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CONVERGENCE AND DIVERGENCE: A NEW APPROACH, NEW DATA, AND NEW RESULTS

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Recently, Penn World Tables include new data that enable calculation of total factor productivity in addition to output for a large set of countries. We use these new data to examine convergence and divergence across countries by applying a new approach, which differentiates between the dynamics of output and of productivity. Our empirical results lead to two main new contributions to the literature. The first is on the interpretation of "β-convergence" in "growth regressions." It means that output per worker in each country converges to productivity but does not imply convergence across countries, since productivity tends to diverge from the global frontier. The second contribution is to the literature, which finds that income gaps across countries are due mainly to differential technology adoption. This paper shows that the gaps in technology are not only large but keep growing over time.

Keywords: Economic Growth, Divergence, Convergence, Global Frontier, Technology Adoption

1. INTRODUCTION

Do income levels across countries converge or diverge over time? This question haunts empirical research on economic growth in the last four decades, and it has received many answers, some of them conflicting. There are two main papers that survey the vast literature on cross-country growth empirics. The first is Durlauf et al. (2005), hereafter DJT, which surveys the research until the early 2000s and

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devotes much space to convergence and divergence. The second survey is Johnson and Papageorgiou (2020), hereafter JP, which surveys later research on growth empirics and focuses mainly on convergence and divergence.[1] Together, the two surveys cite more than 650 articles. This raises the question whether it is still possible to contribute to such vast literature.

This paper presents an attempt to add to this already rich and dense area of research a new idea, a new approach, which leads to new results. The idea is that instead of examining the dynamics of output per capita, or of output per worker, as most of the previous literature does, we examine the joint dynamics of output per worker and of total factor productivity (TFP). We can do it because the new Penn World Tables (PWT), from version 8.0 on, include for many countries not only data on output, but on capital stocks and labor inputs as well, as explained in Feenstra et al. (2015). These new versions of PWT enable us to calculate for many countries the series of productivity, which enable us to study the joint dynamics of output and productivity.

As a result of this new approach, supported by the new data, we reach some new findings, of which we focus here on two. The first is that for each country output per worker follows the path of productivity of that country in the long run, and the rate at which output converges to productivity is around 2%. The second finding is that productivities of many countries do not follow the global productivity frontier and even diverge from it continuously. These two findings lead us to change interpretations of many results in the literature.

We base our empirical analysis on the DJT theoretical model of economic growth in a small open economy, with labor augmenting TFP. Its main assumption is that the equilibrium ratio between output per worker and productivity, called "efficiency output per worker," should converge to a constant. DJT did not present direct tests of this assumption, as PWT data on productivity were not available by then. Actually, our paper is the first that tests directly this main assumption of DJT, by using data on productivity.

We begin with calculation of "labor augmented total factor productivity," denoted LATFP, from the newest version of PWT, version 9.1.[2] The calculation uses the series that follow national accounts, which are best suited to study variation over time. We then examine in two ways how output per worker follows LATFP. First, we show that the ratio between the two variables, the "efficiency output per worker," indeed converges to a long-run constant for each country. Second, cointegration tests of output per worker over LATFP yield a coefficient of cointegration of 1. Both tests show that the rate of convergence of output per worker to LATFP is around 2%.[3] If output per worker converges to productivity, then the question of convergence or divergence of output boils down to convergence or divergence of productivity. This is exactly the next stage in our empirical analysis.

To examine convergence or divergence of productivity across countries, we run cointegration tests of productivity on the global frontier, represented by US productivity, both in natural logarithms. If the coefficient of cointegration d is equal to 1, the ratio between productivity of a country and the frontier remains stable, but if d is lower than 1, the ratio between productivity and the frontier declines continuously and the country diverges away from the frontier. The tests show that many countries fail to follow the global frontier fully. The average d is lower than 1 and is quite low for many countries, in Central and South America, in South Saharan Africa, and in Middle East and North Africa (MENA). Only for Eastern European and East Asian countries productivities grew faster than the frontier, as their d is above 1.

The next step in the empirical analysis is to split productivity into two, human capital and technical change. We use data of human capital from PWT 9.1, calculated from data on education attainment by using the method of "development accounting." We then divide LATFP by the level of human capital and get an estimate of the country's level of technology. This enables us to test the dynamics of both human capital and technology. We find that human capital of each country converges to a finite level, but levels of technology tend to diverge from the frontier even more than LATFP. In other words, many countries do not fully follow the technology frontier.

We next relate these results to the literature on convergence and divergence to clarify their novelty and contribution to the literature. There are two main lines of research that examine convergence with results that seem to contradict each other. The first and most prominent one is "growth regressions," which began with Barro (1991).[4] Its main result is β-convergence, namely that the rate of growth of output per worker is negatively related to the initial level of output per worker, if some additional variables are controlled for. These are called "explanatory variables," since they represent various theories of economic growth, like human capital, geography, institutions, and more.[5] A common critique on growth regressions is that the choice of explanatory variables is quite arbitrary.

What are the most common interpretations of the result of β-convergence? Barro (2015) makes it clear that it tells us about convergence of countries to one another in the long run: "Convergence-rate parameters are important to pin down because they provide guidance on how fast countries like China and India are likely to catch up to richer countries. The convergence rate may also reveal how fast a poor African country could develop…" Another common and less ambitious interpretation of β-convergence is that countries do not necessarily converge to the same steady state, but each converges to its own steady state.[6] However, this interpretation also implies some convergence between countries, since it means that the distribution of output per worker across countries should converge to a limit distribution of steady states.

The second line of research on convergence and divergence examines the distribution of output per worker across countries and how it evolves over time. A well-known such test examines the dynamics of the standard error of this distribution and is called the σ-convergence test. It usually finds that the standard error increases over time, as in Quah (1996) and Pritchett (1997).[7] The common interpretation of these tests is that levels of output per worker across countries tend to diverge over time, which seems to contradict the β-convergence results.

The results of this paper lead to rethink the interpretation of β-convergence results. They show that output per worker in a country converges to productivity of this country, but productivities diverge continuously from the frontier. Hence, the interpretation of β-convergence should not be that countries converge to one another, nor that each converges to its own steady state. Both such interpretations contradict our result that productivities tend to diverge over time. However, our results support the interpretation that output in each country converges to its own productivity, which fits β-convergence, as suggested in DJT. Our critique of growth regressions is therefore not on the tests themselves but on their economic interpretation of convergence or conditional convergence.[8]

Furthermore, our results enable us to reconcile the findings of β-convergence and σ-divergence. They show that output converges to productivity, which is compatible with β-convergence, but that productivity gaps across countries increase over time, which is compatible with σ-divergence. Our approach also adds a number of results to the distribution research. First, it identifies divergence of individual countries and not just the whole distribution. Second, our measure of the degree of divergence is continuous. We also contribute to the literature of growth regressions, by estimating the rate of convergence of output to productivity without using any explanatory variables, which drew much critique in the literature.[9] This adds robustness to the result of 2% rate of convergence, although with a different interpretation.

The results of the paper also contribute to the literature on differential technology adoptions across countries. Klenow and Rodríguez-Clare (1997) find that technology accounts for much of the differences in output per worker across countries. Dowrick and Rogers (2002), Comin and Hobijn (2010), Comin and Mestieri (2018), and Alesina et al. (2018) report on similar results. This paper shows that these differences are not only large but might be increasing over time for many countries. Hence, it supports theories of differential technology adoption, like Krugman (1979), Parente and Prescott (1994), Zeira (1998), and Acemoglu et al. (2006).

Our empirical analysis is also related to "time series tests," which began with Bernard and Durlauf (1995, 1996) and continued with Hobijn and Franses (2000), Phillips and Sul (2007b, 2009), Deckers and Hanck (2014), Stengos and Yazgan (2014) and others, especially in using cointegration tests. However, it differs from these papers in its basic new approach, which differentiates between the dynamics of output per worker and the dynamics of productivity. There is a single recent paper that studies convergence of productivities (Bergeaud et al. 2019), but it does not study together the joint dynamics of output per worker and productivity, as we do. It is also limited to 17 countries, which are developed and tend to converge to the frontier.

Note that there is recently renewed interest in convergence and divergence, as reflected in the surveys by Jones (2016), Barro (2015), and JP. Rodrik (2011, 2013), Madsen (2008), and Madsen and Timol (2011) look at new data, mainly on sectors and industries. This paper fits into this new interest by adding a new approach and by using new cross-country data.

The paper is organized as follows. Section 2 presents data on LATFP. Section 3 explains why output should follow LATFP and Section 4 empirically tests this convergence. Section 5 shows that productivities in many countries diverge away from the global frontier. Section 6 splits productivity to human capital and technology and shows that human capital converges to a finite level, while technology diverges significantly. Section 7 examines robustness of the results over different versions of PWT and different periods. Section 8 summarizes the paper, while the Appendix discusses some issues, like how to calculate LATFP and growth under adjustment costs to investment.

2. LABOR AUGMENTED TOTAL FACTOR PRODUCTIVITY

2.1. Output per Worker and LATFP

Our point of departure is the standard production function with labor augmenting TFP, used extensively in the literature since Solow (1956). Assume, therefore, like DJT, that production in country j in period t is described by

$$Y(j,t) = G\left[K(j,t), A(j,t)L(j,t)\right]. \qquad (1)$$

where $Y(j,t)$ is the aggregate output, $L(j,t)$ is the input of labor during period t, $K(j,t)$ is the amount of capital used in country j in period t, and $A(j,t)$ is the labor augmenting TFP, hereafter LATFP, or just productivity. The function G is a Constant Returns to Scale (CRS) production function and is assumed to be equal across countries and constant over time.

