

# 1    **Hydraulic design of the Center-Pivot irrigation system for gradually decreasing** 2    **sprinkler spacing**

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## 17 18    **Abstract**

19  
20    Design strategies that enhance modern irrigation practices, reduce energy consumption, improve  
21    water use efficiency and crop yields, are fundamental for sustainability. Although, microirrigation is  
22    currently a widely applied method, Center-Pivot irrigation systems have become very popular on  
23    large farms, thanks to their automation, wide-coverage, and reliability. Different design procedures  
24    have been proposed, even though some aspects have not been solved yet. This paper presents a simple  
25    design procedure for Center-Pivot systems using a gradually decreasing sprinkler spacing along with  
26    the pivot lateral, which makes it possible to set favorable and uniformly distributed water application

27 rates. The sprinkler spacing distribution along the radial direction is derived by considering just one  
28 dimensionless group accounting for the geometric and hydraulic input parameters. According to this  
29 outcome, the results showed that the suggested procedure made it possible to select the sprinkler  
30 characteristics and the pipe diameter, based on the desired input parameters, i.e. the uniform water  
31 application rate and the lateral length. For assigned input parameters, the lateral length is delimited  
32 by a threshold value, indicating that lateral lengths longer than that threshold value require the  
33 modifications of sprinkler flow rate or pipe diameter. Finally, applications based on the proposed  
34 hydraulic design procedure were performed and discussed for two different cases.

35

36 **Key-Words:** Center-pivot, analytical solutions, energy-saving, uniform water application

37

38 **Introduction**

39

40 The development of appropriate technologies and design strategies to enhance modern  
41 irrigation practices, reduce energy consumption, and improve water use efficiency and crop yields, is  
42 imperative for sustainability (Noreldin et al. 2015). In small areas, where the topography is not  
43 necessarily flat, microirrigation is currently considered one of the most efficient and widely applied  
44 methods, since it reduces water losses due to evaporation. Many studies have focused on simple  
45 methods to design drip irrigation systems (Al-Samarmad 2002; Baiamonte et al. 2015; Sadeghi et al.  
46 2015), and to minimize the energy consumption (Baiamonte 2017; 2018a; 2018c). However, many  
47 private companies are investing in the mechanization of irrigation (FAO 2018) to optimize the capital  
48 and operating cost. Especially in the last few decades (Abd El Wahed 2015; Abedinpour 2017), the  
49 use of Center-Pivot irrigation system (*CPIS*) has significantly increased, since it makes farm  
50 management easier, is as efficient as microirrigation but allows much larger coverage, and is less  
51 time-consuming compared to the other irrigation systems (Sui and Fisher 2015).

52 Thus, *CPIS* are replacing currently used irrigation systems in reasonably flat areas thanks to

53 their automation, large coverage, reliability, application uniformity, and ability to operate on  
54 relatively rough topography. In fact, *CPIS* are easily automated, and require much lower labor costs  
55 than mobile sprinkler systems that need to be displaced in different sectors of the farm, according to  
56 irrigation scheduling. Moreover, it is relatively easy to carefully manage soil water levels, thanks to  
57 the semi-automatic operation of Center-Pivots and lateral moves.

58 In the '60s, *CPIS* were equipped with standard high-pressure (greater than 350 kPa) impact  
59 sprinklers. These sprinkler packages had the advantage of good application uniformity when the  
60 system nozzles were appropriately sized and pressure variation along the lateral resulted within  
61 recommended limits. However, in the '70s, the increase of energy costs caused major concern among  
62 producers due to the high-energy requirement of impact sprinklers (Bradley and Kincaid 1997; Islam  
63 et al. 2017). Thus, LEPA (Low Energy Precision Application), and a low-pressure application  
64 package for Center-Pivot systems were also developed in the early 1980s (Schneider 2000).

65 Several studies have been performed on the *CPIS* hydraulics, aiming to increase the uniformity  
66 of the water application rate and to limit the peak of instantaneous precipitation rates, which may  
67 determine soil erosion (Kincaid 2005; Baiamonte et al. 2019). Indeed, center pivots often apply water  
68 faster than the infiltration rate, because the irrigation system must apply enough water as it moves  
69 across the field to meet crop water needs until the next time it irrigates that portion of the field  
70 (Bjorneberg 2013). Reddy and Apolayo (1988) derived a correction factor to be used to estimate the  
71 friction losses along *CPIS*. Scaloppi and Allen (1993) developed analytical equations to characterize  
72 the hydraulics of Center-Pivot laterals with and without the end-gun sprinkler. Faci et al. (2001),  
73 compared the performance of fixed and rotating spray plate sprinklers and stated that the rotating  
74 spray plate sprinklers installed at larger spacing along the pivot's lateral are characterized by higher  
75 uniformity coefficients. As a consequence, smaller local peaks of instantaneous precipitation rates  
76 and the reduction of undesired runoff can be observed, thus limiting erosion processes that occur  
77 when the water application rate exceeds the soil infiltration capacity (Rogers et al. 2017; Baiamonte  
78 2019; 2020a).

Valin et al. (2012) developed a software application, named DEPIVOT, which makes it possible to design the *CPIS*, according to five sub-models. Users can verify initial target design values, and then compare alternative sprinkler packages until appropriate conditions are established.

Towards the aim of optimizing water use efficiency, de Almeida et al. (2017) proposed a new system called localized mobile drip irrigation (IRGMO), in the attempt to combine the practicality and benefits of a Center-Pivot system with the efficiency and water-saving capacity of drip irrigation systems. de Almeida et al. (2017) also emphasized that plugging phenomena may occur more than with sprinklers due to the small-sized drippers. Instead of common *CPIS*, equipped with drippers (IRGMO) or sprinklers, recently Baiamonte and Baiamonte (2019) analyzed the ‘geometry’ of the irrigated areas of a *CPIS* equipped with rotating sprinkler guns. Due to the larger-sized nozzles, *CPIS* equipped with sprinkler guns could provide a solution to the plugging phenomena.

