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Dynamic Platonism: Mathematics, Gesture, and Philosophy[1]

Dynamic Platonism: An Introduction

This paper introduces and develops the concept of Dynamic Platonism as a novel philosophical framework for understanding the ontology and practice of mathematics.

Unlike traditional ontological Platonism, which posits the existence of eternal, unchanging Forms outside of space and time, Dynamic Platonism emphasizes the fluid, historically situated, and diagrammatically enacted nature of mathematical idealities. Drawing on the foundational insights of Lautman, Cavaillès, and subsequent models such as TSK (Topos of Sheaves over Kripke Models), the authors argue that mathematical objects are not static entities but are dynamically constituted through gestures, diagrams, and formal processes that evolve within specific contexts. This approach seeks to reconcile the apparent tension between invariance and change by conceptualizing mathematical truths as partial invariants – local stability points embedded within a flux of historical and embodied activity – thus challenging the classical notion of absolute, atemporal Forms. The opening part of the paper (Sections 1.1 and 1.2) explores the seemingly paradoxical phrase "Dynamic Platonism", clarifying that it signals a shift away from static, transcendent notions of eternal ideas, toward an understanding of their productive, historically

[1] The content exposed in this article is the result of shared reflections developed by the authors through seminars, conferences, and private discussions. Nonetheless, Francesco La Mantia is the author of sections 0, 1.1, 1.2, 2.6 (including 2.6.1 through 2.6.5), and 4. Charles Alunni is responsible for sections 2.1, 2.2, 2.3, 2.4, and 2.5. Fernando Zalamea authored sections 3.1, 3.2, and 3.3. For the sake of stylistic consistency, quotations have been provided in English, including those for which no official translation is available.

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mediated, and relational character. This conceptual realignment builds on and reinterprets transcendental and dialectical strands in the philosophy of mathematics, portraying idealities not as fixed entities but as generative engines of mathematical development.

Section 1.2 serves as a transitional interlude, bridging the general theoretical framing with a more focused engagement with the philosophy of Albert Lautman, which becomes the core concern of the second major part of the paper (2.1, 2.2, 2.3, 2.4, 2.5). There, Lautman is positioned as a central figure whose dialectical metaphysics articulates the virtual and generative nature of mathematical idealities. His thought, rooted in a creative reinterpretation of Platonic dialectic and influenced by developments in physics and algebraic topology, conceives of mathematical objects as arising from structural tensions and symmetries—mediated by operators such as involution and relations of symmetry and asymmetry.

An extended interlude in Section 2.6 introduces the relevance of Dynamic Platonism in the context of so-called TSK models, thus preparing the conceptual ground for the third part of the paper. This segment (Sections 3.1–3.3) develops a formal articulation of Dynamic Platonism through these models, which integrate sheaf theory, modal and temporal logic, and higher-category structures to express the stratified, relational, and historically embedded dimensions of mathematical thought.

Finally, the concluding section (4.) synthesizes the findings and proposes Dynamic Platonism – particularly as instantiated through TSK models – as a shift in philosophical paradigm: from a static ontology of mathematical objects to a dynamic, context-sensitive, and evolving schema. This reconceptualization invites a rethinking of mathematical objectivity itself as emergent, relational, and historically situated – opening new directions for philosophical reflection and mathematical practice in dialogue with contemporary science.

1.1 Dynamic Platonism: A Mysterious Phrase?

"Dynamic Platonism" is an apparently mysterious phrase. In mathematics, a Platonist is someone who postulates that mathematical objects (roughly speaking, numbers and geometric figures) are abstract entities, idealities existing outside of space and time[2].

More generally, Platonism is traditionally characterized as the hypothesis that assumes the existence of non-temporal and non-spatial entities,

[2] See Øystein Linnebo "Platonism in the Philosophy of Mathematics", *The Stanford Encyclopedia of Philosophy* (Summer 2024 Edition) Edward N. Zalta & Uri Nodelman (eds.), URL = https://plato.stanford.edu/archives/sum2024/entries/platonism-mathematics/.

properties, or facts of some kind[3]. According to this perspective, Dynamic Platonism is nothing more than a self-contradictory monstrosity, or at most, a diversion for minds accustomed to the ephemeral pleasures of the oxymoron. To the extent that it refers to what changes over time[4], "dynamic" is an adjective that poorly suits the a-temporal ontology of Platonism. Yet, upon closer and more thoughtful analysis, at least three considerations allow us to diminish the apparent self-contradictory scope of Dynamic Platonism. These are three observations that show how the portrait of the Platonist just outlined is actually a reductionist and partial caricature. To limit ourselves to the history of the philosophy of mathematics, we must note that:

1. The commitment to the abstract existence of mathematical entities does not exhaust the profile of mathematical Platonism. Alongside an ontological Platonism, there exist non-ontological varieties of Platonism that have to a varying extent detached the realm of mathematical objectivity from ontology[5].

2. Thanks to the divergence between objectivity and ontology, there are those who have immersed mathematical Platonism in a transcendental perspective, more sensitive to the processes of autonomy constitution of mathematical idealities than to the discovery of their "pre-existing reality[6]".

Before this immersion, known in the literature as "Transcendental Platonism[7]" or "Constituted Platonism[8]", a tradition of studies originating from the research of Albert Lautman (1908-1944) and Jean Cavaillès (1903-1944) has emphasized the productive and inherently non-inert nature of mathematical idealities: mathematical beings are not "static beings[9]", but rather generators of mathematical beings[10], producers of idealities that develop within formal theories subject to the historical dialectic of change[11].

What we call "Dynamic Platonism" arises from the recognition of the productive and historically situated character of mathematical idealities. The paper that we propose here aims to clarify the theoretical conse-

[3] See Peter Slowik *The Deep Metaphysics of Space*, Springer, Dordrecht 2016 p. 165.
[4] See Peter Smith *Explaining Chaos*, Cambridge University Press, Cambridge, p. 37.
[5] See Micheal Resnik *Frege and the Philosophy of Mathematics*, Cornell University Press, Ithaca & London, 1980, p. 162.
[6] Jean Petitot "A Transcendental View on the Continuum: Woodin's Conditional Platonism", Intellectica, 15, 2009, p. 99.
[7] Ivi.
[8] Richard Tieszen *After Gödel*, Oxford University Press, Oxford, 2011, p. 102.
[9] Albert Lautman *Les Idées, les mathématiques et le Réel physique*, Vrin, Paris, 2005, p. 223
[10] Ibidem p. 226.
[11] Ivi.

quences of this particular way of understanding the status of mathematical objectivity. Sharing with Transcendental Platonism a pronounced focus on the constitution processes of mathematical idealities, Dynamic Platonism is a consistent form of Platonism (non-ontological) because it assigns to these idealities not only a historical dimension but also a structural invariance similar to the omni-temporality (Allzeitlichkeit) envisaged by Husserl[12] for the objects of logic and mathematics.

According to the father of contemporary phenomenology, these objects are ideal not because they are inscribed in an a-temporal hyper-uranium, but because they are invariant over time (or omni-temporal). For Dynamic Platonism, the omni-temporality of mathematical entities is always local, that is, subject to the formal constraints of historically determined contexts. For example, the theorem that the sum of the interior angles of any triangle is equal to 180° is a solid result of Euclidean geometry that has stood the test of time.

