{k+1/2}\left(I_k^{n+1/2} - I_{k+1}^{n+1/2}\right), \quad (7)$$
$$A_k = \frac{2L_k - R_k\Delta t}{2L_k + R_k\Delta t}; \quad B_k = \frac{2\Delta t}{2L_k\Delta x + R_k\Delta t\Delta x};$$
$$E_{k+1/2} = \frac{2C_{k+1/2} - G_{k+1/2}\Delta t}{2C_{k+1/2} + G_{k+1/2}\Delta t};$$
$$F_{k+1/2} = \frac{2\Delta t}{2C_{k+1/2}\Delta x + G_{k+1/2}\Delta t\Delta x}.$$

The above formulas can be rearranged in the presence of discontinuities and nodes in the network [26]. The discontinuities that may be encountered regarding a change of the way in which the conductor is immersed or a simple change in the characteristics of the cable, such as an increase of the section. Considering the latter case, a series connection, shown in Fig. 4.

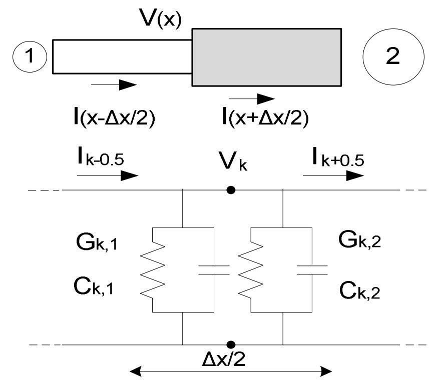

Fig. 4 – Series connection.

By using the law of Kirchhoff to the node where insist two different cells, in the contact section the continuity of the voltage is imposed, for which the sum of the currents is given by:

$$I_{k+1/2}^n - I_{k-1/2}^n = -\left(C_{k,1}\frac{\Delta x}{2} + C_{k,2}\frac{\Delta x}{2}\right)\left(\frac{V_k^{n+1/2} - V_k^{n-1/2}}{\Delta t}\right) + -\left(G_{k,1}\frac{\Delta x}{2} + G_{k,2}\frac{\Delta x}{2}\right)\left(\frac{V_k^{n+1/2} + V_k^{n-1/2}}{2}\right),$$

$$V_k^{n+1/2} = \left(\frac{2C_{k,12} - G_{k,12}\Delta t}{2C_{k,12} + G_{k,12}\Delta t}\right)V_k^{n-1/2} + -\frac{2\Delta t}{2C_{k,12}\Delta x + G_{k,12}\Delta t\Delta x}\left(I_{k+1/2}^n - I_{k-1/2}^n\right), \quad (8)$$

with $C_{k,12} = \frac{C_{k,1} + C_{k,2}}{2}$ and $G_{k,12} = \frac{G_{k,1} + G_{k,2}}{2}$,

The formula used to derive the new value of current does not change.

Fig. 5 shows a ramification. The node has a common voltage in the three branches.

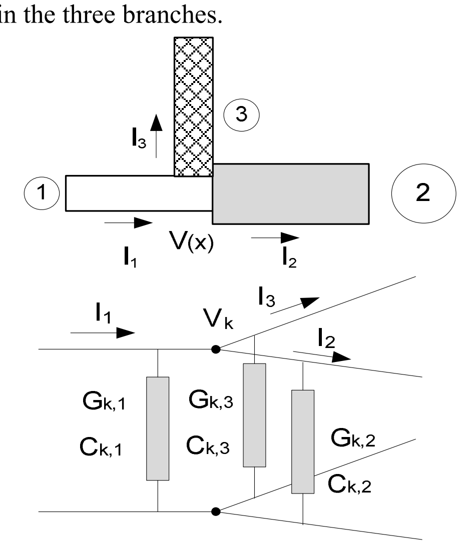

Fig. 5 – Branch connection.