To ensure the uniformity of the water application rate, *CPIS* requires increasing the flow rates along the lateral because the sprinklers farther from the pivot move faster, and therefore their instantaneous application rates must be greater. Thus, the irrigated area under a *CPIS* expands substantially with increasing system length. To irrigate the increased area by maintaining constant application intensity, the manufacturers propose: i) to increase flow rates of equally spaced sprinklers, ii) to gradually decrease the spacing of equal-flow sprinklers along the Center-Pivot lateral (Bradley and Kincaid 1997; Majumdar 2000; Howell 2003), and iii) to use semi-uniform spacing (Allen et al. 2000), which is a combination of the first two methods.

Probably because it is easy to provide from a practical point of view, the most common approach is to have equally spaced sprinklers with increasing flow rates (nozzle sizes) along the Center-Pivot lateral (Howell 2003). However, a variable spacing system is also applied since it makes it possible to arrange the sprinklers at strategic locations on the lateral so that the distribution of water along the lateral is uniform (Chu 1980).

Moreover, the design problem could be complicated by the fact that machines must be designed to match each site and information must be collected to characterize the variability of the soils

105 (Baiamonte et al. 2017; 2019), topography, infiltration rates, microclimates across a field, and  
106 expected crop water use patterns over the season (Evans 2006).

107 In conclusion, although many definitions and design procedures can be found in the technical  
108 literature, a universally accepted design procedure has not been found yet. In fact, the issue of *CPIS*  
109 design is widely debated and there is still a need for simple, yet adaptive designing guidelines for  
110 farmers using these systems, specifically to maximize water use efficiency and minimize the energy  
111 consumption.

112 The objectives of this paper are i) to derive a geometric design procedure for *CPIS* that makes  
113 it possible to set favorable water application rates, according to a gradually decreasing spacing of  
114 equal-flow sprinklers along the Center-Pivot lateral, where parameters can be set according to the  
115 site's needs; ii) to suggest a simplified hydraulic design procedure to determine, for fixed input design  
116 parameters, the maximum length of the lateral, which makes it possible to achieve the desired  
117 uniformity of water application rates in the *CPIS* irrigated area; and iii) to show the reliability of the  
118 proposed procedure with two different cases, by determining the pressure head distribution line  
119 (*PHD*), once the suggested design equations are applied.

120

## 121 **Theory**

122

### 123 *Geometric design of a Center-Pivot*

124

125 Let us assume  $r_0$  (m) as the *CPIS* radius, in which for a given spray distance and spray angle,  $n$   
126 sprinklers,  $j = 0, 1, \dots, n$ , with variable spacing,  $s_1, s_2, \dots, s_n$  (to be determined) are installed (Figure  
127 1). During the lateral rotation, with an angular velocity  $\omega$ , each sprinkler  $j$  irrigates an annulus with a  
128 different width,  $w_0, w_1, \dots, w_n$ . The whole potential irrigated area by the Center-Pivot lateral is denoted  
129 as  $A_0 = \pi r_0^2$ .

130 The annulus area irrigated by each sprinkler  $j$ ,  $A_j$ , is imposed as equal for all of the sprinklers,  
131 and can be expressed by the ratio between the design sprinkler flow rate,  $q_n$  (m<sup>3</sup>/s), and the desired  
132 water application rate  $i$  (m/s):

$$134 \quad A_j = \frac{q_n}{i} \quad (1)$$

135  
136 Once the  $q_n$  value is fixed, Eq. (1) states that for a design water application rate,  $i$ , only a constant  
137 irrigated area  $A_j$ , for each sprinkler installed with variable spacing (Figure 1), will assure a uniform  
138 water application rate.

139 The irrigated annulus area of the first sprinkler ( $j = 0$ ),  $A_1$ , can be expressed according to:

$$141 \quad A_1 = \pi (r_0^2 - r_1^2) \quad (2)$$

142  
143 where the radius  $r_1$  (m) equals to (Figure 1):

$$145 \quad r_1 = r_0 - w_0 \quad (3)$$

146  
147 where  $w_0$  (m) is the annulus width corresponding to the first sprinkler ( $j = 0$ ) at the distal end of the  
148 lateral (Figure 1).

149 Substituting Eq. (3) into Eq. (2) yields:

$$151 \quad A_1 = \pi (2 r_0 w_0 - w_0^2) = \pi w_0 (2 r_0 - w_0) \quad (4)$$

152  
153 According to the aforementioned assumption, Eq. (4) can be put equal to Eq. (1), proving two  $w_0$   
154 solutions of the second-degree equation:

155

$$w_0 = r_0 \mp \sqrt{r_0^2 - \frac{q_n}{i} \pi^{-1}} \quad (5)$$

157

158 Among the two solutions, only that with the minus sign has to be considered, since it refers to  
 159 the  $w_0$  annulus width, corresponding to the inner circle of the Center-Pivot irrigated area (Figure 1),  
 160 whereas that with the plus sign refers to the outer circle ( $r > r_0$ ) and it is not of interest for the purpose  
 161 of this study.

162 By dividing Eq. (5) by the *CPIS* radius,  $r_0$ , the normalized width of the first annulus,  $w_{*0}$ , can  
 163 be derived:

164

$$w_{*0} = \frac{w_0}{r_0} = 1 - \sqrt{1 - \frac{q_n}{i r_0^2} \pi^{-1}} \quad (6)$$

166

167 From Eq. (6), it is interesting to notice that the annuli width depends on one dimensionless group,  $q_n$   
 168 / ( $i r_0^2$ ), consolidating the role of the geometry ( $r_0$ ) and of the hydraulic parameters ( $q_n$ ,  $i$ ). Such  
 169 dimensionless parameter also shows that  $q_n$  represents the flow rate that would be needed to irrigate  
 170 a hypothetical squared area of side  $r_0$ , corresponding to a water application rate,  $i$ .

171 Table 1 reports a possible and fairly wide range of variability of the involved input parameters  
 172 ( $r_0$ ,  $i$ , and  $q_n$ ), i.e. their reasonable minimum and maximum values, and those corresponding to the  
 173 derived parameters ( $A_0$ ,  $A_j$ ,  $q_n / (i r_0^2)$ ,  $w_0$  and  $w_{*0}$ ), showing that the extremes of  $w_0$  could be considered  
 174 meaningless for practical applications.

