Creativity in Conceptual Spaces

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Abstract

The main aim of this paper is contributing to what in the last few years has been known as computational creativity. This will be done by showing the relevance of a particular mathematical representation of Gärdenfors's conceptual spaces to the problem of modelling a phenomenon which plays a central role in producing novel and fruitful representations of perceptual patterns: analogy.

Introduction

There is an old tradition going back to Plato for which the phenomena which fall under the concept of creativity are those associated with the acquisition and mastery of some kind of craft (*techne*), rather than with random activity and aimless chance. According to this way of thinking, there is no reason to believe that an unschooled little ant that happens to draw in its course on the sand the first page of the score of the St. Matthew's Passion is engaged in a creative activity.

Indeed, for the supporters of this tradition, including the later Wittgenstein, creativity presupposes the existence of a high level linguistic competence typical of human beings. Here, of course, painting and music making — when seen as profoundly different from doodling or from casual humming —- are considered to be activities involving the use of some kind of articulated visual or auditory vehicles which give expression to feelings, emotions, etc., articulated visual or auditory vehicles which come with a syntax.

If we were successful in our attempt to model analogy within the particular mathematical representation of Gärdenfors's conceptual spaces we have chosen, this, besides scoring a point in favour of the computational creativity research programme (Cardoso, Veale, and Wiggins 2009), (Colton and Wiggins 2012), would also have important consequences with regard to the tenability of the old traditional view of creativity we mentioned above. For, since Gärdenfors's conceptual spaces, as we shall see in what follows, are placed in the sub-linguistic level of the cognitive architecture of a cognitive agent (CA), there would be at least a phenomenon intuitively belonging to creativity which could be represented independently of language.

After a section dedicated to a brief survey of some of the central contributions to the study of the connection between analogical thinking and computation, the paper proceeds to an explanation of how analogy is related to creativity. The article then develops by means of an illustration of the cognitive architecture of our CA in which the nature and function of Gärdenfors's conceptual spaces is made explicit.

A characterization of two conceptual spaces present in the 'library' of our CA — the visual and the music conceptual spaces — is then offered and visual analogues of music patterns are examined. The theoretical points made in the paper are, eventually, illustrated in the discussion of a case study.

Analogical thinking and computation

Human cognition is deeply involved with analogy-making processes. Analogical capabilities make possible perceiving clouds as resembling to animals, solving problems through the identification of similarities with previously solved problems, understanding metaphors, communicating emotions, learning, etc. (Kokinov and French 2006), (Holyoak et al. 2001).

Analogical reasoning is ordinarily used to 'transfer' structures, relational properties, etc. from a source domain to a target domain, and is clearly involved in that human ability which consists in producing generalizations.

Many models for analogical thinking are present in the literature. They are characterized by: (1) the ways of representing the knowledge on which the analogical capability is based, (2) the processes involved in realizing the analogical relation, and by (3) the manner in which the analogical transfer is fulfilled (Krumnack, Khnberger, and Besold 2013).

A known class of computational models for analogy-making are those based on Gentner's (1983) Structure Mapping Theory (SMT). This theory was the first that focussed on the role of the structural similarity existing between source and target domains, structural similarity which is generated by common systems of relations obtaining between objects in the respective domains. The structure mapping theory uses graphs to represent the domains and computes analogical relations by identifying maximal matching sub-graphs (Krumnack, Khnberger, and Besold 2013).

Other models are based on a connectionist approach, for example, we can mention here the Structured Tensor Analogical Reasoning (STAR) (Halford et al. 1994) and its 'evolution' STAR-2 (Wilson et al. 2001), which provide mechanisms for computing analogies using representations based on the mathematics of tensor products (Holyoak et al. 2001); and the framework for Learning and Inference with Schemas and Analogies (LISA) (Hummel and Holyoak 1996) which exploits temporally synchronized activations in a neural network to identify a mapping between source and target elements.

In 1989 Keith Holyoak and Paul Thagard (Holyoak and Thagard 1989) proposed a theory of analogical mapping based upon interacting structural, semantic, and pragmatic constraints that have to be satisfied at the same time, implementing the theory as an emergent process of activation states of neuron-like objects.

