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Catastrophic risks and the pricing of catastrophe equity put options

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Abstract
In this paper, after a review of the most common financial strategies and products that insurance companies use to hedge catastrophic risks, we study an option pricing model based on processes with jumps where the catastrophic event is captured by a compound Poisson process with negative jumps. Given the importance that catastrophe equity put options (CatEPuts) have in this context, we introduce a pricing approach that provides not only a theoretical contribution whose applicability remains confined to purely numerical examples and experiments, but which can be implemented starting from real data and applied to the evaluation of real CatEPuts. We propose a calibration framework based on historical log-returns, market capitalization and option implied volatilities. The calibrated parameters are then considered to price CatEPuts written on the stock of the main Italian insurance company over the high volatile period from January to April 2020. We show that the ratio between plain-vanilla put options and CatEPuts strictly depends on the shape of the implied volatility smile and it varies over time.

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1 Introduction

Natural disasters are extreme events with concentrated impact, in space and time, greatly exceeding human expectations in terms of magnitude and frequency and having profound consequences on the socio-economic system (Turner 1976). Because of the extent of this damage, natural disasters are associated with catastrophic risks. Numerous statistics show the greater frequency of natural disasters. The recent increase would seem to be caused mainly by two factors: (1) climate change and (2) an increase in the level of urbanization of the population (Charpentier 2008 and Kunreuther and Kerjan 2013). As a direct consequence, individuals become aware that they are more likely to face catastrophic risks and, for this reason, there is a growing need for financial instruments to cover the damages associated with these natural disasters.

Financial instruments for the management of disaster risk have been widely discussed in the literature (see Linnerooth-Bayer and Hochrainer-Stigler 2015). Freeman et al. (2003) and Cardenas et al. (2007) studied how developing countries can transfer part of their public-sector natural catastrophe risk to the international reinsurance and capital markets. Jongman et al. (2014) presented a model to assess the flood risk and they discussed the feasibility of flood risk management policies in the European Union.

In 2017, the economic damage associated with natural disasters worldwide exceeded 300 billion dollars, of which almost 90% related only to climate events. In the same year, the incidence of insurance coverage, on a global scale, expressed as the ratio between the share of the insured damage and total declared damage, however, rose slightly more than 40%.

In many countries the management of catastrophic risks has always been entrusted exclusively to the State, which acts as an ex-post guarantor of last resort and therefore willing to take responsibility for remedying the damage. In any case, the demand for precautionary measures against natural risks requested from the State by individuals is marked by a moral hazard problem that induces subjects to request public intervention ex-post rather than adopting ex-ante prevention measures. This behavior is what has led to a low diffusion of insurance coverage in the past.

The diffusion of insurance policies covering catastrophic events, in addition to being infrequent, is also characterized by a problem of adverse selection of policyholders. Indeed, only those most exposed to the risk of natural disasters have requested these forms of protection (Bantwal and Kunreuther 2000). This inevitably leads to a significant increase in insurance premiums and, as a consequence, in the costs of insurance coverage. It should, therefore, come as no surprise that the end result has been, in fact, the creation of a vicious circle of excessively high insurance premiums and low demand for individual coverage and thus, in other words, an insurance market failure in this particular sector (Akerlof 1970).

In order to break this vicious circle, several authors (Hudson et al. 2014 and Mysiak and Pérez-Blanco 2016) have also proposed public intervention that will take on a considerable part of the insurance premiums (e.g., through tax relief and subsidized loans) to reduce the individual cost of insurance coverage. In particular, Mysiak and Pérez-Blanco (2016) proposed a classification of the different forms of partnership between the State and insurance companies on the basis of three aspects: (1) robust or poor regulation of the insurance market by the public operator; (2) mandatory nature of insurance coverage; (3) possibility that policyholders cover part of the risk (mutuality) inversely related to the amount of insurance premiums.

At the moment, (a) countries like France, Switzerland, Spain, Chile and New Zealand are characterized by massive state intervention, by compulsory insurance coverage with premiums set by law and strong mutuality, (b) countries like Japan and Turkey see a lower incidence of aspects 2 and 3 and (c) countries like the United Kingdom and the United States are characterized by a weaker presence of the State in the sector, and by the non-mandatory nature of insurance coverage with premiums that are no longer constant, but calculated from time to time according to the level of risk to be managed.

The reasons for this differentiation in the diffusion of insurance risk management tools to protect against natural disasters are varied and complex. Among these, regulation, the presence–absence of insurance systems that provide for the participation of the State, the nature of the catastrophic risks to be insured, and the culture in the field of prevention must certainly be included. The current ex-post method of state intervention in the management of catastrophic risk is in any case increasingly unsustainable as well as economically inconvenient.

In the face of these shortcomings, the need for proper risk management practices in the field of natural disasters is bound to emerge to reduce the degree of exposure of families and businesses to these risks (OECD 2010).

In this context, the aforementioned awareness of individuals in facing catastrophic risks is greater, as is their need to pay something to protect themselves from the resulting harmful repercussions. Hence, the timid, constant public demand for insurance products aimed at covering the risks in question has increased, especially in some countries of the world. The transfer of catastrophic risk to the insurance industry is now a widespread practice in certain countries particularly vulnerable to natural risks such as Japan, New Zealand, California, Turkey, Israel, Mexico, Chile and, among European countries, in France, Spain, Belgium, Greece, Switzerland, Germany, Netherlands, Austria and Poland, although with different methods, timings and technical formulas. It is still a limited phenomenon in Italy, considering the substantial increase in natural disasters that have occurred in recent years.

The greater availability of data and mathematical models capable of more realistically estimating expected damages is strengthening the supply of such products marketed by insurance intermediaries, due to the possibility of being able to identify more appropriate risk mitigation strategies underlying new insurance policies for catastrophic risks offered to meet this growing demand from the public.

Insurance coverage certainly plays an important role from a macroeconomic point of view, as it can lead to a decrease in the negative effects of natural disasters on public spending and GDP growth (OECD 2017). The benefits would derive from the management of technical and financial risks adopted by insurance companies which, as is known, calibrate their exposure to overall risk by insuring themselves, in turn, against those risks - already suitably diversified in advance - that they have taken on by transferring part of the risk, both through the use of more traditional reinsurance practices and of new hedging instruments.

In any case, the economic damage caused by catastrophic events requires such considerable compensation that not even the insurance market can efficiently fulfill. It is, therefore, necessary to transfer the management of catastrophic risks to the only market with dimensions greater than insurance: the financial market.

In response to the evolution of the risk landscape, insurance companies have in fact developed a variety of new tools and techniques that broaden the limits of insurability through so-called *alternative risk transfer* (ART) solutions and through the financial market, within which they proceed with the issue of insurance linked securities (ILS) (Munich Re 2001). All these instruments are accompanied by high flexibility and risk diversification prices, in some cases, also in terms of payment timing.

One of these financial instruments that is receiving a particularly increased attention in recent years is the catastrophe equity put option (CatEPut), i.e. a financial option that gives an insurance company the right to sell a stock of its share capital to private investors at a predetermined price when a catastrophic event occurs. For this reason, the aim of this paper is to introduce a pricing approach that provides not only a theoretical contribution whose applicability remains confined to purely numerical examples and experiments, but which can be implemented starting with real data and applied to the evaluation of real CatEPuts. Specifically, the contribution of the method is twofold.

