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Corrigendum: Unirationality of Hurwitz Spaces of Coverings of Degree ≤ 5

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We correct Proposition 3.12 and Lemma 3.13 of the paper published in Vol. 2013, No.13, pp.3006–3052. The corrections do not affect the other statements of the paper.

In this note, we correct a flow in the statement of Proposition 3.12 of [1] which also leads to a modification in the statement of Lemma 3.13 of [1]. We recall that in this proposition one considers morphisms of schemes $\mathcal{X} \xrightarrow{\pi} \mathcal{Y} \xrightarrow{q} S$, where q is proper, flat, with equidimensional fibers of dimension n and π is finite, flat and surjective. Imposing certain conditions on the fibers it is claimed that the loci of $s \in S$ fulfilling these conditions are open subsets of S. A missing condition should be added and the correct version of Parts (g) and (h) of Proposition 3.12 should be as follows:

(g) *$\mathcal{Y}_s$ has no embedded components and the discriminant scheme of $\pi_s : \mathcal{X}_s \to \mathcal{Y}_s$ is of pure codimension one and smooth;*

(h) *$\mathcal{Y}_s$ has no embedded components and the discriminant scheme of $\pi_s : \mathcal{X}_s \to \mathcal{Y}_s$ is of codimension one, irreducible and generically reduced.*

The problem is in the claim on page 3024 of [1], that the restriction of $d_{\mathcal{X}/\mathcal{Y}} : (\bigwedge^{max} \pi_* \mathcal{O}_{\mathcal{X}})^{\otimes 2} \longrightarrow \mathcal{O}_{\mathcal{Y}}$ to every fiber of $q : \mathcal{Y} \longrightarrow S$ is an injective morphism. This holds imposing the additional condition that every fiber $\mathcal{Y}_s$ is without embedded components.

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We prove this as follows. By [2] Théorème 12.1.1 part (iii), applied to $q : \mathcal{Y} \longrightarrow S$ and $\mathcal{F} = \mathcal{O}_{\mathcal{Y}}$, and by the properness of $q : \mathcal{Y} \longrightarrow S$, the set of points $s \in S$ such that $\mathcal{Y}_s$ has no embedded components is open in S. Shrinking S we may thus assume that every fiber of $q : \mathcal{Y} \longrightarrow S$ is equidimensional of dimension n and is without embedded components, while the discriminant scheme $\mathcal{B} \subset \mathcal{Y}$ has the property that every fiber of $q|_{\mathcal{B}} : \mathcal{B} \longrightarrow S$ is equidimensional of dimension $n - 1$. Let $s \in S$ and let $y \in \mathcal{Y}_s$. Let $\mathcal{O}_y = \mathcal{O}_{\mathcal{Y}_s, y}$ and let $d \in \mathcal{O}_y$ be the germ of the restriction of the discriminant. Then $(d_{\mathcal{X}/\mathcal{Y}} \otimes k(s))_y$ may be identified with $\mathcal{O}_y \xrightarrow{\cdot d} \mathcal{O}_y$. If $y \notin \mathcal{B}$ then d is invertible. If $y \in \mathcal{B}$ then the minimal prime ideals of $\mathcal{O}_y$ which contain d are of height 1. In either case d is not a zero divisor of $\mathcal{O}_y$ since the associated primes of $\mathcal{O}_y$ are all of height 0. This proves that $d_{\mathcal{X}/\mathcal{Y}} \otimes k(s)$ is injective for every $s \in S$.

Parts (f) and (g) of Lemma 3.13 of [1] should be modified correspondingly as follows

(f) *Assuming Condition (a) holds, in case Condition (e) determines an empty set, $\mathcal{Y}_z$ has no embedded components and the discriminant subscheme of $\pi_\eta : X_\eta \to \mathcal{Y}_z$ is smooth of pure codimension one;*

(g) *Assuming Condition (a) holds, in case Condition (e) determines an empty set, $\mathcal{Y}_z$ has no embedded components and the discriminant subscheme of $\pi_\eta : X_\eta \to \mathcal{Y}_z$ is of codimension one, irreducible and generically reduced.*

Proposition 3.12 is used only in the proof of Lemma 3.13. This lemma is used: in the proof of Proposition 3.19, in Section 4.1, in Section 4.2, in the proof of Lemma 4.9 and in the proof of Theorem 1.2. Whenever used the morphism $q : \mathcal{Y} \longrightarrow \mathcal{Z}$ is a projection $S \times Y \longrightarrow S$, where Y is a complete irreducible variety and S is a certain variety. So, the condition that $\mathcal{Y}_z$ has no embedded components is fulfilled for every fiber $\mathcal{Y}_z \cong Y$.

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