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Multivariate Statistical Analysis for Water Demand Modeling

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Abstract

The actual level of water demand is the driving force behind the hydraulic dynamics in water distribution systems. Consequently, it is crucial to estimate it as accurately as possible in order to result in reliable simulation models. In this paper, a copula-based multivariate analysis has been proposed and used for demand prediction for given return period. The analysis is applied to water consumption data collected in the water distribution network of Palermo (Italy). The approach showed to produce consisted demand patterns and to be a powerful tool to be coupled with water distribution network models for design or analysis problems.

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Keywords: Multivariate analysis; vine copula; water demand modeling.

1. Introduction

Urban development is creating new problems for the management of water distribution systems, which are called upon to respond to a growing drinking water demand, which is also highly variable in space and time. The general goal for any water utility is to supply constantly water to all customers of good quality and under sufficient pressure [1,2]. The reliability of the water distribution system of that utility depends on the combination of different factors that play an important role in the design and management of the system: water demand variability, size and maintenance of pipes, volumes of urban reservoirs. The development of powerful computers made hydraulic engineers able to simulate the behavior of water supply systems for almost any scenario. However, the accurate prediction of pressures, flows and water quality parameters depends strongly on the quality of the input data. Data needed to simulate the behavior of water distribution network, such as pipes friction coefficients, nodal demands, and their temporal variation contain uncertainty, consequently affect our confidence in the outcome of the

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simulation. There has been agreement in the literature that uncertainties in nodal demands and their variation with time is one of the main source of error responsible for discrepancies between measured and model simulated flows and pressures.

Residential water demand is one of the most difficult parameters to determine when modeling drinking water distribution networks. The simulation of a water distribution system is often carried out by assuming averaged values, both in space and in time, of water demands: the spatial averaged values are obtained by clustering the water consumption of users afferent each node of the network, the time averaged values are obtained as the mean of the instantaneous values of the nodal demands. The simulation results obtained by considering these simplifications could be not much reliable for the hydraulically disadvantages zones of the network. Therefore, water demand modeling has been a very active field of study. Researchers have been above all interested in domestic water consumption by households that is the principal rate of the total volume supplied by the water distribution system in urban regions, often equal to 75% [3]. The prediction of water demand can be done on different time scales: short- and long-term forecasting of the municipal water demand is essential to water utilities for system planning, design, and asset management. Short-term forecasting is useful for operation and management of existing water supply systems within a specific time period, whereas long-term forecasting is important for system planning, design, and asset management. The detailed modeling of the hydraulic behavior of drinking water distribution systems could be get by implementing a domestic water demand modeling in one of the several software programs recently developed. Until today stochastic models for instantaneous residential water demand (for references see section 2.1) have been used to obtain realistic demand patterns for the hydraulic distribution network solvers. Several basic parameters related to the residential water usage are necessary to apply these models, such as the frequency, duration and intensity statistics of the demand pulses. This methodology shows a limit in authors' opinion: it neglects the statistical dependence of the parameters characterizing the consumption process. Many approaches are used in hydrology to develop statistical multivariate analysis. Among methodologies present in literature, the method based on the copula, recently introduced in hydrology, is applied in this paper.

This study has two objectives: (1) to propose a procedure based on a multivariate statistical analysis of the main features of the water consumption process at domestic level; (2) to define a more realistic demand patterns with a given return period. The present paper is organized as in the following: in section 2, a brief review of the studies concerning with demand modeling and multivariate statistical analysis is introduced; in section 3, the case study to which the procedure is applied is presented; in section 4, the multivariate analysis of the consumption process and the resulting demand patterns is described; finally, in section 5, the conclusions of this paper are drawn.

2. Literature review

2.1. Urban water demand modeling

Qi and Chang [4] and House-Peters and Chang [5] present an overview of water demand prediction models on various time scales. The time scale for any prediction model is dictated by the purpose for which the prediction model is to be used [6]. Most of the researches on water consumptions carried out in the past started from the need to quantify the global demand, by means of long-term forecasting [7,8,9,10], and to fix a suitable rate structure [11]. New reasons to better characterize the domestic water consumption have lately come out: the need to assure water volumes demanded by costumers and to supply them with sufficient pressure and good quality, have stand out among these [12,13,14,15]. The many approaches proposed to forecast short- and long-term municipal water demands in the past few decades might be grouped into five categories: the regression analysis, the time series analysis, the computational intelligence approach, and the stochastic model.