From a theoretical perspective, we extend the jump-diffusion framework used in the literature of CatEPut, by modeling the underlying stock price dynamics as a more general exponential Lévy process with a diffusive component, an infinite activity jump part, and a finite activity jump term, correlated to the catastrophic loss process. This modeling assumption introduces dependence between the occurrence of major disasters and the stock price.

From a practical perspective, we extend the literature on this subject by developing a calibration procedure based on real data, which make use of (1) time-series of stock log-returns, (2) the capitalization of the insurance company in search of protection from catastrophic losses, and (3) risk-neutral information extracted from quoted European options on the stock. In the empirical analysis we selected Assicurazioni Generali mainly because it is among the main insurance companies in Europe. This insurance company is listed on a major stock exchange and there is an active derivatives market to calibrate the parameters of our model. The study is conducted during the high volatile period from January to April 2020. From the end of February the smile of Assicurazioni Generali became a smirk and, in March, the implied volatility of in-the-money call options was high (i.e. well above 100), indicating that the probability of the occurrence of a catastrophic event was also high. We assess whether our model is able to explain such observed pattern. Although in the empirical study we analyzed only Assicurazioni Generali, similar implied volatility dynamics were observed among other insurance companies across Europe.

The paper is structured as follows. In Sect. 2, we review the most common financial strategies that insurance companies use to hedge the catastrophic risks that they have taken on. In Sect. 3, we describe a catastrophe equity put option pricing model analyzing both the real-world and risk-neutral dynamics of the underlying asset. Section 4 is devoted to empirical analysis and provides more information on the data, the calibration approach and the simulation method implemented, together with the results. Section 5 concludes.

2 Hedging catastrophic risk

This section contains a review of the most common financial strategies that insurance companies use to hedge the catastrophic risks that they have taken on.

The intensification of the frequency of catastrophic events and public awareness of the need to provide ex-ante coverage of the risk deriving from them, also considering the general inadequacy of the response that a State can provide ex-post as a solution to the problems, has led to an ever growing demand from the public for policies to cover catastrophic risks. This has stimulated - and continues to stimulate - insurance companies looking for alternative strategies to traditional reinsurance aimed at covering the risks underlying the policies in question. Among these, insurance securitization, catastrophic bonds (Cat-Bonds) and CatEPuts must be mentioned.

In this section, we provide a description of these less traditional financial products that insurance companies are generally using to hedge catastrophic risk. We will also present a literature review of the main papers that, to the best of our knowledge, have analyzed CatEPuts pricing models and in Sect. 3 we suggest a possible approach to deal with these options.

Insurance securitization is a financial instrument that allows the transfer of the risk taken on by insurance companies to the capital market. By resorting to this instrument, insurance companies include part or all of the risk of catastrophic events within bonds subsequently sold to investors. Therefore, the possible default of these securities, associated with the occurrence of natural events, can no longer damage the other assets and liabilities of the insurance company, i.e. the originating party that constitutes the financial support to guarantee the issue of securities placed on the capitals market, representative of these activities. In particular, the insurance securitization process consists of the following two elements: (1) the transformation of the underwriting cash flows into financial securities exchanged on the market and (2) the transfer of the underwriting risks to the capital markets through the exchange of those securities. In the face of a certain catastrophe risk, a specialized reinsurance company (Special Purpose Vehicle - SPV) is set up. The SPV issues debt securities so that the financial resources obtained are invested by the same company in highly rated securities. Persons who acquire risk protection from the SPV pay a premium that, added to the interest of the securities in which the financial resources are invested, is paid as interest to the holders of securities. If the catastrophic event does not occur, at the end of the period, the SPV reimburses the principal portion of the securities. If it does, the holders of the securities suffer the relative damage and, consequently, risk the partial or integral loss of the principal portion of the bonds.

Typically, there are two main reasons for resorting to the securitization of insurance risks: (1) a greater coverage–absorption capacity — typical of the financial market — of any financial damage caused by natural disasters (as opposed to insurance companies) and (2) the opportunity to make additional investments that allow greater diversification of the portfolio. The latter consideration is linked to the fact that exposure to natural disasters is not necessarly related to the dynamics of financial markets. The insurance securitization technique of catastrophic risk also produces effects at a macroeconomic level that increase its attractiveness. On the one hand, in a market in which the securities are widespread, the securitization and placement of the related bonds redistributes the risk of catastrophe in individual geographical areas or sectors of activity, diluting their negative effects on the entire market. On the other hand, it redistributes the economic and social cost of the catastrophic event over time.

Bouriaux and MacMinn (2009) discussed the developments of insurance securitization and assessed the potential for growth in the insurance-linked securities (ILS) market and in insurance-linked derivatives. In particular, the authors analyzed the motivations of security sponsors and investors to participate in catastrophe linked capital market, and identified the key components of growth and its impediments. They also discussed the technical and regulatory issues that could be crucial to market growth. In this context, they recommended new private and public initiatives aimed at boosting the use and efficiency of catastrophe linked securities and derivatives.

Cat-Bonds are securities that include a clause relating to the risk of a natural disaster. This alternative instrument to the reinsurance contract can take two forms, depending on whether the compensation clause concerns a natural event on which an insurance contract was previously entered into, or the clause concerns an aggregate index that measures the damage possibly caused by a specific source of default risk.

When this form is adopted, we are talking about indexed Cat-Bonds. Like reinsurances, insurance companies transfer part of the risks to the underwriters through Cat-Bonds; indeed, the clause provides that, upon the occurrence of an event or upon reaching a certain value of the reference index, the creditor loses the right to receive, in whole or in part, the lent capital and (or) the agreed interest.

Another possibility that favors the acquisition of this instrument by insurance companies is the delay in payment of the capital and interest due following the occurrence of the unfavorable natural event. The advantage of Cat-Bonds for issuers is that, in the face of the catastrophic event, they undertake to pay higher interest rates than those related to a traditional loan without clause. This is because, in the event of a catastrophic event, a write-down of the security occurs. Therefore, it is expected that a return higher than the market one will be achieved with a certain probability. Thanks to the transfer of part of the risk associated with catastrophic events to the financial market, insurance companies have managed to solve their solvency problems by reducing the cost of supply services to protect against natural risks.

To allow the transfer of a risk taken on through a bond of this type, the insurance company enters into a reinsurance contract with a SPV from which it will purchase a contract that will allow it to partially or totally transfer the risk taken on.

This risk is the same as for catastrophic bonds and the type of hedge reflects the participation of the underwriters in the losses. In detail, this transfer will involve three parties: the insurance company, the SPV and the private investor in the capital market. In other words, the company purchases a financial reinsurance contract from the SPV "written" (or covered by) a specific bond (the Cat-Bond), while the investor will purchase this security at a price set by a special organization (Applied Insurance Research, AIR) composed of professional actuaries, engineers, physicists, meteorologists and financial analysts. If the event does not occur in the geographical area and in the time period indicated in the Cat-Bond contract, the private investor will receive the expected coupons plus the return of the capital (nominal value of the bond corresponding to the maximum loss suffered associated with the damage). Otherwise, the investor will have to give up the invested capital, which will be used to cover the damage caused by the catastrophe.

In the evaluation of Cat-Bonds, the significance and measurability of catastrophic risk are taken into account, in addition to the evaluation parameters used for corporate issues. In recent years, Cat-Bonds have met with favorable reception from the market, due to a strong demand from investors for asset class alternatives unrelated to traditional ones, which allow the risk-return trade-off to be improved in asset allocation decisions.