According to (French 1995), metaphorical language, analogy making and couterfactualization are all products of the mind's ability to perform slippage (i.e. the replacement of one concept in the description of some situation by another related one) fluidly. All analogies involve some degree of conceptual slippage: under some pressure, concepts slip into related concepts. On the notion of conceptual slippage is based Copycat, a model of analogy making developed in 1988 by Douglas Hofstadter et al. (Hofstadter and Mitchell 1994).

In (Kazjon Grace and Saunders 2012), a computational model of associations, based on an interpretation-driven framework, was put forward and applied to the domain of real-world ornamental designs, where an association is understood in terms of the process of constructing new relationships between different ideas, objects or situations.

In (Grace, Saunders, and Gero 2008) a computational model for the creation of original associations has been presented. The approach is based on the concept of *interpretation*, which is defined as "a perspective on the meaning of an object; a particular way of looking at an object" [1] , and acts on conceptual spaces, where concepts are defined as regions in that space. In this context the authors represent the interpretation process as a transformation applied to the conceptual space from which feature-based representations are generated. The model tries to identify relationships that can be built between a *source* object and a *target* object. A new association is constructed when the transformations applied to these objects contribute to the emergence of some shared features which were not present before the application of the transformations.

Creativity and Analogy

It is intuitively correct to say that the use of a stick made by a bird to catch a larva in the bark of a tree is creative, as it is creative the writing of a poem or the introduction of a new mathematical concept. Creativity, indeed, covers a large variety of phenomena which also differ from one another in relation to their different degree of abstractness, i.e., the creativity of the hunting technique of the bird is much less abstract than that displayed by Beethoven in the writing of the fifth symphony.

It is not our intention in this paper even to attempt to give a definition of creativity. What we want to do here is simply to focus on the concept of analogy — the relation in which A is to B is the same as the relation in which α is to β — which is at the heart of much of what we can correctly describe as creative activity.

A traditional model of analogical thinking is provided by the concept of proportion:

$$\frac{A}{B} = \frac{\alpha}{\beta}$$

where A and B are entities homogeneous to each other — like α and β are homogeneous to each other — but A and B are non-homogeneous to α and β. Analogical thinking allows the emergence/recognition of a pattern in a certain environment E which is similar/the same as that which has already emerged/been recognized in another environment E'. Much of the work to be done in what follows will consist in rendering mathematically rigorous what we have called 'pattern', 'environment E', 'analogy as similarity of patterns given in different environments', 'identity of patterns given in different environments', etc. etc.

Let us say that patterns are here understood as relational entities (structures) defined on a given domain.[2] And since a necessary condition for the emergence/recognition of a pattern is the presence of a system of representation, we are going to identify the environment E with such a system, and choose as a model of such a system of representation Gärdenfors's conceptual spaces. Moreover, two patterns π_1 and π_2 given in two different conceptual spaces V_1 and V_2 are said to be 'analogous to one another' if there is an homomorphism between π_1 and π_2, whereas they are said to be 'exemplifying the same pattern' if there is an isomorphism between π_1 and π_2.

A cognitive architecture based on Conceptual Spaces

The introduction of a cognitive architecture for an artificial agent implies the definition of a conceptual representation model.

Conceptual spaces (CS), employed extensively in the last few years (Chella, Frixione, and Gaglio 1997) (De Paola et al. 2009) (Jung, Menon, and Arkin 2011), were originally introduced by Gärdenfors as a bridge between symbolic and connectionist models of information representation. This was part of an attempt to describe what he calls the 'geometry of thought'.

In (Gärdenfors 2000) and (Gärdenfors 2004) we find a description of a cognitive architecture for modelling representations. This is a cognitive architecture in which an intermediate level, called 'geometric conceptual space', is introduced between a linguistic-symbolic level and an associationist sub-symbolic level of information representation.

