How Can Parmenides' tò ἐόν Be Unending but Non-endless?

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Verses DK 28 B8.42-49 of Parmenides' poem are as follows: αὐτὰρ ἐπεὶ πεῖρας πύματον, τετελεσμένον ἐστί πάντοθεν, εὐκύκλου σφαίρης ἐναλίγκιον ὄγκωι, μεσσόθεν ἰσοπαλὲς πάντηι· τὸ γὰρ οὔτε τι μεῖζον οὔτε τι βαιότερον πελέναι χρεόν ἐστι τῆι ἢ τῆι. 45 οὔτε γὰρ οὐκ ἐὸν ἔστι, τό κεν παύοι μιν ἱκνεῖσθαι εἰς ὁμόν, οὔτ' ἐὸν ἔστιν ὅπως εἴη κεν ἐόντος τῆι μᾶλλον τῆι δ' ἦσσον, ἐπεὶ πᾶν ἐστιν ἄσυλον. οἶ γὰρ πάντοθεν ἶσον, ὁμῶς ἐν πείρασι κύρει.1 Moreover, since its limit is most distant, it is completed On every side, similar to the volume of a well-rounded ball. Everywhere balanced equally starting from its center: for it must be Neither at all bigger nor at all smaller here than there. For neither is there nonbeing, which could stop it from reaching What is similar to it; nor is there being so that of being there would be More here and less there, since as a whole it is inviolable. For, equal to itself on every side, it maintains itself in its limits, similarly.

This is a suggestive and enigmatic image, but whose descriptive effectiveness is undermined by an apparently unsolvable contradiction: we have an extreme singular limit (v. 42) within which tò ἐόν maintains itself in its plural limits, 'equal to itself on every side' (v. 49). The same contradiction emerges when we analyze another passage of frag. B8, where two mythological concepts are called into question: since 'it is $θ έμιζ^2$ of τὸ ἐόν to be non-endless', Ananke 'holds it fast within the bonds of the limit [here too in the singular], which confines it on all sides' (B8.30-32).

If we pursue the mythopoeic description of $\tau \dot{o} \dot{\epsilon} \dot{o} v$ in the fragment, we discover other (at least apparent) inconsistencies: $\tau \dot{o} \dot{\epsilon} \dot{o} v$ has to be 'held in fetters' by Dike to be unbegun and unending (B8.11-15) and has to be 'bounded' by Moira, 'for nothing else <either> is or will be besides what is' (B8.36-37). This sounds very

¹ For the text of Parmenides' fragments and *testimonia* we refer to DK vol. 1, 217ff. The English translation (when not otherwise specified) is Laks and Most 2016, v 3-151 (see i 96-97 and 145-147 for the concordances between the two editions). I would like to thank the anonymous referee for offering useful suggestions for the improvement and organization of my arguments.

 $^{^2}$ The meanings of $\theta \dot{\epsilon} \mu \varsigma$ and Themis are not identical, but they are very closely related and in a mythological context can be used more or less interchangeably.

odd to our ears: how can τὸ ἐόν be an absolute, unique whole when 'fetters', 'bonds', 'limits', and 'sides', which rather evoke differentiation, plurality, and (implicit) movement, not only contrast with the claims about unity that emerge from B8 but are also declared to be the *condicio sine qua non* of that unity?³

This question regards not only frag. B8 but also our understanding of Parmenides' doctrine as a whole. Countless pages have been written on the subject, and we shall not even attempt to cover them all. The present survey, rather, will share the hermeneutic perspective inaugurated by Ruggiu 2014, according to which the text of the Parmenidean poem emerges as a unique and complex language, with logical, cosmological, theological, and ontological elements.

From this point of view, I will argue that it is possible to articulate the unity of the Parmenidean poem, i.e., the manner in which the conventionally labelled $\delta\delta\xi\alpha$ and $\lambda\lambda\eta\theta\epsilon\alpha$ sections are consistent with one another, by comparing its cosmology, developed through multiple semantic-linguistic layers, with the physics of the twentieth century. We do not thereby wish to suggest that Parmenides thought as a modern theoretical physicist would, but that modern theoretical physics has rediscovered cosmological categories that were not very unusual before Plato, Aristotle, and Euclid.

We shall walk a well-trodden but still only partially explored path, indeed one that some scholars have carefully avoided, namely, the comparison between Parmenides' and Einstein's cosmological languages. We shall present a new hermeneutical perspective on Parmenides' thought, which will concern the hypothesis that Parmenides' sphere-shaped, 'whole inviolable' tò ἐόν, as described in B8.22-49, could be cosmologically τετελεσμένον ('fulfilled', 'completed', LSJ) by the διάκοσμος described starting at B8.60-61, and by the complex sphere system introduced in frag. B12.⁴

I. A continuous well-rounded whole

Cerri 1999, 66 (cf. also 2009, 337) maintains that the use of term $\dot{\epsilon}$ va λ í γ κιον in B8.42-47 means that Parmenides only compares τὸ ἐόν to a sphere, but not that he means to say that it actually is a sphere. Furthermore, because the expression μ εσσόθεν ἰσσπαλὲς πάντηι cannot refer to the volume of a sphere—in fact all the material points of a spherical body cannot be defined as 'equally balanced in every direction from its centre' due to the principle of the lever—Cerri 1999, 238-239 claims that in those verses the comparison is with a spherical surface, not with a sphere as a whole. Cerri also excludes the possibility that Parmenides is referring to a purely geometric sphere, because this is completely free of weight or rotating force. Parmenides would refer instead to the points constituting the surface of the material sphere he is describing, and to the points physi-

³ Parmenides describes τὸ ἐόν as one (ἕν, B8.6), whole (πᾶν, B8.22), continuous (συνεχές, B8.6 and 30), ceaseless (ἀνώλεθρος, B8.3; ἐὸν γὰρ ἐόντι πελάζει, B8.25), motionless (ἀτρεμές, B8.4; ἀκίνητος, B8.26 and 38), indivisible (οὐδὲ διαιρετόν ἐστιν; B8.22), unbegun (ἀγένητος, B8.3 and 20-21), and unending (ἀτέλεστον, B8.4).

