Collective Dynamics

Congestion Number Analysis of Cross-Flow Dynamics: Experimental Data and Simulations

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Abstract We recently proposed the "Congestion Number" (*CN*) as a metric to evaluate the state of a pedestrian crowd. Such metric, whose definition is based on the gradient of the rotor of the crowd velocity field, appears to provide additional information with respect to traditional metrics based on pedestrian density and flow.

We also published two works on the dynamics of orthogonally crossing pedestrian flows under different density regimes. In the first manuscript we analysed experimental data by using traditional observables such as density, velocity and relative position between pedestrians, along with less explored ones such as body orientation. In the second one we proposed a hierarchy of simulation models to reproduce the cross-flow dynamics, and used the aforementioned observables to compare such models.

Based on theoretical considerations and analysis of real world data, we believe the crossing flow setting to be a good arena to test the *CN* metric, and in this work we perform a *CN* analysis on the empirical and simulation data. Results show that simulation models, which reproduced almost perfectly the density time dependence of the pedestrian crowd, "fail" to reproduce the *CN* one. Actually, models "outperform" the pedestrian crowd when analysed using *CN*. These preliminary results suggest that the *CN* metric may provide useful information not only in crowd assessment but also in model evaluation.

Keywords Pedestrian cross-flow · model-data comparison · crowd evaluation metrics

1 Introduction

Due to the growth of human population and in particular urban centres in the last century the problem of safely planning and managing human flows, including pedestrian ones, has become of great interest and relevance [1–4]. Our ability of studying, reproducing and predicting the dynamics of crowds has been obviously enhanced by new technologies that emerged starting from the second half of the 20th century [5]. In particular the widespread use of numerical computing has made possible to simulate dynamical systems with a large number of components, allowing for the development of crowd models [6,7]. Furthermore, it is now possible to track pedestrians in real time [8,9], and study large data sets concerning pedestrian behaviour in real-world settings [10, 11].

Nevertheless, real time assessment of crowd condition is often still based on possibly inadequate definitions using only crowd density such as the Level of Service (LOS) [12]. Crowd density is obviously extremely important, and the "fundamental diagram", i.e. the relation between crowd density and pedestrian velocity [13], is at the centre of research on pedestrian crowds. Such relation is in general studied in evacuation settings, in which all pedestrians share the same goal. Although the importance of such conditions is not under debate, lately quantitative and theoretical studies have focused on settings in which pedestrian flows cross at different angles [14], and an assessment metric that, while being still relatively simple, is able to deal with such conditions, is needed.

Recently we have introduced some works in such direction: in [15] we proposed the "Congestion Number" (*CN*)[1] a metric defined using the gradient of the rotor of the crowd velocity field, which showed to provide additional information with respect to traditional metrics based on pedestrian density and flow. Furthermore, in [17, 18] we analysed in detail the dynamics of two orthogonally crossing pedestrian flows, and introduced numerical models to reproduce it. In the following, after a short introduction to the aforementioned works, we join them performing an analysis of cross-flows based on the *CN*.

2 CN

Extending the work of [19], in [15] we introduced the following metric in order to identify "instabilities" in a crowd through the presence of spatially close but differently directed "rotating movements". In order to do that, we take in consideration the rotor of the crowd velocity field[2], or better its only non-zero component (orthogonal to the floor) $(\nabla \times \mathbf{v})_z$ and measure the absolute value of its gradient, that we name "Differential Congestion"

$$DC(\mathbf{x}) = ||\nabla[(\nabla \times \mathbf{v})_z(\mathbf{x})]||. \tag{1}$$

The fundamental idea is that such gradient measures how the (rotating) moving direction of the crowd changes, a higher *DC* representing a more unstable crowd.

[1] A different approach, more focused on microscopic dynamics, to identify a pure number describing the state of the crowd was proposed in [16].

[2] The velocity field has to be defined through a proper discretisation scheme in space and time. The choice of the discretisation scheme is an important but not trivial issue which is discussed at length in [15].

In order to define our metric as a pure number, DC is divided by a (very high) reference value, i.e. an “Extreme Differential Congestion” EDC

$$CN(\mathbf{x}) = \frac{DC(\mathbf{x})}{EDC(\mathbf{x})}. \tag{2}$$

Through a series of approximations, whose purpose is to replace differential operators, that depend strongly on discretisation schemes, with differences and averages, we reach the following operational definition

$$CN(\mathbf{x}) - \frac{(\max_{R(\mathbf{x})}(\nabla \times \mathbf{v})_z(\mathbf{x}) - \min_{R(\mathbf{x})}(\nabla \times \mathbf{v})_z(\mathbf{x}))RL}{6 < v >_{R(\mathbf{x})} L}, \tag{3}$$

$R(\mathbf{x})$ being a “Region Of Interest” centred on $\mathbf{x}$.

We verified the following important properties of CN

1. It can differentiate between an organised crowd (low CN) and a not organised one (high CN) regardless of density and velocity. For example, in a bi-directional flow, when the opposite streams separate, we have no change in (global) density and little change in velocity, but a considerable drop in CN.
2. In a real world environment (a portion of Shinjuku Station in Tokyo), high CN areas are different than high-density ones, the former being located where flows with different directions cross (or join).