CatEPuts represent another financial product that insurance companies can use to transfer catastrophic risk to the capital market. Unlike Cat-Bonds, which have a quite recent history, the first issue of CatEPuts dates back to 1996 on behalf of the RLI Corporation. Through CatEPuts, insurance companies acquire the right to sell a stock of their share capital to private investors at a predetermined price in the presence of a catastrophic event. Therefore, a great advantage associated with this financial-insurance instrument is the availability of contingent capital injections allowing the insurance company to see its solvency unaffected as a result of the significant economic damage caused by a catastrophe. Thanks to CatEPuts, the price of shares in the portfolio of insurance companies does not decrease and neither does the price of new issues. This equity fund at a predetermined price represents a buffer that can be used by the insurance company to recover its capital following the catastrophic event during the life of the option.

A disadvantage of CatEPuts is that they generate a sort of fragmentation of the property of the insurance company following the catastrophic event; indeed, the available equity will increase when the put option is exercised with a consequent reduction in the capital owned by existing shareholders.

Given the widespread diffusion of CatEPuts in recent years, several contributions have focused on their pricing. More details are provided for some of the most recent among them, as it is thought this will facilitate understanding of the path that, starting in the next section, will lead to our pricing proposal.

Some authors - including Cox and Schwebach (1992), Cummins and Geman (1995), Chang et al. (1996) - have explored the possibility of structuring a derivatives market in the insurance sector, concluding, however, that insurance futures actually represent an alternative secondary market to the reinsurance market.

In particular, Cox and Schwebach (1992) have argued that a European call on an insurance future is the equivalent of a captive reinsurance or an insurance company owned by a non-insurer (parent company), set up with the specific objective of insuring, exclusively, in whole or in part, the exposure of the parent company and/or its affiliates to the various risks with the stop loss clause. Thanks to insurance coverage, the parent company will be able to protect its capital by limiting its exposure to catastrophic events up to a predetermined and acceptable maximum amount. The stop loss coverage guarantees the captive against losses that may occur in the aggregate, limiting the annual retention to a predetermined amount. Insurance futures therefore represent an alternative secondary market to the reinsurance market.

Chang et al. (1996) used the randomized operational time approach to transform a compound Poisson process into a pure diffusion process (for its higher tractability) and led to the pricing formula of catastrophe call options as a risk-neutral Poisson sum of Black's call prices in information-time. They also assumed that catastrophe futures price changes flow subordinated processes with jumps in calendar-time.

Dassios and Jang (2003) used the Cox process (or a doubly stochastic Poisson process) to model the claim arrival process for catastrophic events and to value stop-loss reinsurance contracts for catastrophic events and catastrophe insurance derivatives. Their main hypothesis is that there is an absence of arbitrage opportunities in the market to obtain the gross premium for stop-loss reinsurance contracts and arbitrage-free prices for insurance derivatives (this condition can be obtained through an equivalent martingale probability measure in the pricing models). When pricing catastrophe linked financial options, it is prudent to develop a model that takes into account both the formation of value and any losses.

Cox et al. (2004) were the first to investigate such a model for pricing catastrophe linked financial options, in particular double trigger put option and a property insurance with a retention which is a function of a commodity price. The double trigger is associated with two main conditions: the underlying equity must be below the strike price and, in addition, a specified catastrophic event must have occurred affecting the insured firm. Jaimungal and Wang (2006) extended the work of Cox et al. (2004) by introducing a framework with stochastic interest rates and losses generated by a compound Poisson process. By modeling the stock price as a geometric jump-diffusion process correlated to the loss process, they obtained explicit formulas for the price of the CatEPut option and for its hedging parameters. A further extension allowing for floating strike prices was proposed by Wang (2020).

Subsequently, Chang and Hung (2009) analyzed the pricing of CatEPuts under the assumptions of both fixed and stochastic interest rates when the price of the underlying asset follows an exponential jump-diffusion process with negative exponentially distributed jumps.

Lin and Wang (2009) used the discounted expected penalty function, formalized for the first time as part of the studies on pricing models in the derivatives market by Gerber and Shiu (1998), for the pricing of American CatEPuts. It is their opinion that the use of this discounted penalty function can lead to a more precise evaluation of a CatEPut.

Chang et al. (2010), using no-arbitrage martingale pricing methodology, dealt with the pricing of Asian catastrophe options with the uncertainties regarding arrival times and related losses within a doubly binomial framework. They performed a stochastic time change from calendar time to claim time and obtained a more efficient estimate of the price of the catastrophe option as a binomial sum of claim time binomial Asian option prices managing to provide a better estimate of the probability of the catastrophic event occurring.

Braun (2011) proposed a two-stage contingent claims approach to price catastrophe swaps, which distinguishes between the main risk drivers ex-ante as well as during the loss re-estimation phase and additionally incorporates counterparty default risk. Catastrophe occurrence is modeled as a Cox process with mean-reverting Ornstein-Uhlenbeck intensity.

Jiang et al. (2013) introduced a catastrophe option pricing model that considers the risk of default of the counterparty that can only manifest itself when the option expires. The prices of the underlying assets are modeled through a jump-diffusion process related to the counterparty loss process and collateral assets. Their conclusion is that counterparty risk significantly affects the option price.

Wang (2016b) proposed a CatEPut assessment model where the counterparty default risk can occur at any time before the expiry date of the option. In particular, the underlying stock price dynamics is affected by catastrophic losses, generated by a Cox process with log-normal intensity, and the assets of the option issuer follow a geometric Brownian motion.

Wang (2016a) suggested a new class of CatEPuts, with payoff depending on the ratio between realized and target variance over the life of the option, where the target variance represents the insurance company expectation of the future realized variance. The author claimed that this kind of options could help insurance companies to raise more equity capital when a large number of catastrophic events occur during the life of the option.

Recently, Bi et al. (2019) proposed a model assuming that catastrophic events and non-catastrophic events both follow Markov modulated Poisson processes and they defined a pricing formula for CatEPuts allowing for correlated jump risk and default risk.

3 Catastrophe put option pricing model

In this section we describe our pricing model. We assume that the underlying stock price dynamics is an exponential Lévy process with a diffusive component, an infinite activity jump part, and a finite activity jump term, correlated to the catastrophic loss process. This construction allows to make the stock price process sensitive to the occurrence of major disasters. We follow the framework originally proposed by Cox et al. (2004), Jaimungal and Wang (2006) and Chang and Hung (2009).

A catastrope equity put option with maturity T has payoff

$$\breve{P}_T = \max\left[K - S_T; 0\right] 1_{L_T > \Upsilon}, \tag{3.1}$$

where S_T is the stock price, L_T is the total loss of the insureds due only to catastrophic events during the life of the option, and K is the strike at which the issuer has to buy the underlying stock if the total loss due to catastrophe is bigger than the level Υ.

