Risk management for sustainable sovereign debt financing

Stavros A. Zenios[1] Andrea Consiglio[2]
Marialena Athanasopoulou[3] Edmund Moshammer[3]
Angel Gavilan[4] Aitor Erce[5]

This version: March 2020

Abstract

We model sovereign debt sustainability with optimal financing decisions under macroeconomic, financial, and fiscal uncertainty, with endogenous risk and term premia. Using a coherent risk measure we trade off debt stock and flow risks subject to sustainability constraints. We optimize static and dynamic financing strategies, and demonstrate economically significant savings from optimal financing compared to simple rules and consols, and find that optimizing the tradeoffs can be critical for sustainability. The model quantifies minimum refinancing risk and maximum rate of debt reduction that a sovereign can achieve given its economic fundamentals, and an extension identifies optimal timing of flow adjustments that allow the sovereign to go beyond these limits. We put the model to the data on a eurozone crisis country, a low-debt (Netherlands), and a high-debt (Italy) country, and document the significance of the stock-flow tradeoff for debt sustainability, identify potential improvements of the Dutch Treasury practices, and non-sustainability risks in the 2019 Italian budget. The model informs diverse policy decisions on sustainable public finance, and is an essential building block of the European Stability Mechanism methodological framework to assess debt sustainability and repayment capacity of member states, especially in the context of financial assistance.

OR classification:
Decision analysis: risk. Sovereign debt sustainability analysis under uncertainty.
Finance: management. Risk management optimization for sovereign debt financing.
Programming: stochastic. Multi-period portfolio optimization with risk constraints.

[1]University of Cyprus, Nicosia, CY, Non-Resident Fellow, Bruegel, Brussels, and Senior Fellow, Wharton Financial Institutions Center, University of Pennsylvania, USA. Corresponding author, zenios.stavros@ucy.ac.cy. [2]University of Palermo, Palermo, Italy. [3]European Stability Mechanism, Luxembourg. [4]Banco de España, Madrid, Spain. [5]Public University of Navarre (UPNA), Spain.

1 Introduction

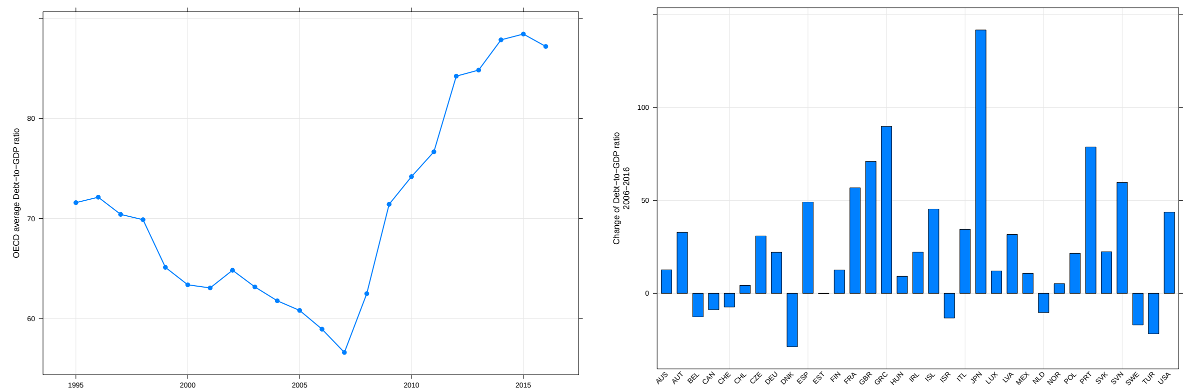
Sovereign debt increased sharply in most advanced economies since the 2008 global financial crisis, with the average public debt growing by 30% of gross domestic product (Figure 1). In the euro area, public debt rose to about 84% of GDP in 2010, decisively contributing to a sovereign debt crisis with five countries requiring financial assistance. In 2018 a new challenge emerged with a disagreement between the Italian government and the European Commission. These episodes, together with the effects of the COVID-19 pandemic, have prompted a renewed interest in debt sustainability analysis (DSA). Policy discussions concern, among other things, the most appropriate DSA variables (debt stock or debt flow), and the thresholds of these variables beyond which debt dynamics are most likely unsustainable. Such thresholds are estimated by international institutions —the International Monetary Fund, the European Stability Mechanism (ESM), or the European Central Bank— and specify rules for accessing official lending. In the words of Klaus Regling, managing director of ESM,

> "Partly in reaction to the European experience, the traditional approach to debt sustainability assessment has evolved. A fresh view on things is one way to improve the analysis of debt sustainability and develop best practices." (ESM debt sustainability conference, Dec. 2018, Luxembourg.)

We provide such a fresh view by developing a normative risk management model. We optimize debt financing decisions with dual sustainability conditions on debt stock and flow, under macroeconomic, financial, and fiscal uncertainty. The model addresses questions like: How do issuance strategies trade off debt financing cost with the risks of debt stock and flow? How do the stock and flow dynamics depend on the sensitivity of interest rates to debt? Can the debt dynamics of a country with given fundamentals satisfy sustainability conditions? If debt dynamics are unsustainable, how and when should adjustments be made to restore sustainability?

The model optimizes the maturity of debt instruments to minimize expected interest costs, constrained by sustainability thresholds on debt stock and flow dynamics. The flow constraint limits the sovereign's gross financing needs, thereby reducing refinancing risks due to excessive funding requirements. The stock constraint sets the minimum pace of debt reduction to avoid exploding stock dynamics (or, the maximum sustainable rate of debt increase for low debt countries), thereby reducing insolvency risk.

We follow the risk management literature and define a coherent measure of refinancing risk, and trace the debt stock-flow tradeoff bounded by sustainability thresholds on the *conditional Value-at-Risk* (CVaR) measure of tail risks. This approach is particularly appropriate for DSA given the need to assess sustainable debt dynamics in a probabilistic way under multi-factor uncertainty. We represent correlated macroeconomic, financial,



Source: OECD, doi 10.1787/a0528cc2-en, accessed March 2018.

Figure 1 – Debt growth of OECD countries.

and fiscal uncertainty by a discrete time- and state-space *scenario tree*, and use stochastic programming to optimize dynamic debt financing strategies. An important feedback effect, which is especially strong for high-debt countries, is between a sovereign's debt and the level and slope of the yield curve. We model this feedback loop using endogenous *risk* and *term premia* that drive issuance decisions and, consequently, the debt level.

Our model contributes to the sovereign debt literature (i) a risk measure in DSA, (ii) the optimization of financing decisions, (iii) simultaneous treatment of debt stock and flow, and (iv) stock and flow sustainability constraints. The model generates sustainability-constrained frontiers of the cost and risks of the financing decisions, and we show that this tradeoff is a significant determinant of debt stock and flow dynamics. We optimize both static (state independent) and dynamic (state dependent) financing strategies, and show that optimal financing can be critical for sustainability.

We test the model on a representative eurozone crisis economy to compare the optimized strategies with several simple rules and consol financing, and document that debt financing optimization matters. Dynamic strategies dominate, but we settle on an intermediate adaptive strategy (time dependent but state independent) which has predictable issuance patterns that can be more practical. Using the adaptive strategy we document the stock-flow tradeoff and show that optimal financing can reduce the likelihood of unsustainable debt dynamics. We also demonstrate the effects of endogenous risk and term premia in creating vicious and virtuous cycles.

We then apply the model to the debt data of the Dutch State Treasury Agency (DSTA) and the Italian Parliamentary Budget Office (PBO) representing, respectively, a low- and a high-debt non-crisis economy.[1] We find that the DSTA practices are near optimal within current policies, we document a premium for the policy of maintaining minimum short term reserves, and identify possible improvements. Using the Italian data we assess the

[1]The calibrations were done in collaboration with SEO Amsterdam Economics as part of a comprehensive assessment of the DSTA funding policies (Hers et al., 2019, pp. 30–36), and the Italian PBO.

sustainability of debt dynamics under no policy changes, under the Italian government plans, and under the 2019 budget agreement with the European Commission (EC), and answer questions of relevance to an independent fiscal council. The 2019 budget improves the debt dynamics, although the probability of sustainability is a low 0.55.

The model quantifies the minimum level of refinancing risks and the maximum debt reduction pace that a sovereign could reach given its economic fundamentals. We extend the model to identify adjustments to gross financing needs to go beyond these limits. In particular, we identify *hot spots* of unsustainable gross financing needs, and compute the minimum size and optimal timing of adjustments to restore sustainability. Results on a crisis economy with non sustainable debt dynamics, show the benefits from adjustments and estimate the cost of procrastination. The model extension is helpful in designing official sector financial assistance programs. We apply it to the Italian data and estimate that additional fiscal effort of about 3.5% of GDP over the next decade can deliver sustainable dynamics with high probability 0.85.

1.1 Related literature

We address a problem of the economics of sustainable sovereign debt financing, drawing from literature on risk management and planning under uncertainty.

Economics of sovereign debt

The literature on what determines sustainable debt levels is extensive (D'Erasmo et al., 2016). Our paper adds to recent contributions which model an active fiscal policy maker, but instead of allowing for endogenous default —see, e.g., Arellano and Ramanarayanan (2012); Ayres et al. (2018); Bocola and Dovis (2019); Chatterjee and Eyigungor (2012); Conesa and Kehoe (2014); Hatchondo and Martinez (2009)— we assume an exogenous process of primary surplus, and focus on the decisions of an active public debt management office to minimize net interest payments while avoiding, if possible, the default risk from breaching some thresholds for market access. This approach has a clear practical advantage in that, by abstracting from the government's utility function for preferences over deficit, we can solve the operational problem of a treasury facing sustainability rules like those imposed by official lenders.

Several papers focus on the role of maturity structures on the tradeoff between borrowing costs and refinancing risks. Cole and Kehoe (2000) find that sovereigns can avoid self-fulfilling refinancing crises by lengthening debt maturity, whereas countries in recession gamble for redemption with shorter maturities (Conesa and Kehoe, 2017), and Arellano and Ramanarayanan (2012) show that maturity shortens with rising interest rates. Barro (2003) shows that a tax smoothing objective leads to contingent consols, or, in the absence of contingent debt, "maturity structure that has no holes", and Angeletos (2002) shows that issuing a perpetuity and investing in short-term reserves smooths refinancing needs and insulates public debt from interest rate risk, without resorting to contingent debt; see Bianchi et al. (2018) for the international setting. The earlier two-period papers consider binary long-short maturity debt, while recent multi-period works use consols with exponentially decaying coupons (Angeletos, 2002; Arellano and Ramanarayanan, 2012; Bocola and Dovis, 2019; Hatchondo and Martinez, 2009), or debt with probabilistic maturities (Chatterjee and Eyigungor, 2012). The binary long-short choice is restrictive, consol financing is rarely used (Meyer et al., 2019), and we are not aware of sovereigns issuing exponentially decaying perpetuities or debt with probabilistic maturities. There is a need for more granular and less stylized models, and this is what our paper does. Also, the maturity structure of legacy debt often has "holes", and by adding to this literature the tradeoff between stock and flow, with a rich set of financing instruments, we deal with this significant practical aspect.

The above papers differ on their maturity recommendations, due to differences in the respective models of default dynamics. For instance, Cole and Kehoe (2000); Conesa and Kehoe (2017) relate the debt maturity structure to rollover risk, Bocola and Dovis (2019) relate maturity to fundamental and non-fundamental (self-fulfilling) risks, and Ayres et al. (2018) relate the maturity structure to good or bad equilibria depending on the distributional characteristics of the feedback between debt and interest rates. Our model is the first *normative* model to optimize over a rich range of maturities to bound both refinancing risk and debt stock dynamics, accounting for the term structure of legacy debt. The significance of studying maturity structures in the absence of contingent consols, as we do, was pointed out in Barro (2003, footnote 5).

For sustainability constraints we use the thresholds of official sector DSA. Recently Dias et al. (2014); Irwin (2015) raised the question on the right variable to use for assessing market access, and Gabriele et al. (2017) show that both debt stock and refinancing needs matter. We contribute to this literature a model of the stock-flow tension.

Risk management

We adopt from the literature on risk management the idea of trading off risk with expected reward. (Regulated industries go as far as setting standards to limit risks in Basel III and Solvency II). Missale (1997) recognizes this tradeoff in sovereign debt management, and Bolder (2003) uses simulations to study the tradeoff embedded in a given financing strategy. Following the seminal work of Markowitz (1952) on optimizing the tradeoff through efficient frontiers, we advance the earlier descriptive approaches by a prescriptive model of the tradeoff, with sustainability conditions.

The quantification of uncertainty in DSA has received attention recently by international organizations and academia (Celasun et al., 2006; Consiglio and Zenios, 2016;

Guzman and Lombardi, 2018). We represent stochastic variables on a discrete time- and state-space scenario tree and define a measure of tail risk of the critical DSA variables.

Planning under uncertainty

We formulate the model using multi-period stochastic programming, which dates back to Dantzig (1963), and received renewed attention in the 1980s with the development of solution algorithms for large-scale problems (Birge and Louveaux, 2011), and a proliferation of financial applications (Mulvey and Ziemba, 1998; Zenios, 2007). Closer to our work, models were developed for the Turkish Ministry of Finance (Balibek and Köksalan, 2010) and the Italian Treasury (Consiglio and Staino, 2012). These works are the precursors to our model. They deal with the short-term problem of debt stock management given an exogenous stochastic yield curve, but do not consider the flow dynamics, economic and fiscal shocks, interest rate endogeneity, or sustainability constraints.

1.2 Contribution

To the best of our knowledge, this is the first DSA model to incorporate a risk measure, looking at both stock and flow dynamics to optimize a new stock-flow tradeoff with dual sustainability conditions. It optimizes *dynamic* financing strategies for the full term structure of legacy debt (and not aggregate stock) with a rich set of instruments beyond long-short maturities. It can also optimize simpler adaptive strategies or static rules. A multi-factor scenario tree represents uncertain economic, financial, and fiscal variables, calibrated to expectations, volatilities, and correlations.

We put the model to the data of three diverse economics to draw policy recommendations. Our experiments corroborate a conjecture by Bohn (1990, p. 1218) that the government's problem would change significantly if debt management affected interest rates, they replicate the recommendation of stylized models to issue at the shortest available tenor when the focus is on cost minimization (Arellano and Ramanarayanan, 2012; Conesa and Kehoe, 2017), and shift from long- to short-term issuance with higher risk tolerance in line with Cole and Kehoe (2000); Conesa and Kehoe (2014, 2017). Whereas literature argues that when legacy debt payments are decreasing and risk tolerance is low, it is optimal to issue at the longest available maturity, we show that several factors may render the long-maturity rule sub-optimal. We obtain the optimal financing deviating from this rule. We also show that, consistent with existing literature, consol financing is the least risky strategy, but it is more expensive than optimized financing with the same risk level. An experiment with the model extension confirms Blanchard et al. (1990) that "delaying adjustment substantially affects the size of the needed policy action".