Outside of Euclidean geometry, however, this result no longer holds, showing how its own omni-temporality is relative or local (i.e., conditional on the axioms of Euclidean geometry). Dynamic Platonism is then characterized as a matrix of "partial invariances[13]", as a generator of idealities sensitive to the history of formal contexts and the plasticity of gestures that modify those contexts in relation to other invariances ready to propagate over time or, if necessary, to give way to other idealities. Hence, in conclusion, the unfolding of a universe in motion, of a field of "quasi-objects[14]" that will be the main domain of inquiry in our article.

1.2 Dynamic Platonism: Moving Toward Albert Lautman

The conception of mathematical idealities as historically conditioned yet partially invariant structures, as advanced in the preceding section, forms the backbone of our formulation of a Dynamic Platonism. Rather than opposing the metaphysical to the historical or the ideal to the empirical, this view contends that ideal mathematical structures evolve within frameworks of intelligibility that themselves are subject to conceptual and scientific transformation. Yet, in adopting such a standpoint, we do not claim to be charting wholly new territory. On the contrary, the intui-

[12] See Edmund Husserl *Erfahrung und Urteil. Untersuchungen zur Genealogie der Logik*, Academia/Verlasbubuchhandlung Prag, Berlin, 1939, pp. 309-3014.

[13] Fernando Zalamea *Synthetic Philosophy of Contemporary Mathematics*, Urbanomic, Cambridge Massachusetts, pp. 345-346.

[14] Alain Badiou *Cours Traité d'Ontologie Transitoire*, Les éditions du Seuil, Paris, 2005, p. 6.

tion that the mathematical ideality is not frozen but mobile, not static but relationally modulated across time and theory, finds an early and profound articulation in the work of Albert Lautman.

Lautman's philosophical program, particularly as elaborated in his Essai sur l'unité des mathématiques et divers écrits, stands as one of the most ambitious and conceptually rich attempts to render intelligible the interplay between the formal apparatus of mathematics and the metaphysical structures that govern their genesis. His vision, we argue, is best understood as a paradigmatic instantiation of Dynamic Platonism avant la letter – a Platonism which neither denies the autonomy of ideal mathematical forms nor separates them from the dialectical processes that engender their manifestation in concrete mathematical theories.

What distinguishes Lautman's contribution, and justifies a pause to reassess his legacy, is precisely the way in which he reactivates Platonic metaphysics in light of contemporary developments in mathematics and physics, especially within the domains of algebraic topology, group theory, and differential geometry. Drawing on a select but crucial set of sources – among them Oskar Becker, Julius Stenzel, and Léon Robin – Lautman develops a reading of Plato that foregrounds dialectic not as a merely logical tool but as a generative process underpinning the emergence of mathematical structures. His selective yet incisive engagement with ancient sources, combined with a deep immersion in the technical currents of his time, allows him to bridge a disciplinary divide often thought unbridgeable: the historical-exegetical study of Platonic thought and the formal practice of modern mathematics.

Lautman's originality lies not only in his attempt to metaphysically justify the fecundity of mathematical invention but also in his articulation of a dialectical schema that mirrors the very operations of scientific thought. In this regard, his reflections anticipate key developments in contemporary philosophy of mathematics, including the categorical formalizations proposed by thinkers such as Lawvere[15] and Zalamea, as well as broader epistemological reorientations that question the dichotomy between structure and genesis.

By positing that ideal mathematical entities arise from the tension between opposing conceptual tendencies – symmetry and asymmetry, continuity and discontinuity, algebra and geometry – Lautman sketches a metaphysics of the virtual, where mathematical truths are not given once and for all but continuously actualized through dialectical mediation. It

[15] See at least Francis William Lawvere & Stephen Schanuel *Conceptual Mathematics: a First Introduction on Categories*, Cambridge University Press, Cambridge ; Francis William Lawvere & Robert Rosebrugh *Sets for Mathematics*, Cambridge University Press, Cambridge.

is in this sense that his work constitutes not only a contribution to the philosophy of mathematics but a full-blown metaphysical system, one in which the ontological status of mathematics is rethought in dynamic, rather than static, terms.

Moreover, Lautman's vision operates reciprocally: just as metaphysical schemas illuminate the structure of mathematical theories, so too do mathematical developments retroact upon philosophical categories, transforming the very dialectic from which they emerge. This recursive relation, wherein philosophical thought is both foundational for and transformed by scientific insight, is perhaps Lautman's most enduring legacy. In foregrounding this dialectical entwinement, his work subverts the notion of philosophy as merely reflective or interpretive in relation to science. Instead, it becomes a participant in the very logic of discovery, an operator within the conceptual fabric of mathematical reason itself.

The next section thus turns to a detailed examination of Lautman's contribution, situating his Dynamic Platonism within the broader intellectual context of early 20th-century mathematics and its metaphysical interpretations. By retracing his engagement with both Platonic dialectic and contemporary scientific practice, we aim to clarify the extent to which his thought not only anticipates but crucially informs current debates about the nature of mathematical ideality and the philosophical stakes of its formalization.

2.1 Albert Lautman's Legacy

Albert Lautman's work is in our opinion the very first use of the notion of Dynamic Platonism (or rather of its conceptual equivalent), outside the specialized field of classical scholars, which is applied to mathematics and physics. Lautman's reference classical scholars, outside the case of Oskar Becker[16], have limited their field of philosophical investigation to ancient mathematics. Only Lautman will consider the contemporary physico-mathematical world.

We will see among other things what the transversal development of his Platonism corresponds to the "renewal" in which he resolutely participates in metaphysics.

[16] On this point, see the seminal work: Oskar Becker *Mathematische Existenz*, Berlin, De Gruyter, 2013 (1973). Note by Francesco La Mantia.

2.2 The Historical Context

Lautman mainly bases his argument on the work of Julius Stenzel, Zahl und Gestalt bei Plato und Aristoteles, Leipzig, 1924, Oskar Becker, “Die diairetische Erzeugung der platonischen Idealzahlen”, Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, B 1, 1931, pp. 464-501 and Léon Robin, Platon, Paris, 1935. It is in the conclusion of the Essai sur l’unité des mathématiques that Lautman reveals his sources:

> We would like to show, before concluding, how this conception of an ideal reality, superior to mathematics and yet so ready to be embodied in their movement, comes to be integrated into the most authoritative interpretations of Platonism [...] All modern commentators of Plato have on the contrary insisted on the fact that the Ideas are not the immobile and irreducible essences of an intelligible world, but that they are linked to each other according to the schemes of a superior dialectic which presides over their coming. The works of L. Robin, Stenzel, Becker have in this respect brought considerable clarity on the dominant role of the Ideas-numbers regarding both the becoming of numbers and that of the Ideas[17].

Who are these authors who bring a new vision of Platonic philosophy? Becker, Stenzel and Robin are just Lautman’s contemporaries, and he needed an absolutely extraordinary philosophical flair to discover their “Dynamic Platonism” which will come to apply in the field of contemporary mathematics and physics.