175 In Figure 2,  $w_{*0}$  calculated by Eq. (6) is graphed in both a semi-log (Figure 2a) and log-log  
 176 plot (Figure 2b) as a function of the dimensionless group,  $q_n / (i r_0^2)$ , illustrating the expected  
 177 monotonically increasing relationship between the two parameters and their already observed  
 178 possible range of variability (Table 1). The fitted power-law, also displayed in the figure, resulted in  
 179 a good approximation of Eq. (6), for  $q_n / (i r_0^2) < 1$ . Figure 2 also shows the pair (black dots)  
 180 corresponding to the applications (run #1 and #2) that will be described later.

181 For different rainfall intensity,  $i$ , Figure 3 plots the annulus width corresponding to the first  
 182 sprinkler,  $w_0$ , versus the sprinkler flow rate,  $q_n$ , for two very different  $r_0$  values ( $r_0 = 100$  m, Figure  
 183 3a, and for  $r_0 = 800$  m, Figure 3b), indicating that too low of a  $q_n$  and too high of a  $i$ , correspond to  
 184 very low  $w_0$  values that are unsuitable for practical applications. Moreover, to be sure that no irrigated  
 185 area occurs between two contiguous annuli, the diameter of the sprinkler irrigated circle,  $d_s$  (Figure  
 186 1b), should be large enough to fully cover the annulus width, thus overlapping the irrigated area and  
 187 improving the water distribution uniformity in the root zone (Al-Kufaishiet al. 2009).

188 Furthermore, according to Eq. (6), the triplet of values ( $q_n$ ,  $r_0$  or  $i$ ) have to be selected  
 189 according to one of the following conditions that needs to be satisfied:

$$191 \quad q_n < \pi i r_0^2 \quad (7a)$$

$$192 \quad r_0 > \sqrt{\frac{q_n}{\pi i}} \quad (7b)$$

$$193 \quad i > \frac{q_n}{\pi r_0^2} \quad (7c)$$

194

195 For example, for the minimum and for the maximum values of  $r_0$  and  $i$  reported in Table 1,  
 196 which were assumed possible in this procedure, the maximum value of  $q_n$ ,  $q_{n,max}$ , derived by Eq. (7a),  
 197 results in the wide range 471.2 -  $2.01 \cdot 10^7$  l/h.

198 The annulus area corresponding to the second sprinkler ( $j = 1$ ),  $A_2$ , can be expressed as:

199

$$200 \quad A_2 = \pi (r_1^2 - r_2^2) \quad (8a)$$

201

202 that can be rewritten by substituting  $r_1$  (Eq. 3) and  $r_2$  (Figure 1):

203

$$204 \quad A_2 = \pi ((r_0 - w_0)^2 - (r_1 - w_1)^2) \quad (8b)$$

205

206 In Eq. (8b), solving the square of the two binomials provides:

207

$$208 \quad A_2 = \pi (r_0^2 + w_0^2 - 2r_0w_0 - (r_1^2 + w_1^2 - 2r_1w_1)) \quad (8c)$$

209

210 The annulus width corresponding to the second sprinkler ( $j = 1$ ),  $w_1$ , needs to provide the same  
211 area of the first annulus,  $A_1$ , with width  $w_0$ , and can be derived by equating the annulus area  
212 corresponding to the first sprinkler,  $A_1$  (Eq. 4), to that corresponding to the second sprinkler,  $A_2$ ,  
213 yielding:

214

$$215 \quad \pi w_0 (2 r_0 - w_0) = \pi (r_0^2 + w_0^2 - 2r_0w_0 - (r_1^2 + w_1^2 - 2r_1w_1)) \quad (9a)$$

216

217 that can be written according to a second-degree equation in the unknown  $w_1$ :

218

$$219 \quad w_1^2 - 2r_1w_1 + (4 r_0w_0 - 2 w_0^2 - r_0^2 + r_1^2) = 0 \quad (9b)$$

220

221 Similar to Eq. (4), two real solutions of Eq. (9b) can be obtained:

222

$$223 \quad w_1 = r_1 \pm \sqrt{r_0^2 - 4 r_0w_0 + 2 w_0^2} \quad (9c)$$

224

225 However, also in this case, only the solution corresponding to the minus sign, i.e. to the inner circle,  
226 is of interest (Figure 1). Eq. (9c) makes it possible to derive the annulus width of the second sprinkler  
227 as well as that of any sprinkler  $j$ , respectively:

228

$$229 \quad w_2 = r_2 - \sqrt{r_1^2 - 4 r_1w_1 + 2 w_1^2} \quad (10)$$

$$230 \quad w_j = r_j - \sqrt{r_{j-1}^2 - 4 r_{j-1}w_{j-1} + 2 w_{j-1}^2} \quad j = 1, 2, \dots n \quad (11a)$$

231

232 As  $w_0$  (Eq. 6), Eq. (11a) is rewritten here, by normalizing  $w_j$  with respect to  $r_0$ :

233

$$234 \quad w_{*j} = \frac{w_j}{r_0} = r_{*j} - \sqrt{r_{*j-1}^2 - 4 r_{*,j-1} w_{*,j-1} + 2 w_{*,j-1}^2} \quad j = 1, 2, \dots, n \quad (11b)$$

235

236 where  $r_{*j}$ ,  $w_{*j}$ ,  $r_{*j-1}$  and  $w_{*,j-1}$  are the normalized radius and annulus width corresponding to the  
 237 sprinkler  $j$  and  $j-1$ , respectively. Eq. (11b) is a recurrence relation that recursively defines a sequence  
 238 of annulus widths, once the initial terms,  $r_{*0}$  ( $= 1$ ), and  $w_{*0}$  are given; Similarly, each further term of  
 239 the  $w_{*j}$  sequence is defined as a function of the preceding terms.