⁴ Cf. B8.42-44, where it is said that τὸ ἐόν is τετελεσμένον ἐστί πάντοθεν.

cally constituting each of the successive concentric spheres, progressively smaller and closer to the center. In this way the spherical metaphor of B8.42-47 would be connected to the complex cosmological system of 'globe-shaped crowns' introduced in frag. B12.⁵

Cerri also assumes that verses B8.42-47 describe an artefact, e.g., of bronze or stone, a material sphere made 'a regola d'arte'. Indeed, the word $\sigma\phi\alpha\tilde{\rho}\rho\varsigma$ can also mean an artefact 'ball' (Hom. *Od.* vi 100), and Plato will use it in *Phaedo* 110b to describe the shape of the earth (see Calder 1958; Morrison 1959; Couprie 2011, 201-212; Brill 2009).⁶ By using this metaphor, Plato is probably more interested in pointing out the geometrical figure of the dodecahedron, which 'the god used for the whole', and not that of an artefact ball (*Tim.* 55c), but if we apply this perspective to the Parmenidean poem it could imply the presence of a sort of demiurge in Parmenides' universe. This is an interesting hypothesis, but if we may imagine a role like this for the $\delta\alpha\mu\omega\nu$ of B12, we can hardly envision a demiurge beyond or beside the sphere-shaped $\tau \delta \dot{c} \delta \nu$.⁷

From another perspective, Ruggiu 2014, 353 emphasizes that verses B8.42-49 describe not a static sphere but a sphere having a dynamic character based on astronomical investigation rather than geometry. Furthermore, in his opinion this metaphor of $\tau \delta \dot{\epsilon} \delta v$ is intended literally in the description of the image of the world in B12, which is founded on a spherical system.

Coxon 2009, 337-338 too thinks that Parmenides 'does not compare Being to a sphere as a figure of solid geometry', and he notes that the phrase $\mu\epsilon\sigma\sigma\delta\theta\epsilon\nu$ iso $\pi\alpha\lambda\epsilon\varsigma$ $\pi\alpha\nu\tau\eta$ 'expresses therefore not a geometrical but a dynamic relation between the whole of Being and its "centre", and indicates that, though indivisible, it is neither a simple nor an inert unity'.

Mourelatos 2008, 248-251 also assumes, albeit from quite a different perspective, that there are some significant analogies ('similarities-with-a-difference', as he calls them) between the verses B8.42-49 and the cosmological descriptions of fragments B10, B11, and B12:

> the proofs of B8 culminated in the comparison of what-is with a 'well-rounded sphere' ...[and] the comparison becomes reified in 'Doxa'. The world is literally a gigantic physical sphere, or a nesting of rings: the 'outermost boundary' has now become the shining 'outermost heaven'; the 'bands' or 'bounds', which expressed the perfection or actuality of whatis, have now been projected as physical 'bands' (the rings or

 5 This is how Cerri 2011, 86 translates the word στεφάναι, the implied subject of the first verses of B12, which we shall analyze below.

⁶ I thank the anonymous reviewer for suggesting that we also take into account this meaning of the word, which 'gives complexity to the image (such as: the whole bears some resemblance to a manufactured artefact)'.

⁷ Mansfeld 2015 suggests that Parmenides' 'different and novel' Eros of B13 'reminds one of the Young Gods of *Timaeus*' and that 'the basic cosmological structure, widely valid till Plato's *Timaeus* and beyond, is now [with Parmenides' poem] in place: a set of elements and one or more Demiurges'.

wheels of the stars).

I for my part think that both the spherical-shaped systems, in B8 and B12, and their reciprocal relationship could be better explained if we abandon the attempt to view them as three-dimensional spaces. This is why I suggest that the cosmos represented by Parmenides through this imagery can be traced, in modern cosmological terms, to a 'gigantic' hypersphere (to echo Mourelatos), a topological object that foresees no other spatial dimension beyond it, rather than to a sphere.

II. The hyperspherical model of the universe

There is no need, as Peterson 1979 and Rovelli 2018 have demonstrated regarding Dante's description of the universe in his *Divina Commedia*, to know Einstein's relativistic theories in order to understand a relativistic spherical, i.e., hyperspherical, whole. To understand it, let us begin with a familiar image: the sphere. It has three dimensions but is defined by circles, so its surface is two-dimensional. Correspondingly, a hypersphere has four dimensional surface (the space we can see). In other words, a hypersphere is enclosed by three-dimensional spheres, so that it has a three-dimensional surface (the way circles, which are two-dimensional, enclose spheres), so that we can view it as an unbounded volume that wraps around itself, much like a sphere can be seen as an unbounded surface that wraps around itself.

This is only an analogy, as Callahan 1976, 94 points out, and one that 'collapses because it is hopeless to imagine what the extra spatial dimension looks like; no one has ever seen it'. A surface has two kinds of geometric properties, intrinsic and extrinsic: physical space has no extrinsic geometry that we know of, because every spatial property we know relates to figures and measurements made in space itself. 'We cannot take an extrinsic view of space by getting outside it and looking back at it', so 'space cannot be analogous to the surface of a sphere, because it has nothing comparable to the sphere's extrinsic geometry'.

But we can still get an idea of what a hypersphere is by means of an analogy elaborated by the theoretical physicist Rovelli, who thinks that it is even more difficult for us to imagine it than for an ancient astronomer, because we are trapped in the rigid Newtonian imagery in which physical space is Euclidean and infinite. The analogy helps us understand that even a mind not accustomed to thinking so abstractly as modern theoretical physics would require can conceive of hyperspherical space: it takes an 'intrinsic geometry' point of view, which is perfectly compatible with the geometry that an ancient astronomer could have elaborated from observations of the sky (see Callahan, 1976), and an isotropic and symmetric conception of the cosmos, such as that contained in Anaximander's explanation of how the earth is suspended in the sky (Rovelli, 2007). Here is the analogy in Rovelli 2017, 94, and his explanation of what it means in cosmological terms (see Callahan 1976 for a more technical explanation):

> the universe can be finite and at the same time have no boundary. How? Just as the surface of Earth is not infinite, but does not have a boundary either, where it 'ends'. This can happen,

naturally enough, if something is curved: the surface of Earth is curved. ...three-dimensional space can also be curved. Consequently, our universe can be finite but borderless. On the surface of Earth, if I were to keep walking in a straight line, I would not advance *ad infinitum*: I would eventually get back to the point I started from. Our universe could be made in the same way: if I leave in a spacecraft and journey always in the same direction, I fly around the universe and eventually end up back on Earth. A three-dimensional space of this kind, finite but without boundary, is called a '3-sphere' [or 'hypersphere'].