The sum of the current is:

$$I_{k,2}^{n}+I_{k,3}^{n}-I_{k,1}^{n}=-\left(C_{k,1}\frac{\Delta x}{2}+C_{k,2}\frac{\Delta x}{2}+C_{k,3}\frac{\Delta x}{2}\right)\cdot \left(\frac{V_{k}^{n+1/2}-V_{k}^{n-1/2}}{\Delta t}\right)-\left(G_{k,1}\frac{\Delta x}{2}+G_{k,2}\frac{\Delta x}{2}+G_{k,3}\frac{\Delta x}{2}\right)\cdot \left(\frac{V_{k}^{n+1/2}+V_{k}^{n-1/2}}{2}\right). \quad (9)$$

with

$$V_{k}^{n+1/2}=\left(\frac{2C_{k,123}-G_{k,123}\Delta t}{2C_{k,123}+G_{k,123}\Delta t}\right)V_{k}^{n-1/2}+ -\frac{2\Delta t}{2C_{k,123}\Delta x+G_{k,123}\Delta t\Delta x}\left(I_{k,2}^{n}+I_{k,3}^{n}-I_{k,1}^{n}\right) \quad (10)$$

and

$$C_{k,123}=\frac{C_{k,1}+C_{k,2}+C_{k,3}}{2}\ ,\ G_{k,123}=\frac{G_{k,1}+G_{k,2}+G_{k,3}}{2}\ . \quad (11)$$

The formula used to derive the new value of current does not change. The cell has an extension of Δx for each of the possible directions ij, for which the contribution of the admittance is to be weighed to the middle step.

IV. Towers and Overhead Lines

The system to be simulated consists of connecting some simpler subsystems. The base subsystem is represented in Fig. 6. A four legs tower is sketched, the guard rope is in the upper edge, below the three-phase arms. To better compensate problems of stability and weight, the high voltage tower has sections which gradually rising in height are reduced: first and third blocks have a varying sections, in the middle the section is constant.

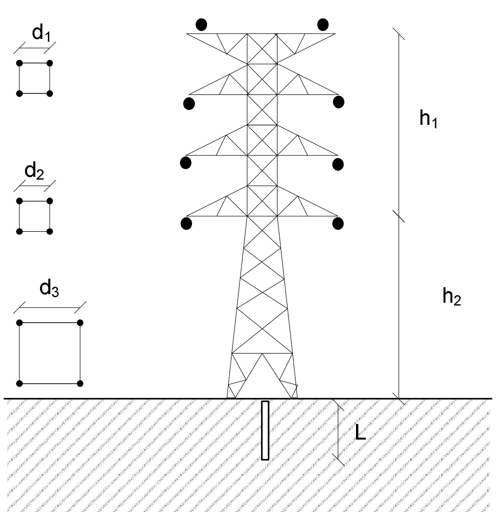

Fig. 6 – Tower and rod.

Each block of the tower is formed by a network of metallic elements, contributing to the characterization of the circuit parameters to be included in the cell model.

Whereas only the vertical elements, for each of them it is possible to determine the impedance value. As reported in [14,15] different are the studies concerning an analytical formulation of the surge impedance of vertical single conductor with circular cross-section, but no formula except Jordan's [15,16] seems to take into account the effect of imperfectly conducting earth.

To simplify the approach, instead of calculating the behavior of each leg of the tower in the multi-conductor system, a single conductor with a mean radius that describes the overall behavior of the system can be used [14]:

$$r_{e1}=\sqrt[n]{r\cdot n\cdot R^{n-1}}, \quad (12)$$

where r_{e1} is the equivalent radius, r is the radius of the leg, R is the radius of circumference inscribing section, n is the number of legs.

In a similar way it is possible to take into account the non-uniformity of the section with height by using an average formula[14, 17,18]:

$$r_{e2}=\frac{r_1 h_1+r_2\left(h_1+h_2\right)+r_3 h_2}{h_1+h_2}, \quad (13)$$

with r_1, r_2 and r_3 average radii obtained with (12).