2.3 Lautman and Plato

The simple survey of Lautmanian quotations allows any reader to take stock of a theory that is not only aware of the radical and irreversible revolution affecting Platonic studies (with Becker, Stenzel and Robin, and I would add what is for me the true founding article by Georges Rodier, “Les mathématiques et la dialectique dans le système de Platon”, Archiv für Geschichte der Philosophie, XV, 1902, pp. 479-490, reprinted in Études de philosophie grecque, Paris, 1926, pp. 37-48, an article that acknowledges the existence of a profound relationship between mathematics and dialectics in the philosopher), which definitively internalizes it in the philosophical and explanatory field of modern mathematics, but which, by this means, broadens its domain. This is the “feedback effect”

[17] Albert Lautman, *Les mathématiques les idées et le réel physique*, Vrin, Paris. 2006, p. 230.

of Lautmanian theory on Platonic philosophy. It should be an object of study for any Platonic philosopher, or any specialist somewhat interested in the dialectical question in general.

Apart from his pertinent reflections on Lautmanian Dynamic Platonism, Emmanuel Barot's enterprise is also exemplary in its rapprochement with Hegelian dialectics[18].

As part of a parallel work on Jean Cavaillès, one will also read with great interest the work of Baptiste Mélès "Pratique mathématique et lectures de Hegel, de Jean Cavaillès à William Lawvere", Philosophia Scientiae, Editions Kime, 2012, From Practice to Results in Logic and Mathematics, 16 (1), pp.153-182. See https://hal.archives-ouvertes.fr/hal-01224100/document

I will take as a representative sample of the Philosophiques issue, a quote from Fernando Zalamea that mathematically expands and specifies his earlier propositions put forward as early as 2006[19]:

> In fact, at a difficult time, the young graduate of the École Normale Supérieure stood up against a "resignation" of philosophy at the expense of language, wished "a higher ambition for the philosophy of science", and imagined a program for understanding modern mathematics – non-analytical, discrete or static – open to a continuous and dynamic synthesis of knowledge (which explains his deep inclination towards a non-reified Platonism, open to movement, following Natorp and Robin). It is astonishing that the logic of sheaves shows – in agreement with the philosopher – that this continuous knowledge of the truth is possible. Thus, by reading Lautman, one comes to "listen to the voice of things" (Grothendieck) and to perceive some conceptual prefigurations which, later, will be translated technically into effective mathematics. According to a General Dialectic which takes into account an authentic pendulum oscillation, an analysis/synthesis of the concept of sheaf can be carried out along some Lautmanian lines.

To conclude on this point, I would add to what has been said that Lautman was rigorously engaged in a process of "desubstantialization" of mathematical philosophy, by a displacement and a complication of the metaphysically foundational relationship form/matter (another Platonic theme).

[18] On this question, see his article in the journal *Philosophiques*, Volume 37, number 1, spring 2010: "Lautman's duality against Hegel's negativity, and the paradox of their formalizations. Contribution to an investigation into the formalizations of the dialectic, pp. 111-148.

[19] For a critical analysis of Zalamea's undertaking concerning this mathematization – translating the dialectic of Lautmanian Ideas and Structure Schemas into the idiom of category theory – see David Corfield's article "Commentaire sur Emmanuel Barot : Lautman", from the journal *Philosophiques* already cited (p. 210).

2.4 Lautman between "Operators of Dialectical Interaction and "Limits Symmetry".

As an example, and among many others, still from the year 1937, Jacques Herbrand's theorem on "fields" presents itself to Lautman as an almost pure case of solidarity between a set of formal operations defined by a system of axioms, and the existence of a domain where these operations are realizable. He notes in this regard:

> It seems that a certain restriction still adheres to this logical schema; genesis only takes place in one direction, from operations to the domain. Now, if we can establish a rigorous appropriation between the domain and the operations definable on it, we can seek to determine both the operations from the domain and the domain from the operations [...]. Our intention being to show that the internal completion of a being is affirmed in its creative power, this conception should perhaps logically imply two reciprocal aspects: the essence of a form being realized within a matter that it would create, the essence of a matter giving birth to the forms that its structure draws [...] In fact, the schema of geneses that we are going to describe within more complicated theories abandons the overly simplistic idea of concrete domains and abstract operations that would possess in themselves a nature of matter or a nature of form. This conception would indeed tend to stabilize mathematical beings in certain immutable roles and would ignore the fact that abstract beings which are born from the structure of a more concrete domain can in turn serve as a basic domain for the genesis of other beings[20].

We will speak here of the result of a certain philosophical axiomatics and its self-application by "reflection": what Lautman affirms of mathematical logic, of the theorems of existence in the theory of algebraic functions, or of the theory of the representation of groups as different domains of transcendental investigation, is "returned" by symmetry on the philosophical device itself, which, at first, finds itself situated in the position of operation - and vice versa. This is an extremely powerful operator of dialectical interaction which, from the domain of physics-mathematics, will induce effects in the philosophical field and activity.

The most impressive place where Lautman's approach appears as almost "prophetic", the domain of his greatest philosophical and intuitive inspiration, the dialectical coupling on which all the promises of his mathematical philosophy are polarized, seem without a doubt to touch on the problem of "symmetry and asymmetry in mathematics and physics". This is the title of one of Lautman's very last texts (1942), first printed as

[20] Albert Lautman, *Les Idées, les mathématiques et le Réel physique*, Vrin, Paris, 2005, pp. 186-187.

a separate issue in the "Actualités scientifiques et industrielles" in 1946, before joining other contributions in the project initiated in 1942 by François Le Lionnais, Les Grands Courants de la pensée mathématique, published in 1948 (reissued in 1998, Les Grands Courants de la pensée mathématique, Paris, Hermann).

I would add that this text could be emblematic of the gap between the Cavaillès and Lautman duo. If, as early as his thesis, Lautman had already shown his interest in mathematical physics – in particular through his attentive study of the texts of Élie Cartan (on the "generalization of the notion of space", "absolute parallelism and unitary theory") or those of Hermann Weyl (on "Riemannian spaces"), but also through his reading of Arthur Eddington's work, Espace, temps, gravitation, or through that of La Structure des nouvelles théories physiques by Gustave Juvet – it was towards the end of his life, before falling in combat, that he oriented all his epistemological activity towards these questions of physics (and this, thanks to his exceptional mathematical background).

His very personal and extremely original contribution mainly concerns the questions of the envelopment of the notions of symmetry and asymmetry. He begins his thematic work with an analysis of Louis Pasteur's pioneering work on cellular asymmetry by "enantiomorphy", "at the origin of all the structural theories of contemporary stereochemistry". Then he moves on to the founding work in physics of Pierre Curie:

> [...] the mixture of symmetry and asymmetry becomes for him a necessary condition of the physical phenomenon in general [...]. To every physical phenomenon is linked the idea of a saturation of symmetry, of a maximum symmetry compatible with the existence of this phenomenon and which characterizes it. A phenomenon can only exist in a medium possessing its characteristic symmetry or a lesser symmetry. If therefore we call the absence of an element of symmetry an element of asymmetry, we can understand how Pierre Curie was able to write: "Certain elements of symmetry can coexist in certain phenomena, but they are not necessary. What is necessary is that certain elements of symmetry do not exist. It is the asymmetry which creates the phenomenon" (P. Curie, "On Dissymmetry in Physical Phenomena", Complete Works, p. 126)[21].

Lautman links this idea of "limit symmetry" to Plato's Timaeus, and in particular to his theory of the receptacle conceived as a "place" that is the Chora. However, this is not a question here of a ceremonial reference (and this is Lautman's prodigious speculative force):

[21] Ibid. p. 266.

