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PAPER

Measurement of the damping coefficient of an elastic rubber band oscillator by a smartphone

Aurelio Agliolo Gallitto , Onofrio Rosario Battaglia and Claudio Fazio

Dipartimento di Fisica e Chimica–Emilio Segré, University of Palermo, Palermo, Italy

E-mail: onofriorosario.battaglia@unipa.it

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Abstract

The use of smartphones as laboratory tools for school physics experiments has recently received attention for the possibility of carrying out a wide variety of didactic experiments with low-cost equipments. This article presents a study on a damped oscillator consisting of an elastic rubber loop and a mass. The investigation of the oscillations was conducted by using a smartphone. The experimental data was interpreted by a simple model, obtaining information on the viscoelastic properties of the rubber material.

1. Introduction

The use of mobile devices as teaching tools for physics schools experiments gives the possibility of carrying out an ample variety of didactic experiments in an introductory physics laboratory [1–5]. The large variety of sensors embedded in a smartphone, controlled by the right software (app), allows users to explore physical phenomena and perform different measurements of physical quantities [6–9] by designing versatile low-cost set-up, where expensive devices are replaced by the smartphone. In this article, we show the use of the smartphone accelerometer sensor (accelerometer), controlled by the free phyphox app [10–12], to carry out measurements of vertical oscillations of a damped oscillator made of an elastic rubber-band loop and a mass. We use a simple model to understand the behaviour of this system [12–14]. The experimental results allow us to obtain information on the viscoelastic properties of the rubber material [15, 16].

2. Experimental setup and results

Measuring the period of a vertical harmonic oscillator is a common exercise in school physics laboratories. Traditionally, this is done by manually timing it by a stopwatch. However, the manual method is not suitable for investigating a damped harmonic oscillator because of the presence of damping, which prevents accurate measurement of the oscillations in time and amplitude. To overcome these difficulties, we take advantage of the potentiality of smartphones as a laboratory tool to gather empirical data.

We record the periodic variation of the damped oscillator consisting of an elastic rubber band and a mass, by utilising the smartphone's built-in accelerometer which is controlled by the phyphox app. Figure 1 displays a photo of the experimental arrangement. The smartphone is suspended by a rubber band loop with a cross area of 1 mm by 3 mm, allowing it to oscillate vertically. The smartphone, which also serves as an oscillating mass, is securely attached to a homemade expanded-PVC support. The smartphone's weigh including the support, is 265 g; the total mass can be increased by hanging additional weights as shown in figure 1.

The phyphox app utilises the accelerometer embedded in the smartphone, a micro-electro-mechanical system (MEMS), to record the device's acceleration. The accelerometer provides the value of the gravitational acceleration, denoted as g, when the smartphone is at rest or moving at constant speed [17]. Nevertheless, the phyphox app offers a dedicated feature called 'Acceleration without g' that allows users to read a zero acceleration value when the smartphone is not being accelerated. This feature utilises a virtual sensor that subtracts the constant acceleration g, typically by using data from other sensors. In addition, this application

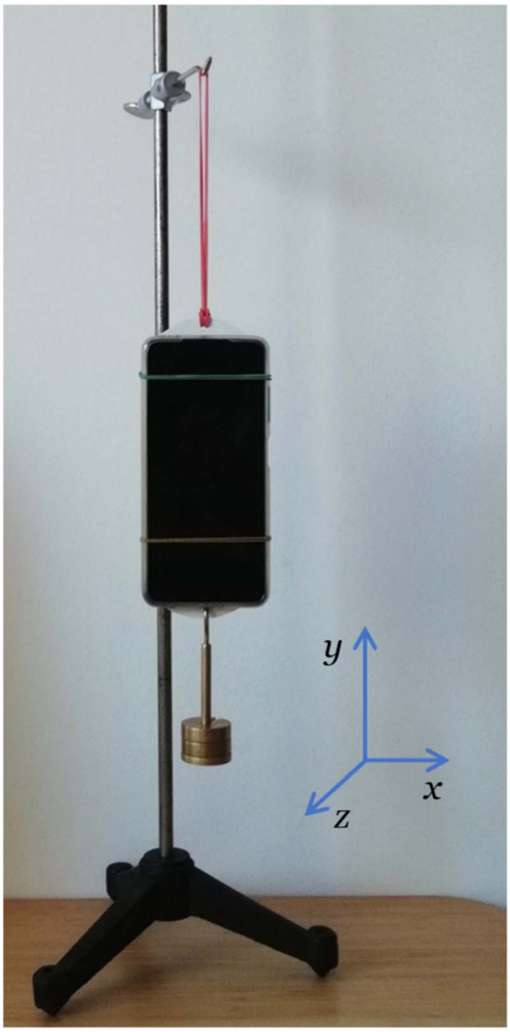

Figure 1. A photo of the experimental setup showing the smartphone hanged to a rubber band loop of cross section of 1 mm times 3 mm on a vertical shaft. The figure also shows the coordinate system.

provides the capability to be remotely monitored from any other network-connected device, as well as to store the resulting raw data in a file for future analysis [10]. Smartphones are equipped with three built-in accelerometers, each aligned with one of the three spatial axes of a standard rectangular coordinate system referred to the device's orientation, as illustrated in figure 1.