This was Einstein's solution to 'the absurdity of an infinite space, and the absurdity of a universe with a fixed border', an absurdity that Greek astronomers and probably Parmenides had already faced, as we will discuss.

Schrödinger 1948, 28-29 was the first to suggest that if we conceive of Parmenides' $\tau \delta \dot{\epsilon} \delta v$ (as it is described in B8.42-49) as a mental image of what the senses yield to us about the real world, we may imagine it as what a modern physicist would be inclined to call 'a simplified, hyperspherical Einstein-universe'. More recently, Casertano 2011, 50-51 insists that 'some Parmenidean doctrines, which could seem paradoxical (i.e., against common opinions), are in fact very close to relativistic physics, developed last century', and he emphasizes some analogies between Parmenides' cosmos and Einstein's 3-spherical (scil. hyperspherical) universe. But Casertano does not pursue this hypothesis, which he has developed from a different perspective.

Comparisons between Parmenides' and Einstein's cosmologies arose early enough. While the scientific world was still trying to understand Einstein's theory of relativity, which, over a century later, 'is [still] not in our bones-we do not feel it', as Greene 2000, 25 points out, Enriques 1921, 77-79, in analyzing the scientific opinion of the Greeks regarding relativity, supposed that Parmenides is probably the author of the concept of the relativity of motion. Examining vv. B8.26-30, Enriques 1921, 82 concluded that Parmenides does not deny $\tau \dot{o} \dot{c} \dot{o} v's$ motion but only its 'absolute motion': the motion of $\tau \dot{o} \dot{c} \dot{o} v$ cannot be defined in relation to something outside of it and, since 'it rests in itself, remaining the same in the same' (B8.34), it can be considered motionless only with respect to itself, not because it has no motion at all. This way, Enriques argued, Parmenides already applied to his universe (considered limited) the concept of the relativity of motion that would be developed by Einstein many centuries later.

This is in fact what Einstein 1950, 41 writes: 'The "principle of relativity" in its widest sense is contained in the statement: ... There is no absolute motion' of things. This does not mean that time and space are not real (as Popper 2002, 148-152 concludes from Parmenides' and Einstein's cosmologies), but that there is no absolute space or absolute time (i.e., there is no infinity or eternity) beyond which the becoming of the universe can be framed. Thus, Enriques was the first to point out that, in light of Einsteinian relativity, Parmenides' ontology does not deny any cosmology.

Even Severino 1998, 221-229, which is generally quite severe against attempts to compare Einstein's theory of relativity with Parmenides' ontology, has to admit that there does exist some enigmatic correspondence between the language of the theory of relativity and that of the truth of being, and that we cannot ignore this enigma.

III. The πεῖρας of τὸ ἐόν

'We should note', De Santillana 1964, 28-29 suggests, 'that Parmenides did not use the already current *peiras* for Limit, but the somewhat archaic *peiras*, which indicated "texture" and "design" more than mere "boundary". To investigate in what sense Parmenides could have used the term $\pi\epsilon$ ipac in his ontology it is worth analyzing what the word $\check{\alpha}\pi\epsilon$ ipoc means in epic and in philosophy.⁸

As Kahn 1960, 232 has pointed out, the noun $\pi\epsilon \tilde{\iota}\rho\alpha\varsigma$ stems from the verb $\pi\epsilon i\rho\omega$, which is negated by the α -privative in the word $\check{\alpha}\pi\epsilon\iota\rho\varsigma\varsigma$. Thus, the true sense of the word $\check{\alpha}\pi\epsilon\iota\rho\varsigma\varsigma$ was 'what cannot be passed over or traversed from end to end', a sense that easily developed into 'immense', 'enormous'. This is how it is used in epic, as a characteristic epithet of earth and sea, even if these are not without limits: in fact Homer and Hesiod both speak of their $\pi\epsilon i\rho\alpha\tau\alpha$ (231).

So the word $\check{\alpha}\pi\epsilon\iota\rho\varsigma\varsigma$ implied concrete bulk, magnitude, and extension. As a result of Anaximander's usage, the term then came to be opposed to $\pi\epsilon\iota\rho\alpha\varsigma$ and accumulated the senses of 'unlimited', 'mathematically infinite', 'qualitatively indeterminate', or 'indefinite', but in Anaximander the $\dot{\alpha}\pi\epsilon\iota\rho\sigmav$ is still 'a huge, inexhaustible mass, stretching away endless in every direction' as the Homeric Earth and Sea, and must be conceived as the substance of the world 'in order that the generation of things may not cease' and as the substance that 'clips the heavens in its vast embrace' (Kahn 1960, 233).

After Anaximander, the conception of the $\dot{\alpha}\pi\epsilon$ iρων as a great cosmic mass encircling the spherical body of our star-studded heaven was a permanent feature of Ionian cosmology (see Kahn 1960, 234). But we find it also in Pythagorean cosmologies, which are of some interest to us, since some ancient commentators associate Parmenides with Pythagorean doctrines.⁹ In particular, Parmenides is associated with Pythagoras through the introduction of three important astro-cosmological theorems: both are said (Diog. Laert. viii 48; A44) to be the first to call the heavens κόσμος, an organized whole, and the earth στρογγύλος ('round'), and (DL ix 23; see A1 and cf. A40a) the first to discover that the stars Eos and Esperos are the same.