240

241 *Sprinkler spacing and lateral length*

242

243 In order to derive an analytical solution to design a Center-Pivot irrigation system, according  
 244 to the above described geometric procedure, the sprinkler spacing needs to be considered. Similar to  
 245 the annulus width, the sprinkler spacing,  $s_j$ , increases from the outer circle versus the pivot-axis  
 246 (Figure 1). The sprinkler spacing can be easily derived by Eq. (11), if the sprinklers are considered to  
 247 be located in the middle of the annulus width (Figure 1). In the following,  $s_j$  is written in dimensional  
 248 and dimensionless terms, respectively:

249

$$250 \quad s_j = \frac{w_{j-1} + w_j}{2} \quad (12a)$$

$$251 \quad s_{*j} = \frac{s_j}{r_0} = \frac{w_{*,j-1} + w_{*j}}{2} \quad (12b)$$

252

253 Of course, Eq. (12a) and Eq. (12b) can be used once the Eq. (11a) and Eq. (11b) have been applied.  
 254 The corresponding lateral length,  $L$  (m), which due to the aforementioned condition (Eq. 7), could be  
 255 lower than the Center-Pivot radius,  $r_0$ , set in Eq. (5) and Eq. (6), is equal to:

256

$$257 \quad L = \sum_{i=0}^n w_i - \frac{w_0 + w_n}{2} = \sum_{i=1}^n s_i \quad (13a)$$

258

259 that once again can also be expressed by normalizing  $L$  with respect to  $r_0$  and denoting the ratio  $L^*$ :

260

$$261 \quad L_* = \frac{L}{r_0} = \sum_{j=0}^n w_{*j} - \frac{w_{*0} + w_{*n}}{2} = \sum_{j=1}^n s_{*j} \quad (13b)$$

262

263 For  $w_{*0} = 0.01$  and for the same two  $r_0$  values already considered in Figure 3 ( $r_0 = 100$  m and  
 264  $r_0 = 800$  m), Figure 4a illustrates the sprinkler spacing,  $s_j$ , versus the number of sprinklers  $j$ , obtained  
 265 by applying Eq. (11a) and Eq. (12a), whereas Figure 4b plots the corresponding cumulated values,  
 266  $\Sigma s_{j.}$ , where the lateral length,  $L$ , expressed by Eq. (13a), is also plotted ( $L = 88.1$  m, for  $r_0 = 100$  m,  
 267 and  $L = 704.6$ , for  $r_0 = 800$  m). Figure 4b indicates that for the considered design parameters, lateral  
 268 lengths,  $L$ , close to the pivot radius,  $r_0$ , ( $L^* = 88.1$  % for both cases), could be designed for a constant  
 269 sprinkler flow rate and uniform water application rate.

270 Similar to  $w_{*0}$ , the sprinkler spacing and the corresponding lateral length (Eq. 12b and Eq.  
 271 13b), have to satisfy the following second condition, which is derived by imposing the radicand in  
 272 Eq. (11b) as positive:

273

$$274 \quad 2 w_{*,j-1}^2 - 4 r_{*,j-1} w_{*,j-1} + r_{*,j-1}^2 > 0 \quad j = 1, 2, \dots n \quad (14)$$

275

276 that states that the lateral length is limited by a maximum real value,  $L_{*max}$ . By solving Eq. (14) with  
 277 respect to  $w_{*,j-1}$ , the quadratic formula provides:

278

$$279 \quad w_{*,j-1} = r_{*,j-1} \left( 1 - \frac{1}{\sqrt{2}} \right) \quad (15)$$

280

281 where again the solution with the minus sign has to be considered. Eq. (15) indicates the limiting  
 282 condition of the normalized sprinkler spacing (Eq. 12b) and that of the lateral length (Eq. 13b). For  
 283 the sprinkler spacing and for the two considered  $r_0$  values, this limiting condition is illustrated in  
 284 Figure 4a (black dots).

285 For a given diameter of the irrigated circle,  $d_s$  (m), and considering that the  $w_j$  values increase  
 286 from the distal end of the lateral toward the pivot, as previously observed, it needs to prevent that  
 287 contiguous annuli are not fully irrigated, thus another condition should be set. Denoting with  $\alpha$  ( $> 0$ )  
 288 the fraction of the overlapping area, the following condition could be additionally considered (Figure  
 289 1b):

$$291 \quad \frac{d_s - w_j}{2} > \alpha d_s \quad (16a)$$

292  
 293 that can be written in dimensionless terms and solved with respect to  $w_{*j}$ , giving:

$$295 \quad w_{*j} < d_{*s}(1 - 2\alpha) = w_{*th} \quad (16b)$$

296  
 297 where  $d_{*s}$  denotes the irrigated circle diameter normalized with respect  $r_0$  and  $w_{*th}$  the corresponding  
 298 normalized threshold value of the annulus width. For example, by assuming  $\alpha = 0.1$ , and  $d_s = 4$  m for  
 299  $r_0 = 100$  m and  $d_s = 12$  m for  $r_0 = 800$  m, Figure 4a shows that the maximum admitted sprinkler  
 300 spacing (black dots) needs to be shortened (white dots), if an overlapping area of ten percent is  
 301 required. Thus, the number of sprinklers have to be less than the admitted maximum reported in Table  
 302 2 ( $n = 49$ ), and equals to  $n = 45$  for  $r_0 = 100$  m ( $w_{th} = 3.2$  m) and to  $n = 15$  ( $w_{th} = 9.6$  m), for  $r_0 = 800$   
 303 m. As a consequence, the lateral length, which is illustrated in Figure 4b, should be also shortened  
 304 (white dots).