When the smartphone is moved a few centimetres far from its resting position and released, it oscillate up and down. At regular intervals of 0.005 s, the three components of the smartphone's acceleration are recorded by the phyphox app. Figure 2 shows a screenshot of the phyphox app during a vertical free motion. The oscillations occur in the vertical displacement along the y-axis. Furthermore, the acceleration values along the x- and z-axis exhibit a negligible deviation from zero.

The measurement uncertainties of the acceleration and time are instrumental uncertainties. Particularly, they depend on the specific sensors integrated into the smartphone and are provided by the phyphox app. In our measurements we used a common Android smartphone and are 0.01 $\mathrm{m/s^2}$ for the acceleration and 0.005 s for the time. These uncertainties may change as the smartphone changes. Figure 3 displays the acceleration values against time for a total mass $m = 440$ g.

Before analyzing the data [18] obtained in the dynamic regime, we give here an estimate of the value of the angular frequency of the undamped oscillator, obtained in the static regime through an estimate of the value of the elastic constant of the rubber band. We obtain a value for the elastic constant of the rubber band loop $k = (43.5 \pm 0.6)$ N/m. With $m = 440$ g, it corresponds an undamped angular frequency $\omega_0 = \sqrt{k/m} = (9.94 \pm 0.07)$ rad/s. The uncertainty associated with the elastic constant is due to the instrumental one, while that of the angular frequency was obtained by error propagation based on the elastic constant value, assuming true the relationship for harmonic oscillators. The angular frequency here reported is compared with the value obtained dynamically.

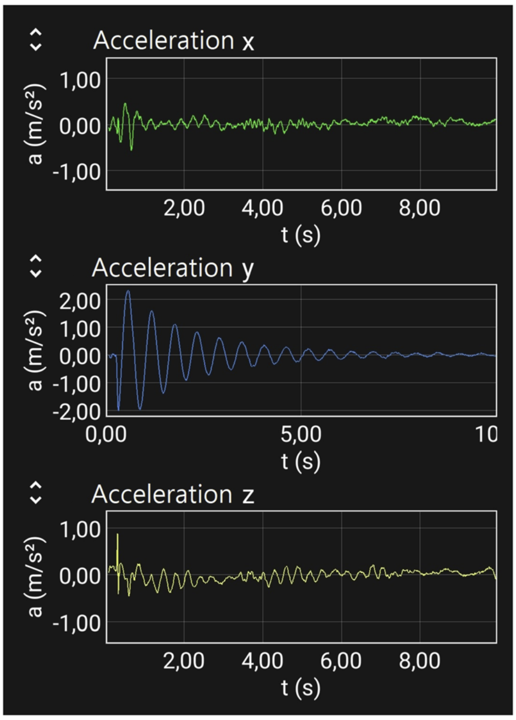

Figure 2. Picture taken from the phyphox app during the damped oscillations, displaying the mass acceleration against time. The *x*- and *z*-axis acceleration values are negligible during the experiment, while oscillations are only observed along the *y*-axis.

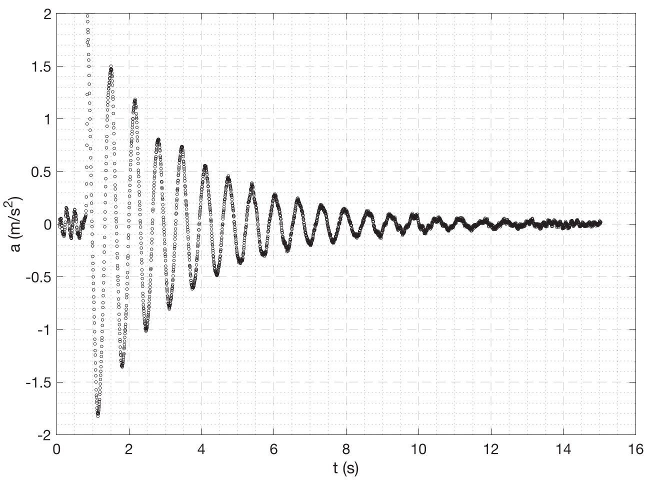

Figure 3. The accelerations along *y*-axis vs time for a total mass $m = 440$ g.

