



ICMI STUDY 25

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Teachers of Mathematics Working and Learning in Collaborative Groups



FEBRUARY 3-7, 2020

LISBON, PORTUGAL

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Teachers of Mathematics Working and Learning in Collaborative Groups

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UNIVERSITY OF LISBON



FEBRUARY 3-7, 2020

CONFERENCE PROCEEDINGS

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Study Themes

ICMI STUDY 25

- Theme A** | Theoretical Perspectives on Studying Mathematics Teacher Collaboration
- Theme B** | Contexts, Forms and Outcomes of Mathematics Teacher Collaboration
- Theme C** | Roles, Identities and Interactions of Various Participants in Mathematics Teacher Collaboration
- Theme D** | Tools and Resources Used/Designed for Teacher Collaboration and Resulting from Teacher Collaboration
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Plenary Lectures

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- Theme A** | **Susanne Prediger** Germany
Content-specific theory elements for explaining and enhancing teachers' professional growth in collaborative groups
- Theme B** | **Masami Isoda** Japan
Producing theories for mathematics education through collaboration: a historical development of Japanese lesson study
- Theme C** | **Konrad Krainer** Austria
Collaborative groups in mathematics teacher education: grasping the diversity of roles, identities and interactions
- Theme D** | **Karin Brodie** South Africa
Resources for and from collaboration: a conceptual framework

COLLABORATIVE TEACHING IN THE ITALIAN “LICEO MATEMATICO”: A CASE STUDY OF CO-PLANNING AND CO-TEACHING

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This contribution is centred on a case study of collaborative teaching, carried out among Upper Secondary School Math teachers. Here we present the context, the forms of the implemented collaboration and the effects that derived from the collaborative teaching. The used investigation tool was the semi-structured interview aimed to induce teachers to gradually reflect on their self and their teaching modus operandi. From the qualitative analysis emerged that collaborative teaching integrated different ways of teaching; it has been a stimulus for a etching “revision” and the possibility to improve the didactic-educational practice. For all teachers this way to define the teaching activities was a valuable “tool” to see mathematics as part of a unified knowledge, creating bridges between scientific and humanistic disciplines, in terms of content and methods.

Introduction

The focus of the paper is referred to the study of contexts, forms and dynamics that can characterize the collaborative teaching. We are conscious that the variety of these elements are important for the way in which the teaching collaboration takes place. In this sense, it could offer different analysis about the teaching/learning phases in classroom and so differed interpretation about the effective of this teaching modus operandi. For this reason we clearly declare, just at the beginning of our paper, the examined context: it concerns a small group of teachers from various disciplines (specifically, Mathematics, Physics, Literature and English), engaged in co-design and Mathematical co-teaching of a Upper Secondary School. The particular chosen context is related to a classroom that attends the first year of the Scientific Upper Secondary School “Benedetto Croce” in Palermo and that, at the time of enrolment, has chosen to join the experimental project of the Mathematical Upper Secondary School, called “Liceo Matematico” (hereinafter referred to, briefly, as LM). This reality is almost new for the Italian school (but maybe also for other countries), it was promoted by the University of Salerno and in few years it has spread to various parts of Italy, coming to live with about one hundred schools on the national territory and leaving a strong imprint of collaboration between Upper Secondary School teachers (teachers of Mathematics and beyond) and University professors belonging to departments of different areas (for further information, we suggest visiting www.liceomatematico.it). Here we present some results related to the terms in which the co-teaching (and the related co-planning of the Mathematics lessons) has been implemented. The research question to which we tried to reply is the following: from the point of view of the Upper Secondary School Mathematics teachers, what kind of possible results can be observed in classroom, defining a Collaborative Teaching approach in this classroom? In order to answer, we decided to conduct a semi-structured and audio-registered interview with almost all teachers attended the LM. According to Egodawatte et al. (2011) there is a strict correlation between the effective teachers collaboration and its effect in the improving of the students learning phase in Mathematics. Referring to their tool (a structured teachers interview), we modified it to better adapt

it to the Italian school context and, specifically, to the socio-cultural context in which the classroom was located. In particular, each teacher was semi-structured interviewed; the interview was organised into several parts and related to the personal background (on the educational and work levels), the phase of didactical planning in the LM, the used collaboration approach with other teachers implemented, the possible benefits that emerged from this choice in terms of co-design and/or effective implementation in the classroom of the “interdisciplinary approach”. In the following paragraph we briefly explicit the theoretical framework and subsequently, the research methodology adopted is discussed. The structure of interviews, mentioned above, is described. Finally, the results of the qualitative analysis related to the collected teachers replies is analysed and discussed.