> This reference to Plato allows us to understand that the materials of the Universe are not so much the atoms and molecules of physical theory as these great pairs of ideal opposites such as the Same and the Other, the Symmetrical and the Dissymmetrical, associated with each other according to the laws of a harmonious mixture [...]. Plato also suggests more [...]. The properties of place and matter are not purely sensible in his work; they are [...] the geometric and physical transposition of a dialectical theory. It could also be that the distinction between left and right, as observed in the sensible world, is only the transposition onto the plane of experience of an asymmetrical symmetry that also constitutes the abstract reality of mathematics[22].

2.5 Operations of Involution

Here, at the height of his power of conviction, it is as if Lautman were making the gesture of his thought flash by a pivot at the very heart of the domain or field, both thematic and operational, of mathematical physics. It is here that it is illustrated as in the purity of a pure diagram of thought, where the virtual power of his schemes would come to be visualized in the eye of the mind that is both mathematician and philosopher. Thought grants itself something like its own "scopic" perception of itself. It is at the very moment when it concretely invests the physico-mathematical domain that the omnipotence of its dialectical device is marked. Philosophy comes here to reveal its habitation of/in science, thus revealing their double power of reciprocal suggestion: for here, Science thinks (and thinks of itself), as inhabited by its philosophical specters. It is in such a moment of suspense, in this "between two worlds" that the artificial objections raised against a supposed "arbitrariness" of his Dialectic come to be cancelled out. It is as if the dialectical operation were suddenly revealed, suddenly made readable to the eye of theory by its own "rise towards the absolute", thus providing it with a sort of "universal covering surface".

This feeling only becomes more acute as Lautman's argument unfolds. In a final exemplary and insistent journey, Lautman brings out the mathematical focus of the entire device: the operation of involution posed as the "universal" operator and core of any dual structure (or principle of "duality") – a notion that is approached in an identical manner by Weyl in the case of the transformation called automorphism for a "zero-dimensional" space (i.e. reduced to the structure of the point):

> We must insist for a moment on the way in which the distinction between

[22] Ibid. p. 267.

> left and right in the sensible world can symbolize the non-commutativity of certain abstract operations of algebra. The fundamental property of symmetry with respect to a plane is that, when performed once, it gives a figure distinct by its orientation from the primitive figure, and that, when repeated a second time, it gives back the primitive figure. It is in this sense that symmetry is called an involutive operation. Let us now consider an algebraic operation involving two quantities X and Y, and which we write (XY), the parenthesis being able to designate an ordinary product, or any other operation defined on the two variables. It is a non-commutative operation if (XY) ≠ (YX) and the most fruitful non-commutativity in mathematics is that where we have (XY) = – (YX). The operation (XY) is asymmetric in X and Y, but it is easily verified that it defines an involutive operation like ordinary symmetry. The expressions (XY) and (YX) are said to be antisymmetric and this word well translates the mixture of symmetry and asymmetry which is thus deeply installed at the heart of contemporary algebra.
>
> The whole theory of continuous Lie groups is based on the non-commutativity of the product of two infinitesimal operations of the group, and it is this theory which, closely associated with the theory of Pfaff forms, expressions with antisymmetric multiplication, allowed Mr. Cartan to discover a profound analogy between the generalized Riemann spaces which intervene in the physico-geometric theories of Relativity, and the space of Lie groups[23].

It is these very fine considerations implying a deep knowledge of the structuring operations of algebras while at the same time referring to an anchoring in the sensible world which are, among all the others, at the origin of the current interest on the part of the true heirs of Lautman – Gilles Châtelet, Alain Badiou or Fernando Zalamea.

2.6 Dynamic Platonism: Moving Toward TSK Models in Mathematical Thought

The preceding analyses have laid a foundational framework for understanding the philosophical and mathematical conceptualization of ideas as living entities in flux rather than static absolutes (§ 1.1 & 1.2)). In doing so, they evoke a transformation of the Platonic tradition – traditionally understood as an ontology of immutable Forms – towards a dynamic and evolutionary model of mathematical thought and its historical development (§ 1.1 & 2.2). This transformation invites a reconsideration of the epistemological structures that underpin mathematical creativity, emphasizing the interrelations between phenomenology, metaphysics, and history.

[23] Ibid. 233.

In this intermediary reflection, we propose to deepen and articulate the dialectical tension between fixity and movement inherent in the Platonic corpus as revisited through the lens of Dynamic Platonism, especially as articulated by Lautman, and further developed in the conceptual apparatus of the TSK models (§ 3). These models integrate the topology of sheaves, the logic of Kripke structures, and the categorical rigor of Grothendieck topoi to form a robust architecture capable of capturing the multifaceted nature of mathematical thought. The discussion will foreground how this architecture offers a coherent metaphysical and epistemic framework where ideas are neither frozen nor purely subjective, but rather dynamically instantiated, stratified, and historically situated.

2.6.1 From Static Ontologies to Dynamic Idea-Structures

As previously noted (§ 1), the classical Platonic ontology posits ideas or Forms as eternal, perfect, and immutable entities existing in a realm apart from the sensory world. This vision has often been critiqued for its apparent detachment from historical and empirical realities. However, the Dynamic Platonism revisited in the French epistemological tradition – following Cavaillès, Lautman, and their successors – disrupts this static interpretation by emphasizing the historically contingent evolution of ideas within the mathematical praxis itself. Ideas are not transcendent monoliths but dynamic structures that evolve, stratify, and interact within temporal and conceptual horizons.

This shift from fixity to fluidity aligns with broader philosophical movements that resist reductive analyticism and static foundationalism. The Dynamic Platonism proposes that mathematical objects embody a creative tension: they are simultaneously invariant in their essential identity and susceptible to transformation through historical development and conceptual reinterpretation (§ 1.1). This dialectical movement reflects a plurality of levels – phenomenological, historical, cultural, and metaphysical – that intersect and generate new conceptual configurations.

2.6.2 Mathematical Thought as a Sheaf: Horizontality and Verticality

To capture this complexity, the notion of the sheaf provides a particularly apt metaphor and formal tool. A sheaf, in its mathematical sense, bridges local and global perspectives by organizing data (or ideas) over a base space in a way that allows for local coherence and global integration (§ 3.). In the philosophical context, this structure metaphorically corresponds to the dual aspects of mathematical thought: the technical and the imaginative.

The base space of the sheaf corresponds to the concrete technicalities of mathematics—the definitions, proofs, examples – while the fibers represent the conceptual images, intuitions, and ideas deployed over this base. This dual perspective echoes Pascal's dictum of reason and heart intertwined, suggesting that mathematical thought is a living organism composed of folded technical precision and unfolded imaginative creativity. The horizontality of sections and the verticality of fibers within the sheaf metaphor captures the interplay of analytical and synthetic reasoning, between segmentation and integration, that constitutes mathematical creativity.

Importantly, this sheaf-theoretic model preserves the plurality of mathematical perspectives, accommodating diverse methods, foundational frameworks, and conceptual stratifications without reducing them to a singular, overarching schema. Thus, the sheaf provides a conceptual space where the multiplicity of mathematical visions coexist and interact dynamically.