305

307

308         Once the geometry of the Center-Pivot is designed, the hydraulic design needs to be performed  
 309 and the pressure head distribution needs to be determined. Towards this aim, the energy balance  
 310 equation, describing the pressure head variations of the  $j$ -th sprinkler,  $h_j$ , along the lateral that lays on  
 311 a flat field needs to be introduced. Under the assumption to neglect the variations of sprinkler flow  
 312 rate along the lateral (Christiansen 1942; Zayani et al. 2001; Provenzano et al 2005), if sprinkler  
 313 spacing would be considered as a constant,  $S$  (m), according to the well-accepted Hazen-Williams's  
 314 equation (Keller and Bliesner 1990), the following equation could be written (Baiamonte 2018a):

315

$$316 \quad h_j = h_{min} + K S H_j^{(-1.852)} + \alpha_L K_L H_j^{(-2)} \quad (17)$$

317

318 where  $h_j$  (m) is the pressure head at the sprinkler  $j$ , and  $h_{min}$  (m) is the minimum pressure head to be  
 319 imposed at the first sprinkler  $j = 0$ . The second term at the right side accounts for friction losses,  
 320 where  $H_j^{(-1.852)}$  denotes the generalized harmonic number of order -1.852, truncated at  $j$ ,  
 321 corresponding to  $H_j^{(-1.852)} = \sum_{k=1}^j \frac{1}{k^{1.852}} / ; j \in \mathbb{N}$  (Baiamonte 2018a), the constant 1.852 is the flow  
 322 rate exponent of the Hazen-Williams's resistance equation, and  $K$  is the corresponding parameter  
 323 accounting for the friction losses (friction head loss gradient):

324

$$325 \quad K = \frac{10.675}{C^{1.852}} \frac{q_n^{1.852}}{D^{4.871}} \quad (18)$$

326

327 in which  $D$  (m) is the inside lateral diameter and  $C$  is a friction coefficient, which is a function of the  
 328 pipe material's characteristics (Keller and Bliesner 1990).

329         The third term at the right side of Eq. (17) accounts for local losses, being  $H_j^{(-2)}$  the  
 330 generalized harmonic number of order -2, and  $\alpha_L$  is the minor loss coefficient multiplied by the kinetic

331 head,  $K_L$ , associated with  $q_n$ . Thus, for a design sprinkler flow rate,  $q_n$ , flowing in the lateral,  $K_L$  can  
332 be expressed according to:

333

$$334 \quad K_L = \frac{8 q_n^2}{g \pi^2 D^4} \quad (19)$$

335

336 For  $\alpha_L = 0$  the minor loss is zero and for  $\alpha_L = 1$ , the minor loss is equal to the kinetic head.  
337 The minor loss coefficient can also be greater than the unity for some components in the hydraulic  
338 design, however, this occurrence is seldom achieved for sprinkler laterals where  $\alpha_L$  usually is in a  
339 lower range (Juana et al. 2002a; Yildirim 2007).

340 It should be noted that Eq. (17) neglects the pressure head variations due to elevation changes,  
341 which may be especially relevant for long lateral lengths. For this reason, most pivots are specified  
342 with pressure regulators at each sprinkler unless the field is very flat. However, this issue is beyond  
343 the purpose of this study, thus the design procedure derived in this work could be applied under flat  
344 field conditions.

345 Another important issue not accounted in the proposed methodology is related to the use of  
346 *CPIS* for crops with different water requirements, either in different seasons or within a single season.  
347 This occurrence could require modifying the design procedure, considering the possibility to impose  
348 variable emitter flow rates along the distribution *CPIS* pipe. However, in the proposed procedure this  
349 occurrence might not be an issue, if low flow rate variations are required, which can be faced  
350 equipping the emitters with pressure regulators.

351 For the variable sprinkler spacing described by Eq. (12) and Eq. (11), Eq. (17) needs to be  
352 reformulated, by relaxing  $S$  as a constant, since it varies with  $j$ :

353

$$354 \quad h_j = h_{min} + K \sum_{j=1}^j j^{1.852} s_j + \alpha_L K_L \sum_{j=1}^j j^2 \quad (20)$$

355

Now, Eq. (20) can be normalized with respect to  $r_0$ , rather than to the constant spacing,  $S$ , as was assumed for drip laterals in Baiamonte (2018a):

$$h_{*j} = h_{*min} + K \sum_{j=1}^j j^{1.852} s_{*j} + \alpha_L K_{*L} \sum_{j=1}^j j^2 \quad (21)$$

where  $h_{*j} = h_j / r_0$ ,  $h_{*min} = h_{min} / r_0$ ,  $K_{*L} = K_L / r_0$ , and  $s_{*j} = s_j / r_0$ . Eq. (21) shows that for the *CPIS* the energy balance equation can be written in dimensionless terms, and that the pivot radius is the scale factor of the design procedure.

Baiamonte et al. (2015) showed that based on the energy balance equation, where the hypothesis to neglect the variation of sprinkler flow rate is assumed, the design problem of sloped drip laterals can be solved by imposing a fixed pressure head tolerance,  $\delta$ . Later, according to any  $\delta$  value, which depends on the desired emission uniformity coefficient of Keller and Karmeli (1974), Baiamonte (2018a) derived analytical solutions to facilitate the design for one-lateral units as well as for rectangular microirrigation units (Baiamonte 2018b). Baiamonte (2018c) also showed that by imposing  $\delta$ , and accordingly the maximum allowable emitter pressure at the inlet and the minimum at the distal end of the lateral, it is possible to minimize the operational energy cost required.

Following a similar approach as described for drip laterals, for the Center-Pivot pipeline, the fixed pressure head tolerance has to be imposed in Eq. (21). In particular, at the inlet ( $j = n$ ) and at the distal end of the lateral ( $j = 0$ ), the normalized maximum and minimum pressure head needs to be set, respectively:

$$h_{*max} = h_{*min} + K \sum_{j=1}^n j^{1.852} s_{*j} + \alpha_L K_{*L} \sum_{j=1}^n j^2 \quad (22a)$$

where  $h_{*max} = (1 + \delta) h_{*n}$ ,  $h_{*min} = (1 - \delta) h_{*n}$ , and  $h_{*n}$  is the nominal pressure head normalized with respect to  $r_0$ . By solving with respect to  $h_{*n}$ , Eq. (22a) can be written as a function of the pressure head tolerance,  $\delta$ :

$$h_{*n} = \frac{1}{2\delta} \left( K \sum_{j=1}^n j^{1.852} s_{*i} + \alpha_L K_{*L} \sum_{j=1}^n j^2 \right) \quad (22b)$$

In Eq. (22a) and in Eq. (22b), the hypothesis to neglect the variation of sprinkler flow rate agrees with the latter assumption of fixing the pressure head tolerance. In fact, along a lateral where drippers or sprinklers are installed, it is under limited pressure head variations established by  $\delta$ , that the ratio between the sprinkler flow rate variation ( $q_{max} - q_{min}$ ) and its average,  $q_n$ , is small (it should be about than 5%, when  $\delta = 10\%$ ), providing a good approximation of this assumption (Wu 1997; Vallesquino and Luque-Escamilla 2002).