3.1 Real-world dynamics

Let $S = (S_t)_{t\geq 0}$ be the stock price process of the insurance company that wants to protect itself from cumulated losses caused by the occurrence of catastrophic events

$$S_t = S_0 \exp(R_t), \tag{3.2}$$

where $R = (R_t)_{t\geq 0}$ is the log-return process under the real-world probability measure P. Define the log-return process of the underlying asset $R = (R_t)_{t\geq 0}$ as

$$\begin{aligned} R_t &= \mu t + \tilde{J}_t \\ &= \mu t + [J_t - t\psi_J(-i)], \end{aligned} \tag{3.3}$$

where $J = (J_t)_{t\geq 0}$ is a P-Lévy process, $\psi_J(-i)$ is its characteristic exponent evaluated at $-i$, and i is the imaginary unit.
Since

$$E\left[\exp(J_t)\right] = \exp\left[t\psi_J(-i)\right],$$

the process $\tilde{J} = (\tilde{J}_t)_{t\geq 0}$ is a P-martingale, the P-expectation of the stock price can be written as

$$\begin{aligned} E[S_t] &= S_0 E\left[\exp(R_t)\right] \\ &= S_0 \exp(\mu t). \end{aligned}$$

Then, we model the process $J = (J_t)_{t\geq 0}$ under the measure P as

$$J_t = \delta W_t + X_t - qL_t, \tag{3.4}$$

where

- $W = (W_t)_{t\geq 0}$ is a standard Brownian motion;
- $X = (X_t)_{t\geq 0} = (B_{G_t})_{t\geq 0}$ is a pure jump Lévy process built by time changing a generalized Brownian motion $B = (B_t)_{t\geq 0}$:

  $$B_t = \theta t + \sigma \tilde{W}_t,$$

  with an independent subordinator $G = (G_t)_{t\geq 0}$ such that $G_t \sim \Gamma(\alpha t, \beta)$, and where $\tilde{W} = (\tilde{W}_t)_{t\geq 0}$ is a standard Brownian motion independent from both $W = (W_t)_{t\geq 0}$ and $G = (G_t)_{t\geq 0}$; thus, the process $X = (X_t)_{t\geq 0}$ can be represented as

  $$X_t = B_{G_t} = \theta G_t + \sigma \tilde{W}_{G_t};$$

- $L = (L_t)_{t \geq 0}$ is a compound Poisson process

$$L_t = \sum_{j=1}^{N_t} Y_j,$$

where $N = (N_t)_{t \geq 0}$ is a Poisson process with jump intensity λ and Y_j-s are independent and identically distributed $\Gamma(\gamma, \eta)$ random variables representing the jumps size;
- $W = (W_t)_{t \geq 0}$, $X = (X_t)_{t \geq 0}$, and $L = (L_t)_{t \geq 0}$ are mutually independent processes;
- q is a conversion factor that represents the percentage drop in the share value price per unit of catastophic loss;
- $\delta, \sigma, \alpha, \beta, \lambda, \gamma$ and η are positive constants, and $\theta \in \mathbb{R}$.

More precisely, the process $X = (X_t)_{t \geq 0}$ is a variance gamma (VG) process (see Schoutens 2003 and Bianchi et al. 2019 and reference therein).

By considering (3.3) and (3.4), the physical log-return process can be written as

$$R_t = m^P t + \delta W_t + X_t - q L_t, \tag{3.5}$$

with characteristic function (see the Appendix)

$$\begin{aligned} \psi_{R_t}(u) = \exp\left\{\left[ium^P - \frac{1}{2}\delta^2 u^2 + \lambda\left(\left(1 + \frac{iuq}{\eta}\right)^{-\gamma} - 1\right)\right] t\right\} \\ \times \left[1 - \frac{1}{\beta}\left(iu\theta - \frac{1}{2}u^2\sigma^2\right)\right]^{-\alpha t}, \end{aligned} \tag{3.6}$$

where

$$m^P = \mu - \frac{1}{2}\delta^2 + \alpha \ln\left[1 - \frac{1}{\beta}\left(\theta + \frac{1}{2}\sigma^2\right)\right] - \lambda\left[\left(1 + \frac{q}{\eta}\right)^{-\gamma} - 1\right].$$

Thus, the stock log-return process $R = (R_t)_{t \geq 0}$ is decomposed into a linear combination of three independent Lévy processes:

- a Lévy process with infinite variation and continuous trajectories, that is, the arithmetic Brownian motion $\tilde{B} = (\tilde{B}_t)_{t \geq 0}$: $\tilde{B}_t = m^P t + \delta W_t$;
- a pure jump Lévy proces with finite variation and infinite activity (i.e. with infinitely jumps in every finite time interval), that is, the VG process $X = (X_t)_{t \geq 0}$;
- a pure jump Lévy process with finite variation and finite activity, that is, the compound Poisson process $L = (L_t)_{t \geq 0}$, which represents the total loss process due to catastrophic events.

We refer to this model having a Brownian, a VG, and a compound Poisson component as *BVG Poisson* model.

Since every linear combination of Lévy processes is still a Lévy process, then the log-return of the stock is a Lévy process. The component $X = (X_t)_{t \geq 0}$ makes the dynamics of the stock price process more consistent with the empirical behavior of market prices compared to jump-diffusion models. In jump diffusion models used in the pricing of CatEPuts, the underlying asset price exhibits continuous trajectories characterized by rare points of discontinuity caused by negative jumps due to the occurrence of catastrophic events. Thus, a jump on the price occurs only in case of a loss due to a catastrophe. As a consequence, in this kind of models the stock log-return distribution shows always a negative asymmetry. Furthermore, the only source of fat tails is the risk of catastrophic losses. However, in the real-world market prices evolve in continuous time through many small jumps and a smaller number of big jumps. The occurrence of big jumps can have several sources not attributable to catastrophic events. Additionally, although rarer than negative, positive huge jumps can occur too. The addition of the component $X = (X_t)_{t \geq 0}$ in the log-return process allows to get a realistic asset price dynamics and a flexible infinitely divisible distribution able to capture different sources of asymmetries and fat tails characterizing the empirical log-return distribution.

3.2 Risk neutral dynamics

Assuming the existence of a riskless asset providing a continuously compounded rate of return, it is possible to show that a geometric Lévy model is arbitrage free and, therefore, that there exists an equivalent martingale measure. However, exponential Lévy option pricing models different from geometric Brownian motion are incomplete. Thus, the equivalent martingale measure is not unique. Since the real-world log-return process contains a Gaussian component, among the possible equivalent martingale measures, we can select the mean-correcting martingale one (see Schoutens 2003), which is simply obtained by changing only the drift parameter m^P to ensure that the discounted underlying price process is a martingale under the risk neutral measure Q, leaving all other parameters and processes not affected by the measure change. More precisely, to get the Q-dynamics of the log-return, m^Q has to be chosen in such a way that

$$
\begin{aligned}
E^Q[S_t] &= S_0 \exp\left(m^Q t\right) E^Q\left[\exp(J_t)\right] \\
&= S_0 \exp\left[\left(m^Q + \psi_J(-i)\right) t\right] \\
&= S_0 \exp\left[(r-d)\, t\right],
\end{aligned}
$$

that is, it is enough to set

$$
m^Q = r - d - \psi_J(-i). \tag{3.7}
$$

Equivalently, to emphasize the correction of the P-drift, it is possible to rewrite (3.7) as

$$m^Q = m^P + r - d - \mu,$$

where r and d represent the continuos risk-free rate and the continuos dividend yield of the stock, respectively. Thus, the risk neutral log-return process $R = (R_t)_{t \geq 0}$ can written as

$$R_t = [r - d - \psi_J(-i)]\, t + J_t,$$

and the Q-characteristic exponent as

$$\psi_R^Q(u) = ium^Q - \frac{1}{2}\delta^2 u^2 + \lambda \left[\left(1 + \frac{iuq}{\eta}\right)^{-\gamma} - 1 \right] \\ -\alpha \ln \left[1 - \frac{1}{\beta} \left(iu\theta - \frac{1}{2}u^2\sigma^2 \right) \right], \tag{3.8}$$

from which it is immediate to get the Q-characteristic function as

$$\phi_{R_t}^Q(u) = \exp\left[t\psi_R^Q(u) \right]. \tag{3.9}$$

Equation (3.9) plays a crucial rule in the pricing of CatEPuts because it will be used to calibrate risk neutral log-return parameters to European market option prices on the underlying stock following the procedure described in Sect. 4.2. Then, these parameters will be necessary to implement the CatEPuts pricing algorithm illustrated in Sect. 4.3.