2.6.3 Kripke Models and Dynamic Topoi

While the sheaf model elucidates the structural complexity of mathematical thought at a given synchronic moment, it remains fundamentally static in its classical formulation. The historical dimension – the diachronic evolution of mathematical ideas—necessitates an enrichment of this framework with a temporal and modal logic component. This is precisely where Kripke models become invaluable.

Kripke's models, initially formulated for modal and intuitionistic logics, provide a natural topology that incorporates a directionality of time or possibility. By situating sheaves over Kripke models, one obtains a dynamic structure capable of representing not only static configurations but their unfolding and transformations through time. The base space gains a topological character reflecting temporal openness towards the future or interpretive modalities, allowing the model to capture the genesis, obstruction, and evolution of mathematical ideas historically.

This union of sheaves and Kripke models yields the SK model: a dynamic conceptualization of mathematical thought as an evolving sequence of sheaves over temporal structures. The SK model acknowledges internal phenomenological changes and external historical transitions, offering a nuanced perspective on mathematical development as both a local sedimentation of ideas and a global historical trajectory.

2.6.4 Towards a Higher Synthesis: Grothendieck Topoi and the TSK Model

Grothendieck's conception of topoi further elevates this conceptual edifice by embedding sheaves within a categorical universe where one considers all sheaves over a given site as a collective whole. This higher-level unity enables a movement from multiplicity back to a new form of unity, wherein archetypes or metaphysical universals are not static Platonic absolutes but flexible structural patterns realized through exactness properties and functorial correspondences.

The TSK model thus integrates three strata: the technical, the phenomenological, and the historical-dynamic, creating a conceptual ecosystem where metaphysics and phenomenology interact within historical temporality. The result is a dynamic Platonic structure – where ideas are archetypes projected onto phenomenological substrata and dynamically modulated through temporal histories (§ 3.2 & 3.3).

This synthesis offers a powerful framework for approaching longstanding philosophical questions concerning the nature of mathematical truth, creativity, and the ontology of mathematical objects. It invites a reconsideration of Platonism, not as a static doctrine of eternal forms, but as a living, elastic metaphysics that accommodates the creative tensions and historical transformations characteristic of mathematical practice.

2.6.5 Philosophical Implications and the Contemporary Relevance of Dynamic Platonism

The dynamic, multi-level framework of the TSK model resonates with contemporary philosophical concerns regarding interdisciplinarity, creativity, and the limits of reductionism. It challenges the analytic reduction of mathematical knowledge to mere formal manipulation by reinstating imagination, intuition, and historical situatedness as essential components of mathematical understanding.

Moreover, the distinction highlighted between interdisciplinarity and trans-disciplinarity underscores the generative potential of transposing specificities across different domains to awaken new creative resonances. Dynamic Platonism, in this view, serves as a philosophical anchor for transdisciplinary thought, illuminating how mathematical ideas can traverse and transform across conceptual boundaries.

The implications extend beyond philosophy of mathematics to epistemology, metaphysics, and even cognitive science, suggesting new paradigms for understanding how knowledge systems evolve dynamically within their historical and cultural milieux.

In summation, the intermediary conceptual space carved out by Dynamic Platonism and instantiated through the TSK models offer a fertile ground for rethinking mathematical thought. It transcends reductive dichotomies of static versus dynamic, local versus global, technical versus imaginative, and phenomenological versus metaphysical. Instead, it proposes a richly stratified, temporally sensitive, and conceptually pluralistic ontology of mathematical ideas that aligns with the evolving nature of mathematical practice itself.

This framework prepares the ground for the subsequent analysis, in which the precise technical and philosophical articulations of the TSK model will be deployed to illuminate the multifaceted dialectics of mathematical knowledge and the role of dynamic ideas within it.

3.1 Dynamic Platonism in TSK Models

Sections 1 and 2 of this paper have examined in detail the concept of Dynamic Platonism and its development within the work of Albert Lautman. At first glance, as already observed (§ 1), the terminology may appear contradictory, bringing together (a) the notion of a Platonism grounded in fixed ideas and (b) a historical, evolutionary, and creative dynamism associated with the practice of mathematics. However, as extensively demonstrated (§ 1 & § 2), such a contradiction is only apparent. The original matrix of Platonism is far removed from a static interpretation—a misreading too often perpetuated in the history of philosophy.

Ideas, far from being immobilized within an unknowable Absolute, move and evolve through a constant flux, shaped by accelerations and halts in historical time (Cavaillès), tensions and mixtures (Lautman), stratifications and horizons (Desanti), projections and injections (Grothendieck), diagrams and blind spots (Châtelet), and logical spaces in fluxion (Badiou).

In particular, Lautman's conception of Dynamic Platonism is fascinating on multiple levels (§ 2). The diversity of mathematical thought, irreducible to any single perspective (e.g., the analytic) or foundational system (e.g., set theory), necessitates a multiplicity of visions to begin to grasp its vast complexity. In our current era – often marked by epistemological disorientation – it is worth emphasizing the critical role of the prefix TRANS, especially in contrast to INTER. Interdisciplinarity, often celebrated uncritically, tends to focus on finding common intersections between distinct domains of knowledge.

Trans-disciplinarity[24], by contrast, seeks to transpose non-shared specificities across different domains, with the aim of awakening (to use Benjamin's typographical emphasis in the Arcades Project) creative resonances through transposition. Whereas INTER helps to explain, TRANS helps to invent[25].

One can trace this awakening of mathematical imagination in the splendid Plato's Mathematical Imagination by Brumbaugh[26]. Through a rigorous study of figures and symbols associated with incommensurability in Platonic imagery, Brumbaugh demonstrates how phenomenological, historical, cultural, and metaphysical dimensions intertwine and mutually reactivate one another through continuous TRANS movements. Years earlier, in his doctoral dissertation, Essais sur les notions de structure et existence en mathématiques (1938)[27], Lautman had already discerned – m building on the work of Robin, Stenzel, and Becker (see Section 2 of this article) – that Platonic number-ideas, far from residing in a fixed Absolute, combine dynamic geometric schemas in which, once again, phenomenological, historical, cultural, and metaphysical levels are interwoven.

The richness of these crossings and amalgamations[28] is manifest not only in modern and contemporary mathematical invention but is also found in embryonic form within a dynamic, open, elastic, and complex Platonism such as the one presented in this paper. One of Lautman's great merits lies in having rediscovered, within the original Platonic texts and their reinterpretation by the German tradition, flexible and sufficiently universal concepts that may be applied to a detailed understanding of the mathematical dialectics of his time[29].

[24] Rosa María Rodríguez Magda, *La sonrisa de Saturno. Hacia una teoría transmoderna*, Barcelona: Anthropos, 1989; Rosa María Rodríguez Magda, *Transmodernidad*, Barcelona: Anthropos, 2004

[25] This text originates from a presentation given at the conference *Diagrams and Gestures. Cross-Disciplinary Perspectives*, held at the Università degli Studi di Palermo on April 6, 2023, and organized by Francesco La Mantia. On that occasion, taking advantage of the remarkable Sicilian context, I contrasted an example of *INTER*—centered on Fibonacci and the golden ratio—with an example of *TRANS*—focused on Serpotta and his extraordinary stuccoes in the Palermitan Oratories. The opposition between number (*INTER*) and form (*TRANS*) served to elucidate Serpotta's free imagination and his anticipation of non-rigid geometries, in line with the admirable analyses of Donald Garstang.

