A new Frequency Domain Measure of Causality based on Partial Spectral Decomposition of Autoregressive Processes and its Application to Cardiovascular Interactions*

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Abstract— **We present a new method to quantify in the frequency domain the strength of directed interactions between linear stochastic processes. This issue is traditionally addressed by the directed coherence (DC), a popular causality measure derived from the spectral representation of vector autoregressive (AR) processes. Here, to overcome intrinsic limitations of the DC when it needs to be objectively quantified within specific frequency bands, we propose an approach based on spectral decomposition, which allows to isolate oscillatory components related to the pole representation of the vector AR process in the Z-domain. Relating the causal and non-causal power content of these components we obtain a new spectral causality measure, denoted as pole-specific spectral causality (PSSC). In this study, PSSC is compared with DC in the context of cardiovascular variability analysis, where evaluation of the spectral causality from arterial pressure to heart period variability is of interest to assess baroreflex modulation in the low frequency band (0.04-0-15 Hz). Using both a theoretical example in which baroreflex interactions are simulated, and real cardiovascular variability series measured from a group of healthy subjects during a postural challenge, we show that – compared with DC– PSSC leads to a frequency-specific evaluation of spectral causality which is more objective and more focused on the frequency band of interest.**

I. Introduction

The heart period (RR interval of the ECG) and the systolic arterial pressure (SAP) are known to dynamically interact in a closed loop, as a consequence of the baroreflex mechanism acting as a feedback of SAP on RR, and of feedforward mechanisms, mainly of mechanical nature, whereby SAP values are influenced by previous RR changes [1]. These cardiovascular interactions are often assessed in the frequency domain, in order to focus on specific oscillations such as those in the low frequency (LF, 0.04-0.15 Hz) and high frequency (HF, 0.15-0.4) bands, using coupling measures like the coherence, or causality measures like the directed coherence (DC) [2]. In particular the DC is widely used, in cardiovascular variability analysis and in many other fields, as a linear frequency domain measure of causal interactions between coupled multivariate dynamic processes. The DC is computed from the spectral representation of a vector autoregressive (AR) process, whose parameters provide the basis to separate the power spectral density (PSD) of a target process into partial spectra related to its own dynamics and to the dynamics of the other processes interpreted as sources [2], [3].

In the practical application of DC where the function needs to be quantified in specific frequency regions, empirical approaches are adopted which consist in taking its maximum value, or taking its average value, within the band of interest [2], [4], [5]. However, these subjective choices may be ambiguous (a DC maximum can be absent within the observed band) or lead to imprecise quantifications (the average DC may be affected by spectral effects of nearby broadband oscillations). To overcome these limitations, the present study introduces a novel method to assess causality in the frequency domain based on spectral decomposition [6], [7]. Our idea is to apply such decomposition to the partial spectra of the PSD of the target process, representing each partial spectrum as the sum of bell-shaped functions with features (power, frequency, spectral bandwidth) related to the type and location (modulus and phase) of the poles of the transfer function which defines the vector AR process in the *Z*-domain. This decomposition leads to calculate, for each AR oscillatory component, a so-called pole-specific spectral causality measure (PSSC).

In this study the new PSSC index is compared with the band-averaged DC, first in a theoretical model simulating cardiovascular interactions along the baroreflex, and then for the analysis of LF spectral causality from SAP to RR in a group of healthy subjects during a head-up tilt test protocol designed to elicit postural stress.