Traditional regression analyses were normally carried out based on statistical estimation of the relationship between water demand and some explanatory variables (i.e., independent variables), such as socioeconomic factors, and assumed that the relationships will continue in the future. Such a regression analysis approach can then be applied for both short- and long-term analyses when a training dataset is available [8, 16, 17, 18, 19, 20, 21, among others]. Time series analysis in water demand forecasting is based on a statistical abstraction of the various trends that inherently contribute to the change of water demand over time. A time series model may inevitably include a long-term trend component, a cyclical component, and a short-term variance component. The time series analysis has been extensively used for short-term water demand forecasting [1,9,10,22,23,24,25,26,27,28,29,30,31,32,33]. The computational intelligence models, including artificial neural networks (ANN), fuzzy logic, and agent-based models, are mathematically suited to simulate complex systems. For example, the ANN were developed for short-term water demand forecasting [34,35,36,37,38,39,40]. ANNs have been offered as effective alternatives to traditional linear modeling approaches because of their ability to explicitly analyze nonlinear time series events. ANNs have been proposed as an improved method for short-term forecasting of peak daily [36,41] and hourly [42] water demand.

The above-mentioned researches deal with water demand modeling at a big spatial scale (e.g. entire network level). At a domestic service level, water demand is sporadic, characterized by sudden demand pulses, and tends to have a stochastic character [13,14], especially when considering time scales on the order of seconds. Therefore, several stochastic models for domestic demand determination were developed. These models include the Poisson Rectangular Pulse model [13,14,15,43,44], the Neyman-Scott Rectangular Pulse model [45] and some more [46].

2.2. Multivariate statistical analysis

The copula function is a new analysis method well-known in the theory of probabilistic metric space before and recently introduced by De Michele and Salvadori [47] in hydrology. The copula function permits separate investigation of the marginal properties and interdependence structures of variables. Therefore, it synthesizes the dependence structure of the variables in the purest and most essential form [48] without assuming that variables are normal or have the same marginal distributions. The application of copulas in simulations of multivariate data, extreme value analysis and modeling dependence structure is becoming popular in hydrological analysis [47, 48, 49, 50, 51, 52]. A historical review and a discussion of major developments in the theory and application of copulas can be found in Schweizer [53] and Kotz [54]. While there is a multitude of bivariate copula, the class of multivariate copulas is still quite restricted. As matter of fact, building higher-dimensional copulas is generally recognized as a difficult problem. The idea of constructing a multivariate dependence model from bivariate copulas as building blocks called pair-copulas goes back to the paper of Joe [55]. He gave the construction of the first pair-copula in terms of distribution functions.

Bedford and Cooke [56,57] realized that there were a significant number of possible pair-copulas constructions (PCC), thus they organized them in graphical way by sequentially designing trees which identify the bivariate copula densities needed to make up a d-dimensional density. It involves only products of bivariate copulas. Since the trees are intrinsically related they called these distributions regular vines (R-vines). Their primary interest was to use vines in the modeling of large networks so they restricted themselves to the case of Gaussian pair-copulas. Aas et al. [58] were the first to recognize that this construction principle can be extended by using arbitrary pair-copulas, since the construction principle has no restriction on the choice of pair-copulas. Vine copulas are flexible models for multivariate dependencies which specify a factorization of the copula density into a product of conditional bivariate copulas. The class of regular vines is still very general and embraces a large number of possible pair-copula decompositions; it includes two simple tree structures, such as line trees and star trees, the first one corresponds to D-vines, while the second one corresponds to C-vines.

3. The case study

The multivariate statistical analysis described in the next section have been applied to water consumption data obtained monitoring eight dwellings located in Palermo (Italy) during the entire 2007. The customers that took part in the consumption monitoring program have been selected according to the following characteristics: family with at least two members; family members of 4-70 years old; one electric household appliance at least (dishwasher or washing-machine); negligible outdoor consumptions; cooperation. The selected eight families were the only that agreed to take part in the consumption monitoring program. Instrument packs to monitor domestic water use were installed on the service line of the secure indoor locations in each of the eight dwellings. The instrument package included a data logger, 4-20 mA impulse sensor. Data loggers were coupled with an input sensor inserted between the base and register head of a multi-jet water meter. The water meter had Q_1 = 15 l/h, Q_2 = 22.5 l/h, Q_3 = 1500 l/h,

Q_4 = 5000 l/h. The input sensor monitored revolutions of a magnet fixed to a positive displacement nutating disc in the measuring chamber of the meter. At each consumption of 0.5 liters, the sensor transmitted a signal to the data logger. Consumptions recorded at each user were reported in a text file where six fields were stored: day, month, year, hour, minute and second at which a use with a volume of 0.5 l occurs. Water demands were downloaded connecting the data logger to a portable pc.