A better definition of the Jordan's formula can be found in [19], in which the image theory can take into account the variation of the ground resistivity, but for sake of simplicity here it is used:

$$Z_{Jor}=60\ln\frac{(h_1+h_2)}{r_{e12}}+90\frac{r_{e12}}{(h_1+h_2)}-60, \quad (14)$$

and $Z_{Jor}=cL,$ with c speed of light.

In order to simulate the behavior of the cascade of towers the parameters of the overhead line have to be set, and in particular the homopolar ones have to be taken into account since the current flows in guard ropes and in the ground as a return way. Longitudinal resistance, per unit length, is given by:

$$R=\frac{\rho_{rope}}{S}+\pi^2 f\cdot 10^{-7},$$

in which ρ_{rope} is a parameter relating to the medium resistivity of aluminum-steel rope, S the equivalent section, and last part takes into account the return path in the ground [20]. A frequency depend material show a linear dispersion, which can be faced by following [22]; here it is used an average value, implemented by approximating the wave of lightning with a triangular signal, splitting the wave of lightning in a Fourier series, and evaluating for each of them a resistance value, Fig.7.

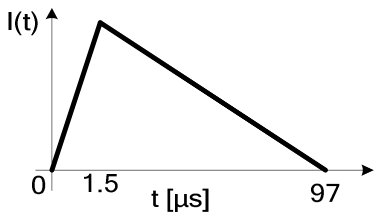

Fig. 7 –Triangular wave, approximating lightning current.

The inductance of service, for a single rope guard, is:

$$L_0 = \frac{\mu_0}{2\pi} \ln \frac{(2H_0)}{r_0},$$

in which r_0 is the average radius of the rope, and:

$$2H_0 = 660\sqrt{\rho_{ground}/f}$$

Again an average value has been used.

The transverse conductance of service takes into account the corona effect, but for the sake of simplicity here is neglected.

In the presence of a voltage, applied to the tower, the rope guard shows a capacity, which may be assessed using the formula similar to that used for the service capacity, so per unit length:

$$C_0 = \frac{2\pi\varepsilon_0}{\ln \frac{(2(h_1+h_2))}{r_0}}.$$

V. SIMULATIONS

The parameters used for the simulation are collected in table I.

TABLE I
DATA ASSUMED IN THE MODEL

Tower material (iron)	9.09 10^{-7}	Ωm
Height h1	28.6	m
Height h2	32.2	m
Distance d1	3.8	m
Distance d2	4.4	m
Distance d3	10.8	m
Tower distance	300	m
Radius guard rope	11.5	mm
Ground resistivity	50.5	Ωm
Rod length	3	m
Rod radius	1.27	cm
Soil breakdown	1.1	kV/cm
Pulse current (maximum)	10	kA
Rise time	1.2	µs
Time to half value	50	µs
Ionization constant τ_1	2.5	µs
Deionization constant τ_2	4.5	µs

Considering a preliminary simulation in which lightning 1.2 / 50 µs, 10kA, directly affects the earthing system, Fig. 7.

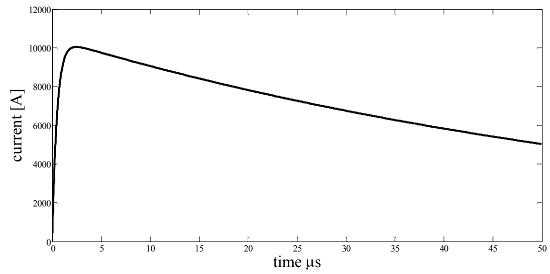

Fig. 8 – Lightning current.

For the system under study, the phenomenon of ionization can help to reduce hazardous situation. In the absence of the phenomenon of the ionization potential of the ground system reaches the peak 150kV, Fig. 9, while this value is lower, 100kV, if the phenomenon takes place. The first voltage value is close to the values that must ensure the insulators, which may be subject to a double voltage, between the power line and support, precluding a good operation, thus leaving that the lightning current flow on the power line, a consequential risk situation for the station that is connected to the line.