II. Methods

A. Directed Coherence in Bivariate AR Processes

Let us consider two jointly stationary, zero mean discrete stochastic processes collected in the vector $Y(n)=[y_1(n)\ y_2(n)]^T$. The causal interactions between the processes may be expressed in a parametric form through the bivariate AR model of order p [2], [3], [8]

$$Y(n) = \sum_{k=0}^{p} \mathbf{A}(k) Y(n-k) + W(n) \ , \qquad (1)$$

where $W(n)=[w_1(n)\ \ w_2(n)]^T$ is a vector of zero-mean uncorrelated white noises with diagonal covariance matrix $\mathbf{\Sigma} = diag\{\sigma_1^2, \sigma_2^2\}$, and where $\mathbf{A}(k)$ is 2×2 coefficient matrix with the coefficient $a_{ij}(k)$ describing the interaction from $y_j(n\text{-}k)$ to $y_i(n)$ (i,j=1,2); here, instantaneous zero lag-effects are allowed in the direction from y_1 to y_2 setting $a_{21}(0)\neq 0$ and $a_{ij}(0)=0$ otherwise. The system properties are evaluated in the frequency domain taking the *Z*-transform of (1) to yield $Y(z) = \mathbf{H}(z)\, W(z)$, where the 2×2 transfer matrix is

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$$\mathbf{H}(z) = \begin{bmatrix} H_{11}(z) & H_{12}(z) \\ H_{21}(z) & H_{22}(z) \end{bmatrix} = [\mathbf{I} - \mathbf{A}(z)]^{-1} = \bar{\mathbf{A}}(z)^{-1}, \quad (2)$$

with $\mathbf{A}(z) = \sum_{k=1}^{p} \mathbf{A}(k) z^{-k}$ representing the coefficient matrix in the Z domain ($\mathbf{I}$ is the 2×2 identity matrix). Making explicit the inverse of the coefficient matrix, each element of the transfer function matrix is represented as (i,j=1,2; $i \neq j$)

$$H_{ii}(z) = \frac{\bar{A}_{jj}(z)}{|\bar{\mathbf{A}}(z)|};\ H_{ij}(z) = \frac{-\bar{A}_{ij}(z)}{|\bar{\mathbf{A}}(z)|}. \quad (3)$$

Then, computing the transfer function on the unit circle in the complex plane ($\mathbf{H}(f) = \mathbf{H}(z)|_{z=e^{j2\pi fT}}$), the 2×2 spectral density matrix of the bivariate process becomes $\mathbf{S}(f) = \mathbf{H}(f)\mathbf{\Sigma}\mathbf{H}^{*}(f)$, where * stands for the Hermitian transpose. This matrix contains the auto-spectra $S_{ii}(f)$ as diagonal terms, and the cross-spectra $S_{ij}(f)$ as off-diagonal terms. Exploiting (2,3), each auto-spectrum can be expanded as the sum of causal contributions to yield

$$S_i(f) \triangleq S_{ii}(f) = \sigma_i^2 |H_{ii}(f)|^2 + \sigma_j^2 |H_{ij}(f)|^2, \quad (4)$$

where $\sigma_j^2 |H_{ij}(f)|^2 \triangleq S_{i|j}(f)$ is the partial spectrum of y_i given y_j (i,j=1,2). From (4), a left-side normalization yields $\gamma_{ii}^2(f) + \gamma_{ij}^2(f) = 1$, where

$$\gamma_{ij}^2(f) \triangleq \frac{\sigma_j^2 |H_{ij}(f)|^2}{S_{ii}(f)} = \frac{S_{i|j}(f)}{S_i(f)} \quad (5)$$

is the squared directed (causal) coherence (DC) from y_j to y_i, a well known function quantifying the dependence of y_i on y_j in the frequency domain. The DC functions defined as in (5) provide a frequency domain picture of the connectivity pattern of the bivariate process, with $\gamma_{ij}^2(f)$ quantifying the normalized coupling strength from y_j to y_i, being 0 when y_j does not cause y_i at frequency f, and 1 when the whole power of y_i at frequency f is due to the variability of y_j.

The most common ways to assess the directed coherence inside a specific frequency band is to take the peak value of the DC assessed within the band, or to average its values over the frequencies of the band.