The model is an essential building block of the ESM methodological framework to assess debt sustainability of member states, mandated in ESM Treaty Article 13-1(b), including the repayment capacity of crisis countries under €295bn assistance programs.

2 Layout of the model

2.1 The economic problem

We consider a sovereign that at period t is endowed with output Y_t, runs a primary balance PB_t, and owes a stock of debt D_{t-1}. The sovereign's *gross financing needs* are given by the *flow dynamics*

$$GFN_t = i_{t-1}D_{t-1} + A_t - PB_t, \tag{1}$$

where i_{t-1} is the *effective nominal interest rate* on debt at $t-1$, and A_t denotes the amortization schedule corresponding to D_{t-1}. The *debt stock dynamics* are given by

$$D_t = (1 + i_{t-1})D_{t-1} - PB_t. \tag{2}$$

The sovereign finances its needs choosing from J debt instruments of different maturities, with *financing decisions* $X_t(j)$ denoting the nominal amount of instrument j issued at t, and X the vector of all financing decisions. The *debt financing equation* satisfies

$$\sum_{j=1}^{J} X_t(j) = GFN_t. \tag{3}$$

The interest on issued debt is determined by the market risk-free rate plus premia idiosyncratic to the sovereign, including term premia for different maturities, and they depend on debt dynamics (Bassanetti et al., 2018; Engen and Hubbard, 2005; Paesani et al., 2006). Our model incorporates a response of interest rates to debt, and following this literature we model premia as a function of debt-to-GDP, $d_t = \frac{D_t}{Y_t}$, and of the maturity of the issued instrument. The interest rate for instrument j issued at t given by

$$r_t(j) = r_{ft} + \rho(d_t, j), \tag{4}$$

where r_{ft} is the term structure of the baseline maturity forward risk-free rate, and $\rho(d, j)$ endogenizes *risk premium* and *term premia* for different maturities j.

The vector X determines the *effective interest rate*

$$i_t = \frac{i_{t-1}(D_{t-1} - A_t) + \sum_{j=1}^{J} r_t(j)X_t(j)}{D_t}.[2] \tag{5}$$

[2] We assume here that the effective interest rate for D_{t-1} and A_t is the same, but our implementation accounts for differences.

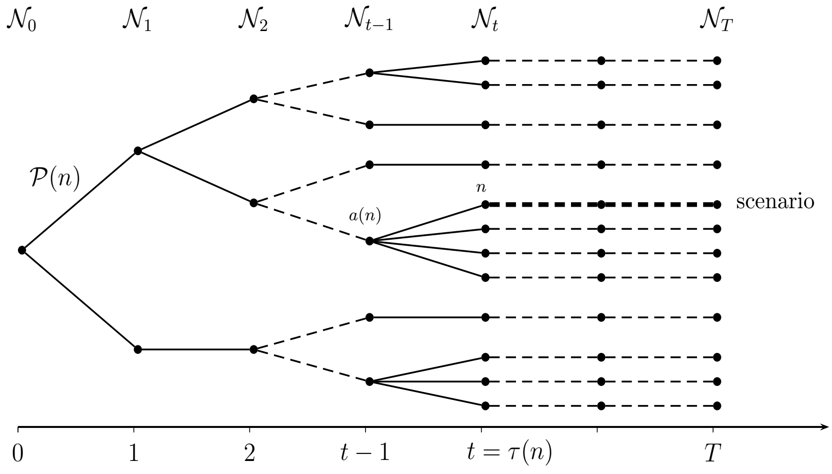


Figure 2 – A scenario tree.

In this paper we model the optimal choice of X for sustainable debt financing. These variables determine the debt dynamics, that determine risk and term premia, which, in turn, influence the maturities to be issued. This *feedback loop* $X \to D \to r \to X$ links stock and flow not only through quantities but also through prices.

2.2 Modelling uncertainty

We model uncertainty using a discrete multi-period scenario tree, see Figure 2. Time steps are indexed by $t = 0, 1, 2, \ldots T$, where 0 is now and T is the risk horizon. Data are indexed by a set of *states* $\mathcal{N}_t$, with each $n \in \mathcal{N}_t$ representing a possible state of the economy at t. $\mathcal{N}$ denotes all possible states during the risk horizon. The number of states at t is N_t, and the total number of states is N. Not all states at t can be reached from every state at $t-1$, and $\mathcal{P}(n)$ denotes the set of states on the unique *path* from the *root state* 0 to n. Each path leading to a terminal state $n \in \mathcal{N}_T$ is a *scenario*. The unique ancestor of n is denoted by $a(n)$, with $a(0) = 0$. A function $\tau(n)$ identifies the time period of states on a path, i.e., $\tau(n) = t, \tau(a(n)) = t-1$,. For state n at t, all information at states m on the path $\mathcal{P}(n)$ is known since $\tau(m) < t$.

Problem data and model variables are indexed by n. Exogenous variables are conditionally known at each state, whereas endogenous variables take state-dependent values. The conditional probabilities of $n \in \mathcal{N}_t$ are denoted by π_t^n, and p^n denote unconditional probabilities of all states $n \in \mathcal{N}$, except the root. Scenario probabilities are the unconditional probabilities of $n \in \mathcal{N}_T$.

This multi-period setup allows us to represent the term structure of legacy debt payments and capture refinancing risk. It also permits finer granularity of financing instruments beyond short-long maturities. On the tree we define debt financing decisions, model the stochastic dynamics of debt stock and flow, and define risk measures of the distributions of both dynamics. At each state we also compute endogenous premia.

3 The optimization model

We develop a parametric optimization model on the scenario tree, to trade off the cost of debt financing with the risk of violating sustainability conditions on debt stock and flow. We discuss the objective function, define the financing variables, and set the constraints. We optimize financing strategies that can be time-dependent or time- and state-dependent. As a special case we obtain optimal time-invariant strategies representing simple rules. The model constrains flow and stock dynamics by exogenous thresholds. Debt flows give a vulnerability signal at any risk horizon whereas stock dynamics reflect long-term insolvency risks, and these variables are interlinked through the endogenous interest rates. The flow risks are constrained by a tail measure of the flow distribution, and for insolvency risks we constrain stock to be on a non-increasing path. The model also accommodates market demand and other policy considerations, such as inter-temporal smoothing of the financing strategy or boundary conditions.

We take correlated stochastic processes for economic output and primary balance as exogenous, and given these stochastic variables we optimize the financing decisions for sustainability. Long-term mean target values for these variables reflect economic policy priorities, and we can parametrize these targets based on auxiliary models so that our model can assess their suitability. (We demonstrate for the case of Italy later, section 6).

3.1 Objective function

The sovereign minimizes the expected interest payments on its debt, subject to constraints on refinancing risks and debt stock dynamics. We measure gross financing needs as a ratio to GDP, to account for economic output uncertainty, and denote this random variable by *gfn*. Denoting by $\Psi(\cdot)$ the risk measure, we write the optimization problem as:

$$\underset{X}{\text{Minimize}} \quad \sum_{\substack{n\in\mathcal{N}_t,\\ \text{for all } t=0,1,2,\ldots T.}} p^n NIP_t^n \tag{6}$$

s.t.

$$\Psi(gfn) \leq \omega, \tag{7}$$

$$\frac{\partial d}{\partial t} \leq \delta. \tag{8}$$

The objective function minimizes expected *net interest payments* (NIP), which is the main relevant variable for the treasury. For instance, the primary goal of the US Treasury Department debt management function is to finance government borrowing at the lowest cost, against acceptable risks to the budget, with a medium and long term view. The same is true for DSTA and the Italian Treasury.[3] Interest payments consist of interest on legacy debt I_t^n, plus interest on debt created by the financing decisions.

To track service payments on endogenously created debt on a path leading to n we exploit the tree structure. Let $CF_t^n(j,m)$ denote the nominal amount of interest payment due at state n of period t, per unit of debt $X_{\tau(m)}^m(j)$ issued at state m of period $\tau(m)$ on the path $\mathcal{P}(n)$. This amount is computed from scenarios of the term structure of interest rates, the terms of the issued instrument, and the premia (cf. eqn. 4). The net interest payments are given by

$$NIP_t^n = I_t^n + \sum_{m\in\mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j)\, CF_t^n(j,m). \tag{9}$$

Net interest payments minus interest on legacy debt is what the sovereign controls through financing decisions. Total NIP/D is the effective interest rate of debt (eqn. 5).

The gross financing needs-to-GDP ratio in (7) takes scenario values $gfn_t^n = \frac{GFN_t^n}{Y_t^n}$, where Y_t^n is the state-dependent economic output. We denote by gfn the random variable over all states $\mathcal{N}$, and (7) bounds the risk on gross financing needs by the risk tolerance ω, as a percentage of GDP.

Eqn. (8) constrains the stock dynamics to be non-exploding, so they remain sustainable. d is a random variable taking scenario values d_t^n, and the debt stock constraint is also implemented using the risk measure; see section 3.3. For crisis countries, official lending comes with strict conditionality on debt stock reduction, and we set $\delta < 0$ targeting a reduction pace according to the lending rules. For low debt countries we allow $\delta \geq 0$. By varying δ and ω we trade off debt stock and flow risks with financing cost.

3.2 Decision variables

Given financing decisions on the tree, $X_t^n(j)$, the debt financing equation (3) becomes

$$\sum_{j=1}^{J} X_t^n(j) = GFN_t^n, \tag{10}$$

[3] The US Treasury policy is stated at https://www.treasury.gov/resource-center/data-chart-center/quarterly-refunding/Pages/overview.aspx, for DSTA see the preface in Outlook 2019 at https://www.dsta.nl/documenten/publicaties/2018/12/14/outlook-2019, and for the Italian Treasury see Section I of Outlook 2017 at http://www.dt.tesoro.it/export/sites/sitodt/modules/documenti_en/debito_pubblico/presentazioni_studi_relazioni/Public_Debt_Report_2017.pdf. All sites accessed in March 2019.

for $n \in \mathcal{N}_t$, and $t = 0, 1, 2, \ldots T$. Using non-negative proportional weights $w_t^n(j)$ we write

$$w_t^n(j) = \frac{X_t^n(j)}{GFN_t^n}, \quad (11)$$

$$\sum_{j=1}^{J} w_t^n(j) = 1. \quad (12)$$

There are three possible strategies to optimize financing decisions. In the simplest case weights are time-invariant $w(j)$, and the financing strategy is fixed for all periods. This is a *fixed-mixed strategy* leading to simple financing *rules.* The second possibility specifies time-dependent but state-invariant weights $w_t(j)$, so that the financing strategy adapts with time but is identical for all states at each period. This is an *adaptive fixed-mix strategy.* The third possibility is a *dynamic strategy* with time- and state-dependent weights $w_t^n(j)$, whereby the issuer implements a decision, waits to observe the state at the next period, implements the optimal decision for that state, and waits again. Decisions are made at $t = 0$ based on all available information, including conditional expectations about the future, and as new information arrives the model makes *recourse* decisions (Birge and Louveaux, 2011). The decision $w_t^n(j)$ is *adapted* to state n from the information at its ancestor $a(n)$, but can not be adapted to states that have not yet been observed, satisfying the *non-anticipativity* property of stochastic programming. Adaptation and non-anticipativity imply that dynamic strategies are implementable.

A dynamic strategy achieves the best results, and our tests demonstrate Pareto improvements when going from simple rules to adaptive fixed-mix and to dynamic strategies. However, we favor the use of adaptive fixed-mix since dynamic strategies can pose implementation problems. In particular, while Treasuries enjoy flexibility in pursuing a financing strategy, they face both demand and supply constraints that may preclude a dynamic strategy. Treasuries tend to pre-commit to annual issuance plans, and they avoid surprising the markets. For instance, the Dutch agency considers "predictability —no undue surprises— an important underlying objective" (op. cit., p. 4), and the Italian and U.S. Treasuries use almost identical language. Dynamic strategies may surprise the markets by changing the debt issue depending on the state of the economy, and adaptive fixed-mix is easier to communicate and can be more practical. Simple rules are sub-optimal, but they are prevalent in standard DSA and we use them for comparison.

3.3 Constraints

We specify the model constraints. For ease of exposition we give state-invariant equations, and introduce the states for a tree implementation in the online Appendix A.

Risk measure and the debt flow constraint

We adapt the CVaR tail risk measure, that satisfies the coherence properties of **?**. CVaR is well suited for DSA, since un-sustainability is a rare extreme event best captured by a tail measure, allowing sustainability assessment with a high probability in line with the practice of the international institutions. CVaR minimization is computationally tractable, using the linear programming formulation of Rockafellar and Uryasev (2002) for discrete random variables on the tree.[4] Using CVaR we bound the expected value of the tail of the distribution of gross financing needs *gfn* (CFaR) at a confidence level α (Zenios, 2007, pp. 58–63) that is acceptable for policymakers. The risk function in (7) is defined as

$$\Psi(gfn) \doteq \mathbb{E}\left(gfn \mid gfn \geq gfn^{\diamond}\right). \tag{13}$$

$gfn^{\diamond}$ is the right α-percentile of the gross financing needs, i.e., the lowest value of *gfn* such that the probability of gross financing needs less or equal to $gfn^{\diamond}$ is greater or equal to α. $gfn^{\diamond}$ is the Value-at-Risk of debt flow. We are constraining the aggregate conditional Value-at-Risk of flow over the tree, and the flow constraints are given in online Appendix A.1.

Debt stock constraint

Debt stock d is stochastic, and constraint (8) can be imposed on the worst case, or on the average, or with high probability. Consistently with flow, we use CVaR to achieve the target pace of debt reduction δ with high probability; see online Appendix A.2.

Debt dynamics

We complete the model with accounting identities, using two sets of recursive equations linking the flow (respectively, stock) at period t to $t-1$. The model finances legacy debt and any maturing debt created endogenously, and we introduce an indicator function $\mathbb{1}^t(j,t')$, with $t' < t$, that takes the value 1 if instrument j issued at time t' matures at t, and 0 otherwise. This function tracks maturing endogenous debt. The amount of issued debt that matures at t is given by $\sum_{t'=0}^{t}\sum_{j=1}^{J} X_{t'}(j)\mathbb{1}^t(j,t')$, and is financed together with the maturing legacy debt and interest. The debt dynamics are given in online Appendix A.3.