3. Damped oscillations

The free vertical motion of an ideal vertical spring-mass oscillator, of elastic constant k and mass m, follows the Newton's second law and it is described by the following differential equation

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\,y(t) + k\,y(t) - mg = 0 \tag{1}$$

which has a periodic solution as

$$y(t) = A\cos(\omega_0 t + \phi) + \frac{mg}{k} \tag{2}$$

where A is the amplitude of the oscillations, ϕ the initial phase, and ω_0 the angular frequency that depends on k and m as

$$\omega_0 = \sqrt{k/m} \tag{3}$$

The position of the mass varies as a function of time as a sinusoidal function around the equilibrium point at $y_0 = \frac{mg}{k} = \frac{g}{\omega_0^2}$, with a constant amplitude and a constant frequency that does not depend on the oscillation amplitude [19]. From equation (2), one obtains the acceleration as

$$a(t) = \frac{d^2}{dt^2} y(t) = -\omega_0^2 A \cos(\omega_0 t + \phi) \tag{4}$$

where $\omega_0^2 A$ describes the acceleration amplitude.

The oscillatory movement we have previously described takes place in ideal systems that continue to oscillate endlessly due to a linear restoring force that follows Hooke's law. Frictional forces in all actual mechanical systems impede motion, preventing infinite oscillation. Friction gradually diminishes the mechanical energy of the system over time, resulting in damped motion. A damped harmonic oscillator refers to an oscillator where there is a frictional force that is directly proportionate to the velocity.

Rubber-band oscillator experiences significant drag and friction, as well as elasticity and restoring force. Therefore, one may use a simplified mathematical model for the Newton's second law to describe a rubber-band oscillator, which is obtained by connecting in series an ideal spring, of elastic constant k, and a damper, with viscous damping, of coefficient β, that exerts a frictional force proportional to the velocity as reported below.

$$F_v = -\beta v_y = -\beta \frac{d}{dt} y(t) \tag{5}$$

The model can be described by the following differential equation

$$m\frac{d^2}{dt^2} y(t) + \beta\frac{d}{dt} y(t) + k\, y(t) - mg = 0 \tag{6}$$

which can be rewritten into the form

$$\frac{d^2}{dt^2} y(t) + 2\gamma\frac{d}{dt} y(t) + \omega_0^2 y - g = 0 \tag{7}$$

where $\omega_0 = \sqrt{k/m}$ is the angular frequency of the undamped oscillator, $\gamma = \frac{\beta}{2m}$ the damping coefficient and g the gravity acceleration.

When $\gamma < \omega_0$, the solution of equation (7) is

$$y(t) = Ae^{-\gamma t} \cos(\omega t + \phi) + \frac{mg}{k} \tag{8}$$

where $\omega = \sqrt{\omega_0^2 - \gamma^2}$.

Equation (8) describes the so-called damped oscillations, since the amplitude $y(t)$ is no longer constant but it decreases exponentially as a function of time [19, 20]. The general mathematical description of damped harmonic oscillator can be found in [21].

From equation (8), one obtains that also the acceleration varies periodically with time as

$$a(t) = Ae^{-\gamma t}[(\gamma^2 - \omega^2) \cos(\omega t + \phi) + 2\gamma\omega \sin(\omega t + \phi)] \tag{9}$$

4. Data analysis

From the experimental values of the acceleration $a(t)$ reported in figure 3, we can determine the damping coefficient by plotting the acceleration values at crest, a_{max}, against time in a semi-log graph, in which the y-axis uses a logarithmic scale. Figure 4 shows the values of a_{max} against t in a semi-log graph.

By considering γ the parameter that determines the decrease in the peak values and fitting this data through a linear regression method we obtained $\gamma = (0.352 \pm 0.016)\text{s}^{-1}$ with correlation coefficient $R = 0.995$.

The value of the damping coefficient γ critically determines the behavior of the system. When, as in our case, the relationship $\gamma < \omega_0$ is verified, the system is *underdamped*: it oscillates (with a slightly different frequency than the undamped case) with the amplitude gradually decreasing to zero. At a relatively low viscous damping, the oscillation motion is preserved but the amplitude of vibration decreases in time and the motion ultimately ceases.