3 Crossing flow dynamics

3.1 Experimental results

In [17] we analyse the results of a series of experiments, organised recruiting students as participants, on the dynamics of two crossing flows. Although the number of subjects (56) and the width of the corridors were fixed, by changing the size of the starting lanes we were able to investigate six different starting lane density conditions, namely[3] $\rho_I = 0.25$, $\rho_I = 0.5$, $\rho_I = 1$, $\rho_I = 1.5$, $\rho_I = 2$ and $\rho_I = 2.5$ ped/m^2. Trajectories were extracted using the PeTrack software [20,21]. Furthermore, for a subset of 9 participants, it was possible to measure also torso orientation by using the inbuilt gyroscope sensors of tablets which were fixed to their body using a bib.

The state of the crowd and its dependence on starting lane density conditions were analysed by studying the probability distributions of the following observables: *pedestrian speed* v, *velocity direction angle* θ^v, *body orientation angle* θ, *difference between* θ *and* θ^v $\Delta\theta$, *relative distance* δ^s and *relative angle* ϕ^s *between pedestrians in the same flow*, and *relative distance* δ^o and *relative angle* ϕ^o *between pedestrians in different flows*. Furthermore, we analysed the time dependence of the global density in the crossing area $\rho(t)$. Our analysis confirmed the emergence of self-organising “stripes” in the crossing area [14].

[3] Here with ρ_I we indicate the density corresponding to when the two flows are separated in the starting areas, while the crossing area density ρ is time dependent and dynamically determined.

3.2 Modelling

In [18], to which the reader is directed for further details, we tried to understand *which are the fundamental components of collision avoidance necessary to describe the observed behaviour* (i.e. to reproduce the observable probability distribution). The purpose of this work was thus not to introduce a new, all-purpose pedestrian model, but to investigate, inside a given framework (second order models) whether, to reproduce the observed dynamics, it is necessary to:

1. introduce body asymmetry (i.e. using an elliptical body shape or a circular one)
2. introduce a collision avoidance process using explicitly body shape and size information (short-range interaction) or if a model using only centre of mass position and velocity (long-range interaction) would suffice[4]

These two conditions create a hierarchy of four models with increasing complexity. The equations of motion for the most complex model read

$$\begin{cases} \ddot{\mathbf{r}} = -k_{v_p}(\dot{\mathbf{r}} - \mathbf{v}_p) - k^v_\theta(\dot{\mathbf{r}} - \|\mathbf{r}\|\mathbf{n}) + (1-\beta(t^c))\mathbf{F}^{long} + \beta(t^c)\mathbf{F}^{short}, \\ \ddot{\theta} = -k_\omega \dot{\theta} - k^\omega_\theta \Delta\theta + \beta(t^c) T^{short}. \end{cases} \tag{4}$$

Here the vector $\mathbf{r}$ identifies the position of the pedestrian's body centre (on the 2D plane), the vector $\mathbf{n}$ the normal to the pedestrian's chest (body orientation), and the angle θ its orientation. $\mathbf{F}^{long}$ [22] and $\mathbf{F}^{short}$ represent respectively the aforementioned long and short interaction acceleration terms (T^{short} being a torque due to the short range interaction), t^c is the time to the next collision and β satisfies

$$\begin{cases} \lim_{t\to 0} \beta(t) = 1, \\ \lim_{t\to\infty} \beta(t) = 0. \end{cases} \tag{5}$$

The constant k_{v_p} represents the tendency to move with a preferred velocity $\mathbf{v}_p$, k_ω to stabilise body orientation, while k^v_θ and k^ω_θ align velocity direction to body orientation ($\Delta\theta$ being the angle between the velocity and body orientation vectors).

Circular body models are obtained by removing all terms related to θ and $\mathbf{n}$, while models using only long-range interaction are obtained by setting

$$\beta(t) = 1, \ \forall t.$$

We verified that the models using (also) $\mathbf{F}^{short}$ reproduce better the empirical observable distributions. On the other hand, the impact of using body orientation on models using $\mathbf{F}^{short}$ was very reduced (at least when comparing to the observed experimental setting and using the very simplified proposed model of body shape). In particular, even when *evaluated on the highest density setting on which they had not been calibrated*, both models reproduced well the time evolution of density in the crossing area $\rho(t)$ (as shown in Fig. 1) and the probability distribution of speed $P(v)$.

[4] The terms short/long range are here used somehow improperly, and they reflect the fact that detailed information concerning body shape and size is needed only in the proximity of a collision.

