



13th Computer Control for Water Industry Conference, CCWI 2015

Sensitivity of regional water supply systems models to the level of skeletonization – a case study from Apulia, Italy

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Abstract

Simulation models supported by state-of-the-art software packages are nowadays available to explore operation rules of regional water supply systems or to select structural alternatives for improving long-term service performances.

Given the, sometimes high, complexity of these systems, model calibration can become a lengthy procedure and many runs are necessary before obtaining convincing results. However, even after calibration, depending on system's complexity and the number of time steps investigated, a single run can take up to several minutes, even on state-of-the-art computers, so that simulation time can become a true bottleneck if such models are to be coupled with metaheuristic optimization techniques, such as genetic optimization.

This paper investigates the possibility to reduce computational time through skeletonization of the models. The regional water supply system of Apulia, Southern Italy, was adopted as a case study and the software package AQUATOR was employed to model the system.

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Peer-review under responsibility of the Scientific Committee of CCWI 2015

Keywords: Regional water supply systems, simulation models, skeletonization level;

1. Introduction

Regional water supply systems commonly provide millions of inhabitants with drinking water in areas of several thousand of square kilometres and include a variety of supply sources, such as springs, wells and large reservoirs.

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The main issues related to this kind of systems lie in finding appropriate, least-cost operation rules for the system in its current configuration, or in understanding the best structural alternative to improve service performances by providing additional capacity in case of breaks and maintenance, or by reducing system's congestions (bottlenecks) through increased pipeline capacity. In this kind of problems, system's performances are measured by operation costs and by reliability indices such as time and volumetric reliability [1], all related to the possibility of meeting demand with the current or future water availability. Emphasis in these models is hence on the water balance on a regional scale and they differ in this from hydraulic models of the network that are generally also concerned with the pressure regime. Another relevant difference lies in the time scale: unlikely hydraulic models, regional supply models are concerned with the variability, be it seasonal or annual, of water resources availability across time. This requires simulating a long sequence of time steps, that can range from one "average" year, in which case the time unit can be one day to many years, depending on the type of problem considered, in which case the time unit can be a decade or one month.

Simulation models supported by state-of-the-art software packages [2], [3], with attractive and user-friendly interfaces that allow building customized schemes, are now available to screen the effect of different operation rules and of structural alternatives. In general, calibration of these models is a lengthy procedure and many runs are necessary before obtaining convincing results. However, even after calibration, depending on system's complexity and the number of time steps investigated, a single run can take up to several minutes, even on state-of-the-art computers. Furthermore, simulation time can become a true bottleneck if such models have to be coupled with metaheuristic optimization techniques, such as genetic or evolutionary algorithms [4], useful to achieve the best solutions in terms of minimum costs and maximum performance, as they are based on an intense search of the decision space through repeated access to the simulation tool.

In the paper we analyse the possibility to reduce computational time through model skeletonization. Skeletonized models have also the advantage to be handier and easier to adjust/change (demand time series and parameters of the elements of the model are greatly reduced in number). As a case study, we use the regional water supply system of Apulia in Southern Italy. The Apulian Aqueduct [5] consists of a large and complex system supplying an area with more than 4,000,000 inhabitants. AQUATOR, a state-of-the-art software package for the simulation and operational optimization of complex water systems with multiple uses, was employed. We started from a scheme that best represents the water system topology and operation, featuring around 100 demand centres and around 350 links (pipelines), that was validated by the technical staff of the regional water utility, and progressively reduced the spatial resolution of the model by aggregating demand centres and lowering the discretization of pipelines.

2. Background on skeletonization

In the field of hydraulic engineering, skeletonization was originally introduced to include in water distribution network models only the parts of the real-world network that have a significant impact on the behaviour of the system, allowing the modeller to obtain reliable and accurate results while lightening time and money burden [6]. Eggener and Polkowski [7] carried out a first study on skeletonization by systematically removing pipes from a model to test the sensitivity of model results. They found that under normal demands, a large number of pipes could be removed without significantly affect pressure heads. Later on, Shamir and Hamberg [8], [9] investigated more rigorously the rules for models' size reduction.