There are four main reasons for assuming that productivity is labor augmenting and not multiplying the production function, as in Solow (1957). One reason is that productivity includes human capital, which should be labor augmenting by its nature. Second, labor augmenting simplifies much of the following analysis but has no effect on the main results, as shown in Appendix A.3. Third, using a labor augmented productivity follows the model of DJT, which is already a classic presentation of such growth empirics. Fourth, if productivity multiplies the production function, we also need to assume a Cobb–Douglas production function, as shown in Appendix A.3, while under labor augmenting productivity, the analysis holds for any CRS production function G.

"Output per worker" in country j at time t is defined as

$$y(j,t) = \frac{Y(j,t)}{L(j,t)} = G\left[\frac{K(j,t)}{L(j,t)}, A(j,t)\right]. \qquad (2)$$

Clearly, output per worker, also termed "labor productivity," is the main variable that measures a country's potential output. Output per worker depends on A directly, but also indirectly, as shown in Section 3, through raising the marginal productivity of capital and thus increasing investment.

2.2. Labor Augmented Total Factor Productivity

PWT 9.1 includes data on output, employment, capital, and the share of labor for a large panel of countries. For output levels, we use the series "rgdpna," namely, real GDP of national accounts at 2011 US dollars (millions).[10] This is the series recommended by PWT 9.1 for comparing output over time, which is the main type of tests we run in this paper. For the labor input, we use the series "emp" in millions of workers. For capital stocks, we use the series "rnna," real capital stock of national accounts at 2011 US dollars (millions), which fits the output series. Labor share is given by the series "labsh." These data enable us to calculate output per worker and also LATFP. There are 182 countries in the data set and its time span is 1950–2017, but not all countries have full data for the entire period. This is available for only 52 countries. In most of our estimations, we focus on 103 countries, for which these data are available since 1970 and for which we can run tests for the period 1970–2017.

The new PWT 9.1 also includes calculation of TFP, but we use our own calculations, for two reasons. First, we prefer to use LATFP. Second, PWT calculate TFP minus human capital, while we separate it from productivity only at a later stage. The calculation of the LATFP is very similar to that of the standard Solow Growth Accounting, as the rate of growth of LATFP is equal to the Solow residual divided by the elasticity of output with respect to labor, η_L. Appendix A.1 shows that the annual rate of growth of LATFP is

$$\frac{A(j,t)-A(j,t-1)}{A(j,t-1)} = \frac{1}{\eta_L(j,t-1)}\left[\frac{Y(j,t)-Y(j,t-1)}{Y(j,t-1)} - \frac{K(j,t)-K(j,t-1)}{K(j,t-1)}\right] + \frac{K(j,t)-K(j,t-1)}{K(j,t-1)} - \frac{L(j,t)-L(j,t-1)}{L(j,t-1)}. \quad (3)$$

Following are three comments on the calculation of LATFP:

1. First, equation (3) holds for cases in which the changes in output, labor, and capital are rather small, so linear differentials are good estimates. In the case of Iraq, there were huge changes in output in some war years (e.g., output declined by 66% in 1991), so application of (3) yields absurd results. In this case, we calculate productivity from data on levels and not rates of change, as in equation (3), assuming that G is Cobb–Douglas.
2. Second, the elasticity of output with respect to labor, $\eta_L(j,t-1)$, is not observed in the data, but under perfect competition it should be equal to the share of labor in output, $s_L(j,t-1)$, as shown by Solow (1957). We have calculated LATFP both by using the actual annual labor share from PWT 9.1 and by using a constant elasticity of 2/3, as suggested by Jones (2016). Due to missing data on labor share, using 2/3 enables us to use a larger sample of countries, 103 instead of 85. Hence, all the results we report are for calculation with 2/3. The results for using the actual share of labor are very similar.
3. Finally, for most of the dynamic analysis, it is sufficient to know only the rate of growth of productivity LATFP, obtained by (3), and not its level. However,

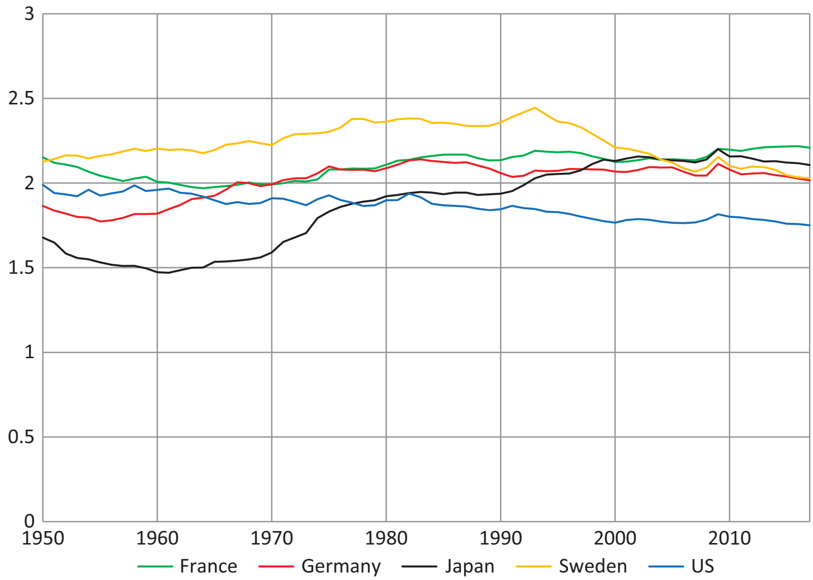

FIGURE 1. Efficiency output per worker for five developed countries.

for calculating the level of efficiency output per worker, we need the level of LATFP as well. We get it by calculating productivity in the first year, assuming that G is a Cobb–Douglas and from that year, LATFP is calculated by chaining.

2.3. Efficiency Output Per Worker

We next follow DJT and define "efficiency output per worker" as the ratio between output per worker to labor augmented productivity LATFP. Hence, efficiency output per worker in country j at time t is

$$y^E(j,t) = \frac{y(j,t)}{A(j,t)} = \frac{Y(j,t)}{L(j,t)A(j,t)}. \quad \textbf{(4)}$$

PWT 9.1 enables us to calculate this efficiency output per worker and examine its dynamics. Figure 1 presents the graphs of efficiency output per worker, $\ln y^E$, for five developed countries in 1950–2017. The countries are the USA, Germany, France, Sweden, and Japan. As Figure 1 shows, efficiency output per worker has been quite stable over time for most countries, and it exhibits convergence to some long-run levels for all countries. Actually, most countries seem to converge to close steady states, which are a little above 2, except for the USA, which has lower efficiency output per worker, probably due to higher productivity.

Figure 2 presents the same variable, efficiency output per worker, for five less developed countries, which are China, India, Kenya, Colombia, and Turkey. These are countries from different global regions, East Asia, South Asia, Sub-Saharan

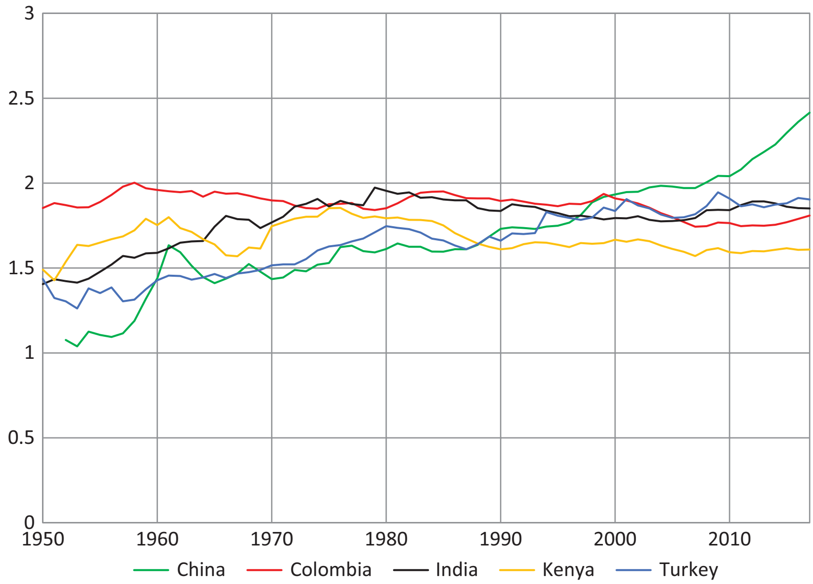

FIGURE 2. Efficiency output per worker for five less developed countries.

Africa, Central and South America, and Middle East and North Africa, respectively. Although Turkey belongs to the Organization for Economic Co-operation and Development (OECD), it is one of the lower income countries in this group. The pattern in Figure 2 is similar to that of Figure 1, as efficiency output per worker in each country tends to converge to some long-run level. However, in these developing countries, efficiency output per worker is generally below 2. An exception is China, for which the efficiency output per worker is climbing faster than in other countries, and has already passed 2. That is possibly because China is still catching up, so its output might grow faster than its productivity.