[26] Robert S. Brumbaugh, *Plato's Mathematical Imagination*, Bloomington: Indiana University Press, 1954.

[27] Albert Lautman, *Essai sur l'unité des mathématiques et divers écrits*, Paris : 10/18, 1977; Albert Lautman, *Les Idées, les mathématiques et le Réel physique*, Paris: Vrin, 2005.

[28] Reference texts: Henri Poincaré, *L'invention mathématique*, Paris: Institut Général Psychologique, 1908; Lautman, *passim*; Alexandre Grothendieck, *Récoltes et semailles*, manuscrit, 1983-1986; Alexandre Grothendieck, *La clef des songes*, manuscrit, 1987.

[29] Jean Dieudonné, with his characteristic grace and generosity, recalls in the preface

In what follows, we will present a new model – the TSK model – for articulating the complexities of mathematical thought (§ 3.2). We will then employ this model to provide a robust framework for Lautman's Dynamic Platonism (§ 3.3).

3. 2 Dynamic Platonism: A Sheaf-Theoretic Approach

A simple idea – following Grothendieck's precept on the fundamental importance of simplicity in mathematics – has guided the development of our TSK models for mathematical thought[30]. The core intuition is to conceive mathematics as an activity where reason and heart intertwine (a homage to Pascal), and where a technical base (definitions, proofs, examples) supports an imaginative space (concepts, images, intuitions) that unfolds upon it. For this purpose, the notion of a mathematical sheaf (Jean Leray, Oflag XVIIA officers' camp, 1940–45) proves especially appropriate and flexible (see Figure 1). A sheaf consists, quite simply, of two topological spaces E and X, and a well-behaved projection p (a "local homeomorphism") that enables the reconstruction of the deployed space E from local gluings and stratifications over the base X. The fibers of the sheaf are the pointwise inverse images p-1(x), x ∈ X, while the sections are the inverse images of open neighborhoods p-1(O), Oouvert ⊆ X. Understanding a sheaf thus involves combining vertical (fiberwise) and horizontal (section-wise) perspectives, with the aim of determining whether and how local sections can be coherently glued into a global section. Sheaves pervade virtually every domain of mathematics. Two paradigmatic examples include the sheaf of germs of holomorphic functions (Riemann–Poincaré–Cartan) and the structural sheaf of a ring (Galois–Dedekind–Grothendieck).

to the 10/18 reedition of Lautman's work how Lautman's vision stood at the very forefront of its time and even surpassed Dieudonné's own (formidable) mathematical training.

[30] The models were conceived and developed within our *Seminar on the Philosophy of Mathematics*, Universidad Nacional de Colombia, Bogotá, 2016-2017. A resulting essay is Fernando Zalamea, *Modelos en haces para el pensamiento matemático*, Bogotá: Universidad Nacional de Colombia, 2021. We refer the reader to this volume for extensive details and examples.

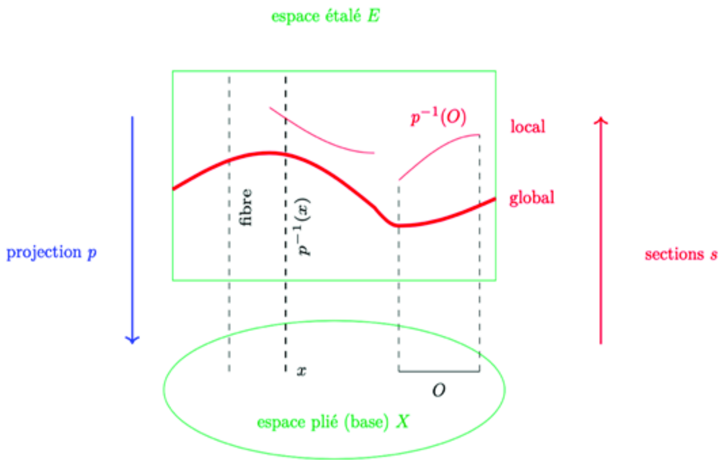

Figure 1
A sheaf: spaces, projections, fibers, sections

Following the line of thought developed by Cavaillès, Lautman, Desanti, and Châtelet, the vast variety of phenomenological levels in mathematics naturally leads us to understand mathematical thinking as a conceptual bundle, within which a dialogue unfolds among diverse partial horizons (see Figure 2).

According to this model (S), the base space contains mathematical techniques: definitions, theorems, examples. In the unfolded space reside the concepts, images, notions, and ideas that project onto these techniques. In this way, mathematics simultaneously combines differential (analytic) techniques and integral (synthetic) conceptualizations, situated on levels and stratifications that are irreducible to one another.

Both the deployed imagination (heart) and the folded technique (reason) are necessary for the rich and complex development of mathematics.

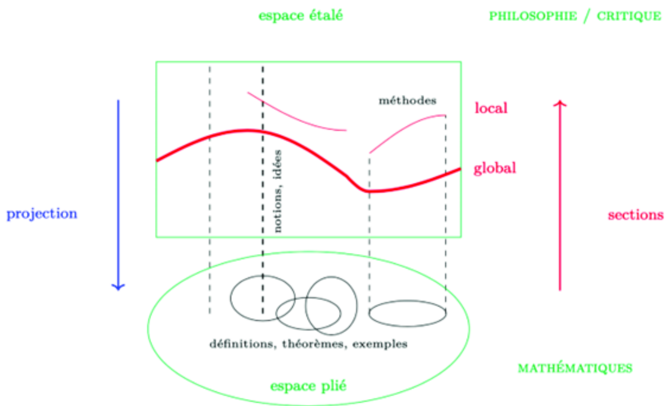

Figure 2
Model F: Mathematical Thought (Fixed) as a Sheaf

The model (S) captures the stratifications, levels, transitions, and obstructions of mathematical thought, grounded in a precise understanding of its phenomena. However, at first glance, its structure is static, fixed in time, and does not account for the dynamic aspects of mathematical development. A sense of historical evolution must therefore be incorporated into the model.

To this end, we can draw on a Kripke model (a concept introduced by a very young Saul Kripke around 1960) and situate it on the basis of a sheaf, equipped with the natural topology induced by the model. This gives rise to the topology of order, with an orientation toward the future in the case of an intuitionistic model, or to the topology of an arbitrary relation, allowing for past-future interpretative dynamics in the case of a modal model.

By employing Kripke models, we can introduce a dynamic register of mathematical phenomena, situating them within a temporal model that is either intuitionistic (development) or modal (contraction). In the following model (see Figure 3), we envision a sequence of sheaves deployed over an intuitionistic Kripke model. We denote this dynamic model as (SK).

This model addresses issues related both to internal history (phenomenological "internal" sections over synchronic periods) and to external history (phenomenological "external" sections over diachronic periods, tracing transitions toward the future or perceiving obstructions toward the past).