4 Empirical analysis

4.1 Data

In this subsection we describe the data used in the empirical analysis. We considered the main Italian insurance company, that is Assicurazioni Generali (ticker I:G). We obtained from Thomson Reuters Datastream daily dividend-adjusted closing prices from January 2nd, 2019 to April 15, 2020 and market capitalization from January 2nd, 2020 to April 15, 2020. Furthermore, implied volatilities were extracted from European call and put options written on the selected stock during the high volatility period from January 2nd, 2020 to April 15, 2020 with one month maturity and with moneyness between 80% and 120%. As risk-free interest rate we took the one-month Euribor rate for the calibration and the interest rate swap with maturity one year for the simulation study. Since we considered dividend adjusted closing prices, we assumed that $d = 0$. By an empirical test it follows that under this assumption on dividends the put-call parity continues to be fulfilled. We selected a single option maturity since as observed by Carr et al. (2007) and Guillaume (2012), Lévy processes are suited to replicate option prices for one single maturity, but are generally not able to reproduce

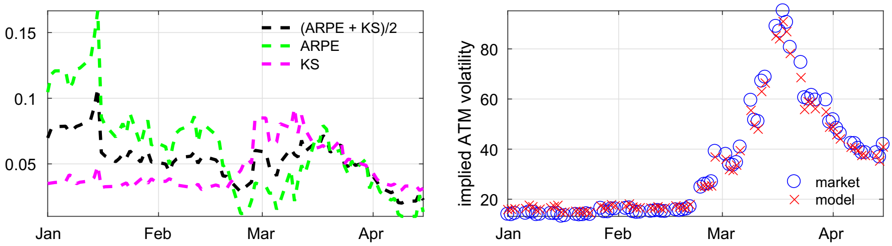

Fig. 1 On the left panel, we report the implied volatility calibration error (ARPE) computed accross all moneyness, the estimated KS distance and the average between the two error measures. On the right panel we report market and model at-the-money implied volatilities. The calibration was conducted for each trading day between January 2, 2020 to April 15, 2020

quoted option prices for the whole set of quoted maturities with sufficient precision, particularly during high volatility periods.

4.2 Calibration

To estimate risk neutral parameters we use both log-returns and European option prices on the stock representing the undelying asset of the CatEPut. Looking at equation (3.8) it is evident that it is impossible to estimate the parameters q and η separately. Since $Y \sim \Gamma(\gamma, \eta)$, then $qY \sim \Gamma\left(\gamma, \frac{\eta}{q}\right)$. Defining $\tilde{\eta} = \eta/q$, the risk neutral log-return characteristic function becomes

$$\phi^Q_{R_t}(u) = \exp\left\{\left[ium^Q - \frac{1}{2}\delta^2 u^2 + \lambda\left(\left(1 + \frac{iu}{\tilde{\eta}}\right)^{-\gamma} - 1\right)\right]t\right\} \times \left[1 - \frac{1}{\beta}\left(iu\theta - \frac{1}{2}u^2\sigma^2\right)\right]^{-\alpha t}.$$

Then, we calibrate the set of risk neutral parameters

$$\widehat{\Theta}^Q = (\delta, \theta, \sigma, \alpha, \beta, \lambda, \tilde{\eta}, \gamma) \tag{4.1}$$

minimizing the distance between model and market implied volatilities (see Chapter 11 in Bianchi et al. 2019) and such that the real-world parameters minimize the Kolmogorov-Smirnov distance of stock log-returns (see also Tassinari and Bianchi 2014 and Bianchi and Tassinari 2020). More precisely, on the catastrophe put option evaluation day, model parameters are calibrated by minimizing the average relative percentage error (ARPE) under Q and the Kolmogorov-Smirnov distance (KS) under P, that is

$$\min_{\Theta^Q}\left(ARPE(\Theta^Q) + KS(\Theta^P)\right), \tag{4.2}$$

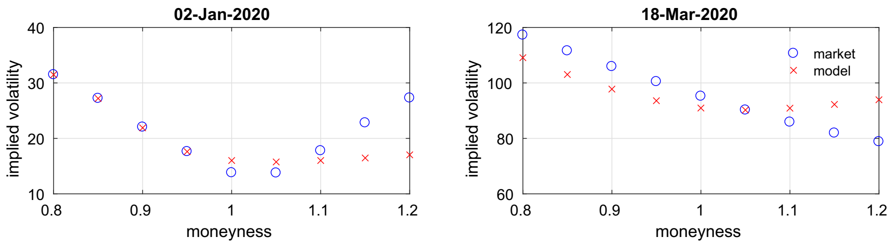

Fig. 2 Calibration errors on January 2, 2020 and March 18, 2020

where

$$ARPE(\Theta^Q) = \frac{1}{\text{number of observations}} \sum_{T_n} \sum_{K_m} \frac{|iVol^{market}_{T_n K_m} - iVol^{model}_{T_n K_m}(\Theta^Q)|}{iVol^{market}_{T_n K_m}}, \tag{4.3}$$

in which $iVol^{market}_{T_n K_m}$ ($iVol^{model}_{T_n K_m}$) denotes the market (model) implied volatility of the option with maturity T_n and strike K_m, and Θ^Q is the vector of the risk neutral parameters. Furthermore, $KS(\Theta^P)$ in (4.2) indicates the KS distance given the set of parameters Θ^P derived from the risk-neutral parameters Θ^Q by means of the mean-correcting martingale measure. This calibration approach is more robust since at each step considers also observed log-returns. The calibration conducted using only option implied data may be problematic. Here the numerical errors are controlled by construction. The algorithm is implemented by following the pricing method for standard vanilla options proposed in Carr and Madan (1999) and the fast Fourier transform needed to find the KS distance (see also Chapter 11 in Bianchi et al. 2019).

In the left panel of Fig. 1 we report the daily timeseries of the ARPE, the KS distance and their average value of the calibration conducted between January 2, 2020 to April 15, 2020. In the right panel of Fig. 1 the daily timeseries of the market and model at-the-money implied volatility over the same observation period is represented. The calibration error in fitting the one-month volatility smile is on average around 6% and the KS test rejects the null hypothesis only in a few cases (the p-value is on average almost 0.7). In Fig. 2 we show the calibration errors at two different trading dates. While the first date is selected during a calm period (January 2, 2020), the second is selected during a stress period (March 18, 2020). The shapes of the implied volatilities smile are different at the two calibration date: this is also reflected on the values of the estimated risk-neutral parameters as shown in Fig. 3 as well as in the simulated path shown in Sect. 4.3.