Regarding the generation of the physical world, according to Aristotle (*Meta*. 1091a15-18), the Pythagoreans think that after 'the one', i.e., 'the limit' 'had been constituted' ($\dot{\omega}\varsigma$ τοῦ ἑνὸς συσταθέντος) at the center of the universe from the ἄπειρος, the nearest part of the latter was first drawn in and completed by the limit (εὐθὺς τὸ ἔγγιστα τοῦ ἀπείρου ὅτι εἴλκετο καὶ ἐπεραίνετο ὑπὸ τοῦ πέρατος).

⁸ I thank the referee for suggesting Kahn's study of the term $\pi\epsilon$ ipac as related to $\check{\alpha}\pi\epsilon$ ipov.

⁹ Cf. DL ix 21 (A1); Strab. vi 1.1 (A12); Procl. *In Parm.* 619.5-8 (A4); Iambl. *VP* 166 (A4); Simpl. *In Phys.* vii 1-3.

Aristotle also explains this process in his *Physics* (as he himself says in *Meta*. 1091a18-20, where he directs the reader to a study of $\varphi \dot{\varphi} \sigma_{\zeta}$): according to the Pythagoreans, all around the cosmos there is an indefinite 'void' ($\kappa \epsilon v \dot{\varphi}$), which is not emptiness but like an $\check{\alpha}\pi\epsilon\iota\rho\varphi\sigma\pi v\epsilon\tilde{\upsilon}\mu\alpha$ that enters heaven itself, which, as it were, inhales it. This 'void' within heaven becomes what 'distinguishes the natures of things, as if it were like what separates and distinguishes the terms of a series' (*Phys*. 213b22-26).

McKirahan 2010, 110 argues that the Pythagorean cosmos is generated by the same process that generates numbers, 'so in a sense numbers are the *kósmos*'. Consequently this cosmos is an arrangement of distinct individual things, kept separate by the void, which 'performs an analogous function in the ordered realm of discrete, whole numbers, separating each from the rest and guaranteeing to each its identity and uniqueness' (see also Philips 1966, 61, Burkert 1972, 37, and Hermann 2004, 99-101). The void resembles the air we breathe, so that even the process of the ceaseless creation of the order of cosmos is metaphorically conceived as the breathing of the *cosmos* itself, i.e., a physical process: as McKirahan 2010, 102 points out, 'the [Pythagorean's] idea that unlimited breath surrounds the *kósmos* recalls Anaximenes..., and the picture of the *kósmos* growing by inhaling this breath is at home among early Ionian ideas'.

The void contains both 'limit' and 'unlimited', but we must not think of Pythagorean 'limit' as a formal grid that brings order to shapeless matter. The $\ddot{\alpha}\pi\epsilon\iota\rho\sigma\varsigma$ is not a 'neutral' substratum: as De Santillana 1964, 26 notes, the Pythagorean 'unlimited' 'had been assigned the role of a field, or filling, but it carried within it all the determination of limit, since it was the field of all positions'. Furthermore, an assumption that arises from Pythagorean cosmologies is that numbers are physical existents ($\tau \dot{\alpha} \pi \rho \dot{\alpha} \gamma \mu \alpha \tau \alpha$; cf. Arist. *Meta.*, 987b28). I believe that the Pythagoreans were aware of the paradox that the idea of a physical universe with an unlimited yet finite structure implies.

De Santillana 1964, 26-27 points out that the dualism 'limit/unlimited' leads the Pythagorean towards the same aporia encountered by the Ionian physicists:

It is curious to note that the Pythagorean movement, which had aimed from the beginning at discovering the principle of form in nature, should have wrecked itself on a rock so much like the one the Ionians had struck. To have order, harmony, and form in the world presupposed a formal substratum which should have no form itself, but be the bearer of all form, exactly as the hydrodynamic universe of the Ionians had been a quest for a material substratum which should be sufficiently neutral in its own intrinsic properties to be modifiable into all the kinds of matter in the world.

We may also agree with De Santillana that Parmenides, 'standing at the confluence' of the Ionian and Pythagorean doctrines, 'realized that the two problems were in fact one'.

In my view, these cosmologists-the Ionians and Pythagoreans-understood

the concept of 'unlimited', regardless of what they called it (water, air, or, literally, $\check{\alpha}\pi\epsilon\iota\rho\circ\varsigma$, as Anaximander and the Pythagoreans), as a further physical dimension of their cosmos, one no less true than the others. We can compare this concept to what modern physicists call space. I refer specifically to physicists because their concept of space is not dictated by common sense, for which space is a kind of 'box' or a kind of stage where the world is staged. Indeed, as far as we know, it is Aristotle (see *Phys.* iv 2.209b4-33), in an open controversy with previous thinkers (including Plato), who first defines place as 'the limit of each body' and compares it to a container, specifically to a vase, specifying that it is by no means matter or form, because it is separated from the bodies it limits.

This was a pivotal concept also in Newtonian physics, but 20th-century theoretical physics has demonstrated that the conception of space as an immaterial container separated from the things it contains does not capture the true nature of space. And Zeno's paradox about space (see DK29 B5, A24) demonstrates that ancient cosmologists were already aware of the incoherence of imagining the cosmos as a gigantic neutral container of things.

The type of epistemological questions that Ionians and Pythagoreans had to face is analogous to the question that the two-dimensional protagonist of Edwin Abbott's novel *Flatland* poses to the Spaceland inhabitants, i.e., to us: 'Suppose a person of the Fourth Dimension, condescending to visit you, were to say, <Whenever you open your eyes, you see a Plane (which is of Two Dimensions) and you infer a Solid (which is of Three); but in reality you also see (though you do not recognize) a Fourth Dimension, which is...a true Dimension, although I cannot point out to you its direction, nor can you possibly measure it>'.

I believe that the Ionians and Pythagoreans were investigating the existence of this further dimension, and that they were also aware that it was a true dimension of which, however, it was not possible to find external measures because we cannot get outside it. They therefore knew that the only features that we can find of the cosmos in its wholeness are internal ones, that is, those that we can deduce by investigating it from the inside, with the significant difference that, in their epistemological view, these characteristics were not geometric but physical, and that they ordered the world.