The basic principle of skeletonization is to account for the effect of the portions of the system that are not modelled, within the parts of the system that are included in the model. There are no general or absolute criteria for determining whether a pipe should be included, but the level of skeletonization mainly depends on the model's purpose. For regional water studies, a broad level of skeletonization is generally sufficient, while for distribution networks design or water quality studies many more elements of the system have to be considered to accurately model the real-world system and engineering judgment plays a central role in finding the most suitable solution. As a general rule, any element capable of influencing the behaviour of the system should be included in the model, for instance large water demand centres, points of known conditions, critical points with unknown conditions, large-diameter pipes, pipes that complete important loops, pumps, control valves, tanks, etc. [6].

When pipes and nodes are removed from a model based on a given criterion, such as pipe diameter, it is necessary to keep track of the demands that were assigned to the removed nodes and to account for the hydraulic

capacity of the pipes removed. When dead-end branch pipes are removed, the removal has no effect on the carrying capacity of the remainder of the system. Similarly, removing two or more pipes in series, by combining them into one, has generally a negligible effect on model performance, but it is necessary to determine the attributes of the resulting pipe (length, diameter and roughness coefficient) [10], [11], and to split the demand of the removed nodes between the two nodes at the ends of the resulting pipe, so as to not alter conveyance capacity and the head loss of the series of pipe removed. Finally, when removing parallel pipes (namely two pipes with the same beginning and ending nodes), one of the pipes is considered to be the dominant pipe and its length and either its diameter or its roughness coefficient is adopted for the new equivalent pipe [6].

3. Methodology

3.1. Modelling of the water system in Aquator

The simulation and optimization of regional water resources systems' operation can be performed through mathematical models developed by means of specific software packages. An updated survey of these models is available in [3]. In this study, we used Aquator, a software package for the simulation of complex water systems with multiple uses, which allows modelling and interfacing natural river networks and water supply networks, with an accurate reproduction of their topology. Aquator works on a daily time step and can simulate the operation of the water system within the current time step, i.e. turning water resources availability into supply to the demand centres, either on the sole basis of its linear optimization algorithm (*Aqua-Solver*) and other predefined rules, or being also guided by user-defined rules. The software adopts an arcs (pipelines) and nodes (junctions, plants and sources) paradigm and water is used to meet demands using an algorithm that seeks to minimize cost in the current time step, while preserving the state of resources [2].

Given their aim and scope, Aquator models do not attempt to reproduce the full hydraulic behaviour of the systems, but they rather focus on water mass balance. The pipelines are mainly characterized by a maximum conveyance capacity ($10^3 \text{ m}^3/\text{day}$), a loss rate ($10^3 \text{ m}^3/\text{day}$) and by fixed and variable costs (€/MI), if available and applicable. Similarly, plants are essentially characterised by the following parameters: treatment plants: maximum capacity ($10^3 \text{ m}^3/\text{day}$), fixed loss and loss rate (%), fixed and variable costs (€/10³ m³); pumping stations: maximum capacity ($10^3 \text{ m}^3/\text{day}$), fixed and variable costs (€/10³ m³); wells: maximum capacity ($10^3 \text{ m}^3 / \text{day}$), fixed (€) and variable costs (€/10³ m³); hydropower stations: maximum capacity ($10^3 \text{ m}^3/\text{day}$), fixed (€) and variable costs (€/10³ m³), unit revenue (€/10³ m³).

3.2. Skeletonization of Aquator Models

Starting from a very detailed Aquator scheme that best represents water system's topology (links, plants, demand centres and water resources) and operation, in the simplification process we followed the basic principle of skeletonization of hydraulic models (see section 2), i.e. to account for the effect of the portions of the system removed from the model within the parts of the system remaining in the simplified model.