The cognitive architecture (see figure 1), is composed by three levels of representation: a *subconceptual level*, in which data coming from the environment are processed by means of a neural networks based system, a *conceptual level*, where data are represented and conceptualized independently of language; and, finally, a *symbolic level* which

[1](Grace, Saunders, and Gero 2008), Section2, page2

[2]For the special case represented by mathematical patterns see (Oliveri 1997), (Oliveri 2007), ch. 5, and (Oliveri 2012).

makes it possible to manage the information produced at the conceptual level at a higher level through symbolic computations. The conceptual space acts as a workspace in which low-level and high-level processes access and exchange information respectively from bottom to top and from top to bottom. The description of the symbolic and subconceptual levels goes beyond the scope of this paper.

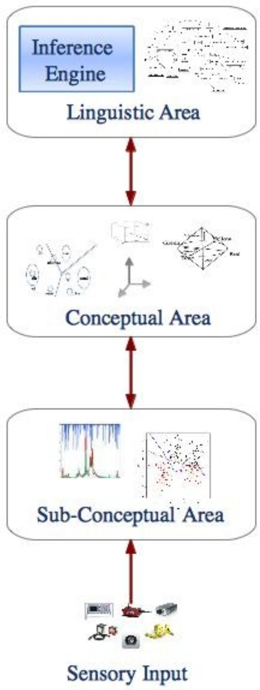

Figure 1: A sketch of the cognitive architecture

According to the linguistic/symbolic level:

> "Cognition is seen as essentially being *computation*, involving symbol manipulation (Gärdenfors 2000)".

whereas, for the associationist sub-symbolic level:

> "Associations among different kinds of information elements carry the main burden of representation. *Connectionism* is a special case of associationism that models associations using artificial neuron networks (Gärdenfors 2000), where the behaviour of the network as a whole is determined by the initial state of activation and the connections between the units (Gärdenfors 2000)".

Although the symbolic approach allows very rich and expressive representations, it appears to have some intrinsic limitations such as the so-called "symbol grounding problem", [3] and the well known A.I. "frame problem".[4] On the other hand, the associationist approach suffers from its low-level nature, which makes it unsuited for complex tasks, and representations.

Gärdenfors' proposal of a third way of representing information exploits geometrical structures rather than symbols or connections between neurons. This geometrical representation is based on a number of what Gärdenfors calls 'quality dimensions' whose main function is to represent different qualities of objects such as brightness, temperature, height, width, depth.

Moreover, for Gärdenfors, judgments of similarity play a crucial role in cognitive processes. And, according to him, it is possible to associate the concept of distance to many kinds of quality dimensions. This idea naturally leads to the conjecture that the smaller is the distance between the representations of two given objects the more similar to each other the objects represented are.

According to Gärdenfors, objects can be represented as points in a conceptual space, knoxels (Gaglio et al. 1988) [5], and concepts as regions within a conceptual space. These regions may have various shapes, although to some concepts — those which refer to natural kinds or natural properties — correspond regions which are characterized by convexity.[6]

For Gärdenfors, this latter type of region is strictly related to the notion of prototype, i.e., to those entities that may be regarded as the archetypal representatives of a given category of objects (the centroids of the convex regions).

One of the most serious problems connected with Gärdenfors' conceptual spaces is that these have, for him, a phenomenological connotation. In other words, if, for example, we take, the conceptual space of colours this, according to Gärdenfors, must be able to represent the geometry of colour concepts in relation to how colours are given to us.

However, we have chosen a non phenomenological approach to conceptual spaces in which we substitute the expression 'measurement' for the expression 'perception', and consider a cognitive agent which interacts with the environment by means of the measurements taken by its sensors rather than a human being.

Of course, we are aware of the controversial nature of our non phenomenological approach to conceptual spaces. But, since our main task in this paper is characterizing a rational agent with the view of providing a model for artificial agents, it follows that our non-phenomenological approach to conceptual spaces is justified independently of our opinions on perceptions and their possible representations within conceptual spaces

Although the cognitive agent we have in mind is not a human being, the idea of simulating perception by means of measurement is not so far removed from biology. To see this,

[3] How to specify the meaning of symbols without an infinite regress deriving from the impossibility for formal systems to capture their semantics. See (Harnad 1990).

