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## Multi-stage linear programming optimization for pump scheduling

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### Abstract

This study presents a methodology based on Linear Programming for determining the optimal pump schedule on a 24-hour basis, considering as decision variables the continuous pump flow rates which are subsequently transformed into a discrete schedule. The methodology was applied on a case study derived from the benchmark Anytown network. To evaluate the LP reliability, a comparison was made with solutions generated by a Hybrid Discrete Dynamically Dimensioned Search (HD-DDS) algorithm. The cost associated with the result derived from the LP initial solution was shown to be lower than that obtained with repeated HD-DDS runs with differing random seeds.

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### 1. Introduction

In recent years, much research has focused on optimizing pump operation schedules; with increasing energy prices, the cost of electricity used for pumping represents the single largest part of the total operational cost in water distribution systems. The scheduling of pumps is frequently undertaken in near-real time, in order to minimize cost and maximize energy savings, however this requires a computationally efficient algorithm that can rapidly identify an acceptable solution.

Several optimization techniques have been applied to obtain solutions to the pump-scheduling problem: linear (Jowitt and Germanopoulos, 1992), non-linear (Yu et al., 1994) and dynamic programming (Lansley and Awumah, 1994), heuristics (Ormsbee and Reddy, 1995) and evolutionary computation (Savić et al., 1997; McCormick and Powell, 2003; van Zyl et al., 2004, Lopez-Ibáñez et al., 2008). Most of them, either greatly simplify the complex water distribution system or require significant time to solve the problem, limiting their real-time capabilities. An interesting literature review on optimal water distribution control models was provided in Price and Ostfeld (2013). According to the number of variables and objectives considered, optimizing the pump-scheduling problem may become very complex, particularly for large networks.

The scheduling of pumps is frequently undertaken in near-real time, in order to minimize cost and maximize energy savings, however this requires a computationally efficient algorithm that can rapidly identify an acceptable solution.

In the literature several methods have been proposed. Lansley and Awumah (1994) have determined the optimal pump operations considering the energy and the pump maintenance costs using dynamic programming showed good

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results for real-time application. Unfortunately, this method is impractical when there are more than three reservoirs; nevertheless, this limitation can be overcome where large systems consist of a number of small subsystems which are hydraulically independent. The model performs off-line hydraulic simulations in order to develop the functions describing the network hydraulics and the energy consumption into the dynamic programming algorithm.

In water distribution systems (WDSs), characterized by multiple reservoirs, more sophisticated techniques for generating optimized pumping schedules have been applied. Yu et al. (1994) proposed a non-linear programming based method where a generalized, reduced gradient technique was used to calculate optimal strategies for reducing the number of full-network simulations. The method does not require any network simplification and it can be used for near-real time application, even if the simulator efficiency improvement is needed. McCormick and Powell (2003) outlined a hydraulic network linearization for two stage Simulated Annealing (SA) algorithm. Although this technique is able to find a near global optimal solution, it is time consuming and, as a consequence, its application is often limited to off-line optimization problems. Nevertheless, the authors have demonstrated that linear programming can be a viable part of the solution process and that it can accelerate SA optimizations. A similar approach was undertaken by van Zyl et al. (2004), in which they coupled a Genetic Algorithm (GA) to two hill-climbing search algorithms for improving the local GA search once close to an optimal solution. Although these efforts employ evolutionary optimization techniques, operating directly on hydraulic simulation, these cannot cope with near-real time use. Conversely, Linear Programming (LP), for example, has been shown to be an appropriate technique for this application (Jowitt and Germanopoulos, 1992). The advantage of the LP model is that it can be solved quickly but requires that both objective function and constraints be linear. It assumes near-linear operating conditions in pumping stations as well as within the network. The linear model can be used for systems with multiple pumping stations, but the resulting accuracy and reliability can be quite poor. However, in recent literature, particular attention has been paid to the applicability of LP to the pumping scheduling optimization problem.