[4] The model can be implemented with a worst-case risk measure, but worst case analysis produces very conservative solutions. The widely used VaR measure is suitable, but it is not coherent, and, importantly, its optimization requires mixed integer nonlinear programming that could be intractable.

Maturity smoothing and boundary conditions

We introduce inter-temporal smoothing of the financing strategies to satisfy the predictability criterion. If M_j denotes the maturity of the jth instrument, the *weighted average maturity of issued debt* at t with adaptive fixed-mix financing is given by

$$WAMI_t = \sum_{j=1}^{J} w_t(j) M_j, \tag{14}$$

and inter-temporal changes can be bounded by parameter λ,

$$\left| WAMI_t - WAMI_{t-1} \right| \leq \lambda. \tag{15}$$

Smoothing applies to adaptive fixed-mix and dynamic strategies, since fixed-mix is smooth.

Finally, we can specify boundary conditions $WAMI_0 = k_0$, $WAMI_T = k_T$. For instance, the sovereign may want to start with a WAMI close to its legacy value k_0, and end at the risk horizon with its historical average k_T; see online Appendix A.4.

Model specification

The model consists of the decision variable definitions, the objective function, the risk constraints, debt dynamics, smoothing and boundary conditions, with the endogenous yield curve of online Appendix A.5. The full model on a tree is given in Appendix A.6.

4 Model calibration

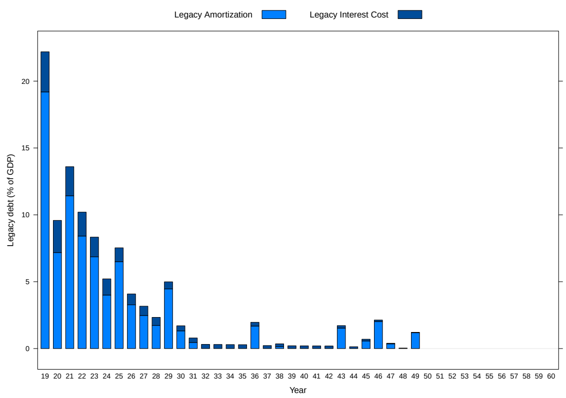
We calibrate the model on three diverse cases: a representative euro area country facing a liquidity crisis, Netherlands with low debt levels and fiscal balance, and Italy with high debt levels but not (yet) facing a liquidity crisis. We use these calibrations to illustrate model performance and inform policy decisions on sustainable public finance.

The first case has legacy debt typical of eurozone crisis countries, with macroeconomic and fiscal projections reflecting euro area averages estimated by the European institutions, and they are, at the time of writing, the eurozone policy targets. The debt-to-GDP ratio is high, signalling a potential solvency crisis, and large amounts of legacy debt maturing early create an imminent liquidity crisis. Our baseline calibration has ex post sustainable long run debt dynamics, but we also test a *non sustainable* variant by lowering long term expected growth while keeping all other parameters fixed. We then calibrate the model to the Dutch and Italian data. For the crisis country and Italy we follow the international institutions and optimize over a set of 3-year (short-term), 5-year (medium-term) and 10-year (long-term) bonds. For the Netherlands we follow DSTA, and optimize over a 1-year treasuries proxy for the money markets, and 3-, 5-, 10-, 20-, and 30-year bonds.

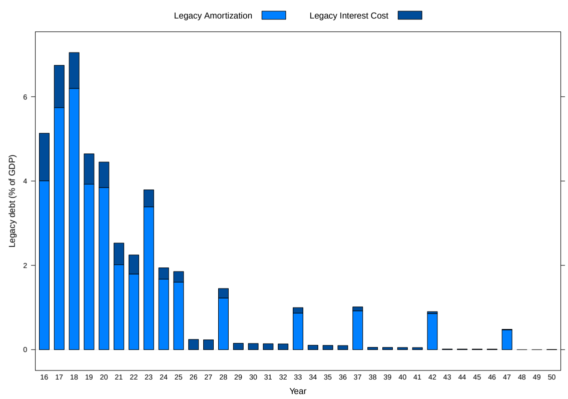
We use GAMS (GAMS Development Corporation, 2016) on a Dell Precision Intel Xeon QuadCore with 32Gb of memory, running Linux, with solvers BARON to fit the trees and CONOPT to solve the model. The adaptive fixed-mix and fixed-mix models are nonlinearly constrained, and are solved within minutes for 64-scenario trees, but beyond that it may take a few hours. The stochastic programming models on 1024-scenario trees are solved within minutes.

4.1 Legacy debt

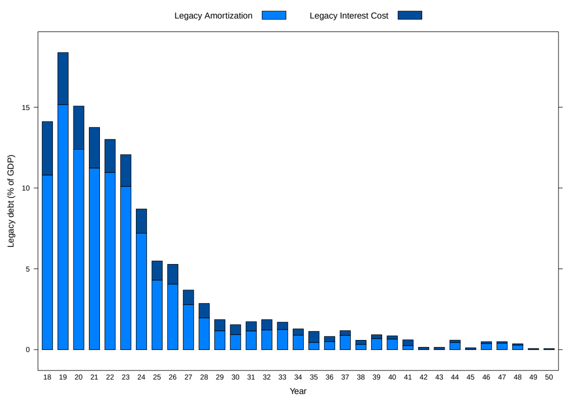
We display in Figure 3 the legacy debt service profiles. For the crisis country (Panel A) we use 2019 data, for Netherlands (Panel B) we use 2016 data for a three-year retrospective analysis, and for Italy (Panel C) we use 2018 data on which the budget agreement was based. The legacy debt stock for the crisis country is 100% of GDP with 20% maturing within a year, for Italy it is 130% of GDP with 15% maturing within a year, for the Netherlands it is slightly below 50% of GDP with 6% maturing annually during the first three years. The legacy debt of the crisis country and Netherlands has maturity holes.



(a) Eurozone crisis country



(b) Netherlands



(c) Italy

Figure 3 – Legacy debt service profiles.

4.2 Yield curve

Risk and term premia (eqn. 4) take the functional form

$$\rho(d_t, j) = a_j + (1 + b_j)\hat{\rho}(d_t), \tag{16}$$

where a_j and b_j are maturity-specific constants (term premia), and $\hat{\rho}(d)$ is the effect of debt stock on interest rates,

$$\hat{\rho}(d) \doteq \hat{\rho} \left[\frac{d_{max} - d}{1 + \exp\left(d_{max} - d\right)} - \frac{d_{min} - d}{1 + \exp\left(d_{min} - d\right)} \right]. \tag{17}$$

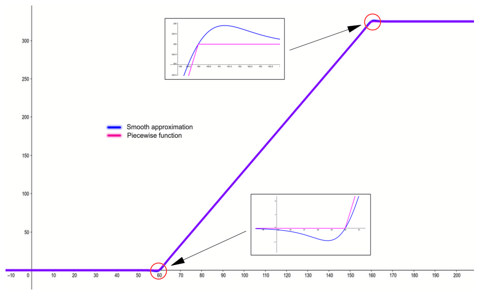
Eqns. (16)–(17) generate yield curves that shift and twist with debt changes. If $b_j = 0$ for all maturities, changes in debt level cause parallel shifts. When b_j is higher (lower) for long- than short-term debt, a debt increase causes curve steepening (flattening) around the baseline maturity rate. The scenario-dependent yield curve is in online Appendix A.5.

Equation (17) is a smooth nonlinear approximation of the piece-wise linear function in Figure 4 (Panel A). It specifies zero premium for debt ratios below the 60% level of the Stability and Growth Pact, which then grows linearly with slope 3.25 up to a ceiling of 325 basis points. The first two piecewise segments are calibrated using panel data from 23 EU countries over the period 1995–2016 (Gabriele et al., 2017), with baseline the 5-year yield. The ceiling of 325bp is imposed by us, since, as has been observed during the eurozone debt crisis, at this spread the sovereign is cut off the markets with its debt ratio exceeding 160%, and funding costs are stabilized by official sector support. The ceiling is never active in our tests, but it is essential when applying the model to countries with debt exceeding 160%, such as Greece. We report the calibration in Figure 4 (Panel B).[5]

4.3 Scenarios

We use the multi-factor moment-matching model of Consiglio et al. (2016) to fit yearly-discretized trees of forward rates, GDP growth, and primary balance. The trees converge to the long-term expected values, matching the means, standard deviations, and correlations of Table 1, estimated from eurozone historical data. For computational tractability we calibrate a tree of 256 scenarios over five years. Each scenario is extended past the fifth year until the risk horizon using cyclical dynamics with normally distributed random shocks. For the crisis country, long-term expected growth and primary balance are provided by the ESM, and we take r_{ft} to be the 5-year forward Euribor rate. Macroeconomic variables converge to their long-term averages after a 6 to 7-year business cycle, with a deep recession followed by strong recovery. The model is calibrated for 2019–

[5] The 5-year bond has zero values for the parameters of eqn. (16), since this is the baseline maturity and carries no term premia but only a risk premium (cf. eqn. 17).



(a) Endogenous premium

Equation (16)				Equation (17)	
Coefficient	Bond maturity			Coefficient	Value
	3	5	10		
a_j	-35	0	72	$\hat{\rho}$	3.25
b_j	-0.13	0	0.13	d_{min}	60
				d_{max}	160

(b) Calibrated parameters for the yield curve

Figure 4 – Risk premium as a smooth approximation of a piece-wise debt function.

	Growth	5-year rates	Primary balance
	Correlations		
Growth	1.00		
5-year rates	-0.20	1.00	
Primary balance	0.25	-0.03	1.00
St. Dev.	0.75	0.85	0.15
Long-term mean (Crisis country)	3.5	3.25	1.00
Long-term mean (Netherlands)	3.8	3.40	0.0
Long-term mean (Italy-IMF)	2.3	3.25	1.5

Table 1: Data to calibrate the scenario trees.

2059, beyond 2049 when legacy debt matures. For Netherlands we use GDP growth, primary balance, and risk-free rate data from the DSTA, and the tree converges to long term projections from the same source (Hers et al., 2019, p. 31). For Italy we use the 5-year forward Euribor, and take as baseline the five-year macroeconomic and fiscal projections from the 2018 IMF World Economic Outlook, with alternative projections from the Italian Ministry of Finance and the EC, converging to Italy's long-term averages. For Netherlands and Italy the model is calibrated to 2050. The scenarios are illustrated in online Appendix B.

5 Model at work

Testing the model on the crisis economy we document the significance of optimizing debt financing and of the stock-flow tradeoff. We compare optimal dynamic, adaptive fixed-mix, and fixed-mix strategies, with several benchmark rules and consol financing. We highlight a significant tradeoff between debt stock and flow embedded in a financing strategy, implying that optimized debt financing can have an impact on sustainability. We also illustrate the implications of interest rate endogeneity.

We then test the model on the Dutch data, to assess retroactively the DSTA financing practices and policies. We show that some policies come with a cost and identify potential improvements. Finally, using Italian data we assess the 2019 budget and contribute to the debate between the Italian government and the EC on the risks for the country's public debt outlook, including sustainability concerns.

We set the CVaR parameter $\alpha = 0.95$, and trace the tradeoff as ω varies from a very large value to the smallest possible value for which the model has a feasible solution. For the eurozone crisis country we do not always impose the stock risk constraint (8), to identify unsustainable debt dynamics. For Netherlands, this constraint is never binding, and for Italy it is a critical constraint. We report the effective cost of issued debt (NIP) and the CFaR for increasing risk tolerance ω, and provide fan charts for the debt stock and flow dynamics, showing the median and selected percentiles. In particular, we examine whether the flow dynamics stay within acceptable thresholds,[6] and that stock dynamics are non-exploding or decreasing from an initial high value.

5.1 Eurozone crisis economy

5.1.1 The significance of optimized debt financing

We compare the optimized strategies with several simple rules. Optimization theory tells us that results can be better with dynamic strategies, compared to fixed-mix or adaptive fixed-mix, and in this section we look at the economic significance of the improvements and whether optimized strategies do materially better than the rules used in practice. We also compare with consol financing and shed some light on the performance of consols.

Figure 5 shows the cost-risk tradeoff with our optimized strategies, and the results with three simple rules —issuing always long (using 0-0-100 weights for the 3-, 5-, and 10-year bonds), medium (0-100-0), or short term (100-0-0) instruments. We also test the *benchmark* of international institutions issuing in all tenors with a fixed weighted average maturity of about five years (40-40-20), and consol financing with coupon payments decaying exponentially at the rate of 2.5%. We observe that dynamic strategies are

[6] We use 15% of GDP for crisis and 20% for developed economies, as used for official lending and estimated from empirically observed market capacity to refinance a sovereign's debt

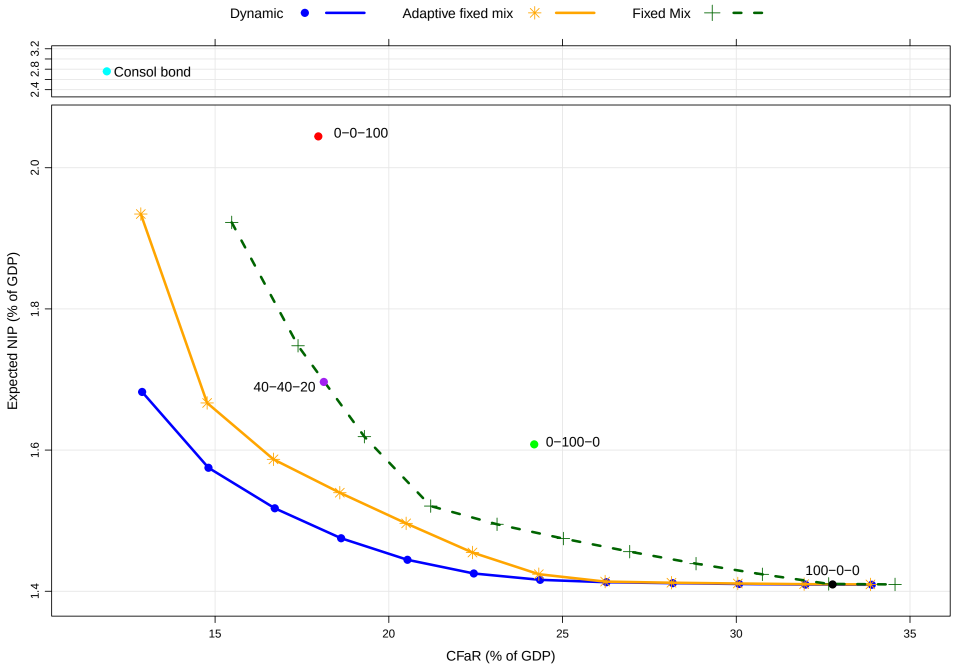


Figure 5 – Expected net interest payment and risk for different financing strategies.

uniformly best performing, and the 40-40-20 benchmark, the simple rules, and the consol, underperform vis-á-vis the adaptive and dynamic strategies. Cost savings increase with higher risk tolerance, up to an economically significant 1% of GDP.