Since Aquator models are constituted of mass balances and decisions on allocations are taken on the basis of cost minimization and on a set of rules that are, however, topology-neutral, the skeletonized model shall respect the following criteria: equivalent global conveyance capacity of sub-systems; equivalent cost of sub-systems; unaltered global capacity and equivalent cost of aggregated wells; unaltered total water loss within the sub-systems; unaltered total demand of aggregated demand centres.

Where "subsystem" indicates a part of the water system subject to simplification.

In particular, in the case of two or more parallel pipes (Fig. 1), the equivalent pipe will have a conveyance capacity equal to the sum of the parallel pipes' conveyance capacities and loss rate equal to the sum of the parallel pipes' loss rates.

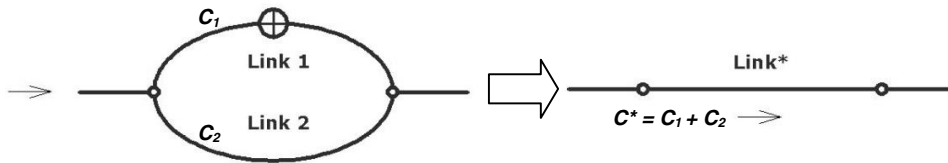


Fig. 1. Pipe equivalent to parallel pipes with conveyance capacities C_1 and C_2 , respectively - the crossed circle indicates a pumping station.

If one of the parallel pipes features a minor pumping station or hydropower plant, as in figure 1, it is necessary to attribute a cost or a revenue to the equivalent pipe. Such equivalent cost (or revenue), c^* , can be assessed as $c^* = c_1 \cdot C_1 / \sum_i C_i$, where C_i are the conveyance capacities of the parallel pipes and c_1 is the unit cost associated to the plant.

In case of two or more pipes in series, the equivalent pipe should have a conveyance capacity equal to the smallest conveyance capacity of the pipes in series and a loss rate equal to the sum of the loss rates of the pipes in series. Caution should be paid if demand centres to be aggregate are present. In Aquator water losses along a link are independent from the water volumes allocated to the demand centre, and are met first, before demands. In the case of Fig. 2, for instance, where the demand centres are aggregated and concentrated in correspondence of the upstream node (node A), there is no need to introduce an equivalent pipe. In fact, the loss rates of the pipes in series will be added to the loss rate of the upstream link, since the water volume available for allocation to the aggregated demand centres is affected by the upstream losses.

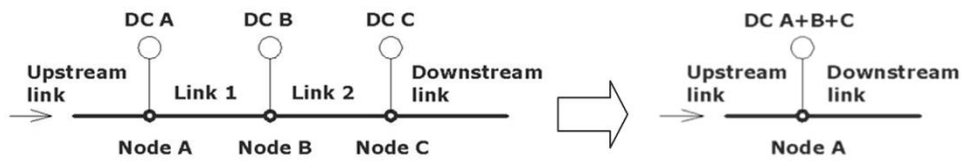


Fig. 2. Aggregation of demand centres in correspondence of an upstream node (node A).

When dead-end branch pipes are removed, the removal has no effect on the conveyance capacity of the remainder of the system, it is just necessary to aggregate the demand centres in one single demand centre and to add the loss rates of the dead-end branch pipes to the loss rate of the upstream link (Fig. 3).

Ideally, the skeletonized model should lead to the following results: same amount of water allocated as in the complete model; same total cost of resources and plants as in the complete model; same use of individual resources as in the complete model.

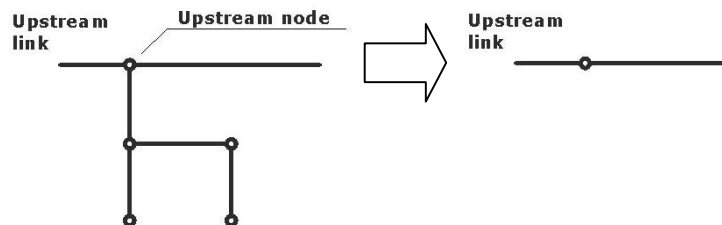


Fig. 3. Removal of dead-end branch: it is necessary to aggregate the demand centres in one single demand centre and to add the loss rates of the dead-end branch pipes to the loss rate of the upstream link.