Pasha and Lansey (2009) formulated the LP optimization problem, linearizing the pumping station relationships by using the relationship between energy, pump flow, user demand and tank water levels. In particular the energy consumed has been approximated as a linear function of the pumping station flow and the initial tank level; the LP model was then tested on a single tank system, although the authors stated that it could be easily extended to more complex systems. Further investigation into the use of LP algorithm has been reported in Giacomello et al. (2013). Here, a fast, hybrid optimization method was developed, coupling LP with a greedy algorithm which was chosen as the local search method. The former solves a “reduced complexity” hydraulic model, then the latter the “full complexity” hydraulic model: the greedy algorithm performing a search starting from the pumping schedule identified by the LP method. They also demonstrated that the hybrid method, when compared to the GA optimization method, is capable of solving the real-life pump scheduling problem in a much more computationally efficient manner.

In this study, a methodology based on LP has been developed for determining the optimal pump schedule. The resulting model does not guarantee the identification of the global optimum solution of the pump scheduling problem, due to the inaccuracies introduced by linearization. However, it can provide a solution of sufficient quality to be applied in practice.

The methodology was applied on a case study derived from the benchmark Anytown network (Walski et al., 1987). In order to evaluate the reliability of the LP, a comparison was made with solutions generated by a Hybrid Discrete Dynamically Dimensioned Search (HD-DDS) algorithm (Tolson et al. (2009)) which is further addressed below.

## 2. Problem formulation

As mentioned above, the pump scheduling problem can be formulated as optimization problem which aims to minimize the energy costs, while keeping within physical and operational constraints. The optimization period is divided into a number of discrete control intervals which must be such that a meaningful definition of the problem is obtained. The structure of the electricity tariff and the system component characteristics inform this selection: the smaller the interval, the greater the accuracy of the analysis. However, the number of decision variables and constraints increases significantly with the number of control intervals defined, leading to increased computational and memory requirements for the solution (Jowitt and Germanopoulos, 1992). To reduce the total number of variables, a single decision variable for each pump station and time interval, that relates the particular set of pumps in operation during that period, can be developed (Ormsbee and Reddy, 1995).

According to these considerations, in this study, the objective function (Eq. 1) was defined in terms of pump station discharges  $Q_i$ , in place of a single pump status, considering both the network hydraulics and the electricity tariff embedded into the coefficients  $c_i$ , as described below. The optimization period was divided into intervals of one hour. The maximum and minimum water levels into the tank (Eq. 2), as well as the limit of the pump station duties (Eq. 3), were considered. Further constraints ensure that the tank level at the end of the optimization period is not lower than the level at the beginning of the next period (Eq. 4) and the tank mass balance over each control interval (Eq. 5) is satisfied. The resulting optimization problem can be then formulated as:

$$\min \sum_{t=0}^T c_t \cdot Q_t \quad (1)$$

Subject to:

$$S_{\min} \leq S_t \leq S_{\max} \quad (2)$$

$$Q_{\min} \leq Q_t \leq Q_{\max} \quad (3)$$

$$\sum_{t=0}^T Q_t = \sum_{t=0}^T q_t \quad (4)$$

$$Q_t \cdot \Delta t + (S_t - S_{t-1}) \cdot A = q_t \cdot \Delta t \quad (5)$$

where  $Q_t$  are the unknown pump station discharges,  $c_t$  the objective function coefficients,  $q_t$  the known demand,  $A$  is the tank surface area,  $S_t, S_{t-1}$  are the tank water level at time  $t$  and  $t - 1$  respectively;  $\Delta t$  is the optimization control interval (often fixed to 1 hour),  $S_{\min}, S_{\max}$  are the lower and upper bound referred to the tank water level while  $Q_{\min}, Q_{\max}$  are those related to the pump station discharges.

The system of equations above (Eqs. 1-5) represents a linear model that can be easily solved as a linear programming problem. The optimal pump station discharges and the tank levels for each time step were calculated, the latter not being explicitly considered as decision variables at this stage of the analysis. In order to provide a schedule the resulting pump station discharges were transformed into discrete pump combinations to provide a similar rate on a 24-hour basis. If the discharge failed to match exactly that of a combination of pumps, some error will result in the approximate cost and tank water level; but these small differences will not significantly modify the optimal solution cost. An extended period simulation was subsequently undertaken in order to verify the feasibility of the obtained pump schedule.