The performance of the simple rules is in line with existing literature, given the upward sloping yield curve in our calibration. When the sovereign focuses on cost minimization, it issues at the shortest available tenor (Arellano and Ramanarayanan, 2012; Conesa and Kehoe, 2017) and the optimal financing strategy coincides with the 100-0-0 rule. When the profile of legacy debt is decreasing and risk tolerance is low, issuing at the longest available maturity is the optimal strategy (Cole and Kehoe, 2000; Conesa and Kehoe, 2014), and we see that the 0-0-100 rule has the lowest risk among the simple rules and the benchmark. However, this rule is inefficient compared to the optimized strategies.

The gap on expected NIP of the long-maturity rule to the fixed-mix strategy with the same risk is a noteworthy 0.35% of GDP, and we perform three tests to explain this gap. First, the long-maturity rule is derived from models with binary maturity structures, and by restricting our model to work with two maturities only, the gap is reduced to 0.15% GDP. Hence, the finer granularity of our model pushes the frontier down creating a gain over the simple rule. Second, and more importantly, we run the model with $\alpha = 0.5$ (i.e., working with the median risk, instead of the tail) and find that the gap is reduced to zero when using two maturities only, and almost zero when working with three maturities. The long-maturity rule is derived from models without tail risk, and what may be efficient when we ignore the tail, is sub-optimal when we optimize for extreme events. Third, we run the model on the eurozone crisis country after we smooth its legacy debt by a negative exponential, and using only two bonds we find the gap reduced to 0.11% GDP. Hence, the stylized assumption of smoothly declining debt is another reason why the long maturity rule is inefficient for legacy maturities with holes.

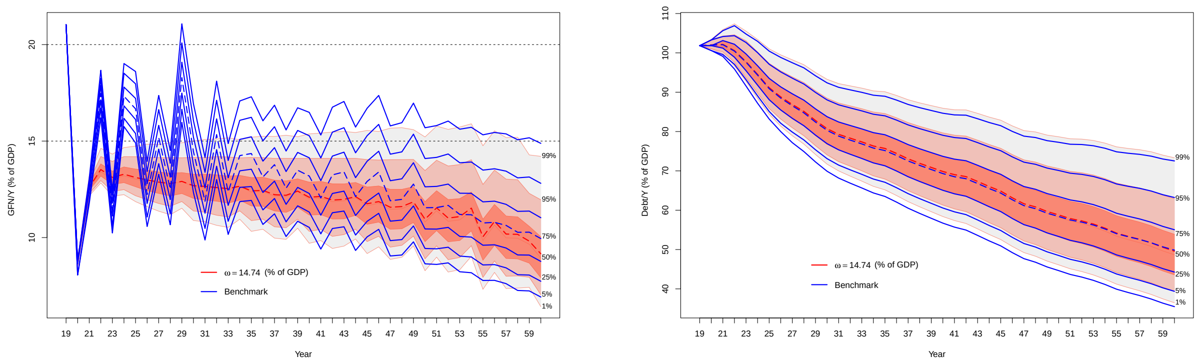
Overall, the flexibility of our model with respect to assumptions leading to the simple rule explain the gap. The long maturity rule is inefficient in our risk space mainly because it does not address tail risk. This result highlights the significance of tail risk and the value-added of a normative model that speaks when stylized assumptions do not hold.

Interestingly, the 40-40-20 benchmark lies on the fixed-mix efficient frontier. This empirical rule, obtained through a non-analytical institutional decision making process, is the optimal fixed-mix strategy for crisis economies under current eurozone growth and primary balance targets, with an intermediate risk preference ($\omega = 18.18$). We consider this an important finding: it validates the prevalent strategy as prudent and optimal in a static context, but improvements are possible with adaptive or dynamic strategies.

We find, consistently with existing literature, that consol financing has lower risk than simple rules, the benchmark, or the optimized fixed-mix (see "Consol bond" in Figure 5). However, it is more costly. (We assume that the consol is priced at zero spread over the benchmark 5-year rate, biasing the comparison in its favor since, in general, consols will command a premium due to their long effective maturity.) Adaptive fixed-mix and dynamic strategies can achieve the same level of risk like the consol, at lower cost by 0.5–1% of GDP. A frontier of adaptive fixed-mix optimal portfolios that include a consol (not shown), starts from the consol risk level and converges to the 100-0-0 low cost portfolio.

Overall, our model broadens the choices of a debt issuing sovereign so that optimized financing strategies dominate simple rules for several reasons: incorporating tail risk, finer granularity on available maturities for debt financing, and legacy debt with holes. They deliver the low risk financing of consols using traded instruments at lower cost. However, the gains from optimization diminish as the risk tolerance increases. For highly indebted countries, the emphasis should be on reducing tail risk to avoid potential unsustainability problems. This is the focus for the crisis country and Italy, but even for the Netherlands, with its low debt level, the current practices of the DSTA reveal a low risk tolerance. From the efficient frontiers of Figure 5 we conclude that low risk countries with access to liquid markets can chose a risk tolerance of 20-25% of GDP, and countries that can face funding difficulties should operate close but below 15%. In both cases the gains from optimization are significant.

To compare the debt dynamics of the benchmark and the adaptive fixed-mix, we make a Pareto move from point 40-40-20 to the adaptive fixed-mix frontier. We move to the left reducing ω from 18.18 to 14.74. We show the benchmark and the optimized debt dynamics in Figure 6. The gross financing needs level and volatility are reduced (Panel A), but debt stock declines somewhat more slowly (Panel B). Optimized strategies are



(a) Gross financing needs **(b)** Debt stock

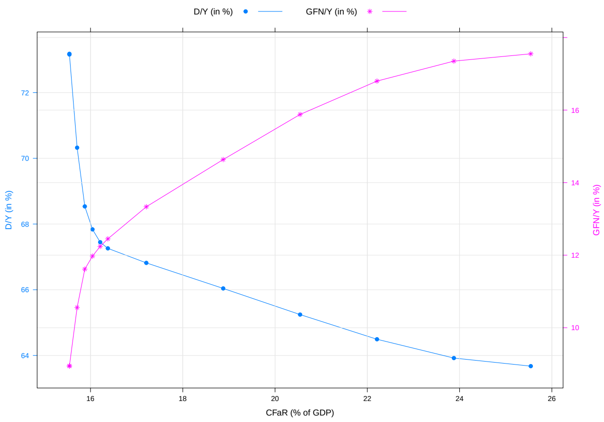
Figure 6 – Debt dynamics with the optimal adaptive fixed-mix strategy and the benchmark.

especially relevant for sovereigns with debt sustainability concerns. We observe (Panel A) that the optimal strategy keeps financing needs below the 15% threshold after 2019 with probability at least 0.95, whereas the benchmark violates the threshold six times until 2032 with probability 0.5, and a 0.05 chance of more violations until 2049 when legacy debt expires. Spikes in gross financing needs can derail sovereign debt management, for instance, because temporary liquidity problems weigh on long-term solvency. An optimal financing strategy can smooth out the spikes, and these results demonstrate the effectiveness of the model in addressing debt sustainability. (Additional tests on optimized financing are given in online Appendix C.1.)

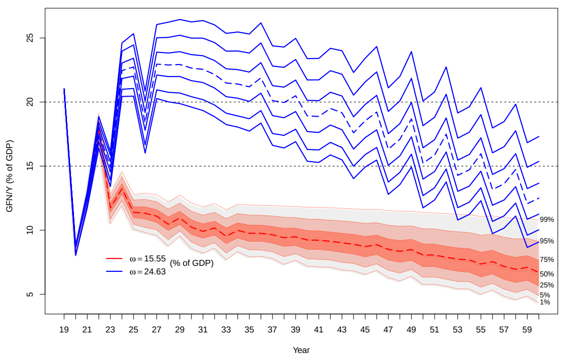
5.1.2 Tradeoff between debt stock and flow dynamics

The financing strategy impacts both stock and flow. Going from cost minimization to refinancing risk minimization, we obtain diversified financing strategies with less volatile gross financing needs, but stock declines more slowly. Varying ω from the lowest possible value for which the model has a feasible solution, to high values such that constraint (7) is not binding, we quantify a stock-flow tradeoff that is new in the literature.

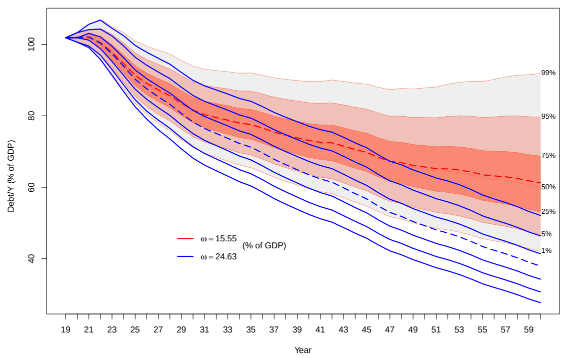
We illustrate this tradeoff in Figure 7. Panel A shows the average debt stock and gross financing needs over the tree, when varying risk tolerance. Panels B–C show the stock and flow dynamics for minimum risk ($\omega = 15.5$) and minimum cost ($\omega = 24.6$). Lower risk tolerance smoothes the gross financing needs (Panel B) but worsens the stock dynamics (Panel C). The stock-flow tradeoff operates through the maturity-cost channel: improving gross financing dynamics using long bonds increases the effective interest rate of debt when the yield curve is upward sloping. The average WAMI over the tree increases by about 5 years, from 3.1 for low cost to 8.2 for low risk, with an increase 0.8% of effective interest rate. The inverse relation between stock and flow in this example is a result of



(a) Debt stock and gross financing needs, averaged over the tree



(b) Gross financing needs



(c) Debt stock

Figure 7 – Trading off stock and flow dynamics for different levels of risk tolerance.

the upward sloping yield curve.

Conceptual tradeoffs are pertinent for policymaking as long as they have material effects, and for the crisis country we find the tradeoffs to be economically significant. The higher effective interest rate for low risk financing implies an increase of 9.3% to debt stock (averaged over the tree), and a drop by 8.4% of average gross financing needs. We also note some marked nonlinearities. When risk tolerance is high, the flow risk constraint is barely binding (if at all), and a reduction in ω has relatively small effects on issuance, cost, flow, and stock dynamics, which are mostly driven by cost minimization. In contrast, for low values of risk tolerance, the same reduction in tail risk implies larger changes in the variables of the problem. For instance, reducing ω from 26 to 19 increases effective rates marginally (0.2%) and debt stock by 2% of GDP, but reducing ω from 17 to 15.5, triples the impact on costs (0.6%) and increases debt by 7% of GDP. The magnitude of these effects depends on the calibrated economy, and running a controlled experiment we find that optimization helps more when the stock of legacy debt is larger or its maturity shorter (online Appendix C.2).

5.1.3 Risk management implications of interest rate endogeneity

Endogenous rates can create both vicious and virtuous cycles. The effect can be severe for countries verging on unsustainability, and to illustrate we lower the long term growth rate to 3%, creating non sustainable debt dynamics, and run the model with and without endogenous premia. The effective cost of debt is higher by 81bp with endogenous interest rates due to the premia, and to make the two runs comparable we add 81bp to the risk-free scenarios when testing without endogenous premia. The debt dynamics with and without endogeneity are shown in online Appendix Figure C.3. Gross financing needs increase significantly with endogenous interest rates. Without endogeneity, financing needs stay below the threshold after the third year, whereas with endogenous premia the sustainability threshold is breached with probability 0.5 until year five. Debt stock increases with probability 0.75 with endogenous premia, in the long run range 60–130% GDP. Without endogeneity it remains in the range 70–110% GDP and is non-increasing at the 0.5 level. The higher upside with endogeneity is a manifestation of the vicious cycle, and the lower downside of the virtuous cycle.

Our results corroborate a conjecture by (Bohn, 1990, p. 1218) that "if debt management affected interest rates, the qualitative nature of the government's optimization problem would change significantly". Ignoring interest rate endogeneity biases stock and flow dynamics downwards and may lead to erroneous DSA conclusions. Abstracting from moral hazard considerations, mitigating interest rate endogeneity is positive for both cost and risk management. This is one of the aims of financial assistance programs, and our model establishes whether this aim can be achieved.

5.2 Netherlands

We evaluate the DSTA practice of issuing debt with maturity anchored at a "7-year benchmark and deviations", the policy of raising a minimum amount from money markets, and the predictability policy. Netherlands' legacy debt amortizes over a long horizon, and under the assumed scenarios, debt-to-GDP ratio declines to a long term average of about 30% and gross financing needs remain below 11%. Debt dynamics are clearly sustainable. We use the model to assess DSTA policies and obtain the frontiers of Figure 8.