So they knew that any attempt to describe the universe as it would appear if we could observe it from outside could only be a mere hypothesis. (This also applies to modern theoretical physics; indeed there are many theories about the true shape of the universe.) Astronomical observation can only offer internal descriptions of the cosmos. So even if they could be quite certain that the cosmos had a spherical shape, they had not yet developed a topology to describe it geometrically. They might have thought that this knowledge was perhaps the exclusive area of the gods. This is the reason why they still employed mythological schemes to describe the cosmos in its wholeness, such as religious theo-cosmogonies, and then developed them according to their cosmological hypotheses.

That is why, for example, we find Dike representing the rules governing the astronomical motion of the Sun, in Homer (*Il.* v 749ff. and viii 393ff.), Anaxi-

mander (DK 12 B1), and Heraclitus (DK 22 B94), although the cosmological frames of reference are very different from case to case. We can say the same for other deities we encounter in Ionian and Pythagorean fragments and *testimonia*, such as in Parmenides' poem. In fact, even if, as Curd 2011, 127 remarks, 'Parmenides removes the power of knowing from the divine and places it in the human capacity for thought', he still uses a mythopoietic framework to describe to ċóv, probably because this was the technical language spoken by his audience.

Modern cosmologists encounter the same epistemological difficulties that afflicted those ancient cosmologists: every physical theory, even the general theory of relativity, deals with matter and its spatial properties, and every property we know relates to figures and measurements made in space itself, because we cannot take an extrinsic view of space by getting outside it. Even imagining the Big Bang as a cosmic explosion is in fact 'a little misleading', as Greene 2000, 83 explains: when a bomb explodes, 'its contents are ejected into the surrounding space', but 'in the big bang, there is no surrounding space'.

Thus, what modern physicists provide are also hypothetical schemes, including the hyperspherical model. These are not mythological, of course, but they perform that same function of letting us visualize what we cannot actually see: the universe in its wholeness. Just as the mythological schemes of those ancient cosmologists were not consistent with their common sense (in fact they would be assimilated to fables when their mythopoeic codes are lost), so too modern cosmological schemes, even if they are described through geometric language, are not consistent with the geometry we know from daily experience.

If the idea of an unlimited cosmos is paradoxical for the ancients, the finiteuniverse picture is open to a strong objection: in being finite it must have a limiting boundary, but that is impossible, because a boundary can only separate one part of space from another, so that something beyond the cosmos is implied. We know that Greek astronomers already put forward this absurdity, as Simplicius' *testimonium (in Ph.* 467.26) on the Pythagorean Archytas demonstrates:

> <If I arrived at the furthest point of the heavens, could I stretch out my hand or a stick towards what was outside, or not?> Now, it would be very strange indeed not to be able to extend it. ...Therefore each time he will go in the same way towards the limit (π έρας) that is chosen each time and will ask the same question, and if each time there is something else to which the stick can go, it is clear that it is an unlimited (ἄπειρος) [body] (DK 47 A24).

Could the metaphor of the sphere in B8.42-49 be Parmenides' attempt to resolve this cosmological aporia with which ancient astronomers—and probably not only the Pythagoreans—were struggling already before Archytas?¹⁰

¹⁰ Although there can be no doubt that Parmenides came into contact with Pythagorean and Ionian doctrines, I believe that any statement that the cosmos has a beginning and its parts are separate, together with any assumption that reality is based on opposite principles, is incompatible with the Parmenidean doctrine of $\tau \delta \dot{\epsilon} \delta v$.

Several scholars (including Montagnino 2018) have, from their particular perspectives, emphasized how Parmenides' doctrine of tò ἐόν is closely interconnected with, if not a consequence of, his astronomical observations and discoveries, and thus with his cosmological theories.¹¹ Since Parmenides was considered one of the foremost astronomers of his time, it should come as no surprise that he used geometry in his observation of the heavenly vault: 'modern scholars of Parmenides keep on discussing his philosophical thought and his poem as if it were bereft of a scientific-astronomical dimension, as if—literally it had not had a dense section set out as a sort of map of the heavens', Cerri 2011, 93-94 writes, and adds that 'every interpretation of his thought effacing or marginalizing such a scientific dimension is manifestly inadequate to answer for the whole of the evidence we possess'.

In B4.2 we read: 'you will not cut off $\tau \dot{o}$ έ $\dot{o}v \tau \sigma \ddot{v}$ έ $\dot{o}v \tau \sigma \ddot{c}$ '. How, then, is it possible to imagine any $\pi\epsilon$ īpaç to $\tau \dot{o}$ έ $\dot{o}v$? To answer this question we have to follow Kahn's arguments, according to which a $\pi\epsilon$ ĩpaç, rather than designating an end, can be conceived as a way to pass from one end to another, thus as a sort of connection. We may thus hypothesize that in Parmenides' metaphor of the sphere, the $\pi\epsilon$ ĩpaç $\pi \dot{v}\mu \alpha \tau ov$ (B8.42) of $\tau \dot{o}$ έ $\dot{o}v$ can be conceived as its outermost limit. This limit can still be exceeded, but only in the direction of $\tau \dot{o}$ έ $\dot{o}v$ itself, so that one can return to $\tau \dot{o}$ έ $\dot{o}v$ while entering it from another side of the $\pi\epsilon$ ĩpaç $\pi \dot{v}\mu \alpha \tau ov$. This would be possible in a hyperspherical cosmos, as we have seen above. If this hypothesis is correct, even the outermost limit would only be a connection, not a boundary, so that ἐ $\dot{o}v$ ἐ $\dot{o}v\tau$ ι $\pi\epsilon\lambda \dot{\alpha}$ ζει (B8.25) in every sense, since 'as a whole it is continuous' (B8.25), viz., unbounded, and 'inviolable' (B8.48).

IV. A continuous interweaving of astronomical στεφάναι

To compare Parmenides' sphere-like cosmology and the hypersphere of 20th century physics, let us start from B12, which we have inherited from Simplicius' *In Phys.* 39.12 = B12.1-3; *In Phys.* 31.10 = B12.2-6. The first verses are:

αί γὰρ στεινότεραι πλῆντο πυρὸς ἀκρήτοιο,

αί δ' ἐπὶ ταῖς νυκτός, μετὰ δὲ φλογὸς ἵεται αἶσα

έν δὲ μέσῷ τούτων δαίμων ἢ πάντα κυβερνῷ

πάντων...