Let's consider now the model described in the previous section. Figure 5 shows a plot of the acceleration a as a function of time for a mass of 440 g. Symbols are experimental data and the continuous red line is obtained by a non linear fitting using equation (9). The γ and ω values are obtained from the curve that best fits the experimental acceleration data and are $(0.368 \pm 0.003)\text{s}^{-1}$ and $(9.725 \pm 0.003)\text{s}^{-1}$, respectively.

It is worth noting that the γ value here obtained by the non linear fit agrees with the value previously obtained by only assuming an exponential decrease of the amplitude.

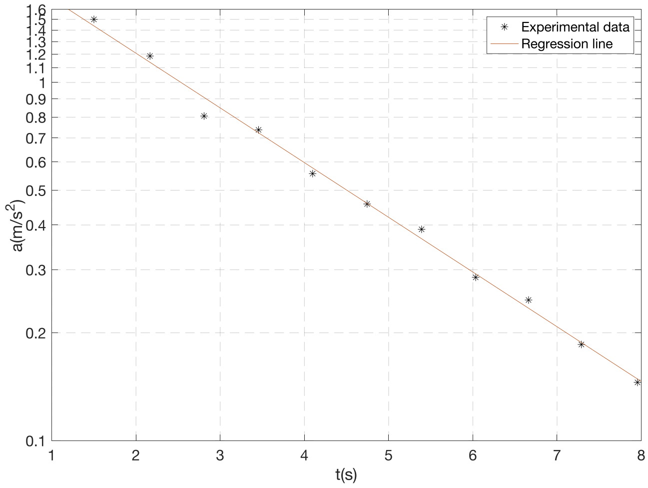

Figure 4. Semi-log graph of the acceleration values, measured at the crest, a_{max} versus time for a mass of 440 g.

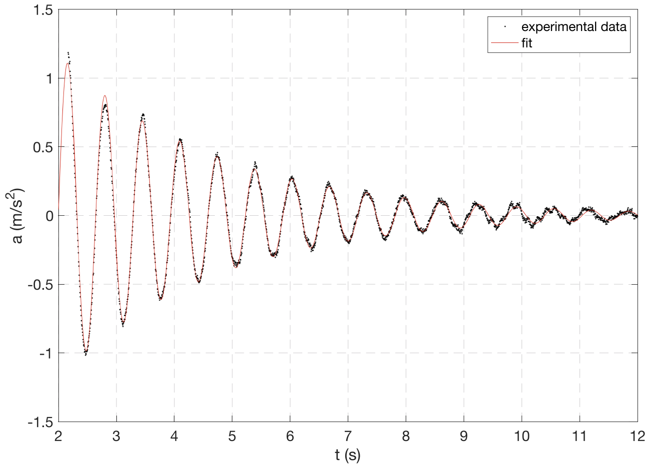

Figure 5. Plot of a versus time. Symbols are experimental data; the continuous red line is a plot of equation (9), with $\gamma = 0.368\ s^{-1}$ and $\omega = 9.725\ s^{-1}$.

5. Discussion

Elastic rubber materials exhibit both viscous and elastic properties when undergoing deformation. This is called *viscoelasticity*.

Unlike purely elastic materials, in a viscoelastic substance the viscous component dissipates energy when a force is applied, then removed. Hysteresis is observed in the force-deformation (stress-strain) curve, with the area of the hysteresis being equal to the energy lost per cycle [22].

The viscoelastic properties of elastic rubber are related to molecular rearrangement. When a force is applied, parts of the long polymer chain composing the elastic rubber change positions. This movement or rearrangement is called *creep*. Polymers remain a solid material even when these parts of their chains are rearranging in order to accompany the internal stress, and as this occurs, it creates a back stress in the material. When the external force is removed, the back stress will cause the polymer to return to its original form [23].

Viscoelastic materials, such as amorphous polymers, semicrystalline polymers, biopolymers and even the living tissue and cells, can be modeled as linear combinations of springs and dashpots (or damper, a mechanical device that resists motion via viscous friction) [16]. In particular, the authors of [15] demonstrated that the dynamical behavior of a mass-rubber band oscillator cannot be described by a simple damped harmonic oscillator model but requires a more elaborate model taking properly into account viscoelastic elements. However, recently, the author of [16], demonstrated that the mechanical properties of rubber materials can be described by static elastic modulus (elastic coefficient), viscoelastic degree and viscoelastic coefficient. Here, we will not discuss about the models developed to account for the viscoelastic properties, since it goes beyond the