4. Case Study: Apulian regional water supply system

The Apulian regional system consists of a large and complex system supplying an area with more than 4,000,000 inhabitants and multiple use, such as industry and irrigation. This complex supply system has been developed over about one hundred years, starting from the Canale Principale (the Main canal), a 200 km, long, 6 m wide, open channel, connecting Sele and Calore springs in Campania, with an average flow of around 4,5 m³/s, (Sele and Calore are two rivers with their mouth in the Tyrrhenian sea) to the entire region, up to Salento, the heel of the Italian boot. When it was completed, at the beginning of the XX-eth century, the main canal was considered, by extension and capacity, among the most important works of hydraulic engineering of its age.

The carbonate aquifers have constituted for centuries the only source of water supply for Apulia. In fact, with the exception of the Ofanto and Fortore rivers, the Apulian territory is devoid of major rivers. Hence, Apulia had to obtain supplies from sources that fall in other regions. Besides Sele and Calore springs, other important sources have been added more recently and they also fall outside Apulia: Pertusillo and Monte Cotugno (also named Sinni) reservoirs, for instance, are located in Basilicata. Overall, the system features five reservoirs, all of which multipurpose, with a total active capacity of around 900 Mm³, supplying water also for irrigation and industrial districts. Another important water resource for both municipal supply and irrigation is the regional aquifer from which Acquedotto Pugliese (also termed AQP in the following), the utility operating the system, draws through about two hundred wells located throughout the region. The Apulian water resources system consists of five main schemes named Sele-Calore, Fortore, Pertusillo-Sinni, Locone, and Ofanto.

5. Simplification of Aquator models

We have built models for the evaluation of skeletonization from a Zero scheme, representing the water system's *status quo* (Fig. 4). In Aquator this model features a total of 716 elements, 108 of which are demand centres and 354 are links (pipelines). This scheme has been validated by the technical staff of AQP that has acknowledged that the distribution of flows, water allocations and costs provided by the model reflect the actual management process.

The model runs on a daily basis, turning the hydrologic input into water allocations to the demand centres. The hydrologic input is that of an average water year and consists of average monthly inflow totals to the five system's reservoirs and average monthly spring yields, which the model breaks into equal average daily values. As far as groundwater resources are concerned, besides a number of minor wells spread throughout the region, the main source is represented by Salento peninsula aquifer.

5.1. Schemes

Three different skeletonized schemes have been considered. Scheme 1 corresponds to the highest level of simplification: following the general criteria set out in section 3.2, the Zero scheme was inspected in order to find components that could be simplified (e.g. loops, parallel pipes, pipes in series, etc.). Skeletonization was performed looking at network's topology, pipe characteristics (conveyance capacity, losses and costs), plants (water treatment, pumping and hydropower) and distribution of demand centres. Only looking comprehensively at all these elements, one may expect to obtain a simplified scheme that is functionally equivalent to the initial one.

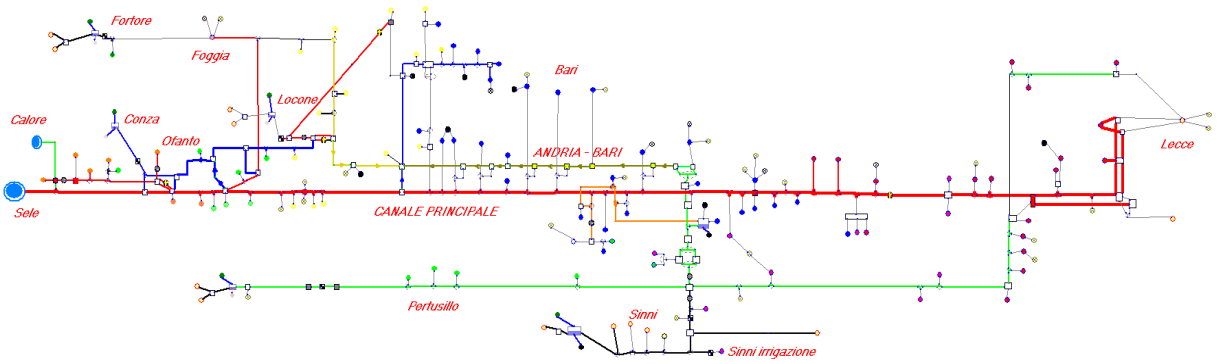


Fig. 4. Zero Scheme representing the water system's *status quo* (716 components).