In Fig. 3 we show the timeseries of the estimated parameters. Note that the parameter λ represents the annual expected number of catastrophic events. To reduce the number of parameters in the optimization problem, at each calibration date we assumed δ equal to the monthly standard deviation estimated on the timeseries of last 250 observed daily log-returns. The Gaussian part of the the process is kept fixed during the calibration phase. Additionally, to mitigate the risk that big negative jumps due to the occurrence

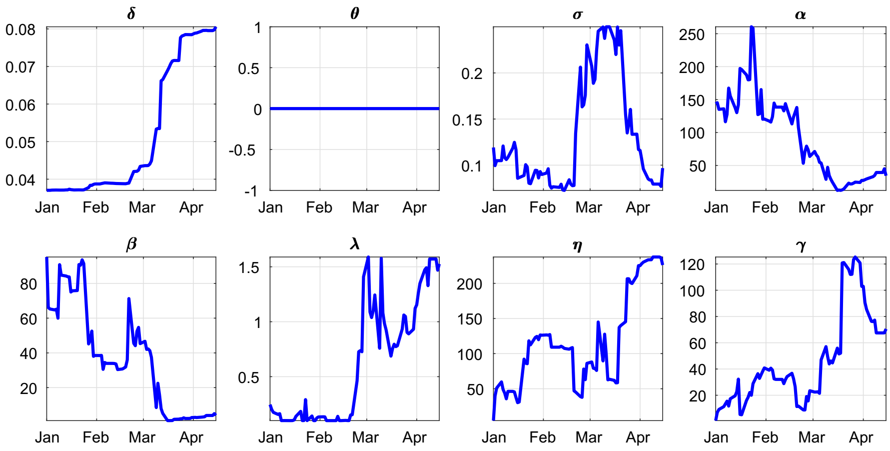

Fig. 3 Risk-neutral estimated parameters. The calibration was conducted on a daily basis for each trading day between January 2, 2020 to April 15, 2020

of cathastrophes are captured by both the VG and the compound Poisson component, we fix θ equal to zero, that is we consider a symmetric VG component and, as shown in Sect. 3.1, the skewness of the price process is driven only by the compound Poisson part. By following the results in Sect. 3.1, we set

$$m^P = \omega + \lambda \frac{\gamma}{\eta},$$

where ω is the annualized empirical mean computed over the 250 last daily log-returns.

While the knowledge q is irrelevant and only the value of $\tilde{\eta}$ matters to evaluate European option, to price a catastrophic put option q is fundamental because it allows to infer η from $\tilde{\eta}$. Following the approach proposed in Jaimungal and Wang (2006) and applied in the related literature (see Chang and Hung 2009 and Burnecki et al. 2019), the trigger level of losses is assumed to be a multiple ν of the expected loss size conditional on the occurrence of a catastrophic event, that is

$$\Upsilon = \nu E[L] = \nu \frac{\gamma}{\eta}. \tag{4.4}$$

The parameter ν is the trigger ratio level, which represents the ratio of the trigger level to the expected loss amount conditional on the occurrence of a catastrophic event.

In the application, we set $\nu = 1$ and Υ as a portion p of the market value of the company capital seeking protection on the day of issue of the catastrophe options, that is

$$\Upsilon = pC_s, \quad s \leq 0, \tag{4.5}$$

where C_s represents the market capitalization of the company at time s. In the application without loss of generality we set $s = 0$. By considering equations (4.4) and (4.5) we can write

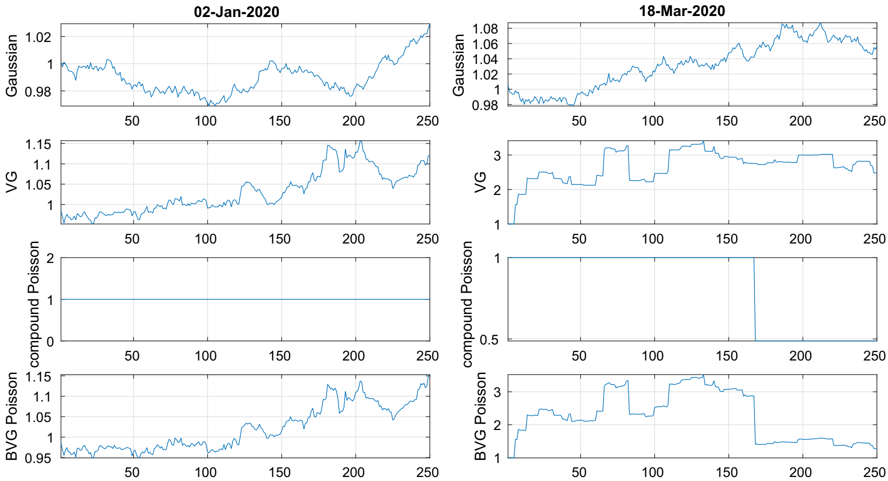

Fig. 4 Simulated trajectories of the Gaussian, VG and compound Poisson component and the overall price process over a five-year horizon (1250 trading days) based on the risk-neutral parameters estimated on January 2, 2020 and March 18, 2020

$$\eta = \frac{\gamma}{pC_s}, \tag{4.6}$$

and, since

$$qE[L] = \frac{\gamma}{\tilde{\eta}},$$

we obtain

$$q = \frac{\gamma}{\tilde{\eta} pC_s}. \tag{4.7}$$

In the empirical study in Sect. 4.3, we consider p equal to 0.25 and, on the basis of the estimated parameters on the market capitalization C_s of the company at the valuation date s, we compute the value of q. It should be noted that the choice of the parameter p influences the definition of catastrophic events.

4.3 Simulation

To price a CatEPut it is necessay to determine its discounted expected risk neutral payoff

$$\breve{P}_0 = \exp(-rT)E^Q\left[\max\left[K - S_T; 0\right] 1_{L_T > \Upsilon}\right].$$

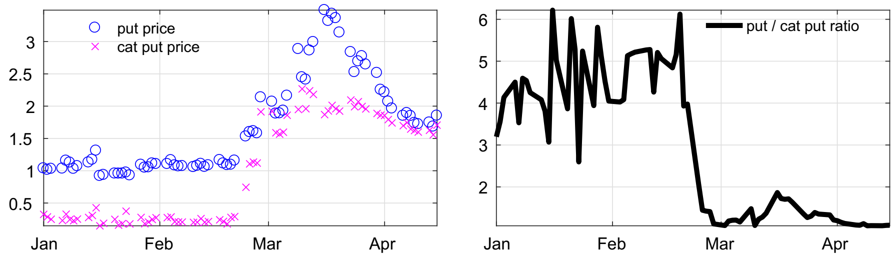

Fig. 5 Monte Carlo prices of at-the-money put and cat put options with one-year maturity between January 2, 2020 to April 15, 2020

To reach this task, we compute the option price by means of Monte Carlo simulation

$$\breve{P}_0 \simeq \exp(-rT)\left\{\frac{1}{MC}\sum_{k=1}^{MC}\max\left[K-\hat{S}_T^{(k)};0\right]1_{\hat{L}_T^{(k)}>\Upsilon}\right\}, \tag{4.8}$$

where MC is the numbers of scenarios (50,000 in our empirical exercise), $\hat{S}_T^{(k)}$ and $\hat{L}_T^{(k)}$ are the simulated values of the underlying stock and of the total catastrophic loss, respectively, at the option maturity, in the k-th scenario, under the probability measure Q.