[4] Having to give a complete description of even a simple robot's world using axioms and rules to describe the result of different actions and their consequences leads to the "combinatorial explosion" of the number of necessary axioms.

[5] The term 'knoxel' originates from (Gaglio et al. 1988) by the analogy with "pixel". A knoxel k is a point in Conceptual Space and it represents the epistemologically primitive element at the considered level of analysis.

[6] A set S is *convex* if and only if whenever $a, b \in S$ and c is between a and b then $c \in S$.

consider that human beings, and other animals, to survive need to have a fairly good ability to estimate distance. The frog unable to determine whether a fly is 'within reach' or not is, probably, not going to live a long and happy life.

Our CA is provided with sensors which are capable, within a certain interval of intensities, of registering different intensities of stimulation. For example, let us assume that CA has a visual perception of a green object *h*. If CA makes of the measure of the colour of *h* its present stereotype of green then it can, by means of a comparison of different measurements, introduce an ordering of gradations of green with respect to the stereotype; and, of course, it can also distinguish the colour of the stereotype from the colour of other red, blue, yellow, etc. objects. In other words, in this way CA is able to introduce a 'green dimension' into its colour space, a dimension within which the measure of the colour of the stereotype can be taken to perform the role of 0.

The formal model of a conceptual space that at this point immediately springs to mind is that of a metric space, i.e., it is that of a set X endowed with a metric. However, since the metric space X which is the candidate for being a model of a conceptual space has dimensions, dimensions the elements of which are associated with coordinates which are the outcomes of (possible) measurements made by CA, perhaps a better model of a conceptual space might be an n-dimensional vector space V over a field K like, for example, $\mathbb{R}^n$ (with the usual inner product and norm) on $\mathbb{R}$.

Although this suggestion is interesting, we cannot help noticing that an important disanalogy between an n-dimensional vector space V over a field K, and the 'biological conceptual space' that V is supposed to model is that human, animal, and artificial sensors are strongly non-linear. In spite of its cogency, at this stage we are not going to dwell on this difficulty, because: (1) we intend to examine the 'ideal' case first; and because (2) we hypothesize that it is always possible to map a perceptual space into a conceptual space where linearity is preserved either by performing, for example, a small-signal approach, or by means of a projection onto a linear space, as it is performed in kernel systems (Scholkopf and Smola 2001).

The Music and Visual Conceptual Spaces

Let us consider a CA which can perceive both musical tones and visual scenes. The CA is able to build two types of conceptual spaces in order to represent its perceptions. As reported in (Augello et al. 2013a) (Augello et al. 2013b), the agent's conceptual spaces are generated by measurement processes; in this manner each knoxel is, directly or indirectly, related to measurements obtained from different sensors. Each knoxel is, therefore, represented as a vector $k = (x_1, x_2, ..., x_n)$ where x_i belongs to the X_i quality dimension of our n-dimensional vector space. The Conceptual Spaces can also be manipulated according to changes of the focus of attention of the agent (Augello et al. 2013a) (Augello et al. 2013b), however the description of this process goes beyond the scope of this paper and will not be described here.

Visual conceptual space

According to Biederman's geons theory (see (Biederman 1987)), the visual perception of an object is processed by our brain as a proper composition of simple solids of different shapes (the geons). Following Biederman main ideas, we exploit a conceptual space for the description of visual scenarios (see fig. 2) where objects are represented as compositions of super-quadrics, and super-quadrics are vectors in this conceptual space.