For a fixed pressure head tolerance, the above equations show the mathematical formulation of the principle of conservation of energy for the lateral of a Center-Pivot, so that the pressure head variation is balanced by the sum of friction and minor losses.

For a fixed pressure head tolerance, a sprinkler flow rate, and a lateral inside diameter, the use of Eq. (22b), together with Eq. (11b) and Eq. (12b), makes it possible to determine the inlet pressure head that is required in the hydraulic design,  $h_{max} = (1 + \delta) h_n$ .

For the *CPIS* design problem, in which the inside parameter,  $D$ , is unknown, once the other parameters are imposed, by neglecting local losses ( $\alpha_L = 0$ ), as generally assumed for the pivot laterals, Eq. (22b) can be made explicit with respect to  $K$ ,  $q_n$  or  $D$ , respectively:

$$K = \frac{2\delta h_{*n}}{\sum_{j=1}^n j^{1.852} s_{*j}} \quad (23a)$$

$$q_n = \left( \frac{2\delta h_{*n}}{\sum_{j=1}^n j^{1.852} s_{*j}} \frac{C^{1.852} D^{4.871}}{10.675} \right)^{1/1.852} \quad (23b)$$

$$D = \left( \frac{10.675}{C^{1.852}} \frac{q_n^{1.852}}{2\delta h_{*n}} \sum_{j=1}^n j^{1.852} s_{*j} \right)^{1/4.871} \quad (23c)$$

404

405 Eqs. (23a-c) provide simple solutions to the Center-Pivot lateral design. In particular, Eq.  
 406 (23b) can be used if the inside diameter is fixed and the sprinkler flow rate is unknown, whereas Eq.  
 407 (23c) can be used when the sprinkler flow rate is fixed and the inside diameter of the lateral has to be  
 408 determined.

409 From Eq. (23a), the following important dimensionless group could also be derived:

410

$$\frac{\delta h_{*n}}{K} = \frac{1}{2} \sum_{i=1}^{n_1} j^{1.852} s_{*j} \quad (24)$$

412

413 This parameter was recently considered (Baiaumont 2020b) to link the kinetic energy fraction  
 414 and the equivalent length method for trickle irrigation design when local losses are considered, and it  
 415 was usefully applied to analyze the relative weight of local and friction losses with respect to total  
 416 losses in the lateral design.

417 In this context, Eq. (24) lets the role of all of the hydraulic variables ( $h_n$ ,  $q_n$  and  $D$ ) be  
 418 consolidated and provides a compacted graphical representation for a Center-Pivot design  
 419 relationship. For different values of  $w_{*0}$  (Eq. 6), in Figure 5a, the dimensionless group ( $\delta h_{*n} / K$ ), by  
 420 Eq. (24), is plotted as a function of the number of sprinklers,  $j$ . The figure shows that, for a fixed  
 421 pressure head tolerance,  $\delta$ , and for a fixed sprinkler  $j$ , at increasing  $w_{*0}$ , the higher the pressure head  
 422 is, the lower the discharge or the higher the diameter (i.e.  $K$ ) will be, as is expected in the design of a  
 423 Center-Pivot lateral. The same occurs for fixed  $w_{*0}$  at increasing  $j$ . Figure 5a also illustrates the  
 424 maximum number of sprinklers,  $n_{max}$ , that can be installed along the lateral, according to the  
 425 aforementioned condition expressed by Eq. (15).

426 For selected pipe material's characteristics ( $C = 140$ ), for a fixed pressure head tolerance,  $\delta =$   
 427 0.1, for a nominal sprinkler pressure head,  $h_n = 30$  m, and for a pair of  $r_0$  (100 and 800 m) and  $q_n$

428 (312.6 and 400.1 l/h) values, Figure 5b plots the inside lateral diameter,  $D$ , versus the number of  
429 sprinklers,  $j$ , indicating that larger diameters are needed at increasing  $j$ . Moreover, for a fixed number  
430 of sprinklers, diameter values that are required for a longer lateral ( $r_0 = 800$  m), are much larger than  
431 those for the shorter one ( $r_0 = 100$  m). Of course, very small diameters correspond to very high  
432 annulus widths that are meaningless for practical purposes.

433

## 434 Applications

435

436 In this section, by neglecting local losses (Eq. 24) and by relaxing the condition described by  
437 Eq. (15), i.e. by assuming that the diameter of the circle irrigated by the sprinkler is large enough to  
438 cover the annulus width (Figure 1b), two theoretical applications were made, according to the design  
439 variables reported in Table 2 (run #1 and run #2). Moreover, by using (Eq. 17) the pressure head  
440 distribution line (*PHD*) was determined. Then, for different values of the exponent  $x$  of the sprinkler  
441 pressure-head flow rate relationship ( $q = k_s h^x$ ), including  $x = 0$ , which was assumed in the suggested  
442 design procedure, and  $x = 0.5$ , which is commonly accepted for sprinklers (Li and Kavano 1998), the  
443 *PHD* was checked by applying the step-by-step (*SBS*) method.

444 The *SBS* procedure was considered as a test procedure here, since it has the greatest level of  
445 accuracy, because the basic hydraulic equations (i.e. the motion equation and the continuity equation)  
446 between consecutive sprinklers are repetitively applied. In particular, to investigate the influence on  
447 the retained hypothesis to assume the exponent of the flow rate–pressure head relationship equal to  
448 zero ( $x = 0$ ), the *SBS* was applied for different  $x$  values ( $0 \leq x \leq 0.5$ ). This is because in the *SBS* the  
449 simplified assumption,  $x = 0$ , can be fully relaxed.