A four steps process was used to transform the raw input signals into archived residential water consumptions: Step 1 involves data retrieval; Step 2 involves data correction (signal repeated removing, putting water demand reading in chronological order) and water uses separation; in step 3, leaks and ultra-low demands were censored; in step 4, the volume of each pulse was uniformly distributed over the duration of the pulse. A sparse matrix collected flow values (l/sec) having as number of columns the seconds in a day and as number of rows the days during which consumption data were recorded (changing for each user).

4. Consumption data analysis

In this paper the vine copula method has been used to build the 3-D copula for the main variables of domestic water consumption: the total daily volume, V_d; the daily peak coefficient, K_p, expressed as the ratio between the maximum consumption in a given time step, V_{max}, and the total daily volume; and time to peak, T_p. As first step of the analysis, the related marginal distributions (F_{Vd}, F_{Kp}, F_{Tp}) and the transformed variables (V, K, R), approximately uniformly distributed in [0, 1], were identified for the triplets (V_d, K_p, T_p). The related marginal distribution of each variable was obtained by fitting several distribution functions to the empirical CDF and by carrying out a K-S test ad goodness-of-fit test in order to choose the best distribution. All the three variables (V_d, K_p, T_p) show a good fitting with the GEV distribution. The parameters of the GEV marginal distributions for user 1 are showed in Table 1.

Table 1. Parameters of the GEV marginal distributions of (V_d, K_p, T_p) for user 1

		k	μ	σ
F_{Vd}	GEV	-0.23	0.17	0.37
F_{Kp}	GEV	0.37	0.04	0.13
F_{Tp}	GEV	0.45	0.08	0.37

As second step, the statistical dependence between the three variables (V, K, T) was evaluated by estimating the Kendall's τ_k rank correlation of each couple of variables V-K, V-T and K-T. For user 1, V-K and V-T showed a negative correlation, with Kendall's τ_k values equal to -0.42 and -0.12, respectively; only the pairwise K-T had a positive correlation, with τ_k equal to 0.07. Furthermore, the correlation was higher for V-K and V-R, and lower for K-T. Then, the vine copula method was used to build the 3-D copula for the variables (V, K, R). In the three-dimensional case there are no differences between a C- or a D-vine, only the ordering of variables can be changed. Fig. 1 shows the possible schemes for composing a 3-D vine copula. In the second tree, the two conditional CDF values are calculated for all triplets (V, K, R).

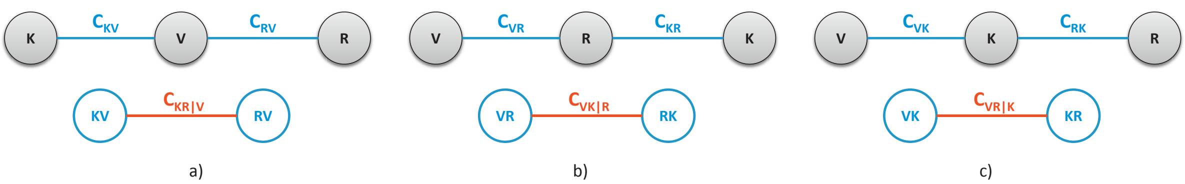

Fig. 1. Possible structures of a 3-D vine

These "conditioned observations", which are again approximately uniformly distributed in [0, 1], are then used to fit another bivariate copula, e.g. $C_{KR|V}$, $C_{VK|R}$ or $C_{VR|K}$. Considering the 3-D vine structure of Fig. 1a) the full density function c_{VKR} of the three-dimensional copula is thus given by:

$$c_{VKR}(v,k,r)=c_{KR|V}\left(F_{K|V}(k,v),F_{R|V}(r,v)\right)\cdot c_{KV}(k,v)\cdot c_{RV}(r,v) \quad (1)$$