3. CONVERGENCE OF EFFICIENCY OUTPUT PER WORKER

As Figures 1 and 2 show, the efficiency of output per worker for each country tends to converge to a long-run level. We next show how this convergence is theoretically explained by capital accumulation. First, rewrite efficiency output per worker as follows:

$$y^E(j,t) = \frac{Y(j,t)}{A(j,t)L(j,t)} = G\left[\frac{K(j,t)}{A(j,t)L(j,t)}, 1\right].$$

Note that K/AL is the ratio between the capital–labor ratio and productivity, so we call it similarly the "efficiency capital–labor ratio" and denote it by k^E. We can therefore write the efficiency output per worker as dependent on the efficiency capital–labor ratio:

$$y^E(j,t) = G\left[k^E(j,t), 1\right]. \tag{5}$$

The accumulation of capital is driven, in the Neoclassical Growth Model, by the gap between profitability, namely the marginal productivity of capital, and the cost of capital, which is usually the sum of the interest rate and the rate of depreciation.[11] In the long run, the marginal productivity of capital should be equal to the long-run cost of capital, which is constant in all standard growth models. It is constant in a closed economy, where the cost of capital in the long run is equal to the subjective discount rate plus the rate of depreciation, $\rho + dep$. It is constant in a small open economy as well, where it is equal to the global interest rate plus the depreciation rate, $r^* + dep$.[12] Note that the Marginal Productivity of Capital (MPK) is homogenous of degree 0 and is therefore equal to

$$MPK(j,t) = G_K\left[K(j,t), A(j,t)L(j,t)\right] = G_K\left[\frac{K(j,t)}{A(j,t)L(j,t)}, 1\right] = G_K\left[k^E(j,t), 1\right].$$

Hence, the long-run efficiency capital–labor ratio should be constant as well, at a level denoted $k^E(j,\infty)$. Therefore, if capital accumulation is gradual and not immediate, the efficiency capital–labor ratio should converge gradually to this long-run value. Equation (5) implies that the efficiency output per worker also converges to a long-run level, denoted by $y^E(j,\infty)$, due to continuity of G:

$$y^E(j,t) = G[k^E(j,t), 1] \underset{t\to\infty}{\longrightarrow} G[k^E(j,\infty), 1] = y^E(j,\infty). \tag{6}$$

Hence, the period-to-period dynamics of this convergence of efficiency output per worker can be described by the following log-linear approximation, as in DJT:

$$\ln y^E(j,t) - \ln y^E(j,t-1) = b(j)\ln y^E(j,\infty) - b(j)\ln y^E(j,t-1). \tag{7}$$

The parameter $b(j)$ measures the rate of convergence of efficiency output per worker to its long-run value. Since PWT 9.1 enables us to measure y^E, we can estimate equation (7) directly.

There are two possible mechanisms that can explain the assumption that capital adjustment is gradual and not instantaneous, as DJT assume. One mechanism holds in a closed economy, where capital accumulation is bounded by savings.[13] The second mechanism is that investment is gradual if there are adjustment costs to investment. This mechanism works in open economies as well, where investment can exceed savings, as shown in Appendix A.2. We believe that the open economy model is better suited to compare economic growth across countries than the closed economy model.

Note that calculation of TFP uses data on output and of capital per worker. Appendix A.4 shows that convergence of the efficiency output per worker to a constant is equivalent to convergence of the capital–output ratio to a constant as well. This is related to the famous "Kaldor Stylized Fact" that capital–output ratios are quite stable over time and across countries. This is not surprising, as Appendix A.4 shows that this is also a result of convergence of marginal productivity of capital to a long-run constant. We focus here on the convergence of efficiency output per worker, as it fits better our general approach of separating dynamics of output per worker and of productivity.

DJT connect equation (7) to growth regressions by substituting equation (4) in equation (7) and calculating the average growth rate of country j over T periods:

$$\begin{aligned}\frac{\ln y(j,T)-\ln y(j,0)}{T} = {} & \frac{\ln A(j,T)-\ln A(j,0)}{T}+\frac{1-[1-b(j)]^T}{T}\ln A(j,0)\\ & +\frac{1-[1-b(j)]^T}{T}\ln y^E(j,\infty)\\ & -\frac{1-[1-b(j)]^T}{T}\ln y(j,0). \end{aligned} \quad (8)$$

Growth regressions estimate this equation, but without observing productivity A. They derive b from the regression coefficient of initial output per worker, $\ln y(j,0)$, assuming that b is equal across countries.[14] In such regressions initial productivity $A(j,0)$, the average rate of growth of productivity $T^{-1}\left[\ln A(j,T)-\ln A(j,0)\right]$ and $y^E(j,\infty)$ are unobserved. To overcome this, such regressions assume that the rate of growth of productivity is equal across countries. They also use additional variables to control for the missing values of A and $y^E(j,\infty)$, which are called "explanatory variables." This is the standard structure of β-convergence tests. Unlike these tests, the next section estimates b directly from equation (7), using the new data on productivity and without using explanatory variables.[15]

4. EMPIRICAL TESTS OF CONVERGENCE OF OUTPUT TO PRODUCTIVITY

4.1. Convergence of Efficiency Output Per Worker

We next test empirically the dynamic equation (7). We estimate it over all the countries in the sample in the years 1970–2017. Note that equation (7) has one unobserved variable, long-run efficiency output per worker $y^E(j,\infty)$. Hence, the constant might differ across countries, as can already be inferred from Figures 1 and 2. We cope with this unobserved variable in two ways. One is by adding country-fixed effects to the regression and the other by controlling for two variables that are supposed to be negatively related to long-run efficiency output per worker. These variables are the average rate of growth of LATFP over the entire period and the average rate of population growth. The reason is that if productivity and population rise more rapidly, capital has to be adjusted at a higher rate and as a result it has a lower steady-state level.[16]

Table 1 presents the results of the various regressions of equation (7), where b is the coefficient of lagged $\ln y^E$. Column (1) in Table 1 presents the results of a pooled regression, where gA is the average rate of growth of productivity and gN is the long-run population growth. The results of the regression are as expected, the coefficients of gA and gN are significantly negative, and the coefficient b is highly significant. In this pooled regression, we assume that b is the same across

TABLE 1. Growth regressions of efficiency output per worker

Dependent variable: difference of $\ln(y^E)$						
Variable	(1) Pooled	(2) Panel Het. b	(3) Panel Same b	(4) Pooled	(5) Panel Het. b	(6) Panel Same b
Lagged $\ln(y^E)$	-0.022^{***}	-0.055^{***}	-0.037^{***}	-0.017^{***}	-0.029^{***}	-0.022^{***}
	(0.002)	(0.005)	(0.008)	(0.002)	(0.003)	(0.007)
Constant	0.024^{***}	0.042^{***}	0.029^{***}	0.021^{***}	0.024^{***}	0.019^{***}
	(0.003)	(0.003)	(0.005)	(0.002)	(0.002)	(0.004)
gA	-0.284^{***}			-0.283^{***}		
	(0.048)			(0.046)		
gN	-0.127^{**}			-0.142^{***}		
	(0.049)			(0.047)		
Country FE	N	Y	Y	N	Y	Y
Data	Annual	Annual	Annual	Smoothed	Smoothed	Smoothed
R^2	0.059			0.133		
R^2 within			0.030			0.034
Obsecvations	4841	4841	4841	4841	4841	4841
Countcies	103	103	103	103	103	103

Notes: We denote significance of 1% by ***, of 5% by **, and of 10% by *. FE, Fixed Effects.

countries. The estimated coefficient b is 2.2%, very close to the famous "Iron Law" of 2% of Barro (2015).

Regression (2) presents the results of a panel estimation with country-fixed effects and without control variables. The coefficients of convergence b are estimated for each country separately, and their average is around 5% and significantly higher than 0. The b coefficients are quite similar across countries and regions. For example, their average across OECD countries is 0.06, their average across Latin American countries is 0.05, their average across Sub Saharan African countries is 0.06, in East Asia it is 0.04, in MENA it is 0.05, and in East European countries it is 0.05. Due to this finding, we assume in the following regression that b is uniform across countries.

Regression (3) is another panel regression with country-fixed effects, this time with uniform b. It shows that efficiency output per worker converges to a long-run country constant at a rate of 3.7%, higher than in the pooled regression. This is not surprising, as Barro (2015) and others noticed that adding country-fixed effects to dynamic regressions tends to raise the measured rate of convergence. One possible explanation to that could be the use of annual non-smoothed data, as the estimated coefficient b might reflect not only long-run dynamics of output, but also short-run cyclical dynamics. Hence, regressions (4), (5), and (6) run the same tests, but with smoothed data, using 5 years' averages of the natural logarithms of efficiency output per worker.[17]

The results of regressions (4), (5), and (6) are similar to (1), (2), and (3), respectively, except that the values of *b* are lower for panel regressions and they are close to 2%. Hence, the estimated rate of convergence of efficiency output per worker is very close to the estimated rate of β-convergence in many studies. Note that the smoothed data are used in annual frequencies and the coefficient *b* is for a one-period lag. We have also reached similar results in regressions with lower frequency, of 5 years, which avoid overlap, and the coefficient is estimated from a 5-year lag.

Note that the estimation of the rate of convergence *b* does not require explanatory variables, as usual growth regressions do. We estimate a very significant coefficient *b*, which is very close to the famous 2%. In comparison, usual growth regressions without explanatory variables yield insignificant *b*. We attribute this to our use of productivity, LATFP, instead of explanatory variables. To examine further that adding explanatory variables is unnecessary, we ran the pooled regressions, (1) and (4), with additional explanatory variables used by Mankiw et al. (1992), namely human capital and the rate of investment. The results did not change much. The coefficient *b* was still significant at 0.028 in one regression and equal to 0.02 in the other regression. Furthermore, adding these explanatory variables does not affect much R^2, which increases from 0.06 to 0.1 in one regression and from 0.13 to 0.18 in the second regression. Hence, our estimation does not really need adding explanatory variables, which we view as a contribution, due to the vast criticism on it.

4.2. Cointegration Tests of Convergence of Output Per Worker to Productivity

Equation (7) describes the dynamics of the efficiency output per worker. Substituting equation (4) in (7) leads to the following dynamic equation:

$$\begin{aligned} &\ln y(j,t) - \ln A(j,t) - \ln y^E(j,\infty) \\ &\quad = [1-b(j)]\left[\ln y(j,t-1) - \ln A(j,t-1) - \ln y^E(j,\infty)\right]. \end{aligned} \tag{9}$$

Equation (9) means that output per worker, in logarithm, converges to the dynamic path of productivity, more precisely to: $\ln A(j,t) + \ln y^E(j,\infty)$. Empirically, equation (9) means that the logarithm of output per worker in each country should be cointegrated with $\ln A(j,t)$, with coefficient of cointegration of 1, with an error correction coefficient equal to $b\,(j)$ and a long-run distance between the two variables equal to $\ln y^E(j,\infty)$. Therefore, we next run cointegration tests of $\ln y(j,t)$ over $\ln A(j,t)$ to find the coefficient of cointegration and the error correction coefficient. Such regressions do not require any information or assumption on the long-run efficiency output per worker as in Table 1. Actually, it can be measured from the cointegration test, though we do not report it.