B. Causal Spectral Decomposition

In this study, we propose to decompose each transfer functions defined as in (3) using univariate spectral decomposition [6], [7] as the sum of q spectral components ($q \cong p/2$), which correspond the poles of the determinant of $\bar{\mathbf{A}}(\mathrm{z})$. The spectral components are described by specific profiles, and have an associated frequency (related to the pole phase) and power (related to the pole residual). The decomposition of the i-j^{th} transfer function and the corresponding complex partial PSD function are obtained as

$$H_{ij}(z) = \frac{\bar{A}_{ij}(z)}{|\bar{\mathbf{A}}(z)|} = \frac{\bar{A}_{ij}(z)}{\prod_k (z - z_k)}, \quad (6)$$

$$S_{i|j}(z) = H_{ij}(z) \sigma_j^2 H_{ij}^{*}(1/z^{*}), \quad (7)$$

where the poles z_k, k=1,...q, are the roots of $|\bar{\mathbf{A}}(z)|$. Then, each function (7) is expanded using Heaviside decomposition with simple fractions relevant to all its poles (i.e., the poles z_k inside the unit circle and their reciprocals $\bar{z}_k = z_k^{-1}$ outside the unit circle, with k=1,...,q), which are fractions weighted by the relevant residuals of $S_{i|j}(z)$ (i.e., $r_k z_k$ and $-r_k \bar{z}_k$):

$$S_{i|j}(z) = \sum_{k=1}^{q} S_{i|j}^{(k)}(z),\ S_{i|j}^{(k)}(z) = \frac{r_k z_k}{z - z_k} - \frac{r_k \bar{z}_k}{z - \bar{z}_k}. \quad (8)$$

After finding the residuals and expanding the partial spectrum in simple fractions, the spectrum of y_i is obtained remembering that $S_i(z) = S_{i|i}(z) + S_{i|j}(z)$ and computing (8) for values of z on the unit circle of the complex plane:

$$S_i(f) = \sum_{k=1}^{q} S_i^{(k)}(f) = \sum_{k=1}^{q} S_{i|i}^{(k)}(f) + S_{i|j}^{(k)}(f). \quad (9)$$

The k-th component $S_{i|i}^{(k)}(f)$ has an associated frequency related to the pole phase, $f(k) = \arg\{z_k\}$, and power related to the pole residuals, $P_{i|i}(k) = r_k$ for real poles and $P_{i|i}(k) = r_k + r_k^{*}$ for complex conjugate poles.

Then, normalizing the spectral components to the total spectrum we can achieve the following decomposition of the causal coherence:

$$\gamma_{ij}^2(f) = \sum_{k=1}^{q} \gamma_{ij}^{2(k)}(f),\ \gamma_{ij}^{2(k)}(f) \triangleq \frac{S_{i|j}^{(k)}(f)}{S_i(f)}. \quad (10)$$

Moreover, we define spectral causality measures by integration over the whole frequency axis, decomposing the variance of the process y_i, λ_i^2, as follows:

$$\lambda_i^2 = \frac{2}{f_c} \int_0^{\frac{f_c}{2}} S_i(f) df = \sum_{k=1}^{q} \int_0^{\frac{f_c}{2}} S_i^{(k)}(f) df \quad (11a)$$

$$= \sum_{k=1}^{q} \int_0^{\frac{f_c}{2}} S_{i|i}^{(k)}(f) df + \sum_{k=1}^{q} \int_0^{\frac{f_c}{2}} S_{i|j}^{(k)}(f) df \quad (11b)$$

$$= \sum_{k=1}^{q} P_{i|i}(k) + P_{i|j}(k) = \sum_{k=1}^{q} P_i(k). \quad (11c)$$

In (11c), $P_{i|i}(k)$is the part of the variance of y_i due to its own dynamics and relevant to k-th oscillation (pole), $P_{i|j}(k)$ is the part of the variance of y_i due to y_j and relevant to the k-th pole, and summing this two parts of the variance we get the part of the variance of y_i relevant to the k-th pole, i.e., $P_i(k) = P_{i|i}(k) + P_{i|j}(k)$. Using these partial variances, we define the so-called pole-specific spectral causality (PSSC) from y_j to y_i, relevant to the k-th oscillation, as:

$$\gamma_{i|j}^2(k) \triangleq \frac{P_{i|j}(k)}{P_i(k)}. \quad (12)$$

With this definition, we have that the sum of the two PSSCs $\gamma_{i|i}^2(k)$ and $\gamma_{i|j}^2(k)$ is unitary, so that the PSSC ranges between 0 and 1, being equal to 0 when the power of the k^{th} oscillation of y_i (i.e. the oscillation at frequency f_k) is totally due to its internal dynamics, and equal to 1 when it is totally due to the dynamics of y_j assessed at the same frequency f_k.