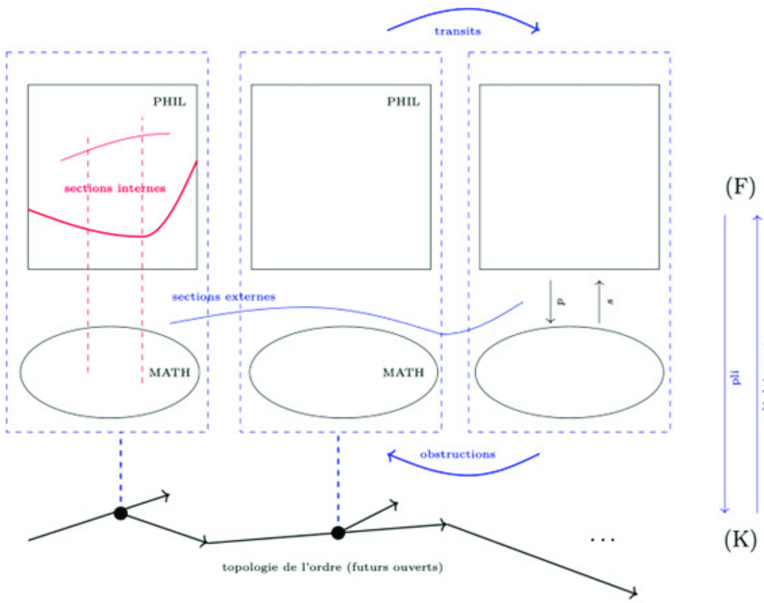

Figure 3
SK Model. Mathematical thought (in its dynamic form) as a sequence of sheaves

Sheaves became central, during the 1950s and 1960s under the French school of Cartan, Serre, Godement, and Grothendieck, to foundational mathematical work in several complex variables, algebraic topology, and algebraic geometry.

The celebrated Riemann–Roch–Grothendieck theorem (1955–1957), which recovers the classical Riemann–Roch equation (with all its geometric, complex, algebraic, and differential depth) as a special case of a projection from the K-theory group (of coherent algebraic sheaves) onto the homology of the underlying space[31], opened a vast panorama for the use of sheaves.

The process of (1) unity → (2) multiplicity → (3) renewed unity – starting from an initial unity at one level, unfolding into multiplicity at a second, and culminating in a new synthesis at a third – has led to extraordinary advances in mathematical thought (many major constructions by Galois, Riemann, Poincaré, Cantor, Hilbert, Gödel, and Grothendieck follow this method).

[31] For a comprehensive treatment of all these topics, see Fernando Zalamea *Grothendieck. Una guía a la obra matemática y filosófica*, Bogotá: Universidad Nacional de Colombia, 2019. 618pp.

A major culmination of this process is the emergence of the Grothendieck topos (1958–1962), in which one begins with (1) a topological space (or its categorical analogue, a site), then considers (2) all sheaves over this space, and finally (3) the unified collection (a category) of all these sheaves.

This complete totality – the topos – unfolds a new structure absent in the base space.

The simple gesture (1)–(2)–(3), embedded in the formation of a topos, is akin to the act of a conductor: beginning with (1) a musical score, (2) the instruments unfold its sounds, and are brought into unity under (3) the conductor's baton.

Only in the combination of all three levels does the full richness of a concert or symphony emerge.

In a Grothendieck topos, archetypes (metaphysical structures), captured by the topos's exactness properties, can be projected onto underlying types (phenomenological structures), which in turn may be reintegrated into higher-level constructions.

This back-and-forth between phenomenology and metaphysics enters into an unexpectedly new sphere of knowledge—made possible by contemporary mathematics.

Our model (TSK) captures this situation in a straightforward manner (see Figure 4).

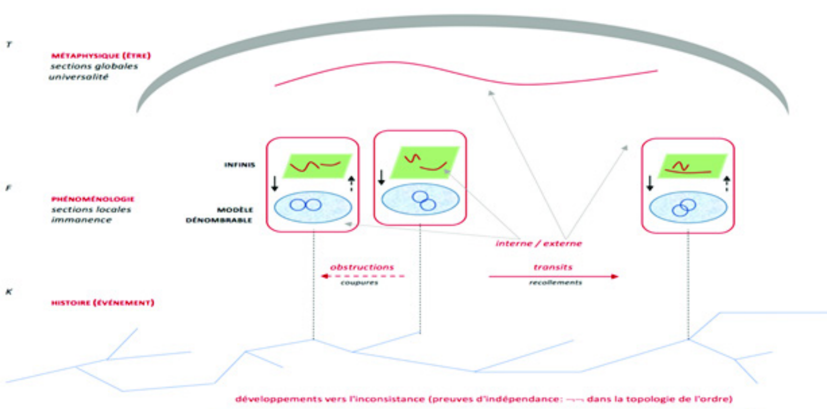

Figure 4
TSK Model: History, Phenomenology, and Metaphysics in Active Dialogue with Mathematics

3.3 Dynamic Platonism in the TSK Models

The (TSK) model incorporates the major elements of Dynamic Platonism, with the sobriety, precision, and exactitude afforded to thought by contemporary mathematics. The (K) level captures the dynamic movement of ideas (Lautman) and their active insertion into the history of mathematics (Cavaillès). The (S) level investigates the stratifications, horizons, and phenomenological layers (Desanti), where mathematical inventions, on one hand, sediment into "sheets," and on the other, erupt at blind spots (Châtelet). The (T) level provides a view of "relative universals," or archetypes (Grothendieck, Badiou), which govern a creative metaphysics of ideas through the projective construction of types.

The combination of these three levels – within the Topoi of Sheaves over Kripke Models (TSK) – offers a unique perspective on the interweaving of history, phenomenology, and metaphysics, as sought by the Dynamic Platonism of the French epistemological school.

The technical instruments employed in the (TSK) models, along with their numerous applications in grasping mathematical thought in action, help us to engage with the fundamental questions posed by Deleuze in the final lecture of his Course on Foucault[32]. In his deep reading of Moby-Dick; or, The Whale[33] (1851) and Pierre; or, The Ambiguities (1852), Deleuze constructs a diagram around Melville's "Whaling Line," wherein dense strata of language are examined, unfolded, and inverted[34] through folds and unfoldings within an "Oceanic Zone" of understanding. Deleuze evokes the mathematical genius of Galois and his proofs as composed of "ellipses, deviations, precipitations, flashes."

Situated between Lautman's "mixtures" and Châtelet's "blind spots," Deleuze proposes a complex mode of approach to a complex reality. No philosophical reduction – especially of the analytic kind – can adequately account for this complexity. The velocity of thought is evoked through mediations, turns, and ramifications. The (TSK) models, particularly attuned to these dynamic elasticities, offer a new point of support for this long-standing French tradition.

[32] Lecture of May 25, 1986, *cf.* Gilles Deleuze, *Curso sobre Foucault. La subjetivación*, Tomo 3, Buenos Aires: Cactus, 2015, pp. 173-204 (the Spanish transcription has not yet been published in French.

[33] Hermann Melville, *Moby-Dick, or The Whale*, Illustrations by Rockwell Kent, London: The Folio Society, 2009.

[34] *Cf.* Pavel Florensky, *La perspective inversée suivi de L'Iconostase*, Lausanne: L'Age d'Homme, 1992.

4. Conclusion: Embracing the Dynamic Ontology of Mathematical Ideas

The exploration of Dynamic Platonism through the lens of the TSK models reveals a profound transformation in how we conceive the ontology and epistemology of mathematical entities. This approach, grounded in the philosophical insights of Lautman and contemporaries, signifies a departure from traditional static notions of Forms and embraces a vibrant, evolving universe of ideas that are both structurally invariant and historically situated. Such a perspective not only aligns with the evolving nature of mathematical practice but also offers a fertile framework for understanding the complex interplay between thought, history, and metaphysical structures.