Combining the bivariate copulas, as in Eq. (1), and substituting the marginal distribution functions F_{Vd}, F_{Kp} and F_{Tp} yields the three-dimensional distribution function of (V_d, K_p, T_p). The full density function f_{VdKpTp} of the distribution of each triplet (V_d, K_p, T_p) is then given by:

$$f_{V_dK_pT_p}(v_d,k_p,t_p)=c_{KR|V}\left(F_{K|V}(k,v),F_{R|V}(r,v)\right)\cdot c_{KV}(k,v)\cdot c_{RV}(r,v)\cdot f_{V_d}(v_d)\cdot f_{K_p}(k_p)\cdot f_{T_p}(t_p) \quad (2)$$

According to Eq. (1) and (2), three bivariate copulas need to be fitted to derive the building blocks of the 3-D vine copula (e.g. in Fig. 1a, the C_{KV}, C_{RV} and $C_{KR|V}$ bivariate copula). The maximum likelihood estimation method (MLE) was adopted to fit a copula from each family investigated for each pair of variables: the copula showing the highest log-likelihood value was selected as best fitting. The copula families investigated include Normal, Student, Gumbel, Frank, Clayton, BB1, BB6, BB7, BB8 and their rotated version. All the possible schemes of 3-D vine copula showed in Fig. 1 were built. The log-likelihood and the Akaike's Information Criterion (AIC) values were evaluated for each of the three vine copula schemes built for identifying the best fitting vine copula model for the analyzed dataset. Table 2 shows copula families, parameters and the Kendall's τ_k rank correlation of the bivariate copula composing the three 3-D vine copula built for user 1 together with the related log-likelihood and AIC values. The best fitting 3-D vine, having the higher log-likelihood value and the lower AIC value, is that showed in Fig. 1a.

Table 2. Copula families, parameters and Kendall's τ_k of the building blocks of the 3-D vine copula constructed for user 1

	3-D vine Fig. 1a			3-D vine Fig. 1b			3-D vine Fig. 1c		
Log-likelihood	106.31			79.22			105.60		
AIC	-204.63			-148.44			-201.20		
	CVK	CVR	CKR|V	CVK	CKR	CVR|K	CVK	CKR	CVR|K
Family copula*	33	40	5	40	10	5	33	10	40
par	0.17	-3.24	2.83	-3.24	1.80	0.97	-0.17	1.80	-4.49
par2	0.00	0.96	0.00	0.96	0.90	0.00	0.00	0.90	-0.90
Kendall's τ_k bivariate copula	-0.08	-0.51	0.29	-0.51	0.22	0.11	0.08	0.22	-0.58

*5 = Frank copula; 10 = BB8 copula; 33 = rotated Clayton copula (270 degrees); 40 = rotated BB8 copula (270 degrees)

After the identification of the best fitting 3-D vine copula model, the analysis focused on the identification of the triplets related to a given return period. The multivariate return period of the triplets (V, K, R) was assessed by means of the copula's Kendall distribution function $K_C(t)$ approach proposed by Salvadori et al. [59]. According to this, the return period T_{KEN3} is given by:

$$T_{KEN3}=\frac{\mu}{1-K_C(t)} \leftrightarrow t_{KEN3}=K_C^{-1}\left(1-\frac{\mu}{1-T_{KEN3}}\right) \quad (3)$$

where μ is the mean inter-arrival time expressed in years (in the case of daily event, μ = 1/365).

The copula's Kendall distribution function $K_C(t)$: I→I is defined as:

$$K_C(t)=P\left(C(v,k,r)\le t\right) \quad (4)$$

where $t \in I$ is the probability level.

According to Eq. (4), after fixing the design return period T_{KEN3}, the corresponding probability level t_{KEN3} can be assessed by means of the inverse of the copula's Kendall distribution function $K_C(t)$. In 3-D this corresponds to an iso-surface, i.e. all triplets (v, k, r) on this surface have the same copula value equal to t_{KEN3}. Kc(t) allows at calculating the probability that a random point (v, k, r) in the unit cubic space has a smaller or larger copula value than a given critical probability level $t = t_{KEN3}$. The Kendall distribution function is an univariate representation of multivariate information as it is the CDF of the copula's iso-surface. Therefore, Kc(t) turns out to be an essential tool for calculating a copula based return period for multivariate events [60].