Theoretical Framework

In literature the term *collaboration* has been declined in different ways in various areas of knowledge, from social psychology to education, from training in general to politics or sociology, just to name a few. Although different definitions of the term can be found within the various disciplines, in the context of teaching the term is used to describe a way in which two or more people work together to achieve a common purpose. When the people involved are teachers of one or more schools, the working context can be characterized, from a theoretical point of view, as a “cooperative learning Community” (Cooper, Boyd, 1997), and/or a “professional learning Community” (DuFour, 2004), and/or a “Community of practice” (e.g. Wenger, 2007). With reference to the latter one, for example, Wenger (1998) stresses the importance of sharing objectives within the Community of practice where the actors involved have common interests and collaborate in order to support the learning of the entire group and of each member of the same group. The common base of the above defined contexts seems to be the definition of a “Community of teachers” in which the individual teacher no longer acts “in isolation” but within a team made up of other educators (internal to the school or belonging to other educational agencies). Working in a collaborative way implies that everyone brings their own experience, their own professionalism, their own thinking to the work team, with a view to sharing and integrating rather than 'summing up' individual contributions. In this way, collaboration can provide opportunities to reflect on one's teaching practice (Thomas, Pedersen, 2003), to encourage a motivated and responsible attitude (Calderhead, Gates, 1993), to share opinions and criticisms of others' practice and to support change (Clark et al., 1996). In this sense, in order to set a fruitful collaboration, a large number of factors play a significant role. According to Jao & McDougall (2016) one of them is to create a working group whose members share the same teaching contexts. Additional factors include beliefs, among the group members and the sense of community (Dunne, Nave, Lewis, 2000) perceived by them. Trust in colleagues is an essential prerequisite for working with a collaborative approach. After the first working experiences with colleagues, this trust could gradually turn into respect, confidence and esteem, and could pave the way for a collaborative dialogue useful both for teachers and students. Finally, it is precisely through collaborative activities that the sense of community mentioned above can be affirmed (Grossman, Wineburg, Woolworth, 2001). Lieberman (1986, p.6) stated: “Contexts, needs, talents and commitments differ, but one thing appears to be constant: school cannot improve without people working together”. His thinking shows the importance of working in groups and of forming groups, within the school environment, to promote the professional development of teachers and the school in general. In another text, he reiterates this

important aspect as follow: “each school needs to establish a collaborative culture as a precondition for its own development” (Lieberman, 1990, p. 9). According to this, several empirical studies (e.g. Harris et al. 2006) shown how a collaborative teaching can be an effective approach to achieve educational goals (in relation to a given classroom context), providing educational resources and practices and providing opportunities for professional development such as co-planning and co-teaching (or cooperative teaching, understood in this contribution as a form of teaching in which two teachers come together to teach together in some course). Collaborative teaching also induces teachers to redefine their teaching practices and supports their mutual interactions (Wilson, Berne, 1999). McLaughlin , Talbert (2001) underlined that in a Upper Secondary school this can mainly be done at the level of the disciplinary departments, as a common ground in which to share norms concerning teaching/learning, student evaluation and classroom experiences (Gutierrez, 1996). Other studies have shown that collaborative teaching works in good (effective) way when the trust within the work team is supported by the presence of educational aim shared by all members of the group (Sydow, 2000) by relationships established previously between teachers and by an explicit management aimed to facilitate the collaboration (Muijs et al., 2006). Our research fits into this research area: as we already mentioned before, we in fact want to investigate how the collaboration between teachers of the LM has been implemented and how it influenced the interaction between teachers in the form and the quality of the teaching/learning process in their classroom. In agreement with Schoenfeld (2002) and Dallmer (2004), giving to teachers the opportunity to develop their skills and to increase their knowledge, could allow them to have more positive feedback on the students performance in Mathematics and so to interpret this in different way. We tried to do this in the context of the LM, where teachers from different departments worked together sharing goals and different approaches. It is not the aim of this paper to investigate about the feedback of collaborative teaching on the students learning, our focus is in fact on the teachers Community.