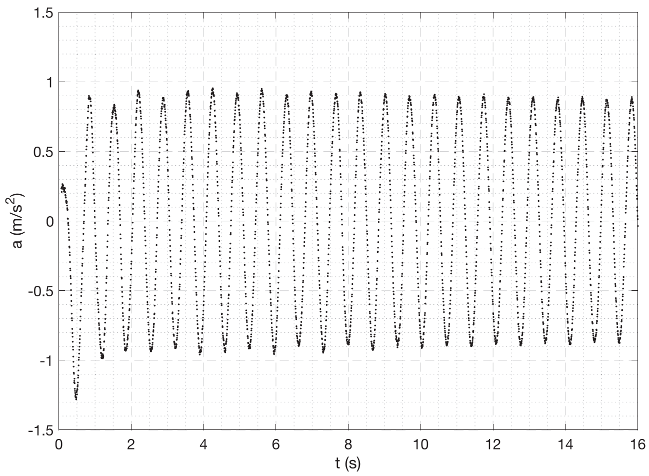

Figure 6. Plot of *a* versus time obtained by substituting the rubber band with a metallic spring.

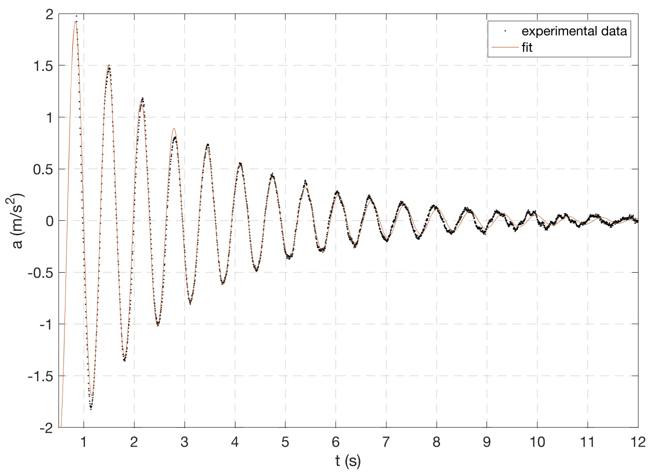

Figure 7. Plot of *a* versus time for a mass of 440 g. Symbols are experimental data; the continuous red line is a plot of equation (9), with $\gamma = 0.39\ s^{-1}$ and $\omega = 9.67\ s^{-1}$.

aim of the present paper. Nevertheless, it is at least important to highlight two aspects emerging from our study. The observed viscosity effects in our measurements can not be attributed to air friction, but rather stems from an inherent characteristic of the elastic material. In order to illustrate this, it is adequate to examine the graph depicted in the figure 6, where the acceleration values over time are acquired by substituting the rubber band in the experimental setup with a metallic spring, while keeping all other components unmodified.

The elastic material utilised in the experiments described here may demonstrate a deviation from the linear force model's predicted behaviour as the amplitude of oscillation and consequently the acceleration amplitude increase. This is shown in figure 7. Specifically, when considering time values about over 6 seconds, it is evident that the fitting curve exhibits a noticeable discrepancy with respect to the experimental data. Moreover, the angular frequency of the underdamped harmonic oscillator calculated as expected by the linear model is $\omega = \sqrt{\omega_0^2 - \gamma^2} = (9.93 \pm 0.07)$ rad/s by substituting to γ the value previously obtained. This value clearly disagrees with the value obtained by the non linear fitting, showing once again that the linear model is not correct for this oscillating system in that regime.

6. Conclusion

A physics experiment was conducted utilising smartphones to gather data with the objective of determining the damping coefficient of an oscillator composed of a rubber loop band and a mass.

Smartphone greatly facilitated our capibility to investigate damping oscillations. Smartphone applications provide an innovative technique of teaching and learning that aims to complement traditional educational methods. This may foster the cultivation of students' enthusiasm for physics [24–28]. Furthrmore, the utilisation of smartphones, as measuring instruments in physics experiments, may facilitate the possibility of constructing versatile and affordable laboratories also for remote learning [28] or lecture experiments, to improve, for instance, student ability to perform error analysis [13].

The examination of the aforementioned phenomenon requires the implementation of experiments, development of theoretical frameworks, and subsequent analysis in relation to the acquired data. The examination of rubber band behaviour in the context of undergraduate studies possesses significant potential in terms of establishing connections between theoretical concepts and practical applications, as well as assessing fundamental models for a widely employed material.

Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: https://sites.unipa.it/griaf/download.

ORCID iDs

Aurelio Agliolo Gallitto https://orcid.org/0000-0001-6617-9056
Onofrio Rosario Battaglia https://orcid.org/0000-0002-0250-0514
Claudio Fazio https://orcid.org/0000-0002-3031-1665

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