The model of scheme 1 (Fig. 5) has 185 components overall, of which 18 are demand centres, 85 links and 9 wells. Both pumping and hydropower stations have been left unaltered. Compared to the zero scheme, featuring 716 components, this implies a component reduction of 75%. Scheme 2 (Fig. 6) was obtained from scheme 1 by reducing the concentration of wells. In scheme 2 the model features 228 components, of which 18 are demand centres, 106 are links and 28 are wells.

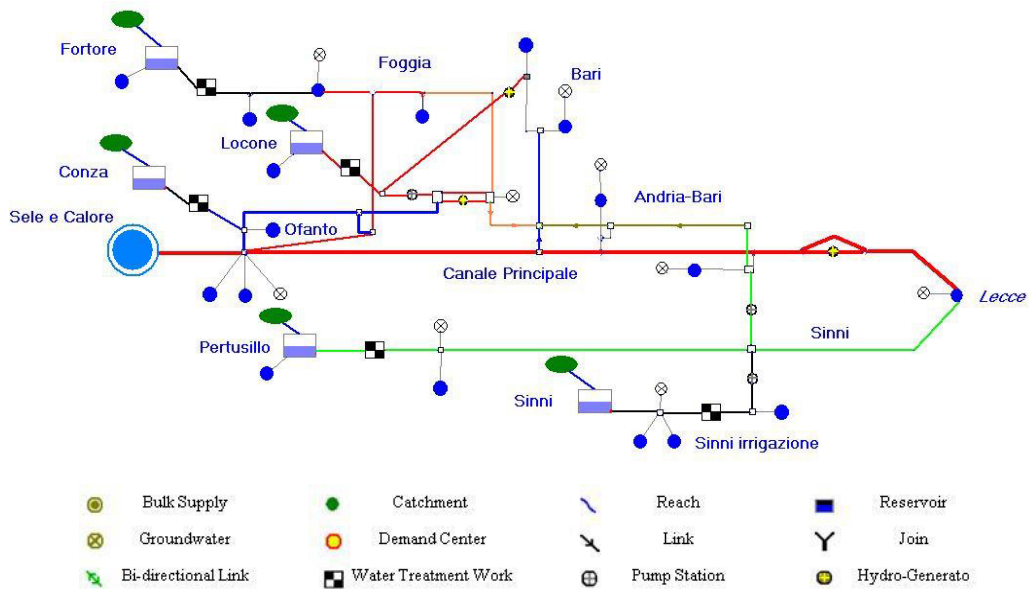


Fig. 5. Skeletonization - Scheme 1 (185 components).