Research methodology

The research moves from the new experimental project of the LM in Palermo, a project that involved first of all in a training course for university teachers and teachers of the Upper Secondary schools of the second degree in which two first mathematics classrooms have been activated. The strong point of this course was to propose lesson and laboratory activities in which different areas of knowledge are integrated with Mathematics and into Mathematics. From these first training ideas, during the 2018/19 school year, a fruitful collaboration between some teachers was born. The forms in which this collaboration was carried out can be divided into two types essentially: workshops, aimed at co-designing the lessons and objectives, and teaching in small groups. For the purposes of the research, it was decided to investigate these forms of collaborative teaching, focusing on the results that teachers have found in their teaching practice and in the students themselves. Specifically, the research to be presented was conducted at a section of LM formed by classroom pupils before a Scientific Upper Secondary School. The classroom council was characterized by the presence of a team of highly qualified teachers who participated with interest and enthusiasm in the design and implementation of ‘interdisciplinary lessons’ carried out in co-teaching, providing their professionalism and their wealth of knowledge and personal experience. The common goal of the teaching team was to encourage a meeting between mathematics and other

disciplines, to support a vision of mathematics as a discipline that integrates with others, supports them and can find in them various and concrete applications.

Collaborative teaching in the form of design and implementation of teaching activities

The team of co-involved teachers actively participated in the co-design of the activities of the LM. The activities were divided into “themes” and for each of them, the classroom council indicated the topics, the laboratory activities, the prerequisites, the objectives to be achieved and the competences and skills that the students would have been acquired. Considering the additional time that some activities required, for logistical reasons or sometimes for inadequacy of teaching technologies in the classroom, during the school year under review (2018/19) was partially implemented only one theme, the theme of the *measure*. Its realization has seen the co-presence in the classroom of teachers from the Department of Mathematics, with the collaboration of the teacher of English in the planning phase. The second part of the *measure* will continue in the next school year and will also see the teacher of Italian, Latin and History with the *agrimensor*.

Survey tool and data collection

The tool used to investigate the results produced by the collaborative teaching under examination was the semi-structured interview, in order to answer the research question. Each teacher was interviewed individually through the use of a recorder; at a later stage, the answers were transcribed and the data processed, as will be better described in the following paragraphs. The interview was designed on the basis of a previous work (Egodawatte et al., 2011) and was formulated in such a way as to adapt to the context of the LM. In defining the parts of the interview, account was also taken of the training that the teachers had received through the university professors of the Department of Mathematics and Informatics of Palermo. This training, in fact, had been an opportunity to present some interdisciplinary contents in which mathematics was linked to the use of artefacts, to the use of ICT (e.g. Geogebra) and in reference to didactic contents that were not exclusively mathematical but, on the contrary, included subjects from other disciplines (e.g. History, Art, Latin). The aim of this training was precisely to introduce the teachers of the various disciplines to create disciplinary links and, consequently, activities in which mathematics could act as a link between the various disciplines. A link between the disciplines would thus have a reflection on the training of the students, on their way of “seeing” mathematics but also on the teachers themselves. No longer a science in its own right, which can be more or less enjoyable than others, but a science that together with the others was born, has evolved and lets itself be discovered in its many facets. Precisely these aspects emerged, to a large extent, in the course of individual interviews. Further aspects that emerged from the qualitative analysis of the interviews conducted will be described in the next paragraph, in which the data collected from interviews with teachers involved in the research will be presented. These are five teachers, one of whom is in Literature, one in English, two in Mathematics and one in Mathematics and Physics. The interview was divided into five parts. The first part, inherent to the teacher's background, aims to collect generic information on the teacher's training, work experience and motivation to teach; the second part aims to induce a reflection on the school context in which he currently teaches, on any changes encountered and on the challenges posed by the school to respond to these changes; the third part concerns the activities of LM specifically (i.e., content, teaching methods, answers from pupils, objectives set by the teacher for the ‘interdisciplinary lessons’ planned); the fourth concerns the

objectives set by the teacher towards the classroom and his opinions on the perception of “success in Mathematics” by the students; the fifth and last part aims to focus on the intra- and interdepartmental collaboration with colleagues, on any effects that may have had on the attitude of students towards mathematics, on the benefits found by such collaborative teaching (in their teaching practice, in professional development, in the sharing of ideas, materials and strategies). Here, reference will be made to the results that emerged from the first four parts, to leave more space for the last.