Since the frequency f_k is associated to a specific causal spectral profile, the corresponding PSSC value is an objective measure of the causal power at that frequency, and the total causal power in a frequency band can be easily obtained summing al PSSC values with frequency inside the band.

III. Theoretical Example

To illustrate the new proposed PSSC measure in comparison with the traditional DC, we realized a theoretical simulation of a bivariate AR process of order p=5, where the process parameters are chosen to reproduce oscillations and interactions typical of cardiovascular variability series [9]. To this end, first we designed the diagonal AR coefficients for y_1 and y_2 in order to simulate the LF (f_1~0.1 Hz), very low

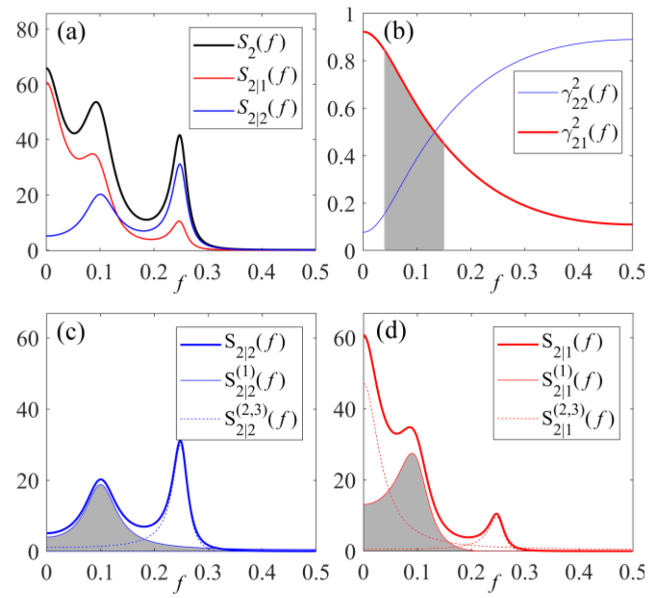

Figure 1. Frequency domain causality analysis performed for the simulated cardiovascular process of Sect III. (a) Theoretical spectral density of the simulated RR process y_2, $S_2(f)$, and partial spectra related to own and causal dynamics $S_{2|2}(f)$ and $S_{2|1}(f)$ obtained as in Eq. (4); (b) DC profiles obtained as in sect. II.A, relating the partial spectra to the total spectrum; (c,d) partial spectral decomposition of $S_{2|2}(f)$ and $S_{2|1}(f)$ yielding the components $S^{(k)}_{2|2}(f)$ and $S^{(k)}_{2|1}(f)$ obtained as in Sect. II.B. Gray areas evidence the LF band (0.04-0.15 Hz) in (b), and the components of the partial spectra associated to f_1=0.1 Hz in (c,d).

frequency (VLF, f_2~0 Hz) and HF (f_3~0.25 Hz) oscillations typical of RR and SAP variability. This was achieved placing poles with modulus ρ_k and phase $2\pi f_k$ in the complex plane; for y_1 we set ρ_1=0.3, ρ_2=0.8, ρ_3=0 to have autonomous LF and VLF rhythms in the simulated SAP process, and for y_2 we set ρ_1=0.8, ρ_2=0, ρ_3=0.9 to have autonomous LF and HF rhythms in the simulated RR process. The poles are the roots of $A_{ii}(z)$, from which the coefficients $a_{ii}(k)$ were obtained for i=1,2. Moreover, we imposed unidirectional interactions from y_1 to y_2, to simulate baroreflex influences, setting $a_{21}(k)$ as the coefficients of a 5th order FIR high-pass filter with cutoff at 0.15 Hz (while letting $a_{12}(k)$=0, k=1,...,5). Tuning of the parameters was performed to yield realistic spectral profiles for the two cardiovascular processes. The spectral density of the simulated RR process, $S_2(f)$, is shown in Fig. 1a.