Prior to solving the system of linear equations, the objective function coefficients  $c_t$  were evaluated as a product of the energy tariff for the slope of the line interpolating the energy consumptions with respect to pumped flow rates resulting from several steady-state simulations performed for each demand pattern and for each pump combination - considering as fixed the initial tank level. This boundary condition is justified by the consideration that the energy consumption is only slightly influenced by variation in tank level, whereas it is more sensitive changes in demand, especially when the pumping station is directly connected to the water distribution network. Besides, these results aim to provide information about the likely system behaviour assuming that the network hydraulics are completely embedded in the energy consumption relationship and thus in the coefficients  $c_t$  of the objective function.

The results of these simulations were also used to identify the possible pump combinations able to supply the required flow rates resulting from the LP model. The model runs were performed by linking the EPANET hydraulic solver (Rossman, 2000) directly to the MATLAB software application; the output is then the pump schedule related to well-defined boundary conditions.

### 1.1. Hybrid Discrete Dynamically Dimensioned Search

In this section a brief description of the Hybrid Discrete Dynamically Dimensioned Search (HD-DDS) optimization is provided. As mentioned above this algorithm was used to evaluate the LP reliability. The HD-DDS is a heuristic global optimization algorithm and the principal advantage, compared with genetic and ant colony algorithms, is that its searching capability (i.e., the ability to find near globally optimal solutions) is good while being significantly more computationally efficient.

In contrast to the majority of evolutionary optimization techniques, HD-DDS operates on a single solution rather than a population of solutions. It operates firstly as a global search by permuting decision variables according to a probability distribution. This search is then coupled to a local search method developed specifically for the pump scheduling problem which is executed at several points in the algorithm. The local search element attempts to improve a pumping schedule through changing the status of a single pump at a given time step and reevaluating the solution. By doing this sequentially, for each of the pumps and time steps, it can be guaranteed that the cost of the pump schedule cannot be improved by switching off any pump at any time during the day without having to simultaneously turn on another pump – and thus increase the cost. The schedule obtained from LP was tested as initial seed solution for the HD-DDS optimization in place of a randomized initialization. The cost associated with the result derived from the LP initial solution was then compared with that obtained with repeated HD-DDS runs with differing random seeds.

### 1.2. Test problem

The methodology was applied on case study derived from the benchmark Anytown network (Fig. 1). It is composed of 19 junctions, 1 tank, 37 pipes and 1 reservoir, representing the only external source, from which four different pumps in parallel supply water to the remainder of the system. The demand varies according to a demand pattern with peak factors ranging from 0.4 – 1.2 as shown in Fig. 2. The daytime tariff cost is set to be twice that charged during the night.

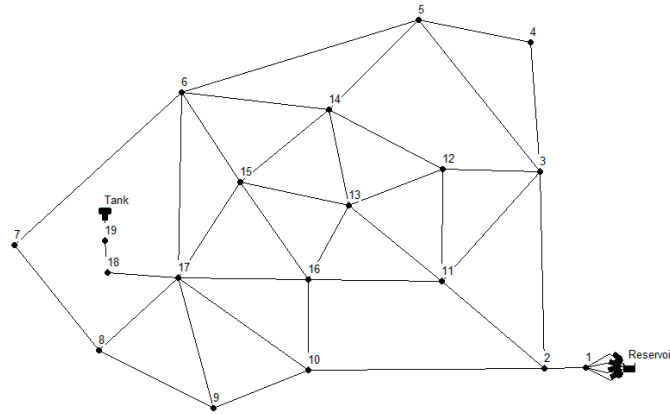


Fig. 1 Test water distribution system: Anytown.