450 For the two different triplet of values ( $i, q_n, r_0$ ), (run #1 and run #2, Table 2), corresponding  
451 to the parameter  $q_n/(i r_0^2) = 0.0625$  and  $w_{*0} = 0.01$  (Figure 2), but differing in  $r_0$  and thus in  $w_0$  (Figure  
452 3a, run #1 and Figure 3b, run #2), first, Eq. (11a) and Eq. (11b) were applied to determine the sprinkler  
453 spacing variation along the lateral (Figure 4a). For both cases, in Figure 4a, black dots indicate the

454 maximum number of sprinklers that can be inserted along the laterals ( $n = 49$ ). According to the  $w_j$   
455 variation (Figure 4a), Figure 6 reports the irrigated circles by varying the annulus width, for run #1  
456 (Figure 6a) and for run #2 (Figure 6b). Since for both runs, the same gradually increased sprinkler  
457 spacing was assumed, the irrigated annuli appear equivalent, because the corresponding  $w_{*0}$  values  
458 only differ by the scale factor,  $r_0$ .

459 Of course, longer lateral lengths could also be designed but according to the present design  
460 procedure, they would require the modification of the sprinkler characteristics or of the inside lateral  
461 diameters. However, the latter issue is beyond the purpose of this work.

462 To determine the dimensionless group,  $(\delta h_{*n} / K)$ , satisfying all of the possible combinations  
463 of the parameters  $\delta$ ,  $h_{*n}$  and  $K$ , Eq. (24) was applied. The latter is displayed in Figure 5a, indicating  
464 that for  $w_{*0} = 0.01$ , i.e. by assuming  $j = n_{max} = 49$ ,  $\delta h_{*n} / K$  equals 307.1 (Table 2). In other words, any  
465 combination of the triplet  $(\delta, h_{*n}$  and  $K)$  providing this value, corresponds to a *PHD* line that lays in  
466 between the minimum,  $h_{min} = (1 - \delta) h_n$ , and the maximum,  $h_{max} = (1 + \delta) h_n$ , admitted pressures.  
467 Interestingly, Figure 5a shows that the same design variables (i.e. for the same  $\delta h_{*n} / K$ ) could be  
468 obtained for different pairs  $(j, w_{*0})$ .

469 By choosing the commonly used value of the pressure head tolerance  $\delta = 10 \%$ , by fixing  $C =$   
470 140,  $h_n = 30$  m, and for the already considered  $q_n = 312.6$  l/h (run #1) and  $q_n = 400.1$  l/h (run #2),  $K$   
471 equals  $9.77\text{E-}05$  and  $1.22\text{E-}05$ , respectively. Thus, the corresponding internal diameter of the lateral,  
472  $D$ , calculated by Eq. (23c), equal to 38.90 mm and 79.51 mm, respectively (Table 2). For  $j = n_{max} =$   
473 49, these  $D$  values can also be observed in Figure 5b where, for run #1 and for run #2, Eq. (23c) is  
474 plotted.

475 In order to validate the aforementioned design, for the derived design variables, for run #1  
476 (Figure 7a) and for run #2 (Figure 7b) by using Eq. (20), the *PHD* was determined. The same figures  
477 illustrate the *PHD* lines obtained by applying the *SBS* method for  $x = 0$ , as assumed in the suggested  
478 design procedure, which verifies Eq. (20) perfectly. It can be observed that the sprinkler pressure

479 head lays in the admitted range between  $h_{min} = (1 - \delta) h_n = 27$  m and  $h_{max} = (1 + \delta) h_n = 33$  m. Moreover,  
480 for the exponent  $x$  of the sprinkler pressure head – flow rate relationship varying in the range 0 – 0.5  
481 (Table 3), *PHDs* obtained by the *SBS* method are also plotted. Note that in Figure 7, the  $x$ -axis  
482 reference ( $x = 0$ ) is referred to the sprinkler where the pressure head is the maximum,  $h_{max} = 33$  m,  
483 whereas in Figure 6, the  $x$ -axis and  $y$ -axis reference (0, 0) were referred to the Center-Pivot axis.

484 Figure 7 shows that using the design parameters obtained by the proposed solution in the *SBS*  
485 procedure, when  $x = 0.5$  is considered, provide slight overestimations of the actual pressure heads,  
486 with differences that increase towards the distal end of the lateral. However, this occurrence does not  
487 invalidate the proposed design procedure. In fact, even a pressure head that is slightly higher than the  
488 admitted minimum,  $h_n (1 - \delta)$ , which should be exactly achieved at the distal end of the lateral ( $j =$   
489 0), would favor the plants' wellbeing, because the pressure head overestimation supplies slightly  
490 higher water volumes, and also limits the admitted range of pressure head variability.

491 For both run #1 and run #2, relative errors, *RE*, in the pressure head in the middle and at the  
492 distal end of the lateral, and coefficient of variation of sprinkler pressure head, *CV*, were calculated,  
493 giving the results reported in Table 3. The *REs* indicate that neglecting the variation of the flow rate  
494 with the pressure head in the design procedure (Eq. 23c), provides slight errors in pressure heads (<  
495 3%), which are in a favorable direction, as previously observed and as also shown in in Table 3 by  
496 the decreasing *CV* as  $x$  increases. Thus, the suggested simplified procedure that lets users design a  
497 Center-Pivot irrigation system under a uniform water application rate, could also be recommended  
498 for the commonly accepted  $x = 0.5$  value.

499

## 500 **Conclusions**

501

502 This paper presents a simple procedure to design Center-Pivot irrigation systems for gradually  
503 decreasing sprinkler spacing along the pivot lateral, and a uniformly distributed water application  
504 rate. The methodology is divided into two parts. First, the gradually decreasing sprinkler spacing

distribution associated with a compacted dimensionless group of geometric and hydraulic input parameters is derived by imposing a uniform water application rate. Then, according to this outcome, a simple formula for the hydraulic design of the Center-Pivot irrigation lateral is derived. The results showed that for assigned input parameters, the lateral length is limited by a threshold value, such that for those lateral lengths longer than the threshold, it is compulsory to modify the sprinkler flow rate or the pipe diameter. Thus, the lateral length limitation described in this study strictly derived by the assumption of a constant sprinkler flow rate. It could be simple and inexpensive to modify sprinkler flow rates by varying nozzle sizes, which could be an issue to be developed in a future work, starting from the results obtained here. Finally, for two different cases, applications and validations of the proposed hydraulic design procedure were performed and discussed. It was shown that the main assumption to neglect the sprinkler flow rate variations provide slight relative errors of the sprinkler pressure head ( $RE < 3 \%$ ) with respect to the values calculated by the accurate *SBS* procedure. Moreover, it was shown that this assumption does not risk supplying less than the minimum water volume requirement, since it provides sprinkler pressure heads that are slightly higher than the admitted minimum, thus laying within the admitted range ( $h_{min} < h_j \leq h_{max}$ ).