In particular, to implement equation (4.8), repeat the following steps for $k = 1, 2, ..., MC$:

- sample a random number $z^{(k)}$ out of a standard normal random variable Z;
- sample a random number $g^{(k)}$ out of an independent gamma random variable $G_T \sim \Gamma(\alpha T, \beta)$;
- sample a random number $\tilde{z}^{(k)}$ out of a standard normal random variable $\tilde{Z}$, independent from both Z and G_T;
- sample a random number $n^{(k)}$ out of an independent Poisson random variable $N_T \sim Poiss(\lambda T)$;
- sample $n^{(k)}$ independent random numbers $v_j^{(k)}$ out of an independent gamma random variable $V \sim \Gamma(\gamma, \eta)$;
- compute

$$\hat{R}_T^{(k)} = m^Q T + \delta z^{(k)}\sqrt{T} + \theta g^{(k)} + \sigma\sqrt{g^{(k)}}\tilde{z}^{(k)} - q\sum_{j=0}^{n^{(k)}} v_j^{(k)},$$

$$\hat{S}_T^{(k)} = \exp\left[\hat{R}_T^{(k)}\right],$$

$$\hat{L}_T^{(k)} = \sum_{j=0}^{n^{(k)}} v_j^{(k)},$$

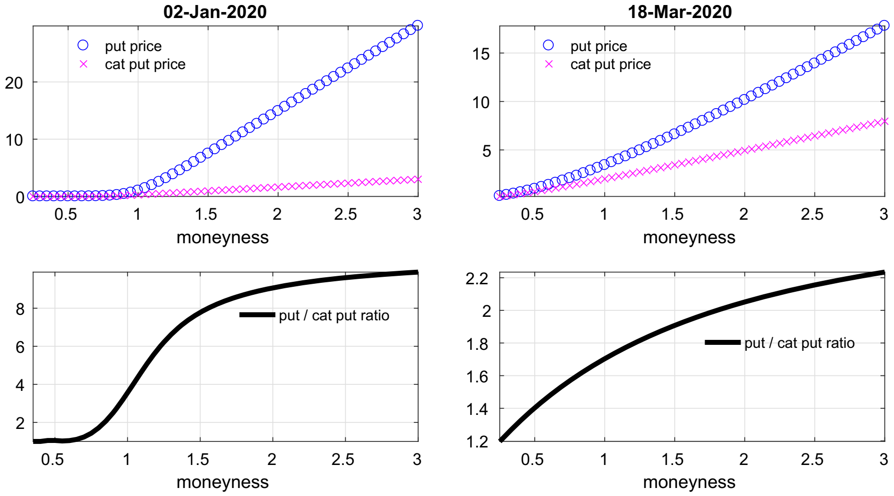

Fig. 6 Monte Carlo prices of put and cat put options with one-year maturity with moneyness between 0.25 to 3 on January 2, 2020 and March 18, 2020

and

$$\max\left[K - \hat{S}_T^{(k)}; 0\right] 1_{\hat{L}_T^{(k)} > \Upsilon}.$$

Then, compute $\breve{P}_0$ as the average value of the simulated option payoffs and discount it using the risk-free rate.

In Fig. 4 we report possible trajectories of the stock price process at two dates. While the first date is selected during a calm period (January 2, 2020), the second is selected during a stress period (March 18, 2020). It is evident the difference in terms of jumps of the compound Poisson component.

On the basis of the risk-neutral parameters estimated in Sect. 4.2, on each trading day between January 2, 2020 to April 15, 2020 we evaluate the price of put and CatEPut options with maturity one year and moneyness between 0.25 and 3. In Fig. 5 we report for both types of option the timeseries between January 2, 2020 to April 15, 2020 of the at-the-money prices. In Fig. 6 we show the behavior on March 18, 2020 of the prices of these two options for different moneyness levels. Additionally, we report the ratio of their prices in order to show the differences among them.

It is interesting to note that the price of the CatEPut increases as the volatility increases. This is a consequence of the trigger event probability that is bigger when the volatility is higher. The ratio between at-the-money put and at-the-money CatEPut sharply decreases from an average value around 5 in the first two months of 2020 to an average value below 2 starting from the end of February, a period from which the smile of Assicurazioni Generali became a smirk and the implied volatility of in-the-money call options was above 100 in March 2020. This means that the option market started quoting a stressed future stock behavior for both the insurance company and the Italian equity market as a whole.

In the third week of February, outbreaks of COVID-19 occured in Veneto and Lombardy regions. In the same period it happened a drastic increase in the intensity of the catastrophic risk (see the behavior of λ in Fig. 3) which produced a fairly sudden reduction in the gap between the prices of the put options and the corresponding CatEPuts (see Fig. 5). A moderate difference between the prices of put options and CatEPuts emerged around mid-March, induced by a reduction in the intensity of the catastrophic risk, which remained high if compared with the average over the first month and half of the year. Probably, this effect was caused by the economic support packages announced by the Italian Government in that period. However, on March 20, the Prime Ministerial Decree was signed, containing new rules for the containment of the contagion throughout the national territory, which provides for the closure of non-essential (non-strategic) production activities. On March 25, further economic activities–not included in the first Prime Ministerial Decree of March 20–were suspended. Simultaneously, a progressive increase in the λ parameter was registered and the price of the CatEPut converged to that of the corresponding put. As already observed in Sect. 4, the choice of the parameter p affects the CatEPuts price behavior, because it directly influences the definition of catastrophic events. Even if in the empirical study we analyzed only Assicurazioni Generali, by considering their implied volatility dynamics, similar results may hold for other insurance companies across Europe.

5 Conclusions

The purpose of this work is twofold. First we provide a detailed description of strategies and products to manage catastrophic risks and review the literature on this topic. Second, we propose a CatEPut pricing model that considers information coming from both the stock and the option market.

From a theoretical perspective, the model extends the jump-diffusion framework used in the literature of CatEPuts, by modeling the underlying stock price dynamics as a more general Lévy process with a diffusive component, an infinite activity jump part, and a finite activity jump term, correlated to the catastrophic loss process. The model produces more realistic stock price patterns compared to jump-diffusion models. The log-return distribution is infinitely divisible and allows for asymmetries and heavy tails. The sources of non-normality are a process with infinitely many jumps (i.e. the variance gamma component) and a process with a finite number of large jumps (i.e. the compound Poisson component) in every finite time interval.

From a practical perspective, we extend the literature on this subject by developing a calibration procedure based on real data, which makes use of (1) time-series of stock log-returns, (2) the capitalization of the insurance company in search of protection from catastrophic losses, and (3) risk-neutral information extracted from quoted European options on the stock.

We conduct an empirical analysis on one of the major European insurance company (i.e. Assicurazioni Generali) from January to April 2020, a period in which the implied volatilities of in-the-money call options were high (i.e. well above 100), indicating that the probability of the occurrence of a catastrophic event was also high. The proposed model is flexible enough to be able to explain observed stock log-returns and one-month option implied volatilities in both calm and stressed periods. Finally, we show that the ratio between plain-vanilla put and CatEPuts strictly depends on the shape of the implied volatility smile and it varies over time.

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Appendix

In this Appendix, we show how to get the characteristic function (3.6) and we provide the main cumulants of the log-return process.