For the narrower ones were filled with unmixed fire,

The next ones with night, and afterward [or: among these]

there rushes a portion of flame.

And in the middle of these, the divinity who steers all things... $(B12.1-4)^{12}$

Parmenides' verses refer to astronomical objects. Theophrastus' notes on the pas-

¹¹ De Santillana 1964, Ruggiu 2014, Cerri 2011, Casertano 2011, Popper 2012, Mourelatos 2011, Graham 2013, among others. For a different viewpoint, see Rossetti 2019, 55-62.

¹² Bollack 1990 discusses whether this is a cosmogonic rather than a cosmological description, but it should not surprise us if a cosmological description is put forth as a cosmogonic narrative: to define a cosmological structure, a myth tells us how it was born (Couloubaritis, 2003 67-68).

sage, handed down by Aëtius,¹³ as well as Cicero's comments, confirm this: both understand that the subject of the verses is στεφάναι.¹⁴ From Homeric poetry onward, στεφάναι and related terms appear in an astronomical context: we encounter the verb στεφανόω in *Iliad* xviii 484-485, where it means that the sun, the moon, and all the τείρεα ('signs in the heaven'; 'constellations'; LSJ) are ornaments τά τ' οὐρανὸς ἐστεφάνωται ('which heaven has all round it'; LSJ, entry στεφανόω).¹⁵ Στεφάναι has been translated as 'rings', 'wreaths', 'bands', 'crowns', or 'globe-shaped crowns', and 'spheres', and we agree with the scholars who understand them as similar to Anaximander's 'wheels' silhouetted against the sky.¹⁶

Aëtius reports that these στεφάναι are made of ἀραιός ('rare') and πυκνός ('dense') texture. He also tells us that for Parmenides, the Milky Way is a 'milky color like' κύκλος (A37), whose blend emerges from a particular ratio of ἀραιός-πυκνός texture (A43a), so we should have no difficulty considering it one of the wreaths. Furthermore, Aëtius identifies 'the most central mixed' of them with a δαίμων, and I agree with Coxon 2008, 363 that the 'equation of the goddess with one of the <mixed rings> is intended to paraphrase fr. 12, 3'.

Coxon 2009, 348, Cerri 1999, 248-252, and Frère 1987, 206-207 demonstrate that ἀραιός and πυκνός are Parmenides' terms: the former is present in B8.57,¹⁷

¹³ Scholars agree that the *testimonium* clearly describes the context of fragg. 9 to 13 (Cerri 2011, 86; Ferrari 2010, 85; Mansfeld and Runia 2009, 399). *Contra* Tarán 1965, 238 ff., who considers Aëtius' report 'untrustworthy'.

¹⁴ Simplicius himself (*In Phys.* 39.14-16 and 31.13-17) does not specify the subject of the first lines of frag. 12.

¹⁵ The anonymous referee has pointed out that the Homeric στεφάναι are also features of an artefact produced by a divinity (Achilles' shield made by Hephaistos), and we have already mentioned that the metaphor of the sphere in B8 could be connected to an artefact. The hypothesis that a maker might be implicit in Parmenides' poem (the δαίμων of 12.3 or the creator of Eros, or even the Θεά) is intriguing: the involvement of a divine maker would constitute a significant divergence from the other *physikoi*. I do not reject this possibility *tout court*, but, as already mentioned, I do not consider the presence of a demiurge in the poem to be very probable, and delving further into this question would exceed our limits.

¹⁶ Cerri 2011, 92, Couprie 2011, 165, Mourelatos 2011, 179, and Tarán 1965, 242-243 think that Parmenides' depiction of the starlit sky resulted in a tentative heavenly map composed of concentric spheres, developed by a notion that had already been outlined by Anaximander. About the location of the wreath 'in the μέσος', where the δαίμων introduced in v. 3 would be settled, Cerri 2011, 93 assumes that it is the orbit of planet Venus; Burnet 1920, 138 and Bollack 1990, 33-47 believe it is the Milky Way; and Coxon 2009, 369 suggests that it is the plane of the zodiac, viz., the ecliptic. We have textual evidence after Parmenides that the word μέσος, when used in cosmological contexts, indicates the 'ecliptic' or the 'equator' (sometimes next to the word κύκλος; cf. the LSJ entry, meaning A.III.f.6.). Yet, there are commentators (Pellikaan Engel 1978, 91-93; Guthrie 1979, 62-63; and Ferrari 2010, 103-106) who assume that the $\delta αίμων$ is not to be identified with an astronomical band but with the core of the Parmenidean cosmos, like the Pythagoreans' (scil. Philolaus') Έστία. This view derives mainly from a testimonium of Anatolius of Laodicea (cf. A44) and is rejected by some scholars (Tarán 1965, 247n49; Burnet 1920, 138; Coxon 2009, 39 and 368; Huby 2011, 101n294), and is not catalogued in the Laks and Most edition.

¹⁷ Diels 1922, i 15 expunges ἀραιός from verse 57 because he considers it a gloss of ἐλαφρός, added by a scholiast. This emendation is maintained in DK 217ff., as against the scholars cited above

and the latter in B8.59. This is confirmed also by Plutarch *Adv. Col.* 13, 1114 A-B and Simplicius *In Phys.* 31.3-7. Mourelatos 2011, 180 remarks that these terms 'are the warp and woof that run through [the] inferential fabric' of the poem's second part,¹⁸ and Ruggiu 2014, 308 suggests that Parmenides does not consider them illusory if they are present mythopoeically in the poem's proem as Nú ξ and 'Hµ $\epsilon\rho\alpha$, in a context, thus, in which there is no mention of the error of mortals.