At the heart of this paradigm shift lies the recognition that mathematical objects are not eternal, immutable entities inhabiting a realm detached from human activity and historical development. Instead, they are dynamic, living schemas that develop, stratify, and metamorphose within the fertile ground of human culture, scientific progress, and phenomenological experience. This conception resonates with Lautman's metaphysical vision, which emphasizes the dialectical tensions, symmetries, and asymmetries that animate the genesis of mathematical ideas. The notion of invariance, therefore, becomes nuanced – localized and contextual, yet capable of extending across diverse formal and conceptual horizons.

The sheaf-theoretic and topos-theoretic formalizations embedded within the TSK models serve to articulate this fluidity with mathematical rigor. Sheaves, as conceptual bundles that organize local data into coherent global structures, metaphorically encapsulate the layered nature of mathematical thought – its technical definitions, intuitive images, and conceptual horizons. When combined with Kripke models and the categorical richness of Grothendieck topoi, these structures embody the dynamic, temporally sensitive evolution of ideas, capturing both their local genesis and their global development through time.

This multi-layered architecture underscores the importance of transdisciplinary thinking – mtransposing ideas across different domains, modalities, and cultural contexts – to foster the creative genesis characteristic of genuine mathematical innovation. It emphasizes that mathematical knowledge is not merely accumulated but continually reconfigured through dialectical processes akin to those Lautman delineates: the tension between form and matter, symmetry and asymmetry, stability and flux. Such a view invites us to see mathematics as a living organism, whose shape and substance are perpetually in flux, responding to internal logical constraints and external historical stimuli.

Furthermore, the conceptual shift from static ontologies to dynamic idea-structures challenges the foundationalist quest for absolute certainties. Instead, it promotes a view of mathematical truth as a historically situated, contextually modulated, and structurally invariant process. This aligns with contemporary philosophical trends that advocate for a pluralistic, process-oriented understanding of scientific and mathematical knowledge, recognizing the role of intuition, imagination, and cultural embedding in shaping mathematical discovery. It also opens avenues for integrating phenomenological insights, emphasizing the lived experience of mathematical practice as a central element in understanding the genesis and development of ideas.

The implications of this dynamic perspective extend beyond philosophy and into the very practice of mathematics. It suggests that mathematical creativity involves navigating a landscape of partial invariances – local truths, formal schemas, and conceptual horizons – that can be recombined, extended, or replaced as contexts evolve. This view fosters a more flexible, resilient understanding of mathematical rigor – one that accommodates the provisional, exploratory, and often nonlinear pathways of discovery. It also underscores the importance of formal tools like sheaves and topoi not merely as technical devices but as conceptual metaphors for the living processes of mathematical thought.

In essence, the synthesis offered by the TSK models and the broader framework of Dynamic Platonism advocates for a reimagining of mathematics as an active, historically embedded enterprise – an ongoing dialogue between ideas and contexts, invariance and flux, form and matter. Such a conception aligns with the philosophical heritage of Lautman and the French epistemological tradition, which refuses to reduce mathematical truth to static absolutes, favoring instead a dynamic, relational, and processual understanding. It recognizes that the evolution of mathematical ideas is both a reflection of human creativity and a manifestation of deeper structural patterns – patterns that are simultaneously invariant and plastic, universal and particular.

This perspective also invites us to reconsider the nature of mathematical objectivity itself. Rather than viewing it as an external, pre-existing realm of perfect Forms, we can see it as a product of collective human activity – a continually reshaped landscape of idealities, schemas, and invariants that emerge, develop, and sometimes fade within the dialectical flow of history. The invariance of certain mathematical results, such as the sum of interior angles of a triangle in Euclidean geometry, becomes a local invariance – valid within particular formal systems – rather than an absolute, unchanging truth. Their persistence over time is thus understood as a form of partial invariance, a stable horizon within a broader flux.

Moreover, this dynamic ontology encourages a more nuanced appreciation of the role of formal structures, such as category theory and sheaf theory, as not merely tools but as conceptual frameworks that mirror the living, relational fabric of mathematical ideas. These structures facilitate the articulation of both the local and global, the static and the dynamic, enabling mathematicians to model the emergence, evolution, and interconnection of ideas in a rigorous yet flexible manner.

In conclusion, the integration of the TSK models within the philosophy of Dynamic Platonism signifies a paradigm shift – one that recognizes the living, evolving character of mathematical ideas and their foundational structures. It advocates for a conception of mathematical objectivity as a dynamic, relational, and historically embedded phenomenon, rich in dialectical tensions and creative potentials. This approach not only advances philosophical understanding but also offers practical insights into the practice of mathematics itself—highlighting the importance of imagination, transdisciplinary exchange, and the ongoing negotiation between invariance and flux. As contemporary mathematics continues to expand its horizons into new territories – be it in higher categories, homotopical methods, or computational paradigms – the philosophical framework of Dynamic Platonism and the TSK models provide a vital compass, guiding us through the complex, vibrant landscape of mathematical thought in the twenty-first century and beyond.

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Titolo italiano

Il saggio esplora il concetto di Platonismo Dinamico come quadro trasformativo per comprendere l'ontologia e la pratica matematica. Superando il platonismo ontologico tradizionale, che postula Forme eterne e immutabili, il platonismo dinamico mette in evidenza il carattere fluido, storicamente situato e diagrammaticamente attuato delle idealità matematiche. Ispirandosi al lavoro di Lautman, Cavaillès e a modelli contemporanei come la TSK, gli autori mostrano come gli oggetti matematici siano dinamicamente costituiti attraverso gesti, diagrammi e processi formali che evolvono in contesti specifici. Questo approccio concilia invariabilità e mutamento, concependo le verità matematiche come invarianti parziali immerse nel flusso della storia e dell'attività incarnata. Integrando intuizioni fenomenologiche di Husserl e Merleau-Ponty con modelli categoriali e topologici, il platonismo dinamico propone un'ontologia sfumata, in cui le idee non sono né fisse né puramente soggettive, ma stratificate, relazionali e temporalmente estese. Tale prospettiva invita a una rivalutazione dell'oggettività matematica, ponendo l'accento sul processo, sull'incarnazione e sulla potenza virtuale delle pratiche diagrammatiche nel ragionamento matematico.

Parole chiave

Dynamic Platonism: Mathematics, Gesture, and Philosophy

This paper explores the concept of Dynamic Platonism as a transformative framework for understanding mathematical ontology and practice. Moving beyond traditional ontological Platonism, which posits eternal, unchanging mathematical Forms, Dynamic Platonism emphasizes the fluid, historically situated, and diagrammatically enacted nature of mathematical idealities. Drawing from the work of Lautman, Cavaillès, and contemporary models such as TSK, the authors highlight how mathematical objects are dynamically constituted through gestures, diagrams, and formal processes that evolve within specific contexts. This approach reconciles invariance with change, viewing mathematical truths as partial invariants embedded in a flux of history and embodied activity. By integrating phenomenological insights from Husserl and Merleau-Ponty with category-theoretic and topological models, Dynamic Platonism offers a nuanced ontology where ideas are neither fixed nor purely subjective but are instead stratified, relational, and temporally extended. The framework invites a reevaluation of mathematical objectivity, emphasizing process, embodiment, and the virtual power of diagrammatic practices in mathematical reasoning.

KEYWORDS