On the asymptotics for $*$-Capelli identities

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Abstract

Let $F\langle X, *\rangle$ be the free associative algebra with involution $*$ over a field of characteristic zero. If L and M are two natural numbers let $\Gamma^*_{M+1,L+1}$ denote the T^*-ideal of $F\langle X, *\rangle$ generated by the $*$-Capelli polynomials Cap^+_{M+1}, Cap^-_{L+1} alternating on $M+1$ symmetric variables and $L+1$ skew variables, respectively.

It is well known that, if F is an algebraic closed field, every finite dimensional $*$-simple algebra is isomorphic to one of the following algebras (see [4], [2]):

· $(M_k(F), t)$ with the transpose involution;

· $(M_{2m}(F), s)$ with the symplectic involution;

· $(M_k(F) \oplus M_k(F)^{op}, *)$ with the exchange involution.

The aim of this talk is to show a relation among the asymptotics of the $*$-codimensions of the finite dimensional $*$-simple algebras and the T^*-ideals $\Gamma^*_{M+1,L+1}$, for some fixed natural numbers M and L. In particular:

$$c^*_n(\Gamma^*_{k(k+1)/2,k(k-1)/2}) \simeq c^*_n((M_k(F), t)),$$

$$c^*_n(\Gamma^*_{m(2m-1),m(2m+1)}) \simeq c^*_n((M_{2m}(F), s))$$

and

$$c^*_n(\Gamma^*_{k^2,k^2}) \simeq c^*_n((M_k(F) \oplus M_k(F)^{op}, *)).$$

Similar results have been found for simple finite dimensional superalgebras in [1] and these extend a theorem of Giambruno and Zaicev [3] giving in the ordinary case the asymptotic equality between the codimensions of the Capelli polynomials and the codimensions of the matrix algebra.

This talk is based on a joint work with A. Valenti.

References

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[4] L. H. Rowen, Polynomial Identities in Ring Theory, Academic Press, New York, 1980.