A numerical evaluation based on a sample of 1,000,000 points simulated from the 3-D vine copula was carried out to calculate the inverse of Kc(t), as no closed form exists for the cumulative distribution function of the 3-D vine copula adopted in this analysis (for more details see Salvadori et al. [59]). Two return period were set, $T_{KEN3} = 2$ years and $T_{KEN3} = 5$ years. According to Eq. (4) and the numerical evaluation of Kc(t), the related t_{KEN3} values were calculated and resulted equal to 0.522 and 0.695, respectively. Thus, from the iso-surface corresponding to C(V, K, R) = t_{KEN3}, 1,000 triplets (V, K, R) were sampled and the corresponding 1,000 triplets (V_d, K_p, T_p) having iso probability were obtained by the inverse marginal distribution. As final step of the analysis, a pattern was statistically assigned to each triplet (V_d, K_p, T_p) taking into account the historical series of consumption. A mass curve (Huff curve) was obtained for each recorded daily consumption pattern as representation of the normalized time versus the normalized cumulative water consumption from the beginning of the day. Then, the Huff curve of the recorded daily consumption pattern that minimized the following objective function [61] was assigned to each statistical triplet (V_d, K_p, T_p):

$$S = \left[\left(\frac{V_{max}}{V_d} \right)_{statistical} - \left(\frac{V_{max}}{V_d} \right)_{historical} \right]^2 \quad (5).$$

The patterns have been finally processed and the percentiles for given return period have been estimated (Fig. 2). The two patterns are similarly shaped: peaks are preserved in the beginning of the morning confirming that this user is typical of working families that are not often at home during the afternoon. The percentiles are consistent demonstrating that the analysis carried out can be efficiently used for water distribution networks simulation.

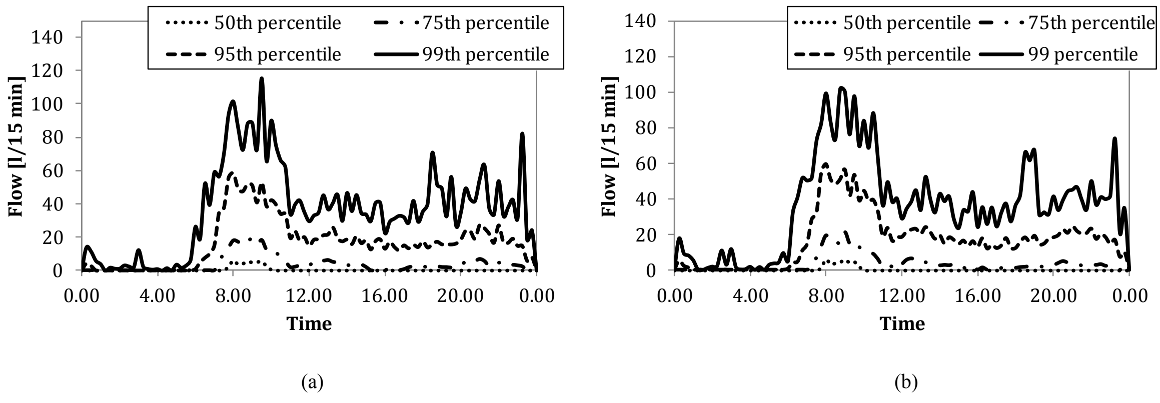

Fig. 2. Water demand percentiles: (a) 2-years return period; (b) 5-years return period

5. Conclusions

The interest in domestic water demand modeling comes from the wish for reaching two main objectives: to analyze the domestic consumption process to aid systems management; to define demand patterns at given return period to aid systems design. Under this point of view, the present paper proposed a statistical methodology for the definition of water consumption patterns based on return period and multivariate probabilistic approach. The method tried to avoid the usual assumption of a constant water demand pattern for network simulation. It is based on a multivariate statistical analysis: a 3-D vine copula was built for the main features of the consumption process at domestic level. The water demand was predicted for given return period by means of patterns that was statistically generated taking into account the historical series of consumption to which the methodology was applied. The analysis of the percentiles of the water demand for given return period showed that the proposed approach produced consisted demand patterns and will be a powerful tool to be coupled with water distribution network models for design or analysis problems.

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