

Chapter 11

Global sensitivity analysis in wastewater treatment modelling

G. Mannina^{1,2}, *A. Cosenza*¹, *M. B. Neumann*^{3,4},
P. A. Vanrolleghem^{5,6}

¹*Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale, dei Materiali – Università di Palermo, Viale delle Scienze, 90128 Palermo, Italy*

²*Department of Earth and Environmental Engineering, Columbia University, 500 West 120th Street, New York, NY 10027, USA*

³*Basque Centre for Climate Change – BC3, Leioa, Spain*

⁴*Ikerbasque, Basque Foundation for Science, Bilbao, Spain*

⁵*modelEAU, Département de génie civil et de génie des eaux, Université Laval, 1065 av. de la Médecine, Québec (QC) G1V 0A6, Canada*

⁶*CentrEau, Water Research Centre of Université Laval, Québec, QC, G1V 0A6, Canada (giorgio.mannina@unipa.it)*

ABSTRACT

Global sensitivity analysis (GSA) is a valuable tool to support the use of mathematical models. GSA allows the identification of the effect of model and input factor uncertainty on the model response, also considering the effect due to the interactions among factors.

During recent years, the wastewater modelling field has embraced the use of GSA. Wastewater modellers have tried to transfer the knowledge and experience from other disciplines and other water modelling fields. GSA has been adopted for

several purposes (optimization, calibration, uncertainty assessment, etc.) with the final aim to improve the use of wastewater models. The purpose of this chapter is to provide the key issues surrounding GSA. Specifically, this chapter aims at identifying, for the most popular GSA methods, their potential use, the critical issues to be solved and current limitations.

Keywords: Computational burden, convergence, modelling, numerical methods, sensitivity analysis, water modelling

11.1 INTRODUCTION

Over the past 30 years the knowledge on wastewater treatment has enormously advanced and matured leading to the setting up of detailed principles for describing the key processes (biological, physical, chemical, hydraulic) occurring inside a wastewater treatment plant (WWTP) (Henze *et al.*, 2008). Over the years, these principles have been codified into mathematical models. The term WWTP model is often used to indicate the combination of activated sludge model (ASM), hydraulic model, oxygen transfer model and sedimentation tank model (Gernaey *et al.*, 2004).

The ASMs introduced by Henze *et al.* (2000) represent the most popular models describing the biological (and chemical) reactions taking place inside the activated sludge tanks. The hydraulic models describe the hydraulic conditions inside the tank and the flow rates (internal recycle, return sludge) throughout the different tanks. Regarding the settler tank, the most popular models suggest the description of the settler as an ideal point (without retention time) or as a one-dimensional layered settler (Takacs *et al.*, 1991). WWTP modelling has several scopes (Gernaey *et al.*, 2004): (i) learning, for example the adoption of a mathematical model can improve the knowledge on the process under study; (ii) design, for example the adoption of a mathematical model allows the comparison of different configuration solutions and design alternatives; (iii) process optimization and control, for example a mathematical model allows the prediction of the WWTP behaviour under different influent conditions and provides the best operation conditions to improve the effluent quality. The WWTP models include tens of model parameters (physical, kinetic, stoichiometric, influent fractionation etc.) for which no universal values exist, thus the correct values are required to be identified for each application. With this regard, the model calibration represents a key step in the mathematical modelling. The model calibration consists of the variation of model parameter values in view of identifying the set that provides the best fit between the model response and the set of data obtained from the WWTP under study (Gernaey *et al.*, 2004). The need to calibrate a model depends on the model purpose. For models aimed at educational purposes the default model parameter values can be adopted (Petersen, 2002). Conversely, in the

case where the model is aimed at process optimization, an accurate description of the processes under study is required and consequently data collection and model calibration are needed (Petersen, 2002). Since WWTP models are often overparameterized with respect to the available measured data, the parameters that are required to be calibrated (in view of improving the model response) are not unique (Brun *et al.*, 2002). Therefore, the approaches and or procedures aimed at the model calibration often adopt sensitivity analysis (SA) in view of selecting the best parameter subset to be calibrated.

One of the most common definitions of SA has been provided by Saltelli *et al.* (2004) who define SA as: “*The study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input*”. This means that the SA allows identification of the model parameters and input variables whose values strongly influence the model response (model output). Conversely, it may be of interest to the modeller to see that although some model parameters may not be very well established they do not significantly contribute to output uncertainty.