We estimate the dynamic equation (9) with a panel cointegration test that uses the Pesaran and Smith (1995) procedure. More specifically, we use MG, the mean group estimator, which allows for heterogeneous coefficients for each country.[18]

Table 2. Cointegration estimation of convergence of output to LATFP

Coefficient	(1)	(2)
Co-integration coefficient	0.915***	0.973***
	(0.039)	(0.041)
Probability of failure to reject coefficient = 1	0.03	0.51
b	0.025***	0.019***
	(0.002)	(0.002)
Period	1970–2017	1950–2017
No. of countries	100	48

Notes: We denote significance of 1% by ***, of 5% by **, and of 10% by *.

Table 2 presents the results of the cointegration estimation. Regression (1) estimates it for the larger set of countries, over the period 1970–2017, and regression (2) for a smaller set of countries, over a longer period 1950–2017. In the estimation for Table 2, as in all the following tables, we use smoothed data. In the cointegration test of the larger set of countries, for the period 1970–2017, we eliminated three outliers, which are countries with cointegration coefficients above 10 or below –10.

The results of Table 2 fit the results of Table 1 and further support the assumption that output per worker follows the long-run path of productivity for each country. The cointegration coefficient is 1 or close to 1 in both regressions, as expected. The probability of failure to reject the null hypothesis that the coefficient is equal to 1 is much higher than 0.05 in the regression of the smaller sample. The probability is slightly below 0.05 for the larger sample, but as shown below most coefficients are around 1. The coefficient *b*, the error correction coefficient, is close to 2% in both regressions, which fits the results of Table 1 as well.

The regressions in Table 2 estimate both the rates of convergence and the cointegration coefficients separately for each country, but the values are quite close. The coefficient of cointegration is between 0.5 and 1.5 for 90 countries out of 100 for the larger sample and in 43 out of 48 countries in the smaller sample.[19] The results with respect to *b* are also close. In the larger sample, *b* is between 1% and 4% for 59 countries out of 100. Concentration of *b* is even higher in the smaller sample, where it is between 1% and 4% for 31 countries out of 48. These results support the assumption made in many empirical studies that *b* is similar across countries.

4.3. Implications for β-Convergence Tests

This section supplies strong empirical support to the DJT assumption that efficiency output per worker converges to a long-run level. We do not need to run standard growth regressions and use explanatory variables, as we use direct data on TFP instead. This convergence is equivalent, as shown by equation (8), to β-convergence, and it also yields a similar rate of convergence, around 2%. However, our results imply that β-convergence should not be interpreted as convergence of one country to another. Its correct interpretation is that output converges to its own productivity path, while the productivity paths can either converge or diverge from one another. Indeed, the next section shows that they diverge for many countries.

5. DIVERGENCE FROM THE GLOBAL FRONTIER

If output per capita in each country follows productivity, then the question of convergence or divergence across countries boils down to whether productivities converge or diverge across countries. We examine it by measuring by how much productivity in each country follows the global frontier. We assume that the USA is the global frontier, since it has led economic growth since the second half of the 19^{th} century. Hence, we examine how productivity of each country is following the productivity of the USA. For that we run cointegration tests of LATFP of each country over US LATFP.

Thus, we test the following relationship between the productivity of country j and the global frontier:

$$\ln A(j,t) - d(j)\ln A(US,t) - a(j) = [1-c(j)]\left[\ln A(j,t-1) - d(j)\ln A(US,t-1) - a(j)\right]. \qquad \textbf{(10)}$$

The coefficient $d(j)$ is the cointegration coefficient between country j and the USA. It means that over time the country follows only d of the annual growth of the productivity of the USA. Hence, if it is equal to 1, the country follows the frontier fully, though it might follow it with a lag. Such a lag is captured by the constant $a(j)$, which is a level difference from the frontier, but at the same rates of growth. If the cointegration coefficient d is lower than 1, it means that country j lags progressively behind the frontier and is actually diverging away from it not only in level but in ratio as well.

Equation (10) is not based on a fully specified model, as are equations (7) and (9) above. It is based in principle on the many theories of incomplete adoption of technologies mentioned in the introduction. It allows for incomplete adoption both in level and over time. Most studies of incomplete adoption of technologies implicitly assume that d is equal to 1. Krugman (1979), Aghion (2004), and recently Gourinchas and Jeanne (2013) assume that productivity adjusts gradually and may not fully catch up with the frontier but will converge to a constant ratio of the frontier. Namely, countries might differ from the frontier but not diverge away from it. Equation (10) expands the possible outcomes and allows d to differ from 1, leaving the final answer to the data. Indeed, our estimations below show that d is smaller than 1 for many countries. We also find that for some countries d is higher than 1, which occurs when a country is catching up with the frontier during the period of estimation.

TABLE 3. Cointegration of productivity over US productivity

Coefficient	1970–2017 (1)	OECD (2)	EA (3)	CSA (4)	SSA (5)	MENA (6)	EE (7)
d	0.592*	0.739***	1.840***	0.149	0.229	−0.021	1.145***
	(0.111)	(0.087)	(0.103)	(0.110)	(0.327)	(0.335)	(0.212)
c	0.064***	0.073***	0.054***	0.085***	0.049***	0.062***	0.054***
	(0.004)	(0.007)	(0.008)	(0.007)	(0.008)	(0.011)	(0.007)
Probability of test $d = 1$	(0.000)	(0.000)	(0.000)	(0.000)	(0.019)	(0.002)	(0.494)
Countries	102	24	16	19	22	16	5

Notes: We denote significance of 1% by ***, of 5% by **, and of 10% by *. CSA, Central and South America; EA, East Asia; EE, Eastern Europe; SSA, South Saharan Africa.

The coefficient c is the error correction coefficient. It measures how fast productivity converges to its long-run path, $a(j) + d(j) \ln A(US, t)$, if the country is not yet on the path. This coefficient is therefore similar to b, but it measures the country's rate of adjustment of its productivity and not of adjustment of its stock of capital, which is measured by b. The coefficient c does not indicate how the country is doing relative to other countries or relative to the frontier, which is measured by d, but only relative to its own long-run productivity path.

5.1. Divergence of Productivity

Table 3 presents the results of cointegration tests for the countries in the sample of 1970–2017. Since this sample has 103 countries and the USA is one of them, the number of countries in the cointegration test is 102. Regression (1) presents the average results for the whole panel. The other regressions test sub-samples of regions. Regression (2) examines the OECD countries, without Mexico, Turkey, and South Korea, which we locate in their respective geographic regions. Regression (3) examines East Asia, while regression (4) tests Central and South American countries. Regression (5) examines Sub-Saharan African countries and regression (6) Middle Eastern and North African countries. Regression (7) tests cointegration for Eastern European countries.

The results of Table 3 show much divergence from the global frontier, namely from the USA, as d is significantly lower than 1. Table 3 also shows that there is a strong regional pattern for following the frontier. For all regions except East Asia and Eastern Europe, the coefficient d is below 1. It is fairly high for the OECD countries, but for the Latin American countries, Sub Saharan African countries, and MENA, it is not significantly different from 0. This means that these countries do not follow most of the progress of the global frontier. Interestingly, the error correction coefficient, which measures catching up with long-run productivity growth, is 6.4%, significant, and quite similar across regions. Note that it is higher than b. Hence, the speed of catching up with technology and with human capital is faster than the speed of catching up with physical capital.

We also examine the results of the cointegration test of productivity on the productivity of the USA for a smaller set of 48 countries, for which data exist for the whole period, 1950–2017. Interestingly, the results are quite close. The average coefficient of cointegration is 0.649. It is lower than 1, though higher than the coefficient in regression (1) in Table 3. That is reasonable, since most of the countries, for which there is data throughout the whole period, are OECD developed countries. The probability of rejecting the test that d is 1 in this cointegration is 0.1. This reflects the fact that the coefficient is indeed higher for this set of countries.

Table 3 also shows that the value of d for East Asia is above 1. This is due to the famous Asian Tigers: Hong Kong, South Korea, Singapore, Taiwan, and recently China, which went through rapid "catching up" over much of the period. Since such a process might involve a gradual rise of the coefficient a from equation (10), it might bias the estimation of d upwards. We therefore treat the high values of d in this region with some caution. This problem of a biased coefficient in time series tests, due to convergence to the long-run path, is already discussed in DJT. We do not constrain d to be between 0 and 1, as implied by economic intuition, following the advice of Eberhardt and Teal (2013). They claim that unconstrained heterogeneous estimation is preferred in such a case, as it reduces the bias of average estimates, by filtering out the noise created by misspecification at the country level.

5.2. Estimation of Productivity Without the Crisis Years

While Table 3 presents the results for 1970–2017, we are aware of the significant economic downturn since 2008. As this great recession had a significant effect on the dynamics of output and productivity, it might bias the estimation of long-run growth parameters. Furthermore, the recession hit many European countries and Japan worse than the USA, so that a cointegration test of such countries on the USA might be further biased by the recession. One way to avoid this short-run effect is to remove the recession years from the estimation, as we do next.

Table 4 presents the results of the cointegration tests of LATFP of 100 countries on LATFP of the USA. The results are in general similar to those of Table 3, but the coefficient d for OECD countries is higher than in the whole period and the hypothesis that it is 1 cannot be rejected. It means that indeed one of the reasons for the low d for OECD countries in Table 3 is the effect of the great recession, mainly on Europe and Japan. The values of d in the regions that lag behind, Central and South America, Sub-Saharan Africa, and MENA are similar in the two tables. The average value of c is 7% in Table 4, which is quite close to the 6.4% in Table 3. One can therefore claim that the results of Table 4 are more representative of the long-run divergence than the results of Table 3.