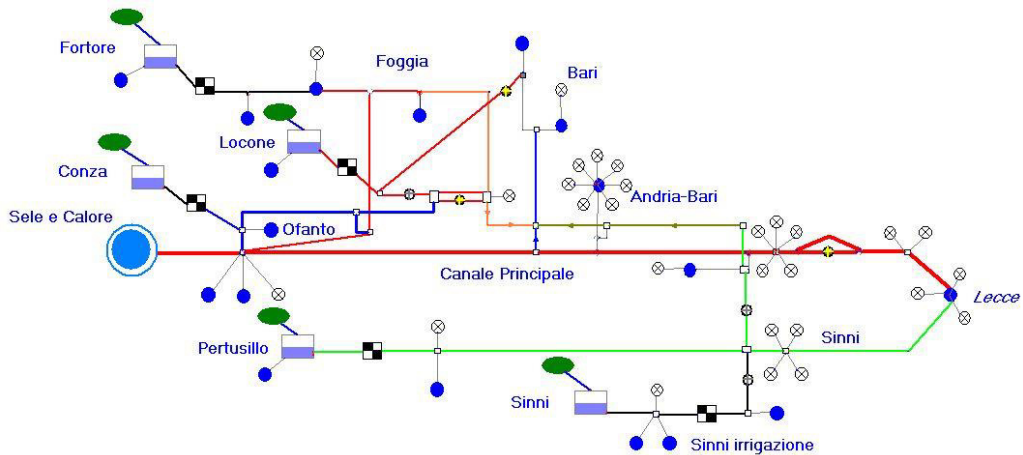


Fig. 6. Skeletonization - Scheme 2 (228 components).

Finally, Scheme 3 (Fig. 7) was obtained from scheme 2 by splitting the demand centre called Lecce (far right in Fig. 6) into four demand centres, DC M1, DC M2, DC M3 and DC M4, each supplied by individual wells. As will be seen, this has improved performances of scheme 3 with a minimum increase of the computational burden. The model of scheme 3 has 236 components, of which 21 are demand centres, 111 links and 28 wells.

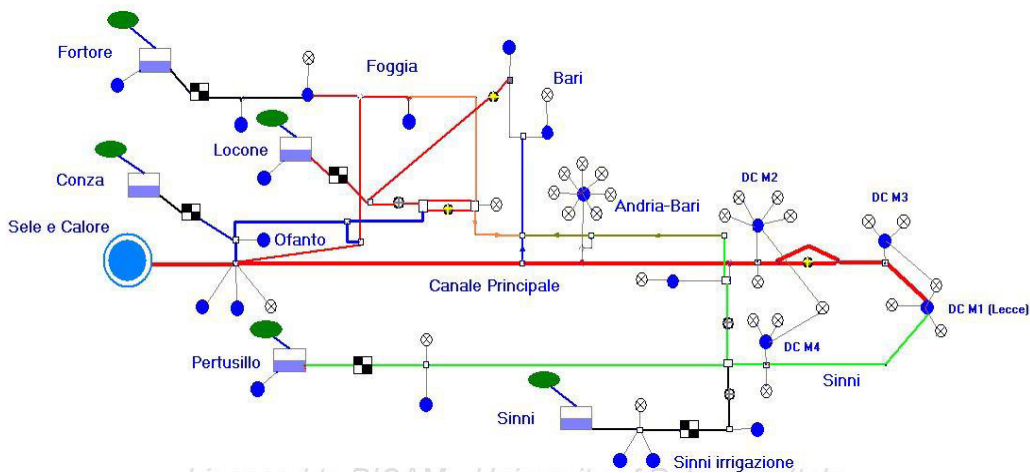


Fig. 7. Skeletonization - Scheme 3 (236 components).

For each scheme, parameters (flows, costs, etc.) of the single aggregated components were calculated from the parameters of the components of the Zero Scheme; subsequently, model calibration was performed. Calibration is necessary because the first runs of an Aquator model generally provide results that can be further improved, especially with reference to the level of unmet demand. This is probably due to the fact that, when performing water allocation, the model balances between the objective of meeting demand and that of minimizing costs: this can result in allocations to demand centres that are less than the feasible ones, given the available water. Reduction of these somewhat hard to justify water deficits requires working on model’s parameters or settings. We found it convenient to act on the Advance Order (AO) of the demand centres. AOs define priority of supply: the higher the rank of a demand centre in the AO list, the likelier that that demand will be met.

Pushing up in the AO list the demand centres with the highest percentage deficits, unnecessary shortages can be reduced and the level of demand met can be raised. In the case of the skeletonized schemes, efforts were directed to making the level of met demand close to that of the Zero scheme. The calibrated scheme 1 was used as starting point for calibration of scheme 2 and the calibrated scheme 2 was used to calibrate scheme 3.