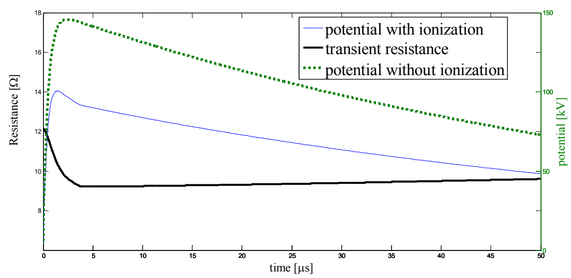

Fig. 9 – Direct stroke on the single rod (no tower simulation) .

The direct lightning strike of the earth system is far from real cases, for which the impedance of the system tower, earth electrode and guard cable, influences the distribution of current and potential.

Generally the impedance of the tower and of the ground electrode is less than the impedance presented by the guard cables, this means that the current path has a reclosure mainly vertical. However, this behavior may overstress the insulators, and failing these, a power line would involved in the discharge, ensuring a further horizontal path that would decrease the characteristic impedance of the same, thus increasing the amount of current that affects the terminal station.

Fig. 10 shows the system under study. A cascade of towers has been affected by 2x10kA lightning strike, only the right line has been taken into account.

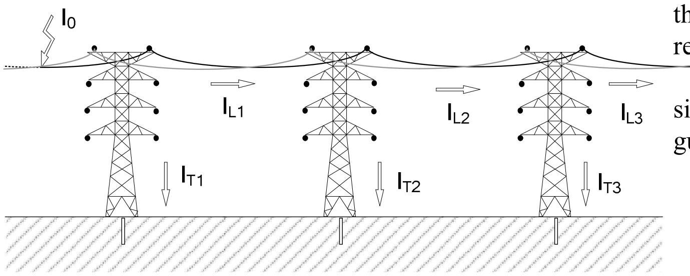

Fig. 10-. Simulated overhead line.

Fig. 11 shows the time profiles of distributed current in the line. In each instant of time it can be seen how the current I_0 is equal to the sum of I_{L1} +and I_{T1} currents, and similarly I_{L1} is equal to the sum of I_{T2} and I_{L3}.

It is possible to see that the current I_{L1} becomes greater than I_{T1} between 22 and 23 µs. Only current I_{T1} shows the same profile of lightning current, the others appear to be flattened, with maximum little accentuated.

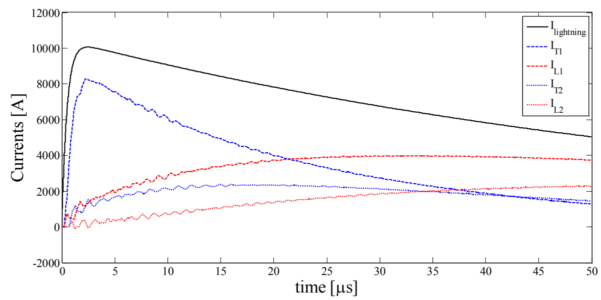

Fig. 11 - Time profiles of currents shown in Fig.10, without considering the ionization phenomenon.

Now we consider the phenomenon of ionization, the current drained from the first tower is sufficiently high to initiate the phenomenon.

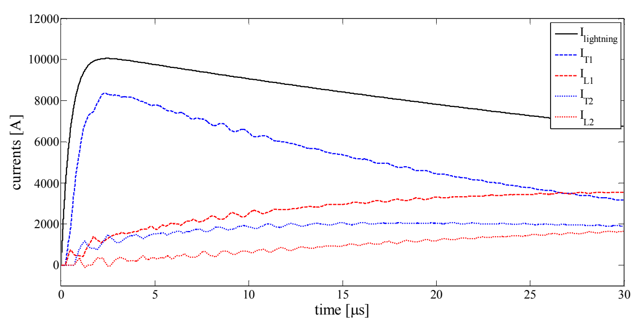

Fig. 12- Time profiles of currents shown in Fig.10, by considering the ionization phenomenon.