Fig. 1 reports the computation of the DC from the partial spectra (a,b), as well as the computation of the PSSC measure from the decomposition of the partial spectra (c,d). The analysis is then focused on the LF band (0.04-0.15 Hz). As the DC from y_1 to y_2 declines monotonically in this band (gray area in Fig. 1b), derivation of an LF value cannot be based on the criteria of taking the maximum. The average value of the DC function within the LF band is $\gamma^2_{ij}(LF)$=0.64. On the contrary, the decomposition of the partial spectra of y_2 given its own dynamics (Fig. 1c), and of y_2 given the dynamics of y_1 (Fig. 1d), evidences clear components with central frequency f_1=0.1 Hz and well-defined power (gray areas); computing the ratio of Eq. (12) yields the PSSC index $\gamma^2_{i|j}(LF)$=0.56.

This simulation evidences that the frequency-specific computation of causality is more objective when it is based on partial spectral decomposition, and suggests that the DC may tend to yield higher estimated values of the causal coupling strength than the PSSC measure.

IV. Cardiovascular Variability Analysis

To verify the practical applicability of the proposed approach, spectral causality was employed to characterize the strength of the coupling directed along the baroreflex (interactions from SAP to RR interval) at rest and during postural stress. Specifically, we considered the beat-to-beat time series of RR and SAP measured from the ECG and the finger arterial pressure signal in a group of 27 young healthy subjects monitored in the resting supine position and in the upright position during head-up tilt [10]. For each subject and condition, stationary windows of RR and SAP lasting N=300 beats were considered for the analysis. The corresponding time series $r(n)$ and $s(n)$, n=1,...,N, were used as realizations of the bivariate AR process (1), with $Y=[y_1\ y_2]^T=[s\ r]^T$. Model identification was performed using vector least squares estimation and setting the model order p according to the Akaike information Criterion [2]. Instantaneous effects were allowed from $s(n)$ to $r(n)$ to account for fast interactions along the baroreflex [3], [8].

In such type of analysis, if cardiovascular variability is assessed in the frequency domain it is recommended to evaluate causality within the LF band, in order to minimize the confounding effects of respiration on SAP and RR that are primarily confined in the HF band [9], [11]. Accordingly, after computing the spectrum of RR as well as its partial spectra and their decomposition, the DC from SAP to RR was averaged in the range 0.04-0.15 Hz to get $\gamma^2_{rs}(LF)$, while the PSSC measure was obtained summing all the components with central frequency in the same range, i.e. computing $\gamma^2_{r|s}(LF)=\sum_{f_k\in LF}\gamma^2_{r|s}(k)$.

The results of LF baroreflex spectral causality analysis are reported in Fig. 2 (DC measure) and Fig. 3 (PSSC measure). Statistical comparison performed by the Wilcoxon signed-rank test indicates that, both at rest and during tilt, the causal coupling from SAP to RR at LF assessed by the DC is significantly higher than that assessed by the PSSC (median: 0.51 vs. 0.13 for supine, 0.63 vs. 0.41 for upright; P<0.0001 in both conditions). Moreover, the causal coupling is significantly higher in the upright position compared to the supine. The increase is detected taking the average DC from SAP to RR within the LF band (P=0.0019), and even more evidently taking the PSSC of the components belonging to the LF band (P<0.0001). These results suggest that, if compared with PSSC, the DC tends to overestimate the strength of the LF causal coupling along the baroreflex, likely as a consequence of the fact that the DC profile within a given frequency band results in part from spectral contributions related to other bands. Moreover, while both DC and PSSC are able to identify the physiologically well-known larger involvement of the baroreflex during head-up tilt [12], PSSC seems to detect the increased LF causal coupling with higher statistical confidence.