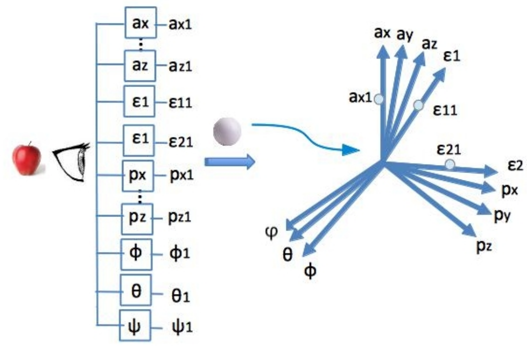

Figure 2: Visual perception and corresponding CS representation

For those who are not familiar with the concept of super-quadric, let us say that super-quadrics are geometric shapes derived from the quadrics parametric equation with the trigonometric functions raised to two real exponents. The inside/outside function of the superquadric in implicit form is:

$$F(x,y,z) = \left[\left(\frac{x}{a_x}\right)^{\frac{2}{\epsilon_1}} + \left(\frac{y}{a_y}\right)^{\frac{2}{\epsilon_2}} \right]^{\frac{\epsilon_2}{\epsilon_1}} + \left(\frac{z}{a_z}\right)^{\frac{2}{\epsilon_1}}$$

where the parameters a_x, a_y, a_z are the lengths of the super-quadric axes, the exponents $\varepsilon_1, \varepsilon_2$, called 'form factors', are responsible for the shapes form: values approaching 1 render the shape rounded.

To see this, let us suppose that the vision system can be approximated and modeled as a set of receptors, and that these receptors give as output, corresponding to the external perceived stimulation, the set of super-quadrics parameters associated to the perceived object. This leads to a superquadric conceptual representation of a 3D world. The situation is illustrated in Fig 2 where an object positioned in the 3D space, let us say an apple, is approximately perceived as a sphere and is consequently mapped as a knoxel in the related conceptual space.

In particular a knoxel in the Visual Conceptual space can be described by the vector:

$$\overrightarrow{k} = (a_x, a_y, a_z, \varepsilon_1, \varepsilon_2, p_x, p_y, p_z, \varphi, \theta, \psi)^T$$

In this perspective, knoxels correspond to simple geometric building blocks, while complex objects or situations are represented as suitable sets of knoxels (see figure 3).

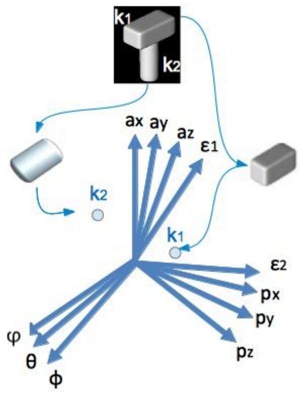

Figure 3: A representation of a hammer in the *visual conceptual space* as a composition of two super-quadrics

Music Conceptual Space

In (Gärdenfors 1988), Gardenfors discusses a program for musical spaces analysis directly inspired to the framework of vision proposed by Marr (Marr 1982). This discussion has been further analysed by Chella in (Chella 2013), where a music conceptual space has been proposed and placed into the layers of the cognitive architecture described in the previous sections.

As reported in (Shepard 1982), it has been highlighted that for the music of all human cultures, the relation between pitch and time appears to be crucial for the recognition of a familiar piece of music. In consideration of this, the representation of pitch becomes prominent for the representation of tones.

In the music CS the quality dimensions represent information about the partials composing musical tones. This choice is inspired by empirical results about the perception of tones to be found in (Oxenham 2013).

We model the functions of the ear as a finite set of filters, each one centred on the i-th frequency (we suppose for example to have N filters ranging from 20Hz to 20KHz at proper intervals. In this manner, a perceived sound will be decomposed into its partials and mapped as a vector $V = (c_1, c_2, \cdots, c_n)$ whose components correspond to the coefficients of the n frequencies that compose the sound $(\omega_1, \omega_2, \cdots \omega_n)$, as illustrated in Fig 4. The supposition is that here we use the discrete Fourier Series Transform, which is commonly used in signal processing, considering not only music but also other time-variant signals such as speech.

The vector $\overrightarrow{V}$ is, therefore, a knoxel of the music conceptual space. The partials of a tone are related both to the pitch and the timbre of the perceived note. Roughly, the fundamental frequency is related to the pitch, while the amplitudes of the remaining partials are also related to the timbre of the note. A similar choice is to be found in Tanguiane (Tanguiane 1993).