TABLE 4. Cointegration of productivity on US productivity over the period 1970–2008

Coefficient	1970–2008 (1)	OECD (2)	EA (3)	CSA (4)	SSA (5)	MENA (6)	EE (7)
d	0.515***	0.830***	1.798***	0.036	0.146	−0.413	1.158***
	(0.147)	(0.158)	(0.250)	(0.110)	(0.368)	(0.573)	(0.288)
c	0.070***	0.070***	0.067***	0.095***	0.053***	0.070***	0.056***
	(0.004)	(0.009)	(0.012)	(0.006)	(0.009)	(0.014)	(0.011)
Probability of test $d=1$	(0.001)	(0.284)	(0.001)	(0.000)	(0.020)	(0.014)	(0.582)
Countries	100	24	16	19	20	16	5

Notes: We denote significance of 1% by ***, of 5% by **, and of 10% by *. CSA, Central and South America; EA, East Asia; EE, Eastern Europe; SSA, South Saharan Africa.

Although the values of d for the OECD countries are higher in Table 4 than in Table 3, they are still below 1 on average. This is puzzling, since these countries are mainly high-income developed countries, and it is reasonable to assume that they follow the global frontier fully. One possible explanation to this low d could be gaps in hours of work. Since 1960, the number of hours worked in Europe declined from 2000 annual hours per worker to 1500 h per worker, while in the USA the number of hours declined from 2000 to only 1800. That could have caused productivity per worker in Europe to rise by less than in the USA. Interestingly, hours worked in Israel declined by less than in Europe and more in line with the USA, from 2000 annual hours to 1850. The coefficient d for Israel is indeed 0.95, much higher than for other OECD countries.

The result that the coefficient of divergence d is significantly lower than 1 for many countries is one of the two major results of the paper, where the other is convergence of output to productivity. It shows that there is significant divergence between countries. Figure 3 further shows that d is indeed indicative of divergence. It plots a scatter of countries with d on the horizontal axis and the average rate of growth of productivity, gA, over the period 1970–2008 on the vertical axis. As Figure 3 shows, the two variables are strongly positively correlated. Hence, productivity tends to grow faster in countries with high d than in countries with low d. The figure further motivates our focus on the parameter d and its importance for understanding growth dynamics of countries over time.

5.3. Divergence of Output Per Worker

If output per worker converges to productivity and if productivity in a country diverges away from the global frontier, then output per worker in that country should diverge from the global frontier as well. Formally, we derive from equations (9) and (10) the following dynamic relationship:

TABLE 5. Cointegration test of output per worker over the US output per worker

Coefficient	1970–2017 (1)	OECD (2)	EA (3)	CSA (4)	SSA (5)	MENA (6)	EE (7)
d	0.785***	0.758***	1.961***	0.339**	0.735**	0.095	1.732***
	(0.108)	(0.159)	(0.225)	(0.123)	(0.297)	(0.221)	(0.192)
EC	0.066***	0.064***	0.043***	0.088***	0.061***	0.068***	0.069***
	(0.005)	(0.012)	(0.005)	(0.007)	(0.009)	(0.010)	(0.026)
Probability of test $d = 1$	(0.046)	(0.127)	(0.000)	(0.000)	(0.373)	(0.000)	(0.000)
Countries	99	24	14	19	21	16	5

Notes: We denote significance of 1% by ***, of 5% by **, and of 10% by *. CSA, Central and South America; EA, East Asia; EC, Error Correction; EE, Eastern Europe; SSA, South Saharan Africa.

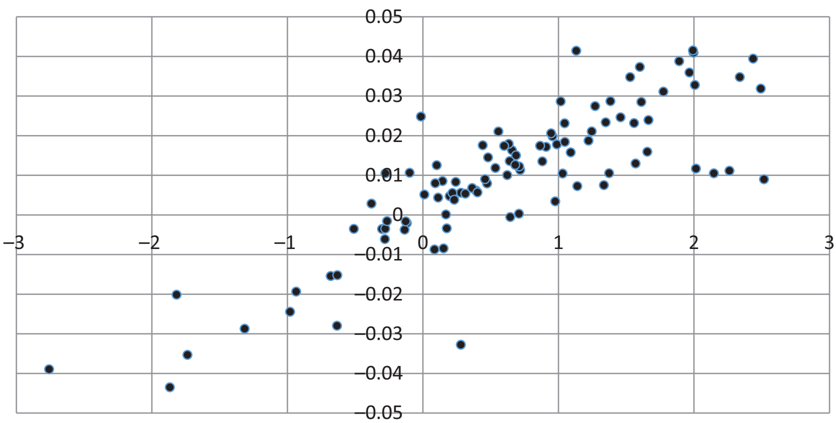

FIGURE 3. A scatter diagram of d versus gA.

$$\ln y(j,t) - d(j)\ln y(US,t) - a(j) - \ln y^E(j,\infty) + d(j)\ln y^E(US,\infty) \xrightarrow[t\to\infty]{} 0. \quad \mathbf{(11)}$$

Equation (11) implies that output per worker $\ln y(j,t)$ and output per worker of the USA should be cointegrated. The coefficient of cointegration should be $d(j)$, the same rate of divergence between productivity and the US productivity. Hence, a cointegration test of output per worker over the global frontier can be an additional test to the coefficient d. Note that estimating (11) does not enable us to identify the rates of convergence b and c, since the error correction coefficient of (11) should be some average of the two. Table 5 presents the results of such tests.

The results in Table 5 reinforce our conclusion above that there is divergence between many countries and the frontier. The average d is significantly lower than 1 for most countries, except for the East Asian and the East European countries. Note that the coefficient d in this table is higher than in Table 3, but it is still

TABLE 6. Cointegration test of output per worker over the US output per worker for 1970–2008

Coefficient	1970–2008 (1)	OECD (2)	EA (3)	CSA (4)	SSA (5)	MENA (6)	EE (7)
d	0.549***	0.646***	1.556*	0.301**	0.103	−0.094	1.875***
	(0.176)	(0.193)	(0.888)	(0.123)	(0.254)	(0.399)	(0.227)
EC	0.072***	0.071***	0.056***	0.094***	0.064***	0.074***	0.069***
	(0.005)	(0.012)	(0.001)	(0.007)	(0.010)	(0.011)	(0.025)
Probability of test $d=1$	(0.010)	(0.067)	(0.531)	(0.000)	(0.000)	(0.006)	(0.000)
Countries	97	22	15	19	20	16	5

Notes: We denote significance of 1% by ***, of 5% by **, and of 10% by *. CSA, Central and South America; EA, East Asia; EC, Error Correction; EE, Eastern Europe; SSA, South Saharan Africa.

much lower than 1. Interestingly, the coefficient *d* for South Saharan Africa is quite high. Table 6 presents the regressions of Table 5 for the period prior to the "great recession," to better identify the long-run effects.

The results of the cointegration of output per worker over US output per worker in the years prior to the great recession are much more similar to Tables 3 and 4 than the results over the whole period in Table 5. This is especially true for the South Saharan African countries, where *d* returns to be close to zero. We do not have a full explanation to this discrepancy between Tables 5 and 6, but it reinforces our tendency to cast some doubts on the results from the great recession.

6. DIVERGENCE OF TECHNOLOGY AND CONVERGENCE OF HUMAN CAPITAL

This paper introduces data on productivity to examine convergence and divergence across countries. Productivity changes over time mainly due to two processes. One is accumulation of human capital, as education expands and reaches larger shares of the population. The second is adoption of technologies, either from the global frontier, or off the shelf, of technologies that were invented in the past. The new method of "development accounting," described in Caselli (2005), enables us to split TFP into these two components, accumulation of human capital and technical change. We use data on human capital from PWT 9.1 and get a time series of human capital for the countries in the sample. Denote the average human capital in country *j* in period *t* by $h(j, t)$. Since human capital is labor augmenting, LATFP should be a multiple of technology and human capital:

$$A(j,t) = T(j,t) \cdot h(j,t).$$

Hence, we present the level of technology in a country as the following residual:

$$\ln T(j,t) = \ln A(j,t) - \ln h(j,t). \qquad (12)$$

We next examine the dynamics of the two variables. Assume first that human capital accumulation is gradual in each country, due to the need to build and expand systems of education. Assume also that human capital in the long run is bounded, by some maximum years of schooling. Hence, adjustment of human capital should follow the standard convergence to a constant:

$$\ln h(j,t) - \ln h(j) = \left[1 - e(j)\right]\left[\ln h(j,t-1) - \ln h(j)\right]. \qquad \textbf{(13)}$$

Here, $h(j)$ is the long-run level of human capital and $e(j)$ is the rate of convergence of human capital. As $h(j)$ is unobserved, we estimate a difference version of equation (13):

$$\ln h(j,t) - \ln h(j,t-1) = \left[1 - e(j)\right]\left[\ln h(j,t-1) - \ln h(j,t-2)\right]. \qquad \textbf{(14)}$$

Table 7 presents the results of estimating (14) over the years 1970–2017, using four different estimations: Fixed Effects, Arellano–Bond, Blundell–Bond, and Pesaran–Smith. The estimation with fixed effects accounts for differences in constants across countries, although the constant should be zero in (14). The Pesaran–Smith estimation accounts for the possibility that the coefficient $1 - e$ is not uniform across countries and is also used to calculate regional averages. We add Arellano–Bond and Blundell–Bond corrected estimations as alternative solutions for the Nickell (1981) bias problem.

The results of all four estimators are quite similar. All tests show that human capital converges to a long-run level and the rate of convergence of human capital is quite high, around 11%. It is much higher than the rate of convergence of physical capital and is even higher than the rate of convergence of productivity. The rate of convergence of human capital is similar across regions, except that it is lower in South Saharan Africa.

If human capital converges to a constant, and productivity A follows the global frontier at a long-run rate d, as shown above, then we can deduce that technology should follow the global frontier at a rate d as well. To examine this conjecture, we estimate a cointegration regression of technology of each country on technology of the USA, where technology is measured as the residual in equation (12). Table 8 presents these regressions for all the countries and for the various regions.