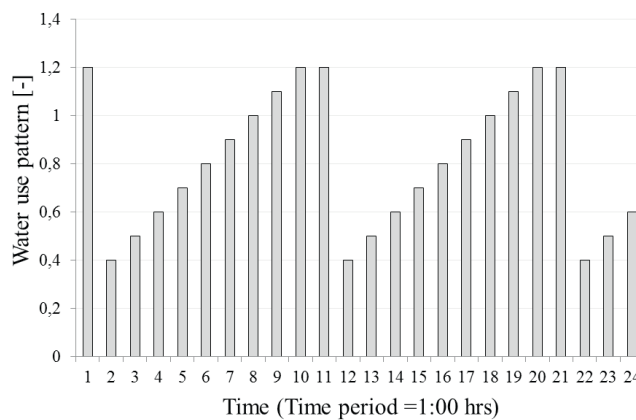


Fig. 2 Water use pattern.

For the analyzed network, all pump combinations have been tested to evaluate the objective function coefficients. As mentioned above, the same initial tank level was arbitrarily set (equal to 50% of the maximum water level) for all of the steady-state network simulations. This assumption is based on the consideration that the variation in the tank initial level has a small influence on the duty point of the pump station compared with that due to variation in demand. For two demand patterns, the pump station discharges and the corresponding energy consumed, resulting from the steady-state simulations with three different tank initial levels (75%, 50% and 25% of the maximum tank water level) were evaluated (Fig. 3). The variation between that of the selected initial level (50% of the maximum level) and the others was about  $\pm 1.3\%$  for the pump flow rate and  $\pm 0.5\%$  for energy consumption.

Fig. 4 shows the energy consumption related to pump station discharge obtained from the steady-state network simulation for all pump combinations and demand patterns. The data were interpolated by linear functions: the slopes of the lines represent the proportional factors between energy and pumped flow. Since the slopes do not vary considerably among the different demand patterns, an average value was considered in this analysis. Hence, the objective function coefficients were evaluated. In order to ensure the comparability of the results, the intercept of the energy function, which represents the potential energy linked to available pump total head, was used to update the cost obtained through the solution of the LP problem.

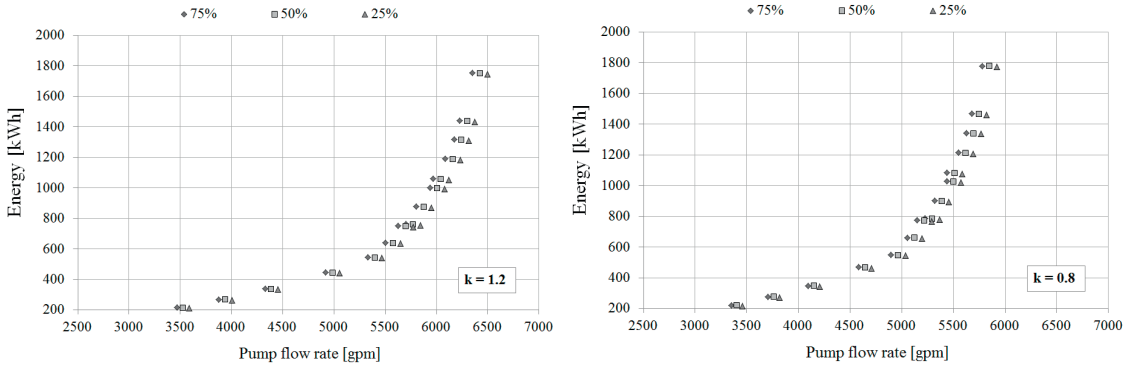


Fig. 3 Energy consumption related to pump station flow rate for all the pump combinations and two demand patterns, obtained fixing three different initial tank water levels (75%, 50% and 25% of the maximum tank water level).