SA can have several aims such as model calibration, model diagnosis, decision-making support, etc. (Mannina *et al.*, 2011a; Butler *et al.*, 2014; Norton, 2015; Mannina *et al.*, 2017). In the field of mathematical modelling, sensitivity analysis represents a very powerful tool as it provides information about how the variation in the outputs of the model can be apportioned to the variation of the model (input) factors (Saltelli, 2000). This can be of interest, for example to understand which of the uncertain (kinetic, stoichiometric or mass transfer-related) model parameters are responsible for most of the uncertainty in model predictions, for example ammonium effluent concentrations. It may then be possible to conduct dedicated experiments (e.g. batch experiments) to reduce uncertainty in the parameters that were found to be causing much uncertainty in the predictions (Dochain & Vanrolleghem, 2001).

Over the past 15 years, the engineering and scientific communities in the environmental water modelling field have improved knowledge on the use of sensitivity analysis (SA). SA has been identified as constituting a crucial step in any environmental modelling exercise (Jakeman *et al.*, 2006; Norton, 2008).

In the sensitivity analysis literature “Factors” is a term widely used and includes model parameters and model input variables. Saltelli (2000) singles out three main classes of sensitivity analysis methods:

- screening methods;
- local methods;
- global methods.

Screening methods are economical and qualitative methods. Local methods provide a measure of how the model output is affected by factor changes at a specific location in factor space by adopting a one-at-a-time (OAT) approach, indeed other factors are usually set to their nominal or mean values.

Global sensitivity analysis (GSA) on the other hand provides information on how the model outputs are influenced by varying factors across many possible locations over a large range of values (Homma & Saltelli, 1996; Saltelli *et al.* 2004).

GSA can be conducted for several reasons: (i) to identify factors that mainly influence specific model outputs of interest (*factor prioritization*); (ii) to select which factors interact with other factors (*interacting factors*); and (iii) to identify non-influential factors (*factor fixing*) (Saltelli *et al.*, 2004). Other possible objectives of GSA not explored in this review include *factors mapping* (to search which factors are responsible for producing outputs in a certain region, e.g. above a threshold value) or *variance cutting* (identify a minimal subset of factors to fix, in order to obtain a prescribed reduction of uncertainty in the output).

By means of GSA, modellers are supported to identify critical regions in the factor space, to establish priorities for research and to simplify models (Saltelli *et al.*, 2008).

GSA methods are classified into (Saltelli *et al.*, 2008):

- (i) global screening methods, for example Morris screening method (Morris, 1991; Campolongo *et al.*, 2007);
- (ii) variance-based methods such as Extended Fourier Amplitude Sensitivity Testing (Extended-FAST) (Saltelli, 2000);
- (iii) regression/correlation-based methods such as the standardized regression coefficients (SRCs) method (Saltelli *et al.*, 2008).

In the environmental modelling field, the majority of SA applications have been local and derivative-based due to the fact that these methods are computationally very efficient (Saltelli *et al.*, 2008). However, local sensitivity analysis (LSA) methods can be misleading in the case of highly uncertain input factors and non-linear relationships between model outputs and factors as they provide information only at the “nominal point”. The main limits of LSA can be overcome by applying GSA. For a priori unknown model behaviour, the GSA should be the preferred method to apply (Saltelli & Annoni, 2010).

Due to the high computational cost of running simulations for environmental models, the dissemination of GSA applications has been limited (Campolongo *et al.*, 2007). Therefore, modellers have often been reluctant to use GSA methods instead of local methods (Saltelli & Annoni, 2010).

During recent years, modellers have started spending considerable time in understanding the potential of each GSA method applied to complex water engineering models (Mannina *et al.*, 2011b; Sin *et al.*, 2011; Benedetti *et al.*, 2011; Mannina *et al.*, 2014; Ramin *et al.*, 2014). An increasing trend of GSA applications has been found in literature (Figure 11.1). From Figure 11.1 one can observe that GSA applications have quickly spread in the water quality and hydrology fields with 321 and 140 documents in 2017. Moreover, an increasing interest in applications even in the wastewater field has occurred in literature (Figure 11.1).