A knoxel in the music CS will change its position when the perceived tone changes its pitch or its loudness or tim-

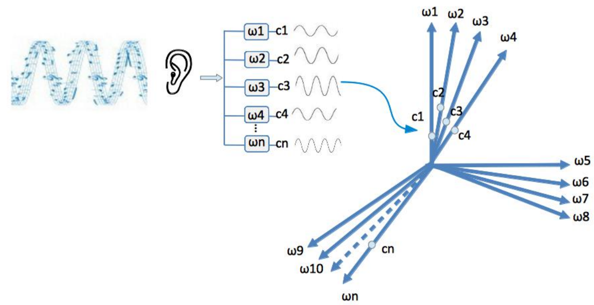

Figure 4: Music perception and corresponding CS representation

bre. In fig. 5 it is shown how the *symbolic level* given by the pentagram representation of a chord is mapped into a *conceptual* space representation.

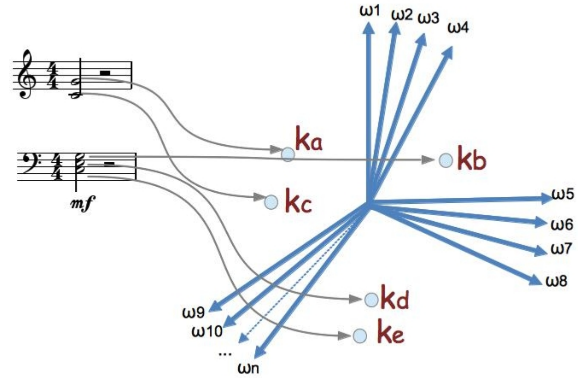

Figure 5: A representation of two chords in the *music conceptual space*.

From Visual Patterns to Music Patterns

A cognitive agent is able to represent its different perceptions in proper conceptual spaces; as soon as the agent perceives visual scenes or music, a given geometric structure will emerge. This structure will be made of vectors and regions, conceptual representations of perceived objects.

Music and visual conceptual spaces are two examples of conceptual representations that can be thought as a basis for computational simulation of an analogical thinking that provides the agent with some sort of creative capability. Knowledge and experiences made in a very specific domain of perception can be exploited by the agent in order to better understand or to express in different ways the experiences and the perceptions that belong to other domains. This process resembles the synaesthesia [7] affecting some people, which allows to perform analogies between elements and experiences belonging to different sensory areas. Analogical thinking reveals similarities between patterns belonging to different domains.

For what concerns the music and vision domains, several analogies have been discussed in the literature. As an example, Tanguiane (Tanguiane 1993) compares visual and music perceptions, considering three different levels and both static and dynamic point of views. In particular, from a static point of view, a first visual level, that is the pixel perception level, can correspond the perception of *partials* in music. At the second level, the perception of simple patterns in vision corresponds to the perception of single *notes*. Finally at the third level, the perception of structured patterns (as patterns of patterns), corresponds to the perception of *chords*.

Concerning dynamic perception, the first level is the same as in the case of static perception, while at the second level the perception of visual objects corresponds to the perception of musical notes, and at the third final level the perception of visual trajectories corresponds to the perception of music *melodies*.

Gärderfors (Gärdenfors 1988), in his paper on "Semantics, Conceptual Spaces and Music" discusses a program for musical spaces analysis directly inspired to the framework of vision proposed by Marr (Marr 1982), where the first level is related to *pitch identification*; the second level is related to the identification of *musical intervals* and the third level to *tonality*, where scales are identified and the concepts of chromaticity and modulation arise. The fourth level of analysis is that at which the interplay of pitch and time is represented.

In what follows we are going to illustrate a framework for possible relationships between visual and musical domains. The mapping is one among many possible, and it has been chosen in order to make clear and easily understandable the whole process. As we have already said, it is possible to represent complex objects in a conceptual space as a set of knoxels. In particular, in the visual conceptual space, a complex object can be described as the set of knoxels representing the simple shapes of which it is composed, whereas in the music conceptual space we have seen how to represent chords as the set of knoxels representing the different tones played together.