The cointegration regressions in Table 8 support the main result of divergence, as they show that technologies of many countries do not converge to the global technology frontier. Actually, Table 8 presents an even grimmer picture than the tests on productivity. The average d over the whole sample is close to 0 and insignificant. Furthermore, it does not differ significantly from zero in all regions, except in MENA, where it is strongly negative. These results seem to be quite extreme, and we think that the reason for that is that our measure of technology, T, might be biased downward for many developing countries, as explained below.

Our measure of human capital is based on years of schooling, weighted by common returns to education, as done in all "development accounting" studies. However, such calculation assumes that all those who acquired education find jobs that fit their skills, while this is not the case in many poor- and middle-income

TABLE 7. Convergence of human capital to a long-run level

Coefficient	Full sample (1)	(2)	(3)	(4)	OECD (5)	EA (6)	CSA (7)	SSA (8)	MENA and EE (9)
$1 - e$	0.888***	0.864***	0.987***	0.894***	0.894***	0.868***	0.869***	0.938***	0.895***
	(0.013)	(0.024)	(0.011)	(0.008)	(0.013)	(0.019)	(0.023)	(0.011)	(0.017)
Probability of test: $e = 0$	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Countries	101	101	101	101	25	16	18	22	20
Method	F.E.	A.B.	B.B.	P.S.	P.S.	P.S.	P.S.	P.S.	P.S.

Notes: We denote significance of 1% by ***, of 5% by **, and of 10% by *. Estimation methods are: F.E.—fixed effects, A.B.—Arellano–Bond, B.B.—Blundell–Bond, and P.S.—Pesaran-Smith. CSA, Central and South America; EA, East Asia; EE, Eastern Europe; SSA, South Saharan Africa.

Table 8. Cointegration test of technology over Global technology

Coefficient	1970–2017 (1)	OECD (2)	EA (3)	CSA (4)	SSA (5)	MENA (6)	EE (7)
d	−0.182	−0.107	0.502	−0.625	0.145	−1.572*	0.726
	(0.261)	(0.223)	(0.658)	(0.539)	(0.723)	(0.880)	(0.816)
EC	0.052***	0.073***	0.074***	0.033***	0.042***	0.045***	0.067***
	(0.005)	(0.015)	(0.012)	(0.008)	(0.009)	(0.010)	(0.011)
Probability of test $d=1$	0.000	0.000	0.449	0.003	0.237	0.004	0.738
Countries	93	23	15	16	22	12	5

Notes: We denote significance of 1% by ***, of 5% by **, and of 10% by *. CSA, Central and South America; EA, East Asia; EC, Error Correction; EE, Eastern Europe; SSA, South Saharan Africa.

countries. There has been rapid expansion of education in recent decades in many countries, but many of the graduates are either unemployed or compromise on jobs of much lower quality. One of the main reasons for the Arab Spring has been the plight of such graduates. This has not been the case at the USA, where the newly educated usually find jobs that fit their human capital. Hence, our measure of human capital is probably higher than the real one for many countries, while not for the USA. As a result, our measure of technology from equation (12) is downward biased for many countries. It reflects not only lower technology, but also inefficiency in creating jobs for skilled workers. That can explain why the estimated values of d in Table 8 are much lower than in Tables 3 and 4 and especially in MENA.

7. ROBUSTNESS OF RESULTS

This paper focuses on separating the dynamics of output per worker and of productivity, LATFP. It shows that while output per worker converges to productivity, productivities tend to diverge across countries. However, these results might depend significantly on the data we use. Although data from PWT on output per capita have been used in most studies on international economic growth and are central in the literature on convergence and divergence, there have been some critiques on it, especially on discrepancies between different versions of PWT, as in Johnson et al. (2013) and Karabarbounis and Neiman (2014). Such critiques might have greater potential from version 8.0 on, as data on capital stocks are harder to collect and can be less reliable, especially in less developed countries. One reason for problems in data quality can be the Purchasing Power Parity (PPP) procedures the data go through.

We try to overcome such difficulties by choosing data of national accounts, which use less PPP procedures and are more suitable to analyze dynamics over time. However, it is still important to examine how our results are affected by

Table 9. A comparison of different versions of PWT

Data set	*b* Panel growth regression	*b* Panel cointegration test	*d* Cointegration 1970–2008 All countries	*d* Cointegration 1970–2008 OECD	*d* Cointegration 1970–2008 CSA
PWT 8.0	0.038	0.031	0.325	0.670	−0.009
PWT 9.0	0.020	0.027	0.367	0.570	−0.163
PWT 9.1	0.024	0.025	0.515	0.830	0.036

Note: CSA, Central and South America.

various data sources. Thus, this section presents two comparisons. First, we compare our results with different versions of PWT and then compare our results with other data sets on economic growth, mainly with longer horizon. First, we examine different versions of PWT, which enable us to calculate productivity, namely PWT versions 8.0, 9.0 and 9.1. Table 9 presents a comparison of the results of these three versions of PWT for a number of parameters examined in this paper.

The first column in Table 9 compares the results for b in the panel regression (5) in Table 1 in this paper, for the three versions of PWT. The second column compares the results for b in the cointegration regression (1) in Table 2. Both columns show that the results of the different versions are not equal, but they are quite similar. The rate of convergence of output per worker to productivity is around 2.5% and is highly significant in all data sets.

The third, fourth, and fifth columns in Table 9 compare the results for d in the cointegration tests for the years 1970–2008, as in Table 4 in this paper. The reason we choose this measure is mainly to be able to compare all three versions of PWT over the same period. The results for the whole set of countries are in column 3, for OECD countries in column 4, and for Central and South America in column 5. Here again, there is some fluctuation between the different versions of PWT, but the main results are robust. The average d for the whole set of countries is around 0.45 and does not differ much from it. The d for the OECD countries differs from 0.57 to 0.83, but in all three versions the probability that it is equal to 1 is zero. The coefficient d for Central and South American countries is close to zero and does not differ much over the various data sets.

Note that we cannot compare previous PWT versions to our results, since they do not enable us to calculate productivities. However, we can compare the tests that we present in Section 5.3, where we run cointegration tests of output per worker in each country on output per worker in the USA. The cointegration coefficient in Table 6 is 0.549 for all countries over the years 1970–2008. The cointegration coefficient for the same regression over the same period, using PWT 7.0, is 0.44. Note that since PWT 7.0 does not require data on capital, the regression covers a larger set of countries, so the results are affected by it. A more interesting comparison between PWT 9.1 and PWT 7.0 is when we compare the same regression for OECD countries, with the same sample and over the same period 1970–2008. Our coefficient of cointegration is 0.65, which the coefficient using PWT 7.0 is 0.83. Clearly, there is a gap between the two coefficients, but both are significantly lower than 1.

We can therefore deduce that there are indeed discrepancies between the different versions of PWT, but these gaps are not too large. The main results of the paper hold in all three versions in Table 9. First, output per worker in each country converges to the country's productivity path, namely to LATFP. Furthermore, the rate of this convergence is around 2.5%, quite similar to the 2% "iron rule" of Barro (2015). Second, productivity in many countries does not fully follow the path of the global productivity frontier. Actually the average rate of following the frontier, or average *d*, is significantly below 1. Interestingly, this is true for OECD countries as well. Even in a cointegration test of output per worker in each country on the US output per worker, the average coefficient in PWT 9.1 is 0.646, it is 0.668 in PWT 9.0, and only in PWT 8.0 it is equal to 1. However, the coefficients of many other countries are much lower than 1.

Finally, we compare our results to another data set, which measures productivity, for a much longer period of time, but over a much smaller set of countries. Bergeaud et al. (2016, 2019) study TFP since 1890.[20] To examine our main divergence result, we use their data set, extended to the years 1890–2018, which covers 23 countries.[21] We run a panel cointegration of TFP of 22 countries over TFP of the USA. Note that this test is equivalent to our test, which uses LATFP instead of TFP, since in our calculation, ln(LATFP) is equal to ln(TFP) times 1.5. The results of the test are that the average *d* is 0.89. The standard error is very small, 0.041, so that the confidence interval at 5% is between 0.81 and 0.96. This means that the interval is below 1. This explains why the probability that *d* is 1 is equal to 0.006, namely less than 1%.

This result does not differ much from ours. The estimate of *d* for OECD countries, which are the countries in the long-run data set, is 0.74 for the period 1970–2017, and 0.83 for the period 1970–2008. These coefficients are quite similar to the coefficient of the long-run regression. Interestingly, we get a much lower estimate of *d* when we run the test for the same 22 countries, using PWT 9.1 data over the years 1951–2017. The coefficient *d* in this period is 0.478, and its standard error is larger at 0.153. Hence, its confidence interval is between 0.18 and 0.78. The probability that the coefficient is 1 is still low at 0.001. Thus extending the period from the last 66 years to a much longer period of 128 years yields a much more precise estimation of *d*, which is of course not surprising. However, using this long-run data limits the analysis to the developed countries, which are close to the global frontier anyway. Using PWT enables us to expand the analysis over a much larger set of countries, and more importantly, to the countries that lag behind.

Finally, we should add some comments on the challenge to our approach of cross country empirics from a new literature that studies convergence of specific sectors and not the whole economies. This literature began with Inklaar and Timmer (2009), Rodrik (2013), and recent contributions are Levchenko and Zhang (2016) and Battisti et al. (2018). They find that there is more convergence in sectors, but it appears mainly in traditional sectors. This is interesting, but of limited effect on the overall growth, due to recent processes in the global economy. The Western economy shifts from traditional sectors, like manufacturing, to sectors that supply global services, like computer and information services, communication, financial services, higher education, and science.