520

## 521 **Data Availability Section**

522

523 All data, models, and code generated or used during the study appear in the submitted article.

524

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526

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532

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646

647 **Notation**

648

- 649  $A_0$  [ $L^2$ ] = area irrigated by the Center-pivot
- 650  $A_1$  [ $L^2$ ] = area of the first annulus  $w_0$
- 651  $A_2$  [ $L^2$ ] = area of the second annulus  $w_1$
- 652  $A_j$  [ $L^2$ ] = annulus area irrigated by each sprinkler  $j$
- 653  $C$  = friction coefficient
- 654  $d_s$  [ $L$ ] = diameter of the sprinkler irrigated circle
- 655  $d_{*s}$  = diameter of the sprinkler irrigated circle normalized with respect to  $r_0$
- 656  $D$  [ $L$ ] = inside diameter of the lateral
- 657  $h_j$  [ $L$ ] = pressure head at the  $j$ -th sprinkler

|     |                                         |                                                                                         |
|-----|-----------------------------------------|-----------------------------------------------------------------------------------------|
| 658 | $h_{min}$ [L]                           | = minimum allowable pressure head                                                       |
| 659 | $h_{max}$ [L]                           | = maximum allowable pressure head                                                       |
| 660 | $h_n$ [L]                               | = nominal pressure head                                                                 |
| 661 | $h_{*j}$                                | = pressure head at the $j$ -th sprinkler normalized with respect to $r_0$               |
| 662 | $h_{*min}$ [L]                          | = minimum allowable pressure head normalized with respect to $r_0$                      |
| 663 | $h_{*max}$ [L]                          | = maximum allowable pressure head normalized with respect to $r_0$                      |
| 664 | $h_{*n}$                                | = nominal pressure head normalized with respect to $r_0$                                |
| 665 | $i$ [L s <sup>-1</sup> ]                | = water application rate                                                                |
| 666 | $K$                                     | = friction head loss gradient                                                           |
| 667 | $K_L$                                   | = kinetic energy associated with $q_n$                                                  |
| 668 | $K_{*L}$                                | = kinetic energy associated with $q_n$ , normalized with respect to $r_0$               |
| 669 | $L$ [L]                                 | = length of the Center-pivot lateral                                                    |
| 670 | $L^*$                                   | = length of the Center-pivot lateral normalized with respect to $r_0$                   |
| 671 | $n$                                     | = number of sprinklers                                                                  |
| 672 | $n_{max}$                               | = maximum number of sprinklers that can be installed in the lateral                     |
| 673 | $q_n$ [L <sup>3</sup> s <sup>-1</sup> ] | = design flow rate of the sprinkler                                                     |
| 674 | $r_j$ [L]                               | = radius of the inner circle of the annulus $w_j$                                       |
| 675 | $r_0$ [L]                               | = radius of the Center-pivot                                                            |
| 676 | $r_1$ [L]                               | = radius of the inner circle of the first annulus $w_0$                                 |
| 677 | $r_2$ [L]                               | = radius of the inner circle of the second annulus $w_1$                                |
| 678 | $r_{*j}$                                | = radius of the inner circle of the annulus $w_j$ normalized with respect to $r_0$      |
| 679 | $s_1$ [L]                               | = sprinkler spacing between the two first sprinklers ( $j = 0$ and $j = 1$ )            |
| 680 | $s_j$ [L]                               | = sprinkler spacing between the sprinkler $j$ and the sprinkler $j - 1$                 |
| 681 | $s_{*j}$                                | = sprinkler spacing between the sprinkler $j$ and the sprinkler $j - 1$ normalized with |
| 682 |                                         | respect to $r_0$                                                                        |
| 683 | $S$ [L]                                 | = constant sprinkler spacing between                                                    |

|     |                   |                                                                                      |
|-----|-------------------|--------------------------------------------------------------------------------------|
| 684 | $w_{th}$ [L]      | = threshold value of the annulus width                                               |
| 685 | $w_0$ [L]         | = annulus width of the first sprinkler ( $j = 0$ )                                   |
| 686 | $w_1$ [L]         | = annulus width of the second sprinkler ( $j = 1$ )                                  |
| 687 | $w_2$ [L]         | = annulus width of the third sprinkler ( $j = 2$ )                                   |
| 688 | $w_j$ [L]         | = annulus width of the sprinkler $j$                                                 |
| 689 | $w_{th}^*$        | = normalized threshold value of the annulus width                                    |
| 690 | $w_{*0}$          | = annulus width of the first sprinkler ( $j = 0$ ) normalized with respect to $r_0$  |
| 691 | $w_{*1}$          | = annulus width of the second sprinkler ( $j = 1$ ) normalized with respect to $r_0$ |
| 692 | $w_{*2}$          | = annulus width of the third sprinkler ( $j = 2$ ) normalized with respect to $r_0$  |
| 693 | $w_{*j}$          | = annulus width of the sprinkler $j$ normalized with respect to $r_0$                |
| 694 | $x$               | = exponent of the sprinkler flow rate-pressure head relationship                     |
| 695 | $\alpha$          | = fraction of the overlapping area                                                   |
| 696 | $\alpha_L$        | = minor loss coefficient                                                             |
| 697 | $\delta$          | = pressure head tolerance                                                            |
| 698 | $\delta h_{*n}/K$ | = dimensionless group of variables of the Center-Pivot lateral                       |