It can be noticed that with the phenomenon of ionization implemented, the current I_{L1} reaches the current I_{T1} at a later time, 27 µs, than that in which it reached in the previous simulation, 22-23 µs. With ionization the maximum current in the tower 1 is 8363 A, without 8288 A; maximum voltage reached with and without is almost the same, 400 kV. The voltage does not vary much, as to vary is only a small part of the subsystem represented by the electrode to earth, which reaches the value of 9.8 Ω, from original 12.1 Ω.

An analysis of sensitivity of the model is performed, by simulating a double current than previously simulated: the guard rope is crossed by 20 kA in each direction.

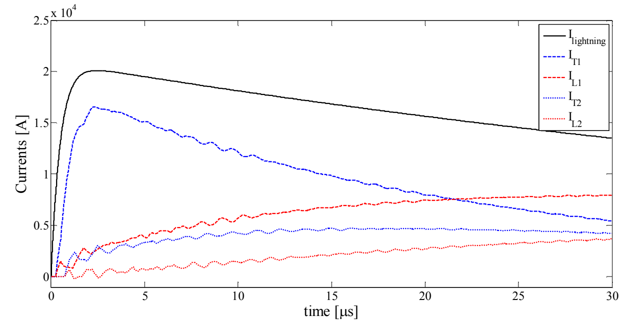

Fig. 13. Time profiles of currents shown in Fig.10, by considering a 20 kA lightning current and neglecting the ionization phenomenon.

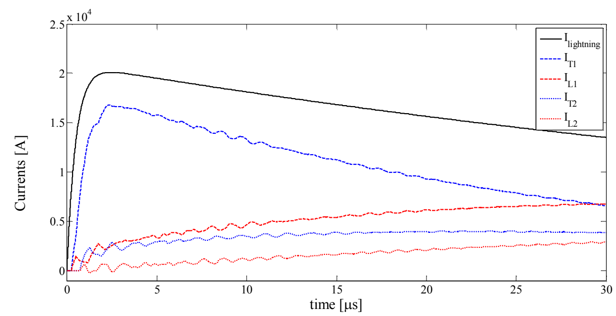

Fig. 14. Time profiles of currents shown in Fig.10, by considering a 20 kA lightning current and implementing the ionization phenomenon.

Comparing the trends of the currents shown in Fig. 13 and Fig. 14, we can observe how the intensification of the phenomenon of ionization (the resistance of the vertical rod 1 reaches the value of 8.1 Ω) involves a considerable decrease of the current on the overhead line, which equals the current injected into the ground for a time of 30 µs. Also the vertical rod 2 takes part in the ionization phenomenon of the ground, and it reaches the minimum value of 11.9 Ω, with a minimum contribution to the overall behavior of the system.

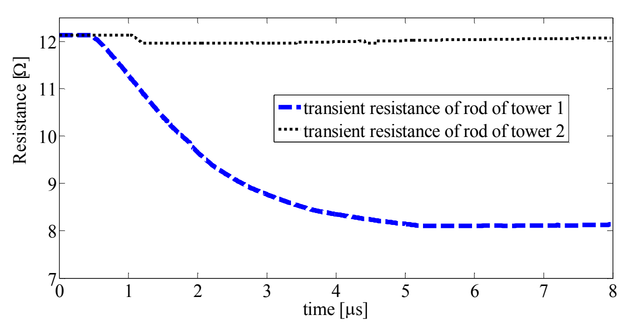

Fig. 14. Time profiles of transient resistance of rods of tower 1 and 2.

VI. CONCLUSIONS

This article describes as the contribution of the ionization of the soil modifies the behavior of an overhead power line. To obtain this information, the technique of finite difference time domain (FDTD) has been applied to Maxwell's equations to describe the behavior of soil subject to the crossing of high currents, and the telegrapher equations to describe the behavior of system tower and overhead line. It has described the difficulties in obtaining an accurate model of the system tower, overhead line and grounding electrode. It has been shown that increasing the lightning current, the contribution of the ionization of the soil tends to further reduce the current on the guard ropes that at the end of the line, will interact with the electric station.

Acknowledgment

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