

MR2817222 Ursescu, Corneliu, A mean value inequality for multifunctions. Bull. Math. Soc. Sci. Math. Roumanie (N.S.) 54(102) (2011), no. 2, 193200, 26E25 (26A24)

The paper is devoted to extend some mean value inequalities from the function setting to the multifunction one. Let (M, d) be a metric space, let F be a multifunctions defined on $D \subset \mathbf{R}$ and taking values in the family of nonempty subsets of M , and let $g : D \rightarrow \mathbf{R}$ be a strictly increasing function. The author proves the following inequality:

$$\frac{\delta(F(b), F(a))}{g(b) - g(a)} \leq \sup_{s \in [a, b] \cap D} \sup_{S \in F(s)} \sup_{t \in (s, b) \cap D} \frac{\delta(F(t), S)}{g(t) - g(s)},$$

where a and b are two points of D with $a < b$ and, if Q and P are nonempty subsets of M , then

$$\delta(Q, P) = \sup_{p \in P} \inf_{q \in Q} d(q, p).$$

An application of the previous inequality to the Dini derivatives of a multifunction is also given.

Reviewed by (*L. Di Piazza*)