To evaluate whether current practices are optimal, we run the fixed-mix model imposing two additional constraints: anchor the WAMI at 8.2 years —the 7-year benchmark plus a deviation of 1.2 years based on the DSTA current portfolio— and issue at least 5% of GDP in the money markets. With anchored WAMI the model optimizes the use of long maturity bonds to counterbalance the target short money-market issue. Portfolios satisfying the benchmark policies (Frontier A) have higher NIP relative to the other frontiers, although the Dutch debt has rather low NIP (0.83–0.86 % GDP), with low associated risk (7.1–8.1% GDP). The frontier exhibits marginal cost savings for increasing risk. The

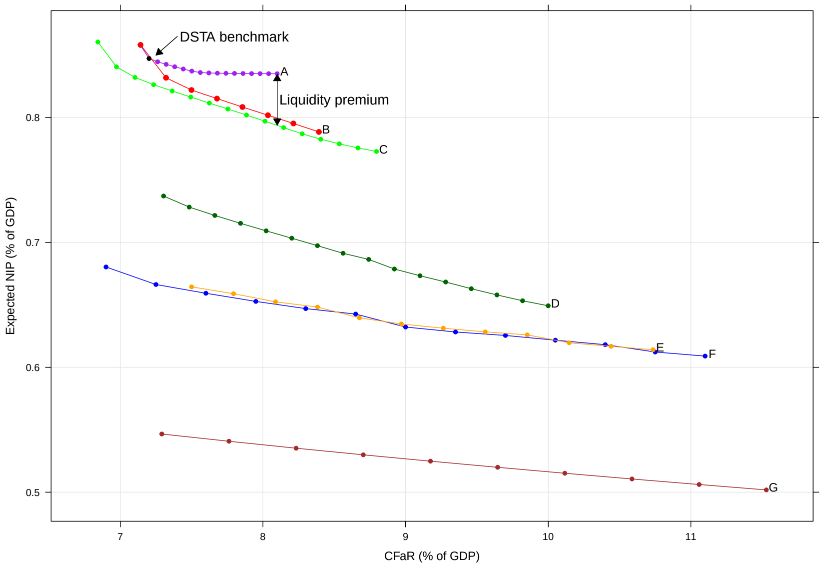


Figure 8 – Cost-risk tradeoff for the policies of the Dutch State Treasury Agency. A- optimal fixed-mix strategy with anchored weighted average maturity at 8.2 years and 5% GDP minimum in money markets; B- optimal fixed-mix without maturity anchor; C- optimal fixed-mix without anchor or money markets minimum; D- adaptive fixed-mix with anchored weighted average maturity and money markets minimum; E- adaptive fixed-mix without maturity anchoring; F- adaptive fixed-mix without anchor or money markets minimum.

DSTA issuance strategy, indicated as "DSTA benchmark", achieves minimum risk, and given the shape of Frontier A, there is not much room for improvement within current policy requirements. One interesting finding from this test is that the model delivers the required maturity by combining the available instruments, whereas DSTA typically issues 20- or 30-year bonds and then uses maturity swaps to stay anchored. The granularity of our model eliminates the need for maturity swaps.

We remove the WAMI anchor and obtain Frontier B showing that it is possible to achieve lower costs at similar levels of risk, except for the most risk averse solutions. Cost savings increase with risk tolerance. The left most part of Frontier B intersects with Frontier A, leading to the conclusion that the DSTA policy of deviations from the 7-year maturity anchor is ex ante cost reducing, provided that DSTA accepts some small risks.

Next, we drop the minimum money markets policy and obtain Frontier C with further, but slight, cost reduction at any given risk level. Issuing in the money markets increases refinancing risks, and the model counterbalances the money markets target with more expensive long term financing to achieve a given risk level. Removing the money markets target we can achieve the same risk with a less expensive portfolio of medium maturities. However, DSTA is expected to have some presence in the money markets, and uses the money markets to bridge funding needs or intervene in the financial sector, so they view a minimum exposure to the money market as a stabilizing factor, and, from this perspective, extending maturity to counter-balance the money-market issuance is optimal. Our model recommends less reliance on money markets and it documents that the money markets target comes with a "liquidity premium", shown in the figure as the distance from Frontier A to C. (The minimum risk portfolio it the left-most point on Frontier C, with WAMI 8.6, which is very close to the target WAMI 8.2 corresponding to the third from the left point, and is somewhat less costly than the DSTA benchmark.)

Overall, within the current policies of issuing a fixed mix of tenors with anchored WAMI and money markets presence, the DSTA benchmark is near optimal. The target WAMI can be achieved without maturity swaps.

We use adaptive fixed-mix to obtain Frontier D (with anchored WAMI and money markets minimum), E (without anchoring, but with money markets minimum) and F (without anchoring or money markets minimum). We notice reduced cost up to 0.2% GDP without worsening risks. Interestingly, the adaptive fixed-mix strategy maintains a money markets exposure, as evidenced by the (near) coincidence of Frontiers E and F. The average WAMI of the least risky portfolio (left-most point of E) is about 9.6 years which is 1.4 years longer than the DSTA benchmark. The adaptive strategy changes the portfolio WAMI, and the model can smooth these changes to satisfy the predictability requirement. Importantly, the model computes the optimal debt issuance strategy ex ante, providing *forward guidance* so that there are no surprises. Recall that the anchor of 8.2 was obtained as deviations from the 7-year target following past DSTA evaluations of its risk framework, so the agency is already engaged in limited adaptation of its issuance strategy. Frontiers E and F show that such adaptations are a good practice, but the position of these frontiers compared to the DSTA benchmark suggests that further improvements are possible.

Finally, we run the dynamic strategy —without anchoring or money markets minimum— to obtain Frontier G, which reduces costs by an additional 0.1% GDP. Examining the minimum risk portfolios issued at different time periods and for different states of the economy we make two observations. First, the issuance starts with short WAMI (4.86 years), and is adjusted as scenarios evolve, to either slightly shorter 3.5 years on average, or stays about the same, or is prolonged to average 10 or 25 years.[7] The model starts issuing very short to take advantage of low initial refinancing rates of the upward sloping yield curve, and subsequently reacts to new information. Second, the shift to longer maturities comes when the GDP growth is lower and debt stock-to-GDP ratio is higher. The model suggests longer issuance strategy as debt increases to minimize rollover risks, which seems appropriate given the country's low debt levels and the fact that at these debt levels the endogenous risk premium remains zero. For a crisis country the situation would have been reversed, namely shorten the maturities if the debt trajectory is very steep to reduce cost and avoid the (vicious) feedback of debt stock on risk premia. This

[7]The issuance strategy differs by state, and there are 128 states after seven periods. We aggregate states by WAMI in the range 2-4 (short), 4-5 (constant), 5-15 and 15-30 (long) years, and we report the average WAMI for each group.

model performance is consistent with *gambling for redemption* (Conesa and Kehoe, 2017) as we demonstrate in the online Appendix C.1.

The dynamic strategy suggests *contingent forward guidance*, whereby the treasury agency communicates ex ante a policy path that depends on the state of the economy. Our example with the DSTA demonstrates that the fully dynamic strategy, and the second-best adaptive fixed-mix, provide significant improvements over rigid simple rules. The issuing agency must weigh the need for predictability with the sub-optimality from not reacting to new information, and decide about adopting a state-contingent strategy. The model documents the savings, and suggests the form of such a strategy.

5.3 Italy

The center-right Italian coalition government that came to power after the March 2018 elections, had campaigned on the promise of large deficits to boost growth, and spreads on the Italian treasury securities more than doubled from the low levels of mid-2016. The proposed deficit and unrealistic growth expectations, raised concerns that Italy was facing unsustainable debt dynamics.[8] Following six months of negotiations with the EC, a compromise agreement was reached in December. EC Vice-President Dombrovskis stated that the new budget is not ideal but it has an important positive element that "it is based on a plausible economic scenario", and he emphasized the need to "put public debt on a clear downwards path".

We use the model to evaluate the compromise agreement by examining if a non-increasing debt path is possible (i.e., eqn. 8 satisfied with $\delta \leq 0$) and gross financing needs stay within the 20% threshold. The Italian case reaches the limit of our model that growth and primary balance are exogenous. It is beyond the scope of our model to endogenize them, because, as we see from this case, they are determined from auxiliary models following protracted negotiations. Instead, we use the model to assess the sustainability of debt dynamics under significantly different exogenous variables obtained under (i) no policy change, (ii) the growth and surplus projections of the Italian government, and (iii) the negotiated EC agreement.

We run the model parametrically combining long-term projections of growth and primary balance, and for each combination evaluate the probability that gross financing needs will not violate the threshold and debt stock will be non-increasing. In Figure 9 we show a heat map of (un)sustainable debt dynamics for combinations of growth and primary balance. Dark green denotes extremely low probability (0.01) of unsustainable dynamics, and red denotes high probability (0.85). On this map we place the alternative long-term projections. The IMF projections, without any policy change, have

[8]See Reuters, Oct. 4, 2018, "'Complete insanity' of Italy debt plans may lead to huge restructuring -euro officials", `https://tinyurl.com/y6dkhe7o`, accessed March 2019.

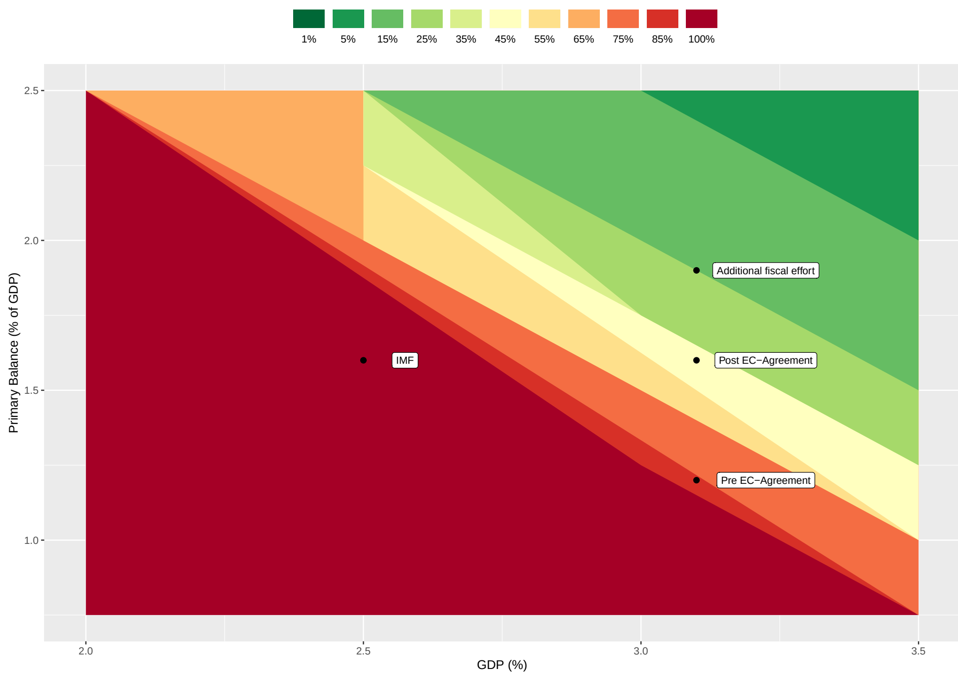


Figure 9 – Probability of the Italian debt violating debt stock or flow thresholds. Color coded are the probabilities of gross financing needs or debt stock violating the thresholds, for different combinations of primary balance and long term growth. Also shown are the results with the 2018 IMF projections for the Italian economy, the projections of the Italian government (Pre EC-agreement), and the projections agreed between the Italian government and the EC (Post EC-agreement).

un-sustainable debt dynamics (dark red). "Pre EC-agreement" corresponds to the Italian government targets, and is an improvement over the no policy change, but these targets were considered implausible by the EC, and there is still a high probability of unsustainable dynamics (red). "Post EC-Agreement" presents further improvements, with probability 0.45 for unsustainable dynamics (lyellow). The sagacity of the agreed policy with 0.55 chance of achieving its objectives is questionable, and additional fiscal effort is needed to push the probability of sustainable dynamics to 0.85 (green), as we discuss next.

6 Adjusting gross financing needs for sustainability

Debt financing decisions may not always restore by themselves the sustainability of debt dynamics, although optimizing renders significant improvements. We address now the issue of restoring sustainability with adjustments of gross financing needs. We extend the model to identify additional efforts that may be needed if the exogenously given primary balance and growth processes fail to deliver sustainable debt dynamics with high probability. We compute optimal adjustments to restore sustainability, but since we abstract from the government's utility function for deficit preferences, the model is silent as to whether the government should want, or be able, to pursue such efforts.

Depending on the assumed economic fundamentals of the country, there is a threshold $\hat{\omega}$ in eqn. (7) below which refinancing risks can not be reduced. Likewise, the lowest feasible $\hat{\delta}$ in (8) delimits policies to reduce debt at a pace beyond the potential of the economy. (The model is always feasible if we allow debt to grow and ignore refinancing risks.) Quantifying these thresholds is an important contribution of our model, especially for debt sustainability negotiations of the official sector with sovereigns in distress. Knowing this minimum (unavoidable) level of exposure to refinancing risks can preclude unrealistic policy targets. We extend the model to answer the following question: How can we ensure that the desired, but seemingly unattainable, targets of refinancing risks and pace of debt reduction are reached? The extension identifies *hot spots* of required adjustments.

We introduce a variable u_t to denote adjustments as a proportion of GDP. If the sovereign manages to save (or to raise) an additional amount $u_t Y_t^n$ at state n of period t, then the debt financing eqn. (10) becomes

$$\sum_{j=1}^{J} X_t^n(j) + u_t Y_t^n \geq GFN_t^n. \tag{18}$$

$u_t Y_t^n$ represents the part of gross financing needs that is not financed by issuing debt. This can be achieved by increasing the primary balance to directly reduce debt, or using external resources such as official sector financing or debt restructuring. To obtain the *minimum* adjustment that meets the desired refinancing risks and stock reduction, we add a penalty term $\mathcal{M} \sum_{t=0}^{T} u_t$ to the objective function (6), where $\mathcal{M}$ is a large constant. The timing of the adjustments marks the hot spots.

We can cap these adjustments, to avoid fiscal efforts that may be overly ambitious (Eichengreen and Panizza, 2016), or large adjustments through privatization that can cause fire sales. We test such variations on a recalibrated eurozone crisis economy with non sustainable debt dynamics (online Appendix C.4). We find that adjustments can keep the gross financing needs dynamics within the threshold with high probability, and decrease stock dynamics. We also find that delaying the adjustments can lead to a longer period of adjustment with significantly higher total effort than what is required if the country does not procrastinate, in line with Blanchard et al. (1990). These findings can guide policy decisions for public finance and operational decisions for official sector support.

We apply this model extension to estimate additional fiscal effort for Italy to attain sustainable dynamics with probability 0.85, shown as the point “Additional fiscal effort” in Figure 9. There are different ways to reach this point from the “Post EC-agreement”. For instance, an annualized increase of primary balance by about 0.5% GDP, for a total of 5% GDP over the next decade, will shift "Post EC-agreement" vertically up towards the dark green area. Running the model extension we find that the 0.85 confidence level can be reached with extra effort of 2.5% GDP, concentrated during the first year. This may be politically untenable and it ignores the shock from such a strong fiscal stance on the economy, so we cap the additional effort by (a politically feasible) 0.3% GDP, and find that a total fiscal effort of about 3.5%, spread evenly over twelve successive years, brings the Italian debt dynamics to the target sustainability level.