This is an important point: while his contemporaries systematically used pairs of opposites between qualities, we should suppose that Parmenides conceived his opposites in a very different way, because the 'matters' $\dot{\alpha}\rho\alpha_i\delta_{\zeta}$ and $\pi\nu\kappa\nu\delta_{\zeta}$ seem to be different by their texture. In fact, Parmenides tells us that they are not two 'forms' ($\mu\rho\rho\phi\alpha_i$) 'separate from each other' ($\chi\omega\rho\lambda_{\zeta}\dot{\alpha}\pi'\dot{\alpha}\lambda\lambda_{\eta}\lambda\omega\nu$) and 'opposite' ($t\dot{\alpha}\nu\tau(\alpha, B8.53-59)$), but rather two $\delta\nu\nu\dot{\alpha}\mu\epsilon\iota\zeta$ (B9.2), which the 'man who knows' ($\epsilon\dot{\imath}\delta\delta\tau\alpha \phi\bar{\omega}\tau\alpha$, B1.3) understands as the necessary unique 'form' (B8.53-54) that constitutes everything (B9).

Aëtius offers us further interesting information on the στεφάναι: they are περιπεπλεγμέναι ἐπαλλήλοι ('intertwined with one another', Laks and Most trans., 'interwoven one around another', Mansfeld and Runia trans.), and Burnet 1920, 144n38 suggests that this means that they are '<crossing one another> as the Milky Way crosses the Zodiac'. De Santillana 1964, 29 is sure that in Aëtius' summary there is 'enough to presuppose that a pattern of spherical symmetry must be intended, i.e. that the text must make astronomical sense, ...and the "crowns" marking an intricate plaited device of tracks over a wide band of heaven'. Probably, as De Santillana points out, Aëtius 'did not understand what he was writing down' (18).

Let us dwell on the expression $\sigma\tau\epsilon\phi\dot{\alpha}\nu\alpha\zeta$ $\pi\epsilon\rho\mu\pi\epsilon\pi\lambda\epsilon\gamma\mu\dot{\epsilon}\nu\alpha\zeta$ $\dot{\epsilon}\pi\alpha\lambda\lambda\dot{\eta}\lambda\rho\nu\zeta$ for a further clue. Burnet 1920, 144n38 suggests that $\dot{\epsilon}\pi\alpha\lambda\lambda\dot{\eta}\lambda\rho\nu\zeta$ is opposed to $\pi\alpha\rho\dot{\alpha}\lambda\lambda\eta\lambda\rho\zeta$, while Tarán 1965, 233 argues that 'in alternate succession' is 'a normal meaning of the word' $\dot{\epsilon}\pi\alpha\lambda\lambda\dot{\eta}\lambda\rho\nu\zeta$. I cannot accept either interpretation, because even if $\dot{\epsilon}\pi\alpha\lambda\lambda\dot{\eta}\lambda\rho\nu\zeta$ means 'continuous succession' or 'sequence', it does not contain in itself the sense of alternation (see, e.g., the LSJ entry). I think Bollack 1990, 29 hits the target by remarking that $\dot{\epsilon}\pi\alpha\lambda\lambda\dot{\eta}\lambda\rho\nu\zeta$ sharpens 'la succession serrée, sans intervalles' of the $\sigma\tau\epsilon\phi\dot{\alpha}\nu\alpha$.

In my view, Parmenides's description in frag. B12 of a continuous physical three-dimensional cosmos should be understood as consistent with the continuous hyperspherical description of τὸ ἐόν in B8.42-49 (cf. also B8.22-33). In fact, τὸ ἐόν is primarily ξυνεχές,¹⁹ i.e., continuous, and the meaning of the concept 'continuous' in Parmenides, as Coxon 2009, 325 points out, is the opposite of the 'Aristotelian sense of "continuous", which is 'divisible *ad infinitum*' (ἄπειρον γὰρ διαιρετὸν τὸ συνεχές, *Phys.* 185b10).

who, following the *mss*. and the earliest editors (Fülleborn, Brandis, Karsten, Mullach, Stein), consider, conversely, $\dot{\epsilon}\lambda\alpha\phi\rho\dot{\phi}\zeta$ to be a gloss of $\dot{\alpha}\rho\alpha\dot{\phi}\zeta$.

¹⁸ This is why I prefer to follow Coxon's and McKirahan's use of 'texture' in translating ἀραιός and πυκνός (Coxon 2009, 348).

¹⁹ This 'is P.[armenides]'s only word for "one" (Coxon 2009, 325).

Conclusion

We began this investigation with Parmenides' statement that $\tau \delta \dot{\epsilon} \delta v$, although unlimited, must necessarily be non-endless, complete. Yet we cannot conceive of any $\pi\epsilon \tilde{\imath}\rho\alpha\varsigma$ of $\tau\delta \dot{\epsilon}\delta v$ as a boundary or delimitation. Yet, the completeness of $\tau\delta$ $\dot{\epsilon}\delta v$ cannot precede it or come after it, and it cannot be outside or beyond it, or even inside it (cf. B8.25 and 44-49), for otherwise we should have to admit the 'non-being'.

I believe that Parmenides figured out a solution like a hypersphere to describe cosmologically his non-endless but unending $\tau \delta \dot{\epsilon} \delta v$. If this is correct, we should imagine the relationship between the complex framework of limits, fetters, and bonds, drawn by Themis and her daughters, and the metaphor of the sphere in B8.42-49, in these terms: the mythopoeic description, paraphrasing Callahan, is Parmenides' extrinsic explanation—an abstraction, since we use the Euclidean geometry of the sphere to imagine a hypersphere—of the extra dimension (which we suppose is physical, according to Aristotle *Cael*. 298b19-23) of $\tau \delta \dot{\epsilon} \delta v$, which no one is able to see, because it is impossible to 'exit' $\tau \delta \dot{\epsilon} \delta v$ and look back at it. The metaphor of the sphere is instead the intrinsic representation of $\tau \delta \dot{\epsilon} \delta v$.