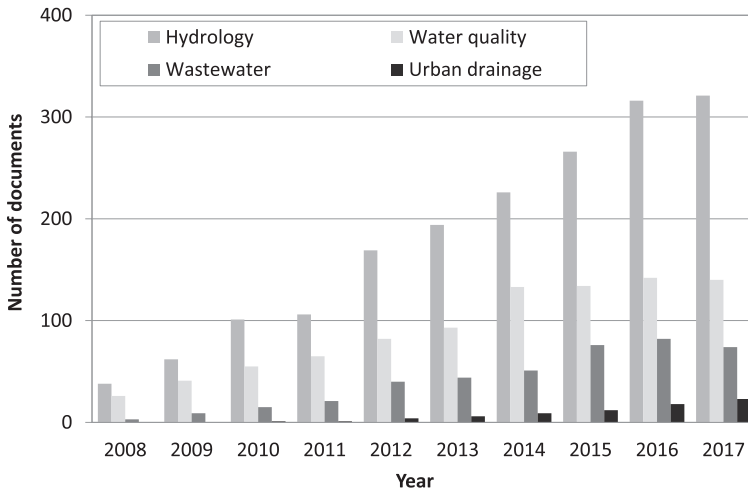


Figure 11.1 Trend of GSA applications found in literature during the last 10 years divided per field of interest (source Scopus, obtained by using the following search words: global sensitivity analysis and hydrology, wastewater, water quality, urban drainage).

Despite the clear advantages of GSA, several questions are still open on GSA: Can the terminology be standardized? Do all model applications require the same numerical settings for GSA? How should modellers test whether the SA has converged?

The purpose of this chapter is to highlight key issues surrounding GSA application in the wastewater modelling field.

Following this introduction, Section 11.2 discusses commonly applied SA methods divided into four main classes: (i) derivative-based LSA; (ii) regression-based; (iii) screening; and (iv) variance-based. Then, the main GSA applications in the wastewater field and some relevant applications in other water modelling fields are provided in Section 11.3. Issues related with the numerical settings in GSA applications (e.g. choice of the criteria for model factor screening in terms of importance or convergence analysis) are presented in Section 11.4. In Section 11.5 details related to the multiple use of GSA methods are presented. Section 11.6 reports a summary and outlook.

11.2 SENSITIVITY ANALYSIS METHODS

In this section a general overview of the features of the most commonly used SA methods is provided. Table 11.1 summarizes the features of each SA method: the ability to cope with interaction among factors and with non-linearity, the computational cost required to employ the analysis, the ability to provide

Table 11.1 Main features of SA methods.

Class	Features					Sensitivity Measures		
	Method	Coping with Interaction	Coping with Non-Linearity	Factor Fixing	Factor Prioritization	Cost of Analysis	Symbol	Definition
Local SA	Derivative-based	yes	no	no	yes	$2n + 1$	$\delta y / \delta x$	Derivative of an output Y with respect to an input X
Regression-based	SRC	no	(no)	no	yes	500–1000	β_i	Standardized Regression Coefficient
Screening	Morris screening	yes	yes	yes	no	$r \times (n + 1)$	μ	Mean of elementary effects
Variance-based	Extended-FAST	yes	yes	yes	yes	$N \times n$	S_i	First-order effect
		yes	yes	yes	yes	$N \times (n + 2)$	S_{π}	Total effect
	Sobol'	yes	yes	yes	yes	$N \times (n + 2)$	S_i	First-order effect
		yes	yes	yes	yes	$N \times (n + 2)$	S_{π}	Total effect

Where: n = number of factors; r = number of trajectories, typically $4 \leq r \leq 10$; N = number of repetitions, typically $500 \leq N \leq 1000$.

information in terms of factors fixing (i.e. how many model factors do not affect the model outputs) and factors prioritization (i.e. how many model factors have a relevant effect on the model outputs variation). The symbols and the definitions in Table 11.1 are in accordance with Saltelli *et al.*, (2008).

In Figure 11.2 are reported the most commonly used sensitivity analysis methods, grouped into qualitative and quantitative methods (Hong & Purucker, 2018).

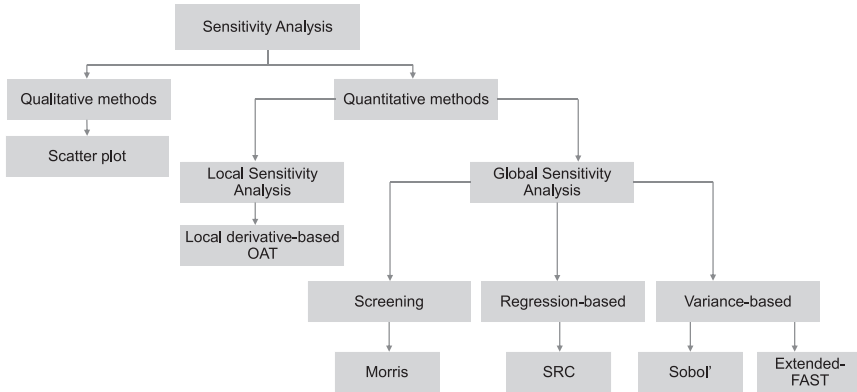


Figure 11.2 Most commonly used sensitivity analysis methods (modified from Hong & Purucker, 2018).