The above are best case estimates. If we account for fiscal multipliers to adjust for the effect of fiscal effort on growth, the extra effort will not shift "Post EC-agreement" up, but leftwards at an angle equal to minus the multiplier. If the multiplier is 1, the shift is at a -45 degree angle and remains in the yellow shaded area. For a multiplier of 0.6 (Blanchard and Leigh, 2013) the country shifts towards the green area. Using the model iteratively with exogenously generated combinations of primary balance and GDP growth scenarios, and accounting for fiscal multipliers, we can obtain more accurate estimates.

7 Conclusion

We develop a risk-based normative model to optimize sovereign financing strategies and trace the tradeoffs embedded in the choice of debt financing, and provide new qualitative and quantitative insights on sustainable sovereign debt dynamics. The model is an essential building block of the European Stability Mechanism debt sustainability methodological framework. It is used to both identify potential (drivers of) vulnerabilities and to inform expert judgement on policy design in the context of financial assistance.

We apply the model to a representative eurozone crisis country, Netherlands, and Italy. From the crisis country we draw lessons about the efficacy of the model in addressing sustainability issues, and from the Dutch and Italian cases we draw policy lessons.

We find that debt issuance optimization can have significant economic effects. For countries with unsustainable dynamics, optimizing the financing strategy may significantly improve debt dynamics, and even satisfy sustainability thresholds by smoothing excessive gross financing needs. We document a tradeoff between debt stock and flow, and demonstrate a significant effect of endogenous interest rates. Dynamic strategies dominate adaptive strategies that, in turn, dominate fixed-mix strategies. All optimized strategies dominate several simple rules and consol financing. Interestingly, the prevailing benchmark of issuing, on the average, 5-year debt, is near optimal in a static setting, but significant improvements are possible with adaptive strategies. Debt financing with consols is a low risk but expensive strategy, and optimized strategies dominate. Adaptive strategies satisfy a predictability criterion, and can provide forward guidance to the market about future debt issuance. Dynamic strategies are less predictable but can provide contingent forward guidance.

A model extension identifies hot spots where adjustments may be needed so that risks can be reduced below what is possible with the assumed country fundamentals. This analysis can inform the terms of official sector lending.

We find that the Dutch State Treasury Agency is following a near-optimal financing strategy within their policy constraints. Their shift towards longer maturities has been in the right direction, although it came ex post and falls somewhat short from the optimum maturity. We quantify a liquidity premium for the policy of issuing debt in the money markets, and show that the model can issue debt of a targeted weighted average maturity without resorting to maturity swaps. We also show how the DSTA can ex ante adjust the maturities using forward guidance to satisfy the predictability criterion, and further improvements are possible with contingent forward guidance using dynamic debt financing. For Italy we show that without policy changes the debt dynamics are unsustainable, and that the 2019 budget agreement with the EC improves the debt dynamics but the probability of sustainability is only 0.55. Additional fiscal effort of about 3.5% of GDP over a decade can render debt dynamics sustainable with high probability of 0.85.

The model can accommodate additional risk factors, such as inflation, exchange rates, and political uncertainty. It can also incorporate contingent debt or "principles-based constraints" (Guzman and Stiglitz, 2016). Using scenarios we can incorporate shocks, such as pandemics, wars, or natural disasters, and the long horizon of the model allows to incorporate emerging climate risks in sovereign financing. Stochastic programming on scenario trees is a versatile and effective tool for debt sustainability analysis, providing a fertile ground for further research on relevant policy questions.

Acknowledgements. We benefited from the comments of Andrew Abbel, Roel Beetsma, Maria Demertzis, Michael Dotsey, Martin Guzman, Patrick Harker, Enrique Mendoza, Juan Rojas, Rolf Strauch, Guntram Wolff, and seminar participants at the Finance Department of the Wharton School, ORFE at Princeton University, Federal Reserve Bank of Philadelphia, Bruegel, Departments of Economics at University of Bologna, Bergamo, Verona, Florence, Rome (Tor Vergata), the Center for Financial Studies at Rensselaer Polytechnic Institute, NY, Norwegian High School of Economics, Norwegian Technical University, DebtCon2 and DebtCon3 conferences at the Graduate Institute, Geneva, and Washington D.C., 2019 EFMA Conference, 2018 European Stability Mechanism conference on debt sustainability, 2019 OECD/World Bank conference on sovereign debt management. We thank, without implicating them, Roel Beetsma and Joost Witteman from SEO Amsterdam Economics, and Cecilia Gabbriellini and Flavio Padrini from the Italian Parliamentary Budget Office, for giving us access to debt data for Netherlands and Italy, and discussing the model results for their countries. We are grateful to Alex Meeraus from GAMS Development Corporation for granting us access to GAMS and for numerous helpful discussions. Two anonymous referees and the Associate Editor made several insightful suggestions that led to additional tests and improved interpretations of the results. Stavros A. Zenios is funded by a Marie Sklodowska-Curie fellowship from the European Union Horizon 2020 research and innovation programme under grant No 655092.

Disclaimer. The views herein are those of the authors, and are not to be reported as those of the European Stability Mechanism.

References

G-M. Angeletos. Fiscal policy with noncontingent debt and the optimal maturity structure. *The Quarterly Journal of Economics*, 117:1105–1131, 2002.

C. Arellano and A. Ramanarayanan. Default and the maturity structure in sovereign bonds. *Journal of Political Economy*, 120:187–232, 2012.

J. Ayres, G. Navarro, J-P. Nicolini, and P. Teles. Sovereign default: The role of expectations. *Journal of Economic Theory*, 175:803 – 812, 2018.

E. Balibek and M. Köksalan. A multi-objective multi-period stochastic programming model for public debt management. *European Journal of Operational Research*, 205: 205–217, 2010.

R. Barro. Optimal management of indexed and nominal debt. *Annals of Economics and Finance*, 4:1–15, 2003.

A. Bassanetti, C. Cottarelli, and A.F. Presbitero. Lost and found: market access and public debt dynamics. *Oxford Economic Papers*, 71:445–471, 2018.

J. Bianchi, J. C. Hatchondo, and L. Martinez. International reserves and rollover risk. *American Economic Review*, 108:2629–70, 2018.

J.R. Birge and F. Louveaux. *Introduction to Stochastic Programming*. Springer Series in Operations Research and Financial Engineering, Heidelberg, 2011.

O.J. Blanchard and D. Leigh. Growth forecast errors and fiscal multipliers. *American Economic Review: Papers & Proceedings*, 103(3):117–120, 2013.

O.J. Blanchard, J-C. Chouraqui, R. Hagemann, and N. Sartor. The sustainability of fiscal policy: New answers to an old question. *OECD Economic Studies*, 15:7–35, 1990.

L. Bocola and A. Dovis. Self-fulfilling debt crises: A quantitative analysis. *American Economic Review*, 109:4343–77, 2019.

H. Bohn. Tax smoothing with financial instruments. *American Economic Review*, 80: 1217–1230, 1990.

D.J. Bolder. A stochastic simulation framework for the government of Canada's debt strategy. Working Paper 2003-10, Bank of Canada, Ontario, CA, 2003.

O. Celasun, X. Debran, and J.D. Ostry. Primary surplus behavior and risks to fiscal sustainability in emerging market countries: A "fan-chart" approach. *IMF Economic Review*, 53:401–425, 2006.

S. Chatterjee and B. Eyigungor. Maturity, indebtedness, and default risk. *American Economic Review*, 102:2674–2699, May 2012.

H. Cole and T.J. Kehoe. Self-fulfilling debt crises. *Review of Economic Studies*, 67: 91–116, 2000.

J.C. Conesa and T.J. Kehoe. Is it too late to bail out the troubled countries in the eurozone? *American Economic Review, Papers and Proceedings*, 104:88–93, 2014.

J.C. Conesa and T.J. Kehoe. Gambling for redemption and self-fulfilling debt crises. *Economic Theory*, 64:707–740, 2017.

A. Consiglio and A. Staino. A stochastic programming model for the optimal issuance of government bonds. *Annals of Operations Research*, 193:159–172, 2012.

A. Consiglio and S.A. Zenios. Risk management optimization for sovereign debt restructuring. *Journal of Globalization and Development*, 6:181–214, 2016.

A. Consiglio, A. Carollo, and S.A. Zenios. A parsimonious model for generating arbitrage-free scenario trees. *Quantitative Finance*, 16:201–212, 2016.

G. Dantzig. *Linear Programmng and Extensions.* Princeton University Press, Princeton, NJ, 1963.

P. D'Erasmo, E.G. Mendoza, and J. Zhang. What is a sustainable public debt? In J.B. Taylor and H. Uhlig, editors, *Handbook of Macroeconomics*, chapter 32, pages 2493 – 2597. Elsevier, Amsterdam, 2016.

D.A. Dias, C. Richmond, and M.L. Wright. The stock of external sovereign debt: Can we take the data at face value? *Journal of International Economics*, 94:1–17, 2014.

B. Eichengreen and U. Panizza. A surplus of ambition: can Europe rely on large primary surpluses to solve its debt problem? *Economic Policy*, 31:5–49, 2016.

E.M. Engen and R.G. Hubbard. Federal government debt and interest rates. In Mark Gertler and Kenneth Rogoff, editors, *NBER Macroeconomics Annual 2004*, volume 19, pages 83–160. MIT Press, Cambridge, MA, 2005.

C. Gabriele, M. Athanasopoulou, A. Erce, and J. Rojas. Debt stocks meets gross financing needs: A flow perspective into sustainability. Working Paper Series No. 24, European Stability Mechanism, Luxembourg, 2017.

GAMS Development Corporation. GAMS-A User's Guide. On line, 2016. URL `www.gams.com/help/topic/gams.doc/userguides/GAMSUsersGuide.pdf`.

M. Guzman and D. Lombardi. Assessing the appropriate size of relief in sovereign debt restructuring. Research Paper 18-9, Columbia Business School, New York, NY, 2018.

M. Guzman and J.E. Stiglitz. *A Soft Law Mechanism for Sovereign Debt Restructuring. Based on the UN Principles.* International Policy Analysis. Friedrich-Ebert-Stiftung, New York, NY, 2016.

J.C. Hatchondo and L. Martinez. Long-duration bonds and sovereign defaults. *Journal of International Economics*, 79:117–125, 2009.

J. Hers, R. Beetsma, J. Witteman, and N. Verheuvel. Independent assessment of the 2016-2019 interest rate risk framework and funding policy. Report commissioned by Dutch State Treasury Agency, SEO Amsterdam Economics, Amsterdam, Feb. 2019.

T.C. Irwin. Defining the government's debt and deficit. *Journal of Economic Surveys*, 29:711–732, 2015.

N. Jobst, G. Mitra, and S.A. Zenios. Integrating market and credit risk: A simulation and optimization perspective. *Journal of Banking & Finance*, 30:717–742, 2006.

H.M. Markowitz. Portfolio selection. *The Journal of Finance*, 7:77–91, 1952.

J. Meyer, C. Reinhart, and C. Trebesch. Sovereign Bonds since Waterloo. NBER Working Paper 25543, National Bureau of Economic Research, Cambridge, MA, 2019.

A. Missale. Managing the public debt: The optimal taxation approach. *Journal of Economic Surveys*, 11:235–265, 1997.

J.M. Mulvey and W.T. Ziemba, editors. *Worldwide Asset and Liability Modeling.* Cambridge University Press, Cambridge, MA, 1998.

P. Paesani, R. Strauch, and M. Kremer. Public debt and long-term interest rates. the case of Germany, Italy and the USA. Working Paper Series 656, European Central Bank, Frankfurt, DE, July 2006.

R.T. Rockafellar and S. Uryasev. Conditional Value-at-Risk for general loss distributions. *Journal of Banking & Finance*, 26:1443–1471, 2002.

S.A. Zenios. *Practical Financial Optimization. Decision making for financial engineers.* Blackwell-Wiley Finance, Malden, MA, 2007.

Online Companion

Risk management for sustainable sovereign debt financing

by

S.A. Zenios, A. Consiglio, M. Athanasopoulou, E. Moshammer, A. Gavilan, A. Erce.

A The scenario optimization model

A.1 Debt flow constraint

Following Rockafellar and Uryasev (2002), we compute aggregate CFaR (cf. eqn. 13) on the tree, denoted by $gfn^{\diamond\diamond}$, using the following linear system

$$gfn^{\diamond\diamond} = gfn^{\diamond} + \frac{1}{1-\alpha}\sum_{n\in\mathcal{N}} p^n z^n \tag{19}$$

$$z^n \geq gfn^n_t - gfn^{\diamond},\ n \in \mathcal{N} \tag{20}$$

$$z^n \geq 0,\ n \in \mathcal{N}, \tag{21}$$

and the flow risk constraint (7) becomes

$$gfn^{\diamond\diamond} \leq \omega. \tag{22}$$

Since $n \in \mathcal{N}$ is equivalent to $n \in \mathcal{N}_t$ for all $t = 0, 1, 2, \ldots T$, it follows that eqn. (20) with time indexed gfn^n_t but time independent z^n, is well defined.

Bounding the aggregate CFaR by a threshold does not guarantee that CFaR will be below the threshold at each time period. It may exceed the threshold at some time t' at the α confidence level of the distribution $gfn_{t'}$, and t' will be a *hot spot*. In several tests of the model on multiple countries we consistently found that the aggregated formulation also limits the risk at every time period. However, an unusually large spike of legacy debt could create a hot spot. In such a case we can impose CFaR constraints at the hot spot to shape risk. We define $gfn_{t'}$ over states $\mathcal{N}_{t'}$ at t', and compute the CVaR for gross financing needs at t',

$$\Psi(gfn_{t'}) \doteq \mathbb{E}\left(gfn_{t'} \mid gfn_{t'} \geq gfn^{\diamond}_{t'}\right), \tag{23}$$

where $gfn^{\diamond}_{t'}$ is the right α-percentile. The disaggregated risk measure can also be formulated using linear inequalities (Jobst et al., 2006; Rockafellar and Uryasev, 2002).

A.2 Debt stock constraint

Changes of d on the tree are given by

$$\Delta_t^n = d_t^n - d_{t-1}^{a(n)}. \tag{24}$$

Δ_t^n is a random variable and to impose the debt stock constraint (8) we follow the conditional Debt-at-Risk (CDeaR) of Consiglio and Zenios (2016), so that debt stock changes are bounded by δ at the α confidence level. Similarly to the flow constraint, we model (8) on the tree using the linear system

$$\begin{aligned}
\Delta_t^n &= d_t^n - d_{t-1}^{a(n)},\ n \in \mathcal{N} && (25)\\
\Delta^{\diamond\diamond} &= \Delta^{\diamond} + \frac{1}{1-\alpha}\sum_{n\in\mathcal{N}} p^n y^n && (26)\\
y^n &\geq \Delta_t^n - \Delta^{\diamond},\ n \in \mathcal{N} && (27)\\
y^n &\geq 0,\ n \in \mathcal{N} && (28)\\
\Delta^{\diamond\diamond} &\leq \delta. && (29)
\end{aligned}$$

$\Delta^{\diamond}$ is the Value-at-Risk, and $\Delta^{\diamond\diamond}$ is the CVaR of debt stock changes.