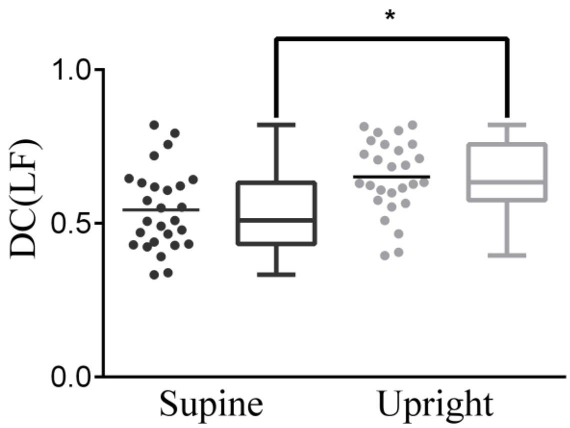

Figure 2. Low frequency (LF) spectral causality analysis of baroreflex interactions based on the directed coherence (DC). Plots depict the distributions across subjects, depicted as individual values and box-plot, of the DC from SAP to RR, $\gamma_{rs}^2(LF)$, computed in the supine (black) and upright (gray) body positions. *, P<0.05 supine vs. upright.

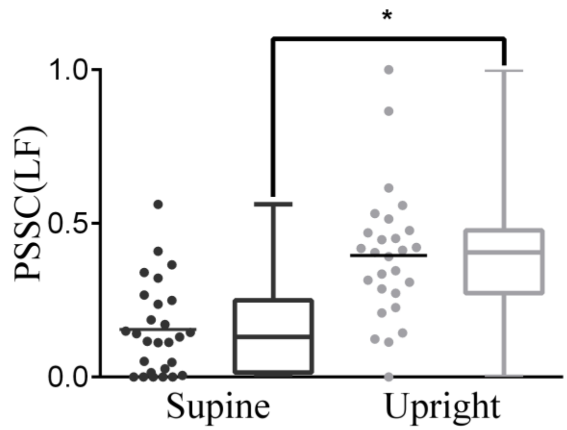

Figure 3. Low frequency (LF) spectral causality analysis of baroreflex interactions based on pole-specific spectral causality (PSSC). Plots depict the distributions across subjects, depicted as individual values and box-plot, of the PSSC from SAP to RR, $\gamma_{r|s}^2(LF)$, computed in the supine (black) and upright (gray) body positions. *, P<0.05 supine vs. upright.

V. Conclusions

This study shows that applying spectral decomposition [6], [7] to the partial spectra derived from the frequency domain representation of bivariate processes [3], [8] provides a means to quantify objectively, at specific well-defined frequencies, the causal contribution of a source process to the power of the target process. Our results document that the resulting PSSC measure can be obtained as the power ratio of well-identifiable spectral components even when the traditional DC does not exhibit peaks in the frequency band of interest. Moreover, while averaging the DC values within an assigned frequency region can be inaccurate because external broad-band oscillations may convey information into the region, the separation obtained by spectral decomposition allows to focus on the components of interest, ultimately resulting in PSSC values which reflect more accurately the amount of causal power that the source contributes to the target. This aspect can have great relevance in cardiovascular variability analysis, where VLF oscillations are often predominant and may thus have a relevant effect on the evaluation of causality in the LF band of the spectrum.

While the methodology is presented here for the bivariate case, it can be readily extended to the multivariate case exploiting the DC representation of vector AR processes [2] combined with multivariate spectral decomposition [7]. Future studies should also be focused on technical developments such as the evaluation of statistical significance of PSSC exploiting approaches already in use for the DC [13], and on practical validations including the comparative analysis of DC and PSSC between different physiological signals, inducing different physiological challenges, or testing pathological conditions [14].

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