In the two spaces will emerge recurrent patterns, given respectively by proper configurations of shapes and tones which occur more frequently. A fundamental analogy between the two domains can be highlighted, concerning the importance of the mutual relationships between the parts composing a complex object. In fact, in the case of perception of complex objects in vision, their mutual positions and shapes are important in order to describe the perceived object: e.g., in the case of an hammer, the mutual positions and the mutual shapes of the handle and the head are obviously important to classify the complex object as an hammer. A the same time, the mutual relationships between the pitches (and the timbres) of the perceived tones are important in order to describe the perceived chord (to distinguish for example, a major from a minor chord of the same note). Therefore, spatial relationships in static scenes analysis are in some sense analogous to sounds relationships in music conceptual space.

Although in this work we are overlooking the dynamic aspect of perception in the two domains of analysis, we can also mention some possible analogies, for example, we could correlate the trajectory of a moving object with a succession of different notes within a melody.

As certain movements are harmonious or not, so in music the succession of certain tones creates pleasant feelings or not.

Visual representation of musical objects: a case study

In what follows, we describe a procedure capable of simulating some aspects of analogical thinking. In particular, we consider an agent able to: (1) represent tones and visual objects within two different conceptual spaces; and (2) build analogies between auditory perceptions and visual perceptions.

At the heart of this procedure there is the ability on the part of the CA of individuating the appropriate homomorphism $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ which maps a knoxel belonging to a n-dimensional conceptual space $\mathbb{R}^n$ — the acoustic domain — on to another knoxel in a different m-dimensional conceptual space $\mathbb{R}^m$ — the visual domain.

For the sake of clarity we simplify the previously illustrated model of both music and visual conceptual representation of the agent. In particular:

- for what concerns the visual perceptions, we consider only a visual coding of spheres: this leads to the assumption that every observed object will be perceived as a sphere or as a composition of spheres by the agent;
- for what concerns the auditory perceptions, we consider only a limited set of discrete frequencies which the agent perceives. All information about pitch, loudness and timbre is implicitly represented in the auditory conceptual space by the Fourier Analysis parameters.

Figure 6 illustrates the mapping process leading from sensing and representation in the music conceptual spaces to a pictorial representation of the heard tone. The mapping is realized through an analogy transformation which let arise a visual knoxel in he visual conceptual space. The analogy process of the agent can be outlined in the following steps:

- the agent perceives a sound (A)
- the sound is sensed and decomposed through Fourier Transform Analysis (A)
- the measurements on the partials lead to a conceptual representation of the perceived sound as a knoxel in the acoustic space (A)

[7] a condition in which the stimulation of one sense causes the automatic experience of another sense

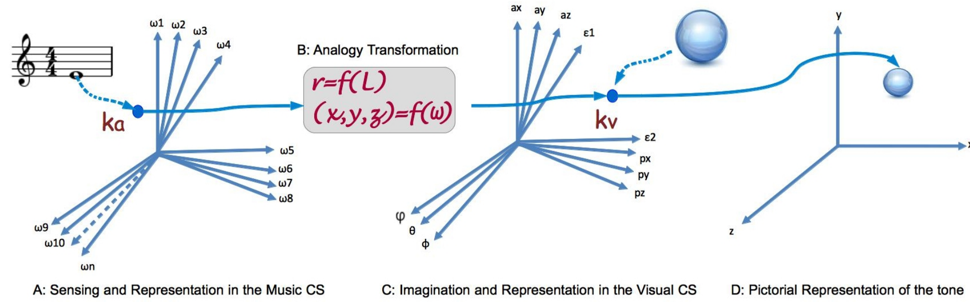

Figure 6: Mapping process leading from sensing and representation in the music conceptual spaces to a pictorial representation of the heard tone

- the knoxel k_A in the acoustic space is transformed into a knoxel k_V in the visual conceptual space (B)
- the mapping lets arise a conceptual representation of an object that is not actually perceived. It is only "imagined" by analogy. (C)
- the "birth" of this new item in the visual conceptual space, is directly related to the "birth" of an image, which, most importantly, is simply imagined and not perceived (D)

Given two conceptual spaces $\mathbb{R}^n$ and $\mathbb{R}^m$, the mapping can be any multidimensional function that realizes the appropriate transformation $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$. The function f can be learned in a supervised or unsupervised way through machine learning algorithms.