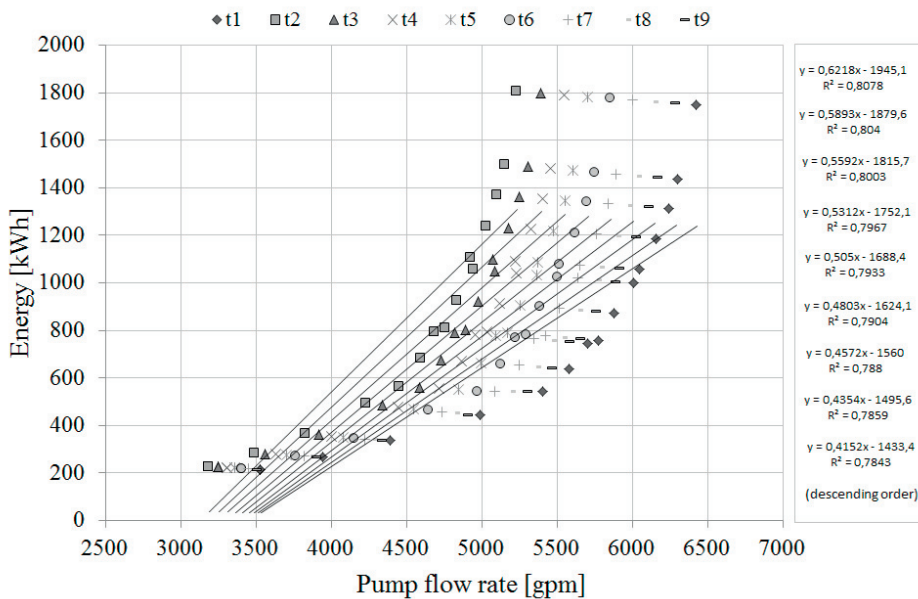


Fig. 4 Energy consumption related to pump station flow rate for all the pump combinations and demand pattern, obtained fixing initial tank level to 50% of its maximum value. The data were interpolated by linear functions.

### 3. Results

The LP solution was quickly identified once all of the information about constraints and objective function coefficients were calculated. Figs. 5 and 6 show the pump station discharge and tank water levels during the optimization period respectively; both the LP model solution and the EPS results related to the derived-schedule are presented. Moreover, the user demand profile and the energy tariff are shown. The tank is filled during the low-cost period while it is allowed to almost empty (minimum level: 7ft) during the more expensive daytime tariff, at which point the pump station supplies nearly the entire users’ demand. The cost associated with the derived-schedule is about £377/day.

The water discharges related to this derived-schedule must be considered as hourly average values: since the minimum value of pump flow rates resulting from the possible combinations (see Fig. 4) was higher than the most of the values of the LP solution obtained. The pump combination (except the “no-pumps running” combination) able to provide the closer flow rate was chosen and then forced to work for less than one hour in order to ensure the mass balance within each LP control interval with a reasonable tolerance (about 10%). Conversely, maintaining the pump state as “on” during the entire control interval can result in an overflow in the tank. In this event, the hydraulic solver (EPANET) turns off the pumps: in this case any control on the water volume pumped into the system is possible. For

the analysed case study the LP 24-hour solution was transformed into a schedule with a 15-minutes interval in order to verify the mass-balance constraint fulfilment.

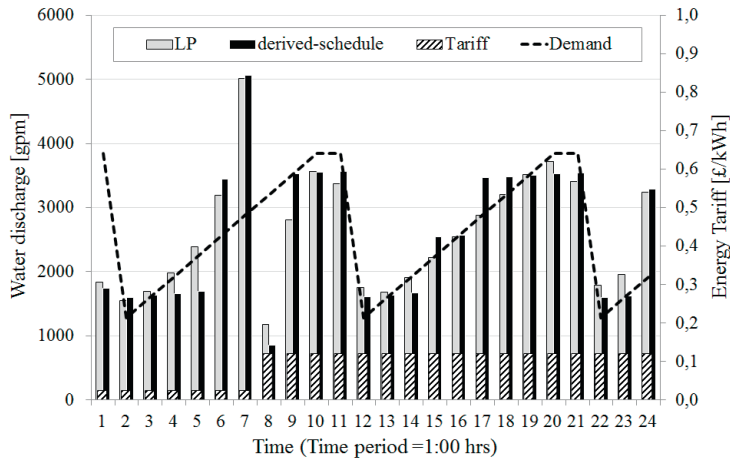


Fig. 5 The pump station discharges from the LP model and from the simulation of the derived schedule, energy tariff and user demand for all optimization control intervals.