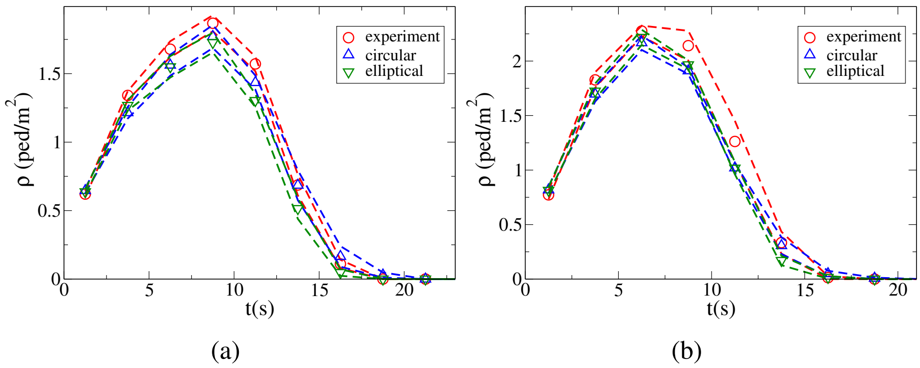


Figure 1 Time evolution of the average density $\rho(t)$ in the crossing area. (a): initial density condition $\rho_I = 1.5$ ped/m^2; (b): initial density condition $\rho_I = 2.5$ ped/m^2. Red circles: experimental data; blue up triangles: circular body model; green down triangles: elliptical body model. Dashed lines show the standard error confidence values computed by treating different experiment repetitions as independent. Refer to [17] for more details.

The models were also able to reproduce the relative angles probability distributions, which are related to the stripe formation self-organisation pattern, although these observables were not used in the calibration process (see [18] for details).

4 CN analysis of cross-flow data

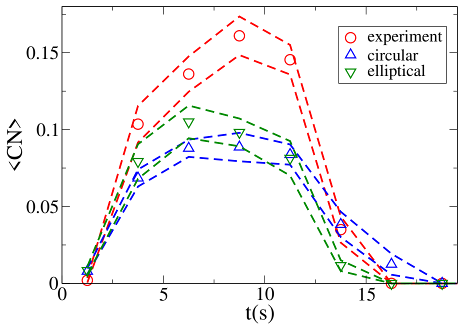
We ran a CN analysis on the cross-flow empirical and simulated data, using the same code and parameters proposed in [15], and in detail we analysed the time evolution of the average value of $\langle CN(t)\rangle$ in the crossing area (refer to [15] for more details).

The results concerning the initial density condition $\rho_I = 1.5$ ped/m^2 (on which the models were calibrated) and the initial density condition $\rho_I = 2.5$ ped/m^2 (on which the models were evaluated in [18]) are shown respectively in Fig. 2 (a) and (b).

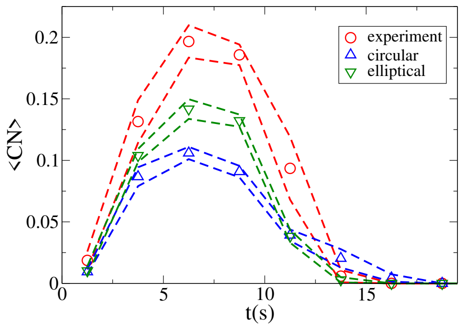
The difference in $\langle CN(t)\rangle$ between empirical and simulation results is very clear, in particular comparing to density (Fig. 1). Excluding the high t tail behaviour in the circular model (agents using the circular model tend to spend a longer time in the environment when compared to actual pedestrians and those using the elliptical model, [18]), models always present a lower $\langle CN(t)\rangle$, and in the $\rho_I = 2.5$ ped/m^2 we even have a sensible difference between the models, with the circular one presenting the lower $\langle CN(t)\rangle$.

5 Discussion and conclusions

These very preliminary results hint again to the ability of the CN to provide an analysis qualitatively different and complementary to the traditional methods based on density and velocity. Nevertheless a clear interpretation of these specific results regarding cross-flow dynamics is still missing and deserves future study. The main open issues are:



(a)



(b)

Figure 2 Time evolution of the average value of $\langle CN(t) \rangle$ in the crossing area. (a): initial density condition $\rho_I = 1.5$ ped/m^2; (b): initial density condition $\rho_I = 2.5$ ped/m^2. Red circles: experimental data; blue up triangles: circular body model; green down triangles: elliptical body model. Dashed lines show the standard error confidence values computed by treating different experiment repetitions as independent. Refer to [15] for more details.

1. Why did the models "outperform" (from a CN view point) the actual pedestrians? Our current guess, that needs further testing, is that the stripe formation pattern described in [14, 17, 18] was stronger in the models than in the empirical data. It has to be nevertheless verified if this is due to an actual dynamical effect, or if it is just related to the noise inherent in real data (if this results to be the case, particular care should be used when performing a CN comparison between models and experiments). Furthermore, even if the effect is dynamical, it may not be due to a better avoidance strategy, but just due to the fact that establishing a self-organisation pattern in space is easier when using 2 dimensional discs with respect to the pedestrians' complex 3 dimensional body (this could also explain why the circular model has a lower $\langle CN(t) \rangle$ than the elliptical one).

2. Assuming the models are actually "outperforming" the pedestrians, would such models be of any use? We obviously want the models to reproduce actual behaviour. Nevertheless, it may be interesting to understand why simulated pedestrians moved more orderly, since their dynamics could provide some hints for better planning of infrastructures, or have applications in robotics and autonomous navigation.

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