Table 1 reports flow and cost parameters for the main plants of the system. Well have extraction costs ranging from 0.10 €/m³ to 0.32 €/m³. Yearly water demand is 866.7 Mm³, 348.0 Mm³ of which for irrigation, 15.9 Mm³ for manufacture and 502.8 Mm³ for municipal supply.

Tab. 1. Flow and unit cost parameters of pumping stations (PS), water treatment plants (WTP) and hydropower plants (HP) of the Apulian system.

Plant	Sinni (PS)	Parco del Marchese (PS)	Locone (PS)	Fortore (WTP)	Locone (WTP)	Pertusillo (WTP)
Max Flow [m ³ /s]	4.80	6.20	1.80	2.40	1.75	4.50
Unit cost [€/m ³]	4.83	9.08	8.38	4.36	7.51	13.21
Plant	Sinni (WTP)	Conza (WTP)	Battaglia (HP)	Barletta (HP)	Monte Carafa (HP)	
Max Flow [l/s]	6.00	1.50	0.45	0.30	1.00	
Unit cost [c€/m ³]	17.13	7.51	-5.51	-1.44	-1.01	

6. Results

Table 2 contains simulation results in terms of demand met (in % of total demand), volumes supplied by the different sources and total costs.

Tab. 2. Comparison of simulation outputs from the reference (Zero) scheme and the three schemes skeletonized with increasing level of detail.

	Zero	Scheme 1	Scheme 2	Scheme 3
Total demand satisfaction [%]	99.43%	99.12%	99.01%	99.99%
Civil demand satisfaction [%]	96.89%	97.00%	96.82%	97.34%
Withdrawals from wells [10 ⁶ m ³]	44.08	59.42	39.43	41.00
Withdrawals from Sele-Calore springs [10 ⁶ m ³]	148.00	148.00	148.00	148.00
Withdrawals from Fortore reservoir [10 ⁶ m ³]	150.09	150.09	150.09	150.09
Withdrawals from Conza reservoir [10 ⁶ m ³]	29.44	36.02	36.02	35.58
Withdrawals from Locone reservoir [10 ⁶ m ³]	12.79	5.10	5.10	5.10
Withdrawals from Pertusillo reservoir [10 ⁶ m ³]	182.48	186.53	186.53	186.53
Withdrawals from Sinni reservoir [10 ⁶ m ³]	319.75	298.76	317.76	321.24
Total withdrawal [10 ⁶ m ³]	886.63	883.91	882.93	887.54
Total cost [M€]	62.46	57.93	62.12	62.77

Results from scheme 1 are poorly convincing since they are not able to reproduce the results of the reference model (zero scheme) in terms of demand satisfaction, distribution of supply source use and total costs. Scheme 1 works poorly because well aggregation was pushed too far. Aggregation of different wells in a single one is performed by summing each well's maximum extraction capacity and using an average unit cost weighed on maximum extraction capacities of each of the wells included in the aggregation process. This cost averaging approach causes wells to be used more intensely in scheme 1 than in scheme 0, with an increase of total costs. The loss of resolution due to the aggregation process brings in this case to a misinterpretation of the actual allocation

process, as reproduced by the reference scheme. This is circumvented in scheme 2 by avoiding aggregation of wells with different unit costs and aggregating only wells with similar unit costs. By doing so, results from scheme 2 are closer to those of scheme zero, although satisfaction of water demand is still slightly less than that of the reference scheme and so are total costs.

Finally, in scheme 3 a better reproduction of system’s performances is obtained by splitting the Lecce demand centre in four demands, each supplied by the system and by the pertaining local wells.

A common feature of all the three skeletonized schemes is that more water is provided by Conza reservoir and less from Locone reservoir compared to the Zero scheme. While it is comprehensible that the model will push the use of Conza, whose water has the same unit treatment costs as Locone, but no pumping costs (Table 1), it is still to understand whether the skeletonization process has left out some bottlenecks that were reproduced in the reference model and that would constrain the amount of water from Conza reservoir.