At present, we superimpose the structure f. In order to make a choice for f we take some inspirations from Shepard in (Shepard 1982).

Many geometrical mappings have been proposed for pitch: the simplest one is that one which use of a monodimensional logarithmic scale where each pure tone is related to the logarithm of its frequency.

However, according to the two component theory (Révész 1954) (Shepard 1982), the best manner to pictorially represent pitches is a helix or 3D-spiral instead of a straight line. A mapping based on this theory is illustrated in fig. 7, where simple sounds are drawn on the helix, as spheres of different sizes, according to their associated loudness.

That mapping allows to complete one turn per octave and reaches the necessary geometric proximity between points which are an octave distant from each other.

The strong point of the uniform helix representation is that the distance corresponding to any particular musical interval is invariant throughout the representation. Each tone can be mapped onto a spiral laying on a cylinder where points vertically aligned correspond to the same tone with different octave. This projective property holds regardless of the slope of the helix (Shepard 1982).

In superimposing f we suppose that when the agent perceives a sound which is louder than another one, this evokes in his mind the view of something that is more cumbersome than another one. We assume that this perceived object has no preferred direction or shape, therefore the easiest way to represent it is a sphere, whose radius can be associated to the loudness of the perceived sound.

The other parameter is the pitch. As soon as the agent perceives different pitches, he tries to visualize them, imagine them, locate them according to the helix whose equations are:

$$x = rcos(2\pi\omega) \tag{1}$$
$$y = rsin(2\pi\omega) \tag{2}$$
$$z = c\omega \tag{3}$$

If we consider a simple tone of given frequency ω , the pitch will be represented by a point $p(x, y, z)$ in the spiral, while its loudness L will be represented by a sphere having centre in $p(x, y, z)$ and a radius whose length r is related to the perceived loudness.

The sphere corresponds to a knoxel in the Visual-conceptual-space, while the perceived tone corresponds to a knoxel in the Music-conceptual-space.

The agent therefore will visually imagine the perceived sound as a sphere whose radius is proportional to the perceived loudness, while its position corresponds to a point laying on the helical line representing all the tones that can be perceived by the agent, and a chord will be imagined as a set of spheres in this 3D space.

Conclusions

We have illustrated a methodology for the computational emulation of analogy, which is an important part of the imaginative process characterizing the creative capabilities of human beings.

The approach is based on a mapping between geometric conceptual representations which are related to the perceptive capabilities of an agent.

Even though this mapping can be built up in several different ways, we presented a proof-of-concept example of some analogies between music and visual perceptions. This allows the agent to associate imagined, unseen images to perceived sounds. It is worthwhile to point out that, in similar

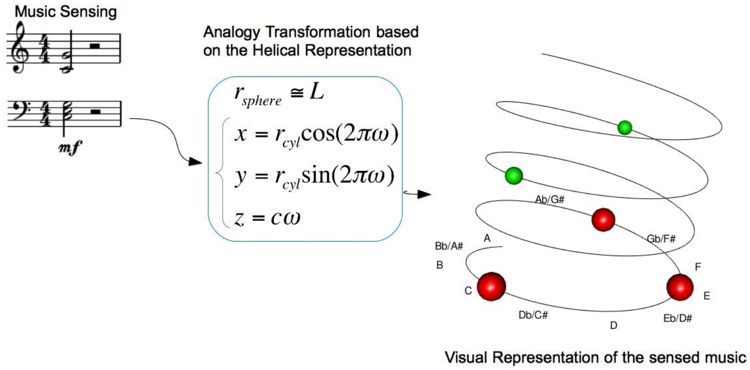

Figure 7: Visual representation of music chords deriving from a "two-component theory" based mapping

way, it is possible to imagine sounds to be associated to visual scenes, and the same can be done with different kinds of perceptions.

We claim that this approach could be a step towards the computation of many forms of the creative process. In future works different types of mapping will be investigated and properly evaluated.

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