A.3 Debt dynamics

We give the accounting identities for debt dynamics, for states $n \in \mathcal{N}_t$ at $t = 0, 1, 2, \ldots T$. On the tree structure we use the state-dependent indicator function $\mathbb{1}^{\tau(n)}(j, \tau(m))$ to keep track of maturing endogenous debt,

$$\mathbb{1}^t(j, \tau(m)) = \begin{cases} 1, \text{ if instrument } j \text{ issued at } \tau(m) \text{ matures at } t = \tau(n), \text{ where } m \in \mathcal{P}(n), \\ 0, \text{ otherwise.} \end{cases}$$

The flow dynamics eqn. (1) is written as

$$GFN_t^n = \underbrace{I_t^n + A_t^n}_{\substack{\text{Legacy service}\\ \text{payments}}} - \underbrace{PB_t^n}_{\substack{\text{Primary}\\ \text{balance}}} \tag{30a}$$

$$+ \underbrace{\sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j)\, CF_t^n(j,m)}_{\substack{\text{Interest payment}\\ \text{of debt financing decisions}}} \tag{30b}$$

$$+ \underbrace{\sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j) \mathbb{1}^t(j, \tau(m))\,.}_{\substack{\text{Principal amortization}\\ \text{of debt financing decisions}}} \tag{30c}$$

I_t^n is the part of $i_{t-1}D_{t-1}$ of eqn. (1) due to legacy debt and (30b) is the part due to endogenously created debt. Similarly, A_t^n is the part of A_t of (1) due to legacy debt and (30c) is the endogenously created part.

The debt stock dynamics (2) can be expressed in terms of flows on the tree,

$$D_t^n = D_{t-1}^{a(n)} + GFN_t^n - \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j) \mathbb{1}^t(j, \tau(m)) - A_t^n. \tag{31}$$

Substituting (30) into (31) we link financing decisions to the effective interest rate on debt, which was the point of departure for our model,

$$D_t^n = D_{t-1}^{a(n)} + I_t^n - PB_t^n + \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j)\, CF_t^n(j,m). \tag{32}$$

Comparing this with (2) we get the *effective cost of debt* i_t at state n as

$$i_t^n = \frac{I_t^n + \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j)\, CF_t^n(j,m)}{D_t^n}. \tag{33}$$

The numerator is the net interest payment optimized in the objective function (6).

A.4 Maturity smoothing and boundary conditions

The absolute value function (15) is not continuously differentiable, but we can stay with linearly constrained optimization by introducing variables v_t^+ and v_t^- to denote, respectively, increase and decrease of weighted average maturity at t. For the adaptive fixed-mix and the dynamic strategies we have, respectively:

$$v_t^+ \geq WAMI_t - WAMI_{t-1}, \text{ and } v_t^- \geq WAMI_{t-1} - WAMI_t, \tag{34}$$

$$v_t^+ \geq WAMI^n - WAMI^{a(n)}, \text{ and } v_t^- \geq WAMI^{a(n)} - WAMI^n. \tag{35}$$

Setting $0 \leq v_t^+, v_t^- \leq \lambda$, these variables get their value equal to the absolute value of WAMI changes and bounded by λ. Scenario dependent boundary conditions at the risk horizon take the form $WAMI_T^n = k_T$, for all $n \in \mathcal{N}_T$.

A.5 Yield curve

Equation (4) on the tree reads

$$r_t^n(j) = r_{ft}^n + \rho(d_t^n, j). \tag{36}$$

The risk and term premia take state-dependent values

$$\rho(d_t^n, j) = a_j + (1 + b_j)\hat{\rho}(d_t^n), \tag{37}$$

where the state-dependent eqn. (17) is

$$\hat{\rho}(d_t^n) \doteq \hat{\rho} \left[\frac{d_{max} - d_t^n}{1 + \exp(d_{max} - d_t^n)} - \frac{d_{min} - d_t^n}{1 + \exp(d_{min} - d_t^n)} \right]. \tag{38}$$

d_t^n is obtained dividing eqn. (32) by the state-dependent economic output Y_t^n.

A.6 Model specification

The complete model on the scenario tree consists of the decision variable definitions, objective function, and flow and stock risk constraints eqns. (6)–(9), (11)–(12), (19)–(22), (25)–(29), flow (30) and stock (32) dynamics, smoothing constraints (14) and (34) or (35) (depending if we optimize an adaptive fixed-mix or a dynamic strategy, respectively), boundary conditions, and the endogenous yield curve (36)–(38).

To optimize dynamic strategies we have a stochastic programming model with time- and state-dependent variables $w_t^n(j)$. For adaptive fixed-mix strategies we use time-dependent and state-invariant $w_t(j)$, and for fixed-mix we use time-invariant $w(j)$.

A.7 Adjusting gross financing needs

Equation (18) holds with inequality because the adjustment u_t is time dependent but state invariant, whereas the total amount raised from a given adjustment is state dependent due to the economic output. This implies that, under some states of the economy, a surplus could be created. We assume that this state-dependent surplus will be used to pay down debt, and (31) becomes

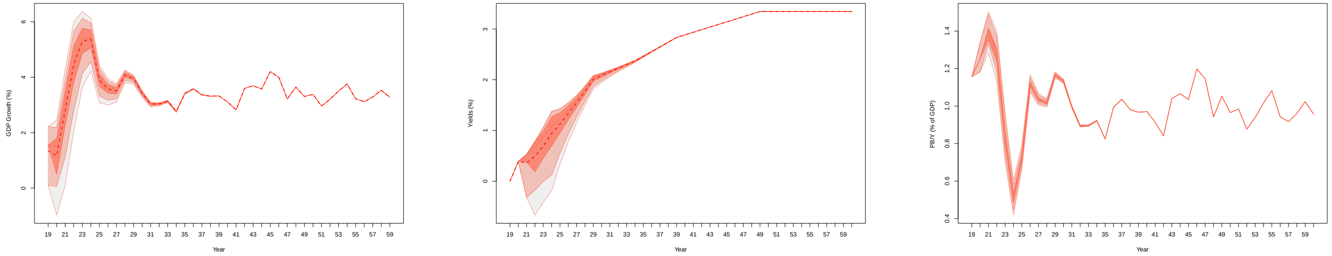
$$D_t^n = D_{t-1}^{a(n)} + GFN_t^n - u_t Y_t^n - \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m (j) \mathbb{1}^t(j, \tau(m)) - A_t^n. \tag{39}$$

A smoothing constraint can be imposed on u_t, similar to the WAMI smoothing. We can also allow for state- and time-dependent adjustments u_t^n, to structure *contingent contracts* for a country under an assistance program.

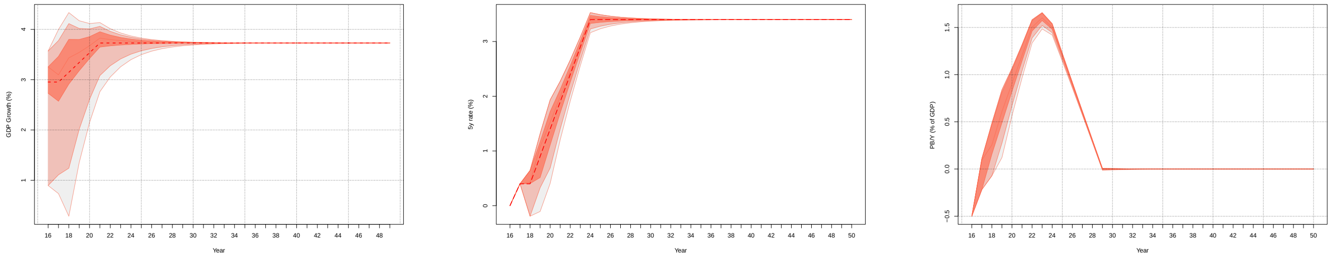
B The calibrated economies

In Figure B.1 we illustrate the scenarios for the macroeconomic, financial, and fiscal variables used in our tests, for the Eurozone crisis country, Netherlands, and the baseline for Italy using the IMF projections. The calibration is discussed in section 4.3.

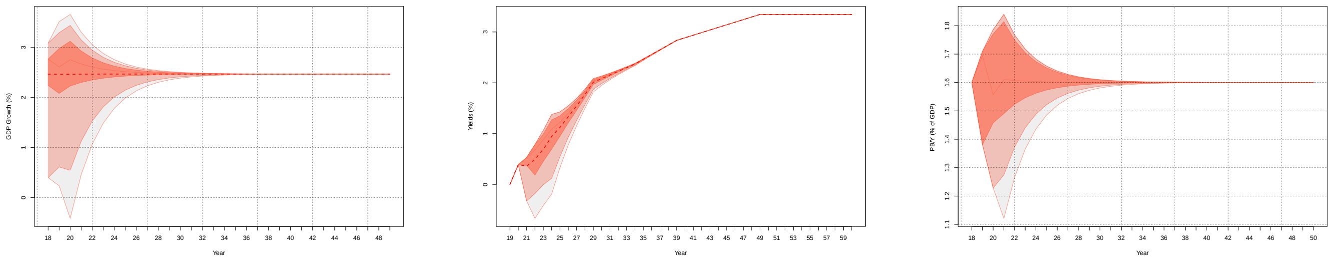
(a) Eurozone crisis country



(b) Netherlands



(c) Italy (IMF)



GDP growth | 5-year rates | Primary balance

Figure B.1 – Scenarios for macroeconomic (GDP growth), financial (5-year forward risk-free rates), and fiscal (primary balance) variables.

C Supplementary results

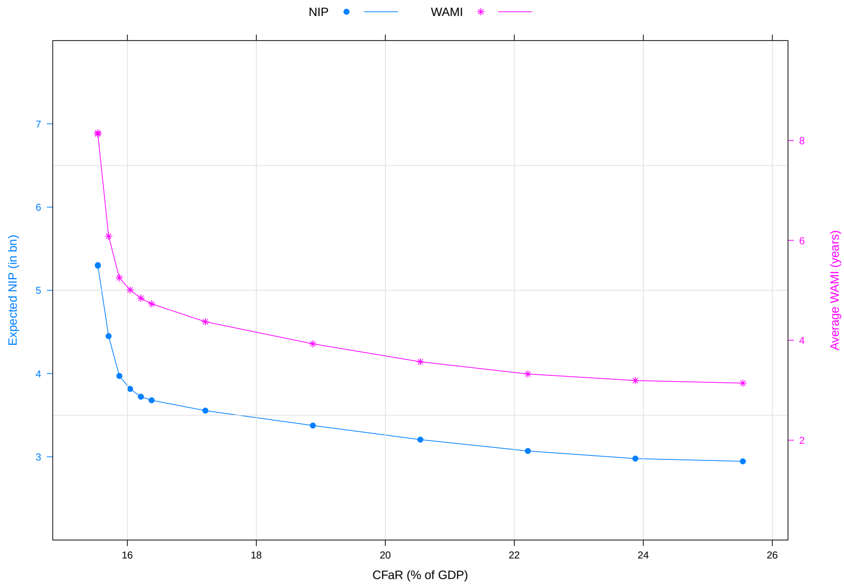
In this appendix we provide supplementary results and illustrate the sensitivity of the model to the problem parameters. The model results change smoothly and in intuitive ways with changes in the input data, signalling a robust model.

C.1 The significance of optimized debt financing

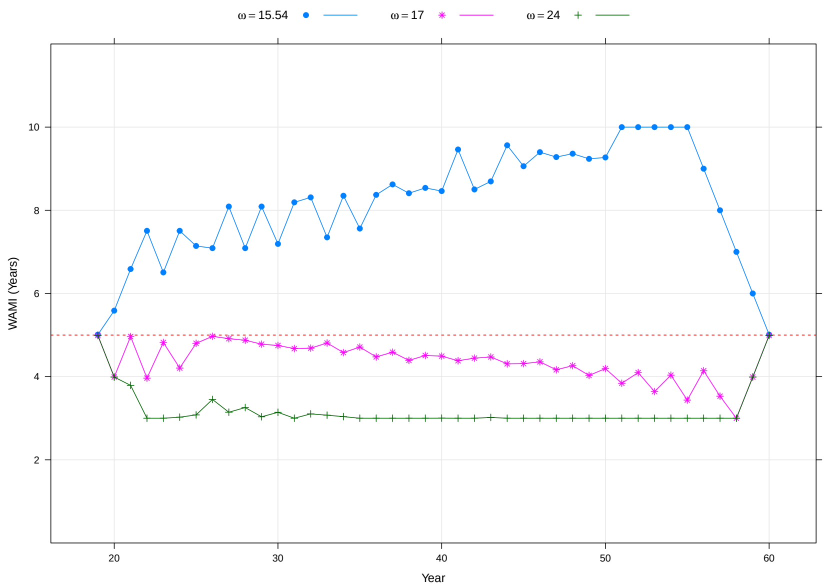
We look in some more detail at the tradeoff between borrowing costs and refinancing risks embedded in the financing decisions. Under an upward sloping yield curve with upward sloping endogenous term premia, reduced financing risks imply higher expected interest payments as risk averse sovereigns issue more expensive long-term instruments (Figure C.1, Panel A). The shift from long- to short-term issuance with higher risk tolerance (Panel B) is in agreement with Cole and Kehoe (2000); Conesa and Kehoe (2014), and can lead to countries *gambling for redemption* (Conesa and Kehoe, 2017). Our model can quantify this tradeoff for any shape of the yield curve.

C.2 Tradeoff between debt stock and flow dynamics

We run a controlled experiment and find that optimization helps more when the stock of legacy debt is larger or its maturity shorter. Ceteris paribus, more legacy debt means worse initial conditions, and this implies both higher average gross financing needs and associated refinancing risks, and higher funding costs due to interest rate endogeneity. The same is true when legacy debt amortizes on a shorter horizon, since this means higher refinancing pressures. In either case the efficient frontier for adaptive fixed-mix strategies shifts up and to the right (Figure C.2) and adaptive or dynamic strategies become more beneficial (Table C.1). The benefits from using more flexible optimal strategies are relatively larger with worse initial conditions, and the table reports the lowest expected interest costs that can be achieved with fixed-mix and adaptive fixed-mix strategies for different initial conditions of legacy debt and its maturity. The lower interest costs of the flexible strategies are more pronounced in the worst scenarios. The same is true when comparing adaptive fixed-mix strategies and the 40-40-20 benchmark by setting the risk tolerance of the adaptive fixed-mix equal to the risk of the benchmark in each of the calibrated economies (ω^*), to make for a fair comparison. Overall, optimizing the financing strategy is more significant for worse debt dynamics, an observation also made by comparing the results of the crisis country with the Netherlands.