Characteristic function

The stock log-return has been defined as

$$R_t = m^P t + \delta W_t + X_t - qL_t.$$

Since $R = (R_t)_{t\geq 0}$ is a linear combination of three independent processes, its characteristic function can be computed as

$$\phi_{R_t}(u) = \exp\left(ium^P t\right) \phi_{\delta W_t}(u)\phi_{X_t}(u)\phi_{-qL_t}(u),$$

and, therefore as

$$\phi_{R_t}(u) = \exp\left(ium^P t\right) \phi_{W_t}(\delta u)\phi_{X_t}(u)\phi_{L_t}(-qu). \tag{5.1}$$

Since $W = (W_t)_{t\geq 0}$ is a standard Brownian motion then

$$\phi_{W_t}(\delta u) = \exp\left(-\frac{1}{2}u^2\delta^2 t\right). \tag{5.2}$$

The process $X = (X_t)_{t\geq 0}$ is a VG process and since it has been defined by changing the physical time with a gamma stochastic time (i.e. with a gamma subordinator), its P-characteristic function can be derived as

$$\phi_{X_t}(u) = \exp\left[t\psi_X(u)\right] = \exp\left[tl_G(\psi_B(u))\right], \quad u \in \mathbb{R},$$

where $l_G(\psi_B(u))$ denotes the composition of the Laplace exponent of the subordinator $G = (G_t)_{t\geq 0}$ with the characteristic exponent of the generalized Brownian motion

$B = (B_t)_{t\geq 0}$. Since

$$l_G(s) = -\alpha \ln\left(1 - \frac{s}{\beta}\right), \quad s < \beta,$$

and

$$\psi_B(u) = iu\theta - \frac{1}{2}u^2\sigma^2,$$

the characteristic function of the process $X = (X_t)_{t\geq 0}$ is given by

$$\phi_{X_t}(u) = \left[1 - \frac{1}{\beta}\left(iu\theta - \frac{1}{2}u^2\sigma^2\right)\right]^{-\alpha t}. \tag{5.3}$$

Taking into account that

$$\phi_{L_t}(u) = \sum_{j=0}^{\infty} \frac{(\lambda t)^j \exp(-\lambda t)}{j!} \left[\phi_Y(u)\right]^j,$$

the characteristic function of a compoud Poisson process $L = (L_t)_{t\geq 0}$ can be computed as

$$\phi_{L_t}(u) = \exp\left[\lambda t(\phi_Y(u) - 1)\right].$$

Since

$$\phi_Y(u) = \left(1 - \frac{iu}{\eta}\right)^{-\gamma},$$

the P-characteristic function of the total loss process $L = (L_t)_{t\geq 0}$ is

$$\phi_{L_t}(u) = \exp\left\{\lambda\left[\left(1 - \frac{iu}{\eta}\right)^{-\gamma} - 1\right]t\right\},$$

and, thus we obtain

$$\phi_{L_t}(-qu) = \exp\left\{\lambda\left[\left(1 + \frac{iuq}{\eta}\right)^{-\gamma} - 1\right]t\right\}. \tag{5.4}$$

Due to independence of the processes $W = (W_t)_{t\geq 0}$, $X = (X_t)_{t\geq 0}$, and $L = (L_t)_{t\geq 0}$, the characteristic exponent of $J = (J_t)_{t\geq 0}$ can be written as

$$\psi_J(u) = \psi_{\delta W}(u) + \psi_X(u) + \psi_{-qL}(u),$$

and therefore computed as

$$\psi_J(u) = \psi_W(\delta u) + \psi_X(u) + \psi_L(-qu). \tag{5.5}$$

By considering (3.4) and (5.5), we get

$$m^P = \mu - \psi_J(-i) = \mu - \psi_W(-i\delta) - \psi_X(-i) - \psi_L(iq), \tag{5.6}$$

with

$$\psi_W(-i\delta) = \frac{1}{2}\delta^2,$$
$$\psi_X(-i) = -\alpha \ln\left[1 - \frac{1}{\beta}\left(\theta + \frac{1}{2}\sigma^2\right)\right],$$

and

$$\psi_L(iq) = \lambda\left[\left(1 + \frac{q}{\eta}\right)^{-\gamma} - 1\right].$$

Substituting equations (5.2), (5.3), (5.4), and (5.6) into (5.1) we get the characteristic function (3.6).

Cumulants

From the cumulant characteristic function of R_t

$$\psi_{R_t}(u) = \left\{ium^P - \frac{1}{2}\delta^2u^2 + \lambda\left[\left(1 + \frac{iuq}{\eta}\right)^{-\gamma} - 1\right] - \alpha \ln\left[1 - \frac{1}{\beta}\left(iu\theta - \frac{1}{2}u^2\sigma^2\right)\right]\right\} t,$$

it is possible to derive the first four cumulants of the log-return distribution on time intervals of lenght t:

$$c_1[R_t] = E[R_t] = \left[m^P + \theta\frac{\alpha}{\beta} - \lambda q\frac{\gamma}{\eta}\right] t,$$
$$c_2[R_t] = var[R_t] = \left[\delta^2 + \left(\theta^2 + \beta\sigma^2\right)\frac{\alpha}{\beta^2} + \lambda q^2\frac{\gamma(\gamma+1)}{\eta^2}\right] t,$$
$$c_3[R_t] = E[R_t - E[R_t]]^3 = \left[\theta\left(2\theta^2 + 3\beta\sigma^2\right)\frac{\alpha}{\beta^3} - \lambda q^3\frac{\gamma(\gamma+1)(\gamma+2)}{\eta^3}\right] t.$$
$$c_4[R_t] = E[R_t - E[R_t]]^4 - 3var^2[R_t]$$
$$= 3\left[\left(2\theta^4 + 4\beta\theta^2\sigma^2 + \beta^2\sigma^4\right)\frac{\alpha}{\beta^4} + \lambda q^4\frac{\gamma(\gamma+1)(\gamma+2)(\gamma+3)}{\eta^4}\right] t.$$

The third and the fourth cumulants contain information about the asymmetry and the heaviness of the tails of the log-return distribution. The skewness is generated by two independent sources: the variance gamma process and the compound Poisson process. The impact of the first source depends on the sign of the parameter θ. Specifically, the contribution to the skewness of the log-return distribution is positive, negative, or null if θ is positive, negative, or null, respectively. The impact of the second source is always negative. This means that our model is able to generate a distribution negatively skewed if $\theta \leq 0$ or if $\theta > 0$ and $\theta\left(2\theta^2 + 3\beta\sigma^2\right)\frac{\alpha}{\beta^3} < \lambda q^3 \frac{\gamma(\gamma+1)(\gamma+2)}{\eta^3}$, positively skewed if $\theta > 0$ and $\theta\left(2\theta^2 + 3\beta\sigma^2\right)\frac{\alpha}{\beta^3} > \lambda q^3 \frac{\gamma(\gamma+1)(\gamma+2)}{\eta^3}$, symmetric if $\theta\left(2\theta^2 + 3\beta\sigma^2\right)\frac{\alpha}{\beta^3} = \lambda q^3 \frac{\gamma(\gamma+1)(\gamma+2)}{\eta^3}$. The fourth cumulant is strictly positive and heavy tails are generated by the joint effect of a process with infinitely many jumps (i.e. the variance gamma process) and of a process with a finite number of jumps (i.e. the compound Poisson process) in every finite time interval.

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