8. CONCLUSIONS

This paper introduces a new idea to the analysis of convergence and divergence in economic growth. The idea is to separate the dynamics of output per capita, or more precisely output per worker, and the dynamics of productivity. What enables us to apply this idea is the arrival of relatively new data that are part of PWT since version 8.0 in 2015. These data sets cover not only output but also inputs of labor and capital. This enables calculation of productivity for many countries over a long period of time, of almost 50 years, using the Solow method of "growth accounting."

The data on productivity enable us to examine two important issues. The first is how output follows the path of TFP over time. We find that it converges to it at an annual rate of around 2%. The second is to examine whether productivities of individual countries follow the global frontier, and we find that in many countries productivity diverges away from the frontier not only in level but in ratio as well. These two main results have important implications to the literature on convergence and divergence.

The first implication is that the result of standard growth regressions, that the rate of growth depends negatively on the initial level of output, cannot be interpreted as indicating convergence of output across countries. It only means that output follows the level of productivity over time. This is interesting, since many growth regressions claim that this is the main result they search, and they add explanatory variables mainly to control for variables they do not measure directly, like productivity. Once productivity becomes available, the need to use such explanatory variables as controls is reduced. However, we can still look at regressions of the rate of growth on various explanatory variables as a way to improve our understanding of economic growth across countries. In this case, there should be more attention to important issues like endogeneity of the explanatory variables. For example, a variable such as the rate of investment, which appears in many growth regressions, is clearly highly endogenous. Similarly, fiscal policy variables are also quite endogenous as well.

The results of this paper also support many claims that there is still significant divergence in economic growth. However, as shown in JP, the dynamics are quite different across regions and even across countries. This is also demonstrated in our paper. This finding indicates that the search for understanding why countries diverge should not be too general. We should look more carefully into specific regions and even into specific country case studies, in order to improve our understanding of such lagging behind the frontier. Actually, our measures show that even the degree of lagging behind differs across countries.

Some recent papers claim that analyzing differences in levels of output across countries are more important than analyzing differences in rates of growth. However, this is not the case, if long-run rates of growth differ significantly across countries over long periods of time, as shown in this paper. Then, the distribution of income levels changes continuously. In other words, countries are poor because they have followed the frontier partially for a long time. This implies that we should refocus our research on differences in growth rates. This view is also supported in the recent survey by Jones (2016).

The empirical result of the paper on divergence of productivity and of output per worker across countries holds of course during the period we study, which is 1970–2017. We hope that this result will change and countries in East Asia, Africa, and Latin America will continue to catch up with the frontier, or that even other countries will join them. However, when we add these last lines to the paper, the world sinks into the Covid-19 Recession and the near future looks gloomy, especially for developing countries. We hope that the recession will end soon and many poor countries will be able to renew their efforts to catch up with the global frontier.

NOTES

1. Actually JP discusses an earlier version of this paper.
2. As discussed in Section 2 and in Appendix A.3, our results do not depend on the assumption that productivity is labor augmenting and they hold for standard TFP as well.
3. Appendix A.2 to this paper presents a growth model with adjustments costs to investment. The Appendix shows that the rate of catching up of output to productivity should indeed be close to 2 percent.
4. Early other papers are Mankiw et al. (1992) and Barro and Sala-i-Martin (1992). Even earlier papers are Baumol (1986) and Kormendi and Meguire (1985). See DJT for a thorough survey of this literature.
5. An important generalization of β-convergence models are ‘varying parameters models’ by Liu and Stengos (1999), Durlauf et al. (2001), Lee et al. (1997, 1998) and others.
6. See Jones (2016).
7. See Barro (2015) and Jones (2016) for recent results of σ-divergence. See also Pesaran (2007) and Henderson and Russell (2005).
8. One important econometric critique on growth regressions is that they confuse convergence with regression to the mean. See Friedman (1992), Quah (1993) and Pritchett and Summers (2014).
9. See Durlauf (2009).
10. This series is chained, so it is also PPP (Purchasing Power Parity) adjusted.
11. We can also add to the rate of interest and the rate of depreciation a country specific risk premium.
12. More precisely, in an adjustment cost model the long-run marginal productivity of capital should also depend on the rates of growth of productivity and population, as shown in Appendix A.2 and as supported by our empirical analysis below. However, marginal productivity of capital of a country still converges to a constant level.
13. This was the assumption used in most growth regressions, as in Mankiw et al. (1992), Barro and Sala-i-Martin (1992) and many others which followed them.
14. A non-parametric study that differs from this assumption is Henderson (2010).
15. We use only two control variables, rate of growth of productivity and of population, in only two of the tests, as explained below in Section 4.1.

16. See Appendix A.2 for a formal analysis in an adjustment costs model.

17. The value of the smoothed variable is the average over the current and previous four years. For the early years, if there is no preceding data, the average is only over the available years.

18. For a survey of the methods of cointegrated panel heterogeneous estimations see Pedroni (2019).

19. Since the MG estimator is unconstrained, even 1 or 2 outliers may strongly affect the average coefficient and the standard errors, as noted by Eberhardt and Teal (2013). We have only 2 extreme values in the 1970-2014 sample and report the results without them, and no extreme values in the 1950–2014 sample. We tried two robustness checks. First, we applied the pooled mean group estimator assuming that the coefficient of cointegration is uniform across countries, for the case of 1970-2014 it came out 0.94. Second, we computed the median of MG estimation coefficients and it is equal to 0.90.

20. Their data are not based on PWT, but rather on the Maddison Project.

21. The countries are Australia, Austria, Belgium, Canada, Switzerland, Chile, Germany, Denmark, Spain, Finland, France, UK, Greece, Ireland, Italy, Japan, Mexico, Netherland, Norway, New Zealand, Portugal, Sweden and the US.

22. In the Appendix we delete the country index j, as we discuss a single country throughout.

23. This open economy model fits exactly the canonical growth model of DJT. It can also be adjusted to the estimations of this paper. An economy that follows the global frontier by a rate d and the global frontier grows at a constant rate gg, then productivity in this country grows at an average rate $g = dgg$.

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APPENDIX A

A.1. "GROWTH ACCOUNTING" IF TFP IS LABOR AUGMENTING

Assume that productivity is labor augmenting, as in equation (1) in the paper:[22]

$$Y(t) = F\left[K(t), A(t)L(t)\right].$$

The differential of the change in output between period $t-1$ and t is described by the following equation, where the derivatives are taken in period $t-1$:

$$\begin{aligned} Y(t) - Y(t-1) &= F_K(t-1)\left[K(t) - K(t-1)\right] \\ &\quad + F_L(t-1)A(t-1)\left[L(t) - L(t-1)\right] + F_L(t-1)L(t-1)\left[A(t) - A(t-1)\right]. \end{aligned}$$

Divide by output at time $t-1$ and get

$$\begin{aligned} \frac{Y(t) - Y(t-1)}{Y(t-1)} &= \frac{F_K(t-1)K(t-1)}{Y(t-1)} \frac{\left[K(t) - K(t-1)\right]}{K(t-1)} \\ &\quad + \frac{F_L(t-1)A(t-1)L(t-1)}{Y(t-1)} \frac{\left[L(t) - L(t-1)\right]}{L(t-1)} \\ &\quad + \frac{F_L(t-1)A(t-1)L(t-1)}{Y(t-1)} \frac{\left[A(t) - A(t-1)\right]}{A(t-1)}. \end{aligned} \tag{A1}$$

Note that the elasticities of output with respect to labor and to capital are

$$\eta_{Y,L}(t-1) = \frac{F_L(t-1)L(t-1)}{Y(t-1)}, \text{ and } \eta_{Y,K}(t-1) = \frac{F_K(t-1)K(t-1)}{Y(t-1)}.$$

The sum of these elasticities is equal to 1, due to CRS. Hence, we get from equation (A1)

$$\begin{aligned} \frac{Y(t) - Y(t-1)}{Y(t-1)} &= \left[1 - \eta_L(t-1)\right] \frac{K(t) - K(t-1)}{K(t)} \\ &\quad + \eta_L(t-1) \frac{L(t) - L(t-1)}{L(t)} + \eta_L(t-1) \frac{A(t) - A(t-1)}{A(t)}. \end{aligned}$$

From this equation, we can derive the rate of change of productivity:

$$\begin{aligned} \frac{A(t) - A(t-1)}{A(t-1)} &= \frac{1}{\eta_L(t-1)} \left[\frac{Y(t) - Y(t-1)}{Y(t-1)} - \frac{K(t) - K(t-1)}{K(t-1)}\right] \\ &\quad + \frac{K(t) - K(t-1)}{K(t-1)} - \frac{L(t) - L(t-1)}{L(t-1)}. \end{aligned} \tag{A2}$$

Under the assumption of perfect competition, $F_L(t-1)A(t-1) = MPL(t-1)$ and hence the elasticity of output with respect to labor is equal to the share of labor in output: $\eta_L(t-1) = s_L(t-1)$. We can rewrite equation (A2) as follows:

$$\begin{aligned} \frac{A(t) - A(t-1)}{A(t-1)} &= \frac{1}{s_L(t-1)} \left[\frac{Y(t) - Y(t-1)}{Y(t-1)} - \frac{K(t) - K(t-1)}{K(t-1)}\right] \\ &\quad + \frac{K(t) - K(t-1)}{K(t-1)} - \frac{L(t) - L(t-1)}{L(t-1)}. \end{aligned} \tag{A3}$$

In the empirical analysis, we examine two options, one is to use the share of labor in PWT 9.0 as a measure for the elasticity $\eta_L(t-1)$, and the second is to assume that it is constant and equal to 2/3, as suggested by Jones (2016).