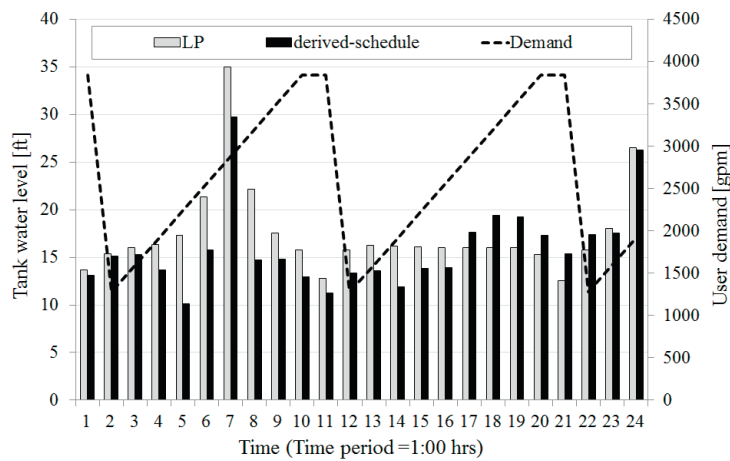


Fig. 6 The tank water level from the LP model and from the simulation of the derived schedule, and user demand for all optimization control intervals.

In order to evaluate the reliability of the LP, a comparison was made with solutions generated by a Hybrid Discrete Dynamically Dimensioned Search (HD-DDS) algorithm. The algorithm runs with a 1-hour interval, for this reason the previous derived-schedule was transformed accordingly; it was simply assumed that the same pump combination working for one hour; the cost associated to this schedule was £479. Ten different optimization runs were performed from different, randomly selected, starting points.

Table 1 shows the solutions generated by the HD-DDS algorithm testing the LP derived-schedule and the randomized initialization as initial seed solution. The cost associated with the results derived from the LP schedule initial solution varied between £375.33 and £393.10 with an average of £384.65. The values were lower than those obtained running the HD-DDS with the randomized initialization. The best improvement was in run 9 where the LP derived-schedule initial solution improved the HD-DDS solution by 11.9%. Therefore, it is interesting to notice that the cost of the schedule with a 15-minutes interval was very close at £377.

Table 1 Comparison between the best solutions obtained from different initial seeds.

Run number	Best solution (£/day)	
	HD DDS with LP initial solution	HD DDS with randomized initial solution
1	392.57	396.54
2	378.46	401.06
3	377.45	402.86
4	387.09	402.50
5	384.48	428.16
6	384.93	418.06
7	387.99	403.65
8	393.10	394.97
9	375.33	426.09
10	385.07	386.68
Average	384.65	406.06

#### 4. Conclusions

In this work, the pump scheduling problem was investigated through a Linear Programming based method. Once the constraints and objective function coefficients were formulated, the decision variables – the pump station water discharges – were quickly obtained solving the linear system of equations which describes the above-mentioned optimization problem. Then, the LP solution was transformed in a discrete schedule able to provide the same rate on a 24-hour basis. This step of the methodology revealed some criticality due to the appropriate selection of the pump combination for each control interval. To overcome this problem the schedule was evaluated with a 15-minutes interval rather than hourly; this different discretization ensured that the mass balance constraint was respected and proved to be a good solution.

The LP methodology demonstrates, therefore, the potential to be adopted to rapidly determine an approximate, though acceptable, solution which may itself then be subject to further optimization. When the schedule with an hourly interval was tested as an initial seed solution into the HD-DDS algorithm, the resulting cost was lower than that obtained using the randomized initialization for ten different optimization runs.

Further studies are needed in order to verify the reliability of the proposed methodology to real-system applications, e.g. considering a more efficient selection of the pump combinations which must be performed for the objective function coefficients evaluation; testing the LP derived-schedules as seed solutions to speed up other optimization algorithm.

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