Finally, Fig. 8 shows how skeletonization dramatically reduces running times: data refer to a 2 GB RAM processor with a 2.10 GHz CPU. Computation times are cut down by around 99%. The same is found with more powerful machines (4GB RAM) where reductions are of the order of 95%. Running time reduction is larger than the reduction of components: a reduction of 75% of model’s complexity in terms of components brings to a reduction of 99% of computing time. Based on this admittedly small sample of simplified schemes, running times vs. model’s number of components seem to follow an exponential law (Fig. 8).

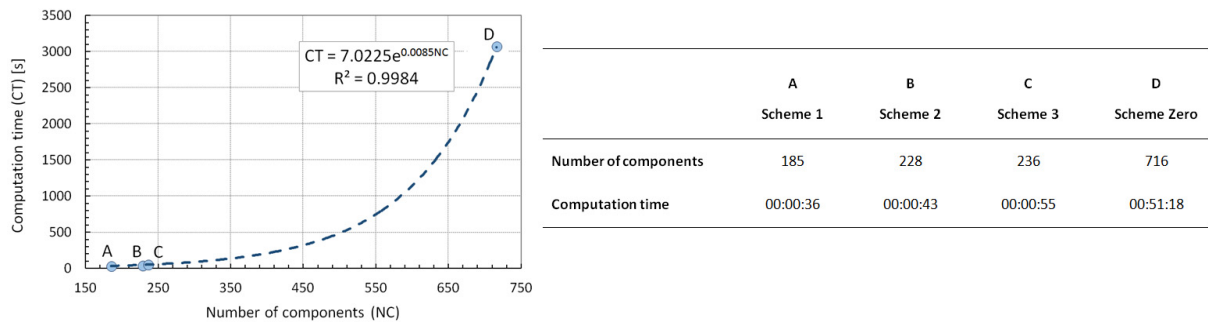


Fig. 8. Computation time (CT) as a function of model’s number of components (NC) and interpolated exponential law.

Therefore, there seems to exist a “complexity threshold” below which the number of components has little influence on computation times. This suggests that the skeletonization level should be customized on the basis of the availability of time and resources of the organization that uses the model. Once an acceptable maximum allowable time for one simulation is agreed upon, the level of model simplification should be consistent with this parameter.

However, there are applications where minimization of running time can be viewed as an important objective to achieve. From this standpoint, running time differences among the simplified models are relevant in themselves: a reduction of components from scheme 3 to scheme 2 by around 3% (from 236 to 228) brings a reduction in computation times of around 22%. These reductions (12 seconds per simulation) can become crucial when the model is to run thousands of times, as is the case when coupling a simulation model with metaheuristic algorithms.

Acknowledgements

This work has been developed in the framework of the MOGESA project, a research project of Palermo University’s DICAM funded by AQP.

7. Conclusions

The paper has examined some issues related to how the degree of complexity of a regional water supply model can influence model’s results and what is the impact of model simplification on computation times. The paper has

provided some criteria to follow when performing skeletonization of such a model: equivalent global conveyance capacity of sub-systems, equivalent costs of sub-systems, unaltered global capacity and equivalent cost of aggregated wells, unaltered total water loss within the sub-systems, unaltered total demand of aggregated demand centres.

As far as outputs are concerned, in the skeletonized model the level of demand met should ideally be the same as in the complete model, total cost of water resources and plants should also be the same as in the complete model and finally the use of supply sources should have the same distribution. Results show that it is possible to build simplified schemes that are consistent with the above criteria and that computation times can be reduced dramatically, by more than 95% compared to the reference model. Based on the few skeletonized models examined, an exponential law was found to fit running times quite satisfactorily, implying that there is a domain of possible models where reduction of complexity will result in little changes in absolute running times. If running time minimization is not seen as a crucial objective, a trade off could be found between computation time and model accuracy by agreeing upon an acceptable running time for a single simulation and adjusting model simplification accordingly. This simplified model could be used to screen a large number of alternative scenarios and the most promising ones could be then further analysed using the complete model.

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