A.2. CONVERGENCE IN A SMALL OPEN ECONOMY WITH ADJUSTMENT COSTS

Consider a small open economy with full capital mobility facing a constant global interest rate r. Output in the economy in period t is described by the Cobb–Douglas production function:

$$Y(t) = K(t)^{\alpha}\,[A(t)L(t)]^{1-\alpha}\,. \tag{A4}$$

where $Y(t)$ is the output, $L(t)$ is the labor, and $K(t)$ is the amount of capital invested prior to t. Capital depreciates at a rate dep. Productivity A and population N increase at constant rates:

$$A(t) = A(0)e^{gt}, \text{ and } N(t) = N(0)e^{nt}. \tag{A5}$$

Here g and n are positive numbers.[23] Each person supplies one unit of labor per period, so $L = N$. Investment has adjustment costs, which are assumed to be quadratic and have CRS:

$$a(t) = \frac{1}{2m}\frac{[K(t+1) - K(t)]^2}{K(t)}. \tag{A6}$$

The parameter m is an inverse measure of the intensity of these costs. Also assume that the economy is open and has full capital mobility. The global interest rate is r.

Due to the constant returns to scale of the production and the adjustment cost functions, the value of each firm is proportional to its capital and marginal q is equal to average q, as shown in Hayashi (1982). Hence, the market value of capital $V(t)$ satisfies:

$$V(t) = q(t)K(t+1).$$

The price $q(t)$ is the economy-wide value of one unit of capital. Denote the wage rate in period t by $w(t)$. Profit maximization by firms leads to the following two first-order conditions. The first is with respect to labor:

$$w(t) = (1-\alpha)K(t)^{\alpha}A(t)^{1-\alpha}L(t)^{-\alpha}. \tag{A7}$$

The second is with respect to capital so that the rate of capital accumulation is

$$\frac{K(t+1) - K(t)}{K(t)} = m\left[q(t) - 1\right]. \tag{A8}$$

We next introduce the equilibrium conditions. Labor market equilibrium requires

$$L(t) = N(t).$$

Due to capital mobility and lack of risk, the returns on capital and on lending are equal, so that

$$q(t)(1+r) = MPK(t+1) + q(t+1) - dep + \frac{m}{2}\left[q(t+1) - 1\right]^2. \tag{A9}$$

This is the condition of equilibrium in the capital market.

To better describe the dynamics of the economy, we transform the dynamic variables to better fit the empirical model. Instead of the price of capital we use: $Q(t) = q(t) - 1$, and instead of capital we use the natural logarithm of its marginal productivity: $x(t) = \ln[MPK(t)]$. From (A9), we get

$$Q(t)(1+r) = \exp[x(t+1)] + Q(t+1) - (r+dep) + \frac{m}{2}Q(t+1)^2. \quad \textbf{(A10)}$$

The dynamics of x are derived from (A5) and (A8):

$$x(t+1) = x(t) + (1-\alpha)\{g+n-\ln[1+mQ(t)]\}. \quad \textbf{(A11)}$$

The equilibrium solution to this dynamic system, (A10) and (A11), is a saddle path, which is described by the function: $Q(t) = Q[x(t)]$, where Q is the monotonic increasing. From equation (A11), we get that the steady-state price of capital is described by

$$Q^* = \frac{g+n}{m}. \quad \textbf{(A12)}$$

Applying it with equation (A10) and using a log-linear approximation, we get

$$x^* = \ln(r+dep) + \ln\left[1+\frac{g+n}{m}\frac{r-(g+n)/2}{r+dep}\right]. \quad \textbf{(A13)}$$

We next turn to connect this model more to the convergence assumption in the paper. Note that efficiency output per worker, $y^E(t)$, satisfies

$$\ln y^E(t) = \frac{-\alpha}{1-\alpha}\,[x(t)-\ln\alpha]. \quad \textbf{(A14)}$$

Hence, efficiency output per worker converges to a steady state $\ln y^E(\infty)$ along the saddle path. This steady-state efficiency output per worker can be calculated from (A12) and (A13) and is equal to

$$\begin{aligned}\ln y^E(\infty) &= \frac{\alpha}{1-\alpha}\left\{\ln\alpha - \ln(r+dep) - \ln\left[1+\frac{g+n}{m}\frac{r-(g+n)/2}{r+dep}\right]\right\}\\ &\cong \frac{\alpha}{1-\alpha}\left[\ln\alpha - \ln(r+dep)\right]. \quad \textbf{(A15)}\end{aligned}$$

Note that although r is the same for all countries, and α and dep are technological parameters that should also be the same for all countries, the long-run efficiency output per worker depends negatively on the rate of growth of productivity g, as found in the paper. But equation (A15) shows that this effect is small in size.

From (A11) and (A14), we derive the dynamics of efficiency output per worker:

$$\ln y^E(t+1) = \ln y^E(t) + \alpha m Q\left[\ln\alpha - \frac{1-\alpha}{\alpha}\ln y^E(t)\right] - \alpha(g+n). \quad \textbf{(A16)}$$

Hence, the efficiency output per worker follows dynamics of convergence similar to those described by equation (10) in the paper. The rate of convergence of y^E in the neighborhood of the steady state is

$$b = (1-\alpha)mQ'(x^*).$$

We next try to estimate the size of b. One way to find it is to calculate the slope of the saddle path at the steady state, $Q'(x^*)$. Another way to estimate the rate b is to examine the dynamics of capital accumulation using a first-order approximation around the steady state. We get

$$\ln K(t+1) - \ln K(t) = n + g + mQ'(x^*)\frac{MPK(t)-MPK^*}{MPK^*}.$$

Hence,

$$b = (1-\alpha)MPK^* \frac{\partial[\ln K(t+1) - \ln K(t)]}{\partial MPK(t)} \cong (1-\alpha)(r + dep)\frac{\partial[\ln K(t+1) - \ln K(t)]}{\partial MPK(t)}. \quad \textbf{(A17)}$$

This equation enables us to reach a rough estimate of b. We can assume, for example, by comparing China today with the USA, that the effect of MPK on the rate of growth of capital should be somewhere between 0.3 and 0.5. According to standard assumptions, $r + dep$ is around 0.1 and $1-\alpha = 0.65$. Hence, the rate of convergence b should be somewhere between 1.7% and 3.2%. Therefore, the open economy model yields a rate of convergence that fits the data well, unlike the closed economy models used in many other growth regressions, as shown by DJT.

A.3. NON-LABOR AUGMENTING PRODUCTIVITY

Assume that productivity is not labor augmenting, but output augmenting as in Solow (1956) and many other studies:

$$Y(t) = A(t)G\,[K(t), L(t)].$$

The long-run equilibrium condition in an open economy is

$$A(t)G_K\left[\frac{K(t)}{L(t)}, 1\right] = r^* + dep. \quad \textbf{(A18)}$$

However, to derive how the capital–labor ratio and output depend on productivity in equilibrium, we need to use a specific production function. Assume that it is the Cobb–Douglas production function. Then, equation (A18) yields the following long-run capital–labor ratio:

$$\frac{K(t)}{L(t)} = \left(\frac{\alpha}{r^* + dep}\right)^{\frac{1}{1-\alpha}} A(t)^{\frac{1}{1-\alpha}}.$$

As a result, the long-run output per worker is equal to:

$$y(t) = \left(\frac{\alpha}{r^* + dep}\right)^{\frac{\alpha}{1-\alpha}} A(t)^{\frac{1}{1-\alpha}}. \quad \textbf{(A19)}$$

Hence, we can define the following variant of "efficiency output per worker":

$$y^E(t) = \frac{y(t)}{A(t)^{\frac{1}{1-\alpha}}}.$$

Clearly, equation (A19) implies that this efficiency output per worker converges in the long run to a constant equal to:

$$\left(\frac{\alpha}{r^* + dep}\right)^{\frac{\alpha}{1-\alpha}}.$$

This means that if productivity is not labor augmenting but output augmenting, output per worker converges not to labor augmenting productivity but to output augmenting productivity in the power of $1/(1-\alpha)$. However, Equation (A3) implies that empirical LATFP is equal, if we use a Cobb–Douglas production function, to the empirical output augmented

TFP in the power of $1/(1-\alpha)$. Hence, the results of the model and the empirical results do not change if we use labor augmenting TFP or output augmenting TFP.

A.4. CONVERGENCE OF CAPITAL–OUTPUT RATIOS

If we apply equation (A2) to calculation of the rate of growth of efficiency output per worker, we get that for each country:

$$\frac{y^E(t)-y^E(t-1)}{y^E(t-1)}=\left[\frac{K(t)-K(t-1)}{K(t-1)}-\frac{Y(t)-Y(t-1)}{Y(t-1)}\right]\frac{1-\eta_L(1-1)}{\eta_L(1-1)}. \qquad \textbf{(A20)}$$

If we adopt the assumption that the output elasticity of labor is constant at 2/3, we get

$$\frac{y^E(t)-y^E(t-1)}{y^E(t-1)}=\left[\frac{K(t)-K(t-1)}{K(t-1)}-\frac{Y(t)-Y(t-1)}{Y(t-1)}\right]\frac{1}{2}.$$

Hence, the rate of growth of efficiency output per worker is half of the rate of growth of the capital–output ratio. This means that convergence of one variable to a long-run constant is equivalent to convergence of the other variable to a long-run constant.

It is important to note that both convergences are a result of the convergence of the marginal productivity of capital to a constant. As shown in Section 3, it leads to convergence of efficiency capital–labor ratio in a country to a constant, which is country-specific,

$$k^E(t)\xrightarrow[t\to\infty]{}k^E(\infty).$$

Hence, the capital–output ratio satisfies

$$\frac{K(t)}{Y(t)}=\frac{K(t)/[A(t)L(t)]}{Y(t)/[A(t)L(t)]}=\frac{k^E(t)}{G[k^E(t),1]}\xrightarrow[t\to\infty]{}\frac{k^E(\infty)}{G[k^E(\infty)]}=\frac{k^E(\infty)}{y^E(\infty)}. \qquad \textbf{(A21)}$$

Therefore, the convergence of the capital–output ratio and of the efficiency output per worker are both the result of the convergence of the marginal productivity of capital to a long-run constant.