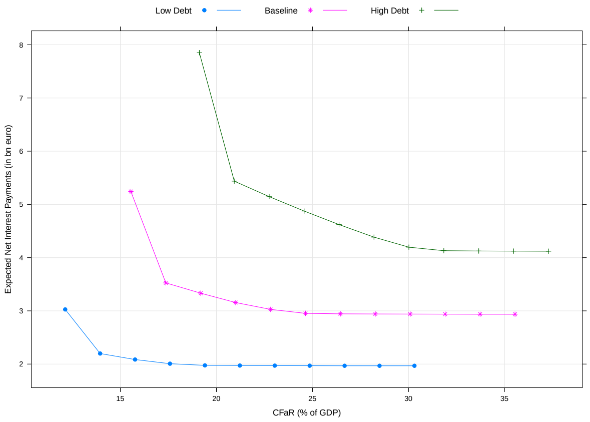


(a) Expected net interest payments and weighted average maturity at issuance, averaged over the tree

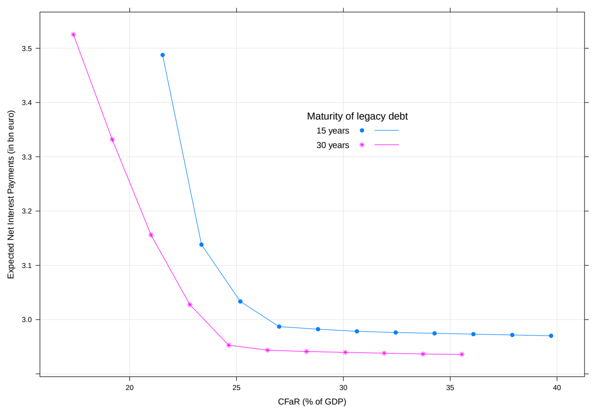


(b) Dynamics of the weighted average maturity at issuance

Figure C.1 – The cost and maturity effect of risk management.



(a) Expected net interest payment and CFaR, for different levels of legacy debt



(b) Expected net interest payment and CFaR, for different maturities of legacy debt

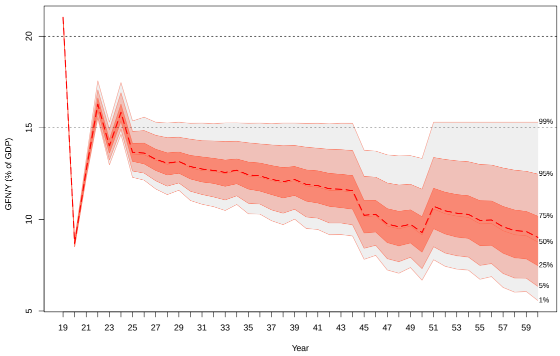
Figure C.2 – Benefits of optimization for varying level and maturity of legacy debt.

	Baseline	High debt	Short maturity
$\omega = 20$			
Adaptive fixed-mix	3.22	5.79	3.86
Fixed-mix	3.57	8.17	5.21
Difference	-0.34	-2.38	-1.51
ω^*	18.19	22.82	22.74
Adaptive fixed-mix	3.41	5.09	3.16
Benchmark 40-40-20	4.12	6.15	4.20
Difference	-0.71	-1.06	-1.05

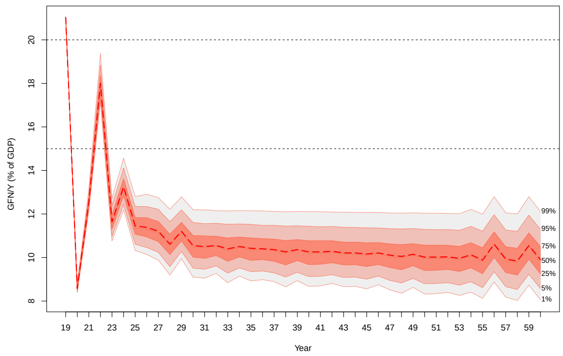
Table C.1: Expected net interest payments for different economies and financing strategies.
(ω^* is the risk tolerance of the optimal adaptive fixed-mix strategies when set equal to the rik level of the benchmark rule in each of the calibrated economies, to make for a fair comparison.)

C.3 Risk management implications of interest rate endogeneity

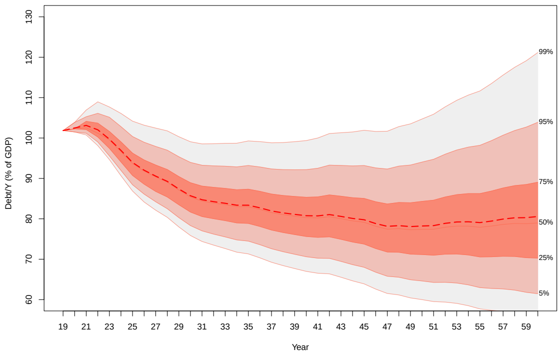
We show in Figure C.3 the debt dynamics with and without interest rate endogeneity, documenting the significant effects of endogeneity. The conclusions from this figure are discussed in section 5.1.3. We also show in Figure C.4 the effect of the parameter $\hat{\rho}$ of the sensitivity of funding costs to debt stock (eqn. 17). The cost-risk tradeoff worsens with higher sensitivity of interest rates to debt stock, creating more scope for optimization.



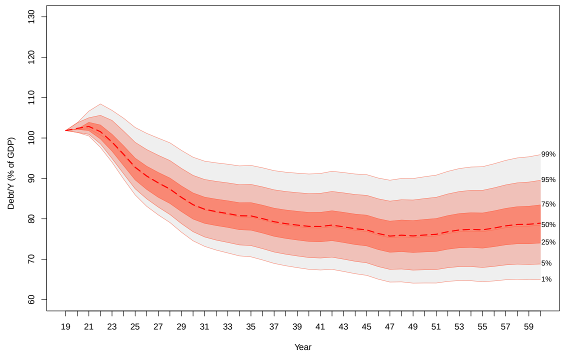
(a) *gfn* with endogenous yield curve



(b) *gfn* without endogenous yield curve



(c) *d* with endogenous yield curve



(d) *d* without endogenous yield curve

Figure C.3 – Effect of endogenous yield curve on debt dynamics.

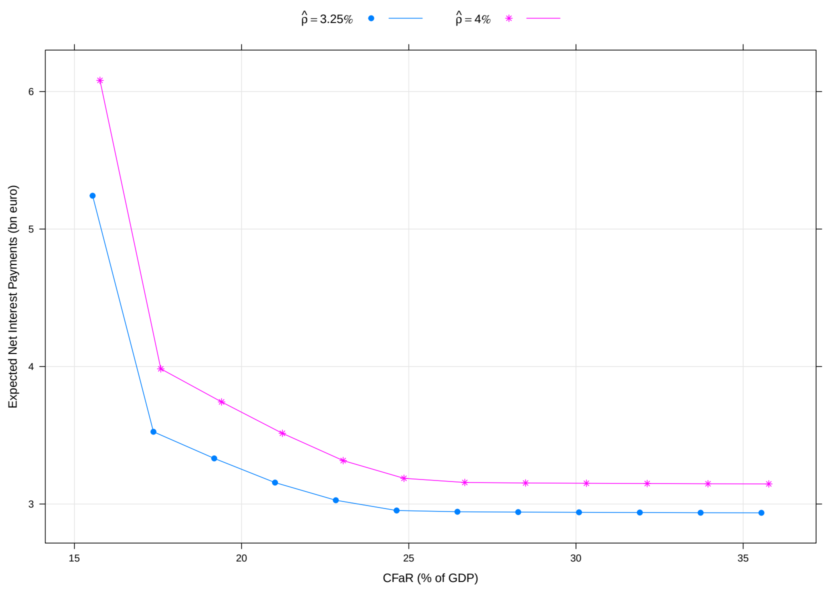


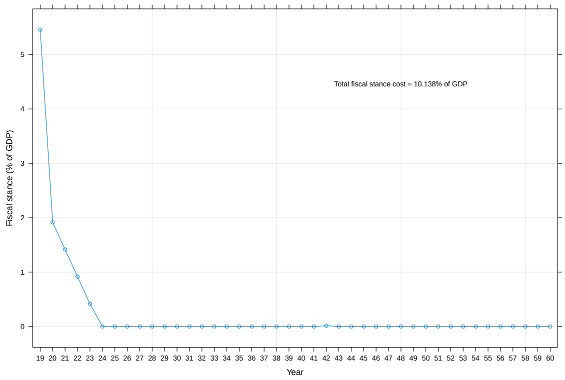
Figure C.4 – Effect on the cost-risk tradeoff of increasing sensitivity $\hat{\rho}$ of funding costs to debt stock.

C.4 Adjusting gross financing needs for sustainability

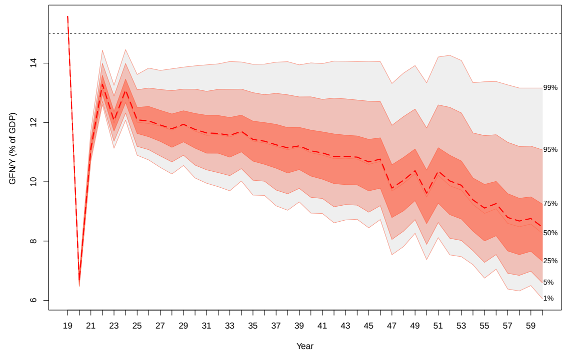
We illustrate the model extension on the recalibrated (unsustainable) crisis country with lowered long-term expected growth of 3% that creates non sustainable debt dynamics. Under this calibration the refinancing risks can not be reduced below the 15% sustainability threshold with high probability, and stock dynamics increase (recall Figure C.3, Panels A and C). Comparing with the minimum risk dynamics of the baseline calibration in Figure 7, Panels B and C, we observe a significant increase of gross financing needs and debt stock, with repeated liquidity crises within the first five years (Panel A in Figure C.3) and more than 0.5 probability of long term insolvency (Panel B in Figure C.3).

We compute minimum adjustments to reduce financing risks below the 15% threshold and show the results in Figure C.5. The minimum achievable risk level is 13.7% using the adjustments shown in Panel A. Panel B shows that, after the first period, the new gross financing needs stay within the threshold with probability 0.99. The model suggests early adjustments of more than 5% of GDP, for a total adjustment of 10.14% of GDP. To address potential concerns that such large adjustments may be economically and politically infeasible, we cap them at 3% of GDP per period, and show the results in Panels C-D. The yearly adjustment is smaller, but it carries on for two more years and the total adjustment is slightly higher at 10.52% of GDP. Gross financing needs remain within the threshold with very high probability. Stock dynamics (not shown) are decreasing.

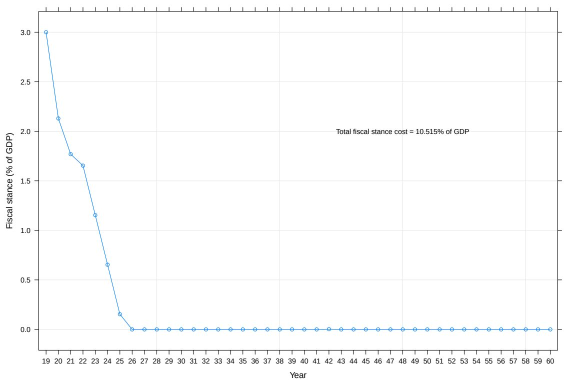
To study the effect of delays, we disallow adjustments in the first year and show the results in Panels E-F. Again gross financing needs remain within the threshold with very high probability, and stock dynamics (not shown) are decreasing. However, the delayed adjustments last for two more years, and the total (12.55% of GDP) is about 2% GDP higher than what is required if the country does not procrastinate.



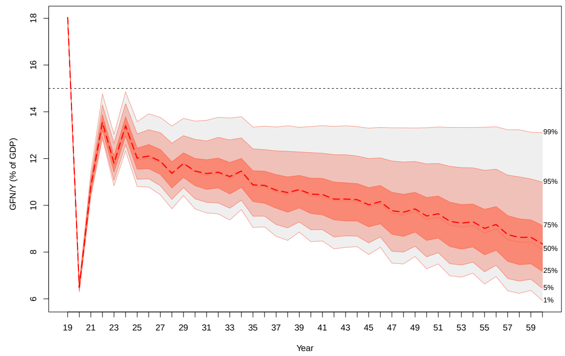
(a) Unrestricted adjustments



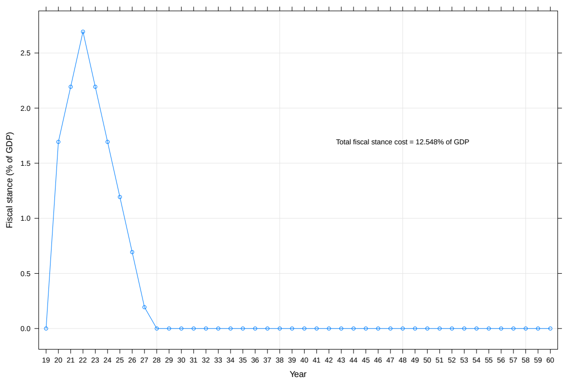
(b) *gfn* with unrestricted adjustments



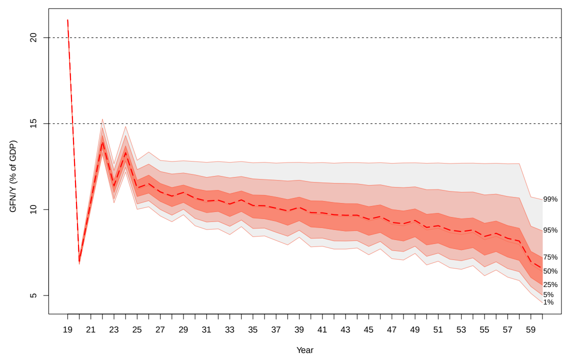
(c) Restricted adjustments



(d) *gfn* with restricted adjustments



(e) Delayed adjustments



(f) *gfn* with delayed adjustments

Figure C.5 – Adjusting gross financing needs for sustainability.