Qualitative analysis of collected data and discussion

The analysis of the data collected during the interviews was qualitative, given the small sample involved. This was possible starting from the semi-structured form of the interview, which left teachers a certain freedom of expression in their answers and clarified, where necessary, the meaning of the questions asked to them. Each researcher listened several times to the recordings of the interviews to analyse the differences and similarities in the answers given to each question; specifically, each defined the categories of answer (e.g. Students Mathematics perception, particular choices about the didactic practices, example of multidisciplinary approaches), in relation to the variables that have been investigated, compared for each question the similar terminologies used by the interviewed and the meaning with which similar terms were used, compared the occurrences of common terms or terms similar to the same thematic field. After this first data analysis (done by each researcher, authors of the paper, separately), the two researchers met with the aim to compare the work done and find out possible common teachers reply. At this stage, these were categorised. We are working on a subsequent “quali/quantitativo” analysis based on the implementation of the data by the atlas.ti software.

Results

The results will be discussed in five sections, how many parts the interview is divided into, with particular reference to the most significant answers to some of the questions in the interview. *Teacher's background:* first of all, the teaching of Mathematics concerns teachers who include in their training curriculum studies in different scientific fields. Each of them has been teaching for at least eighteen years and, during the 2018/19 school year, the whole team was involved in the co-programming of the classroom's ‘interdisciplinary lessons’. *School context:* the transcript of the various interviews revealed some key words more frequent than others, such as “change” - in a purely negative sense. All teachers stated that in the last ten years they have found a progressive deterioration in the basic knowledge of students entering Upper Secondary School, combined - according to one teacher - with the change in learning styles and the worsening of “skills of expression and communication”. Someone else highlighted the fact that the “human material” of the school has changed - referring to the students - not only in the knowledge possessed by the incoming students, but also in their expectations and desires. *LM activities:* in this changing and dynamic context, the school has tried, through the LM, to combine the scientific and humanistic fields, encouraging the development of a “strong literary culture”. *Teacher's objectives and perception of “success in Mathematics” in the students:* when asked what do they think the students of the Mathematical Upper Secondary School consider to be “success in Mathematics”. For some, it is taking a ‘good’ grade, in reference to evaluation. For others, such ‘success’ also consists in understanding the links with the outside world or in being able to connect mathematics with other

fields never imagined before. Two teachers, in particular, believe that the students of the LM perceive as a 'success in Mathematics' the fact of having knowledge or having acquired additional skills compared to their peers, thanks to the experiences made during the LM. *Intra- and interdepartmental collaboration with colleagues*: regarding collaboration with colleagues, all respondents stated that they had previously collaborated with other colleagues to co-design paths that included multiple disciplines (Mathematics and English, Mathematics and Science, English and Science, Latin and Science). Four of them had also spontaneously implemented co-teaching in the past. Since three out of four teachers teach Mathematics and had previously collaborated with each other, the most interesting reflections emerged regarding the collaboration between colleagues from different departments. It was intended to investigate the influence on teachers through the question *What benefits did you receive from working with colleagues in other departments? (E.g. access to ideas, materials, strategies and skills of others, lesson planning, perception of the quality of one's teaching, professional development, professional gratification, ...)* and about the students through the subsequent question *In your opinion, what benefits have the students received (or will they receive in the future) from such intra- and interdepartmental collaboration?* Among the answers to the first question, there is that of a teacher who refers to having expanded the knowledge of individual teachers (for example, in the part of Mathematics and English, there was a marriage between the English language and the "mathematical language"), having increased the awareness of how much is spendable the teaching action in an interdisciplinary vision, having had the opportunity to intersect their teaching practice with other disciplines from the point of view of content and - even more difficult - from the point of view of method. Continuing, the teacher emphasizes that it is more difficult to share the method in an interdisciplinary collaboration and that, at the same time, this collaboration is even more profitable than a collaboration between colleagues in the same department, because seeing new teaching practices can open new horizons and improve the quality of their teaching. Someone else sees benefits in having experimented with new teaching strategies or in having perfected the times and methodologies previously used. Among the answers to the second question, it is noted that the students were given "the opportunity to see knowledge as unitary and not 'parcelled' in the various disciplines" (it was precisely one of the main objectives of the work team), having "learned to look at the classroom council as a community body" whose members design and work together for the classroom, having acquired a greater awareness of the role played by mathematics in the surrounding world, having learned a part of micro-language (useful for the scientific training of a student), seeing more integrated styles of teaching/learning and last but not least, having contributed to the creation of a certain "openness of mind through these multidisciplinary paths". In support of this, one teacher, speaking of hopes she has for her students, says she hopes that these paths, or rather, the 'alternative' vision they offer of Mathematics and Physics, will help students to "learn to think". In his case, this is a hope that can only be confirmed in the long term. Going a little deeper on the subject, with the question *In your opinion, how can collaboration between teachers influence students' attitudes towards mathematics?*, the answers were almost unambiguous. Everyone has said that such collaboration can influence this attitude in a positive way and, in particular, they agree that it can contribute to 'seeing' Mathematics no longer as a discipline in itself, detached from reality and written in a language incomprehensible to many, but as a discipline that has evolved with others, in history and time, which finds applications in various branches of knowledge. From their own answers:

“I think that in the end the pupils' attitude towards mathematics should be more open, considering it not only as a 'closed' discipline but as a discipline capable of permeating all the others, just as it should be for linguistics, which is a discipline that is absolutely transversal to any teaching.”

“I think they can understand that the whole world is mathematical and mathematics enters everywhere, in everyday life, in history, in the history of thought.”

“In the meantime it can give an idea of the unity of knowledge and then clearly they can see mathematics from different points of view in areas where it is not generally so easy to 'see' mathematics. It seems strange, but even the fact that the same language has often been used [...] has allowed us to see how, for example, the language of Statistics and the language of Error Theory in Physics are closely linked: the same term has been used in different fields, giving different facets but showing that the concept is always the same. We have noticed this, for example, with the concepts of 'dispersion', 'error' or 'mean value'. We have seen the 'average value' in the statistical field and the meaning of the 'average' as the 'average value' of a physical quantity, as the best estimate of the true value. [...]”

Conclusions

The research carried out aimed to examine the results of the qualitative analysis of the interviews conducted with the team of case study teachers who collaborated in the forms of co-planning and co-teaching. Given the small sample of only five teachers, we do not intend to generalize the results discussed here, but only to describe the aspects that emerged from the actors involved in the case study. This research also aims to be a contribution to the existing literature as it refers to collaborative teaching in multidisciplinary paths (e.g. Hurd, Weilbacher, 2018) and in a specific common teaching context (Jao, McDougall, 2016) which is that of the Mathematics Upper Secondary School, a project that is affecting more and more school realities in Italy. The innovative aspect was the application of collaborative teaching in a Mathematics Upper Secondary School: looking at the programming of the Mathematics courses from a multidisciplinary point of view, we wanted to promote a collaborative culture among teachers first and an integration of the disciplines then, to "develop a critical view of reality", as desired in the National Indications (Miur, 2010) of the Italian Ministry of Education. The qualitative analysis of the data collected has made it possible to highlight the benefits that the teachers have derived and the results achieved, both on a professional and human level, during the "interdisciplinary lessons" that have served as a framework for collaborative teaching. Specifically, the interviewees said they had increased, through collaboration with colleagues, their knowledge and skills in several areas related to each other (referring to the study from which we draw inspiration here): the achievement of objectives (first of all, that of the team), the success of the students (influenced by the integration of disciplinary content and methods of teaching-learning), professional development (derived from reflections on their own and others' teaching practice), the opportunities arising from co-planning and co-teaching (eg. improve teaching times and strategies). However, work is still in progress on the data collected through the atlas.ti software; this could lead to further interesting results or confirm what has been described above. Certainly, further questions, possibly more specific, or turning to a larger sample would have brought to light other aspects that are not described here. Although considering the limits of the research, the interview allowed us to respond, limited to the case study, to the research question and to derive food for thought from which to investigate collaborative teaching in wider contexts.

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