Let us now draw a comparison between the morphological characteristics of a hyperspherical universe and what Parmenides tells us about to eov:

(1) a hypersphere is a curved unlimited space of finished and homogenous volume;	τὸ ἐόν is 'similar to the volume of a well- rounded sphere' and it is completed but not ended by its limits (B8.42-49)
(2) a hypersphere is immobile, because it is an unbounded volume that wraps around itself: we can imagine a hypersphere as a dynamic succession of concentric spheres, first increasing and then decreasing (Odifreddi 2011);	τὸ ἑόν 'rests in itself' (B8.29), 'motion- less within the limits of its great bonds' (B8.26), which are intrinsic (cf. B8.36- 38, 49), and 'as a whole it is continuous', <i>scil.</i> 'unbounded' (ξυνεχὲς πῶν, B8.25); at the same time the 'wreaths' that in B12 constitute Parmenides' cosmos have been understood as 'intertwined with one another', 'wound around each other' (A37) in a concentric spheres system;
(3) a hypersphere is the set of points at a constant distance from a given point called its center, so we could probably metaphorically say	with Parmenides' words that, in a space like this, everything would be μεσσόθεν ἰσοπαλὲς πάντηι (B8.44);
(4) in a hyperspherical space, no matter what direction one takes and what point one moves from, one will always return to that starting point;	in B5 it is said regarding τὸ ἐόν: 'It is indifferent to me whence I begin, for to that place I shall come back again' (Coxon trans.);

Regarding the last point, it is interesting to note that Proclus cites B5 between the statement that $\grave{c} \diamond \gamma \grave{a} \rho \grave{c} \diamond \tau \tau \pi \epsilon \lambda \acute{a} \zeta \epsilon \tau$ (B8.25), which describes $\tau \grave{o} \grave{c} \diamond \tau a s a$ *continuum* (as is specified in the same verse: $\xi \upsilon \tau \epsilon \chi \acute{c} \tau \pi \delta \tau \dot{c} \tau \tau \nu$), and the other statement we have already seen: $\tau \grave{o} \grave{c} \diamond \tau i s \mu \epsilon \sigma \sigma \delta \theta \epsilon \nu i \sigma \sigma \pi \lambda \acute{c} \varsigma$.²⁰ Thus, paraphrasing Rovelli, if we passed the $\pi \epsilon \tilde{\iota} \rho \alpha \zeta \pi \acute{u} \mu \alpha \tau \circ \nu o f \tau \grave{c} \grave{c} \diamond \nu$ we would not find ourselves elsewhere but still within it.

The center of Parmenides' hyperspherical $\tau \delta \dot{\epsilon} \delta v$ could therefore still be understood in geometric terms, but within a cosmologically intrinsic geometry. Consequently, the center of this cosmos could have been conceived as a constant relation, as a sort of regulatory principle, in respect to which (in the person of the $\delta \alpha (\mu \omega v)$ everything has to be 'equal' to everything else—so that $\dot{\epsilon} \delta v \dot{\epsilon} \delta v \tau t$ $\pi \epsilon \lambda \dot{\alpha} \zeta \epsilon t$ seamlessly (B8.25; cf. B4)—whatever meaning this "equality" (rendered by the expression $\mu \epsilon \sigma \sigma \delta \theta \epsilon v i \sigma \sigma \pi \alpha \lambda \dot{\epsilon} \zeta \pi \dot{\alpha} v \tau \eta t$ in frag. 8 and through the ubiquity of the $\delta \alpha (\mu \omega v \text{ in B12 hypothesized by some scholars) had for Parmenides.²¹$

This becomes an all-alike and unified, entire whole, full of itself, indivisible, unmoving, resting by itself, determinate but without beginning or cessation, and equally balanced in every direction, so that when one reaches its $\pi\epsilon i\rho\alpha\varsigma \pi \omega\mu\alpha\tau ov$ ('outmost limit', B8.42) and keeps going, one will not find oneself somewhere else but will just keep 'turning' along its concentric and intertwined wreaths, so that one returns to one's starting point without ever leaving $\tau \delta \dot{c} \delta v$ (cf. B4.2; B5; B8.3-4, 22-30 and 42-49), because $\mu\eta\delta\epsilon v \delta'$ oùk $\dot{\epsilon}\sigma\tau v$, as the $\theta\epsilon\dot{\alpha}$ states in B6.1-2. This, in my opinion, is what emerges about $\tau \delta \dot{c} \delta v$ from the fragments we have examined, and we could hardly imagine a more fitting description for a hyperspherical universe.

There is considerable evidence for the hypothesis that the dynamic system of concentric and intertwined spheres described in frag. 12 encloses $\tau \delta \epsilon \delta v$ as its cosmological ceaseless 'frame', making it the whole that is described in frag. 8 (analogously to how a complex system of concentric spheres encloses a hypersphere). A 'frame' that is not an external or internal boundary is also not a border at all. This could also answer a question we posed at the beginning of this investigation, namely, how $\tau \delta \epsilon \delta v$ can be unending but non-endless if limits suggest a plurality.

Might Parmenides have referred to this cosmo-ontology in the enigmatic conclusion of the proem, where the goddess says about $\tau \dot{\alpha}$ δοκοῦντα that διὰ παντὸς

²⁰ Proclus cites it *In* Plat. *Parm.*, i 708.16. This is the only source of fragment 5. I do not agree with Coxon 2009, 287 that 'Proclus' account of the context of the fragment and his association of it with the simile of the sphere in fr. 8, 43...[is] unacceptable'.

²¹ It is what Bollack 1990, 39 defines as the hypothesis 'de l'ubiquité'. Cf. Reinhardt 1916, 13; Verdenius 1942, 6; Fränkel 1968, 185; Mansfeld 1964, 164. They consider the δαίμων (who in B12.3 is said to be in the μέσος) to be wherever the ἀραιός and πυκνός principles are in contact, as a sort of medium, because they do not think it possible to determine any fixed location with the information we have. But we do have Simplicius' close paraphrase of frag. 12.3-4 (*in Phys.* 34.15-16), where he describes the δαίμων as the unique common (ἕν κοινόν) cause of 'the whole coming to be' (πᾶσα γένεσις), and this could support the interpretation that Bollack completely rejects: the ubiquity of the δαίμων would represent the ubiquitous order of the cosmos in every single entity.

πάντα περῶντα (B1.31-32)?²² Has the expression διὰ παντὸς anything to do with the concept διά-κοσμος? What does it mean that all things 'pass right through $(\delta i \dot{\alpha})$ ' παντὸς?

These inquiries are worthy of further investigation.

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²² The reader may find a concise but comprehensive survey of the main interpretations of these verses in Maso 2011.

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