The scatter plot is a qualitative method which allows visual identification of the influence of model factors on model outputs and the possible (non) linear or (non) monotonic dependence between an input and output. Therefore, on the basis of this dependence the scatter plot method is usually adopted as a first step of sensitivity analysis to select the most adequate quantitative method. A disadvantage of the scatter plot method is that interpretation of a scatter plot is subjective (Frey & Patil, 2002).

Quantitative methods are classified into local and global. LSA methods allow identification of the relationship between inputs and outputs by changing the input value (model factor value) within a given point while other factors are usually set to their nominal or mean values by using an OAT approach (Figure 11.2).

GSA methods are model independent and therefore do not require a specific relationship between model factors and model outputs. The proper sampling of the entire parameter space with GSA allows handling of non-linear, non-monotonic and non-additive models. According to Saltelli *et al.* (2004) the application of regression/correlation based methods implies the assumption of model linearity. The application of the variance-based and global screening methods does not rely on special assumptions about the behaviour of the model (such as linearity, monotonicity and additivity of the relationship between input factors and model

output) (Saltelli *et al.*, 2008). Specifically, the most common screening method, Morris screening, is mainly effective in identifying important model factors for monotonic models by adopting reasonable computational cost (Table 11.1). The adoption of variance-based methods (e.g. Sobol' or Extended-FAST) is mainly effective for selecting important model factors for all kinds of model structure (linear, non-linear, monotonic and non-monotonic).

For the sake of completeness, it is important to recall the definitions of linear, monotonic and additive models. Specifically, the relationship between Y and an input factor X is monotonic if the curve $Y = f(X)$ is either entirely non-decreasing or entirely non-increasing over all the interval of definition of X . Further, a mathematical model can be defined as monotonic if the rule of monotonicity can be applied for all factors of the model. A model $Y = f(X_1, X_2, \dots, X_k)$ is additive if f can be decomposed as a sum of k functions f_i , each of which is a function only of the relative factor X_i . A mathematical model can be defined as linear if linear equations connect the model state variables and the model factors.

Most local SA methods used in literature are based on derivatives $\delta y / \delta x$ (Saltelli *et al.*, 2008). Despite having a low computational cost, the derivative-based SA methods can provide information of factor prioritization and interaction (Table 11.1). In the following sections, the key features of each SA method reported in Table 11.1 and Figure 11.2 will be discussed in detail.

11.2.1 Derivative-based

The derivative-based local sensitivity method represents the simplest type of SA. The core concept behind this method is the OAT perturbation of the input factors from their nominal values and the evaluation of the effect on the model outputs. This effect can be visually evaluated by comparing the model responses under different input factors perturbation or evaluating a sensitivity measure (Paton *et al.*, 2013). The sensitivity of the i th input factor on the model output y can be represented by the partial derivative $\delta y / \delta x$ evaluated at the nominal value of the factors. Since input factors may have different units of measurements, the partial derivatives are not comparable. In order to make them comparable they can be re-scaled by adopting a scaling factor. In terms of computational demand, this kind of method requires $2n + 1$ runs, where n represents the number of factors. Setting the size of the perturbation is a critical element in its application, leading to sensitivity to non-linearity or numerical inaccuracy (De Pauw & Vanrolleghem, 2006).

11.2.2 Regression-based

11.2.2.1 Standardized regression coefficients

The rationale of regression/correlation based methods is to perform Monte Carlo (MC) simulations of the model output by using a randomly sampled factor matrix.

Multivariate linear regression according to Equation 11.1 is then used relating model outputs (y) to the factors (x_i) (Saltelli *et al.*, 2008).

$$y = b_0 + \sum_{i=1}^n b_i \cdot x_i + \varepsilon \quad (11.1)$$

where n represents the number of factors, b_i are the regression slopes, and ε is the random error of the regression model. For each factor the standardized regression coefficient (SRC = β_i) of the multivariate linear model is calculated according to Equation 11.2.

$$\text{SRC}(x_i) = \beta_i = b_i \cdot \frac{\sigma_{x_i}}{\sigma_y} \quad (11.2)$$

where σ_{x_i} and σ_y represent the factor and the model output standard deviation, respectively. β_i is a valid measure of sensitivity if the coefficient of determination R^2 , which indicates the portion of total variance explained by the regression model, is higher than 0.7 (Saltelli *et al.*, 2008). A high absolute value of β_i indicates a relevant effect of the related i th factor on the model output. The sign of β_i indicates the direction of change. More specifically, the positive sign of β_i indicates that increasing the value of the i th factor increases the model output and *vice versa* in the case of a negative sign (Saltelli *et al.*, 2004). In the case of a linear model, the coefficient of determination R^2 is equal to 1 (Saltelli *et al.*, 2008). The SRC method does not provide information about the interaction among factors. It allows identification of the important model factors (*factors prioritization*) if the model is not too far from being linear (Table 11.1).

In terms of computational demand, the regression/correlation based methods are feasible to be used even for complex models with tens of factors. Indeed, the application of these methods requires a limited number of MC simulations, typical numbers found in literature are between 500 and 1000 (Neumann, 2012). However, the regression/correlation-based methods explore only the first-order effects and do not provide any information about the interaction among factors. Thus, these methods can be used only for factor prioritization in cases when the effects of non-linearity are not too strong ($R^2 > 0.7$).

The SRC method has the advantage of being simple to apply. However, it cannot be applied when the relationship between model factors and model output is non-linear or non-monotonic and when interactions are present (Saltelli & Sobol', 1995).

11.2.3 Screening

11.2.3.1 Morris screening

The Morris screening method represents the most used method belonging to the class of global screening methods. It is based on OAT perturbation of the model factors under investigation (Morris, 1991). However, in comparison to the derivative

based methods, the OAT perturbation is repeated at different locations within the factors range before being averaged. For each perturbation, the elementary effect (EE) is quantified. The EE represents the relative difference of the model output with, $y(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_n)$, and without, $y(x_1, \dots, x_n)$, a perturbation Δ of the i th factor (Equation 11.3).

$$EE_i(x_1, \dots, x_n, \Delta) = \frac{y(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_n) - y(x_1, \dots, x_n)}{\Delta} \quad (11.3)$$

The EE is repeatedly computed (r times) at different locations in the factor space (considering a p -level sampling grid). For each of the n factors, the measure of sensitivity is summarized by the mean (μ) and the standard deviation (σ) of the r EEs. The mean μ represents a measure of the importance of the factor in determining model output, whereas the standard deviation σ indicates whether the factor is responsible for introducing non-linearity or interactions (i.e. whether the sensitivity changes for different locations in the factor space) (Table 11.1). A high value of σ indicates that the model output variation is influenced by non-linearity or interactions. Thus, the Morris screening method is able to detect interactions among factors (Table 11.1). In order to avoid the problem that EEs of opposite sign cancel each other out, Campolongo *et al.* (2007) proposed using the mean of the absolute EEs (μ^*). The main objective of the Morris screening method is factors fixing: factors with a low value of μ^* or σ are considered non important and can be fixed anywhere in the factor space.

Further, as suggested originally by Morris (1991), the line $\mu_i^* = +2(\sigma_i/\sqrt{r})$, where σ_i/\sqrt{r} represents the standard error of the mean, can be used to discriminate between interacting and non-interacting factors. Factors which lie below the line have a linear effect on the model outputs whereas factors above this line have a non-linear effect or are involved in interactions.

As suggested by Campolongo *et al.* (2007) the required number of simulations for the Morris screening application is equal to $r^*(n + 1)$. Typical numbers are $r = 10-20$ and $p = 4-8$ (Campolongo *et al.*, 2007).

11.2.4 Variance-based

Variance-based methods are founded on the variance decomposition theorem. The main interesting features of variance-based methods are: (i) independence of the model structure; (ii) capability to analyse the influence of each factor within its entire range; (iii) capability to quantify the interaction among factors; and (iv) groups of factors can be considered as single factors. However, the main disadvantage of these methods is their computational cost, as they require a large number of simulations per factor (500–1000 according to Saltelli *et al.*, 2005). In the application of variance-based methods, modellers are often interested in two sensitivity indices: the first-order effect index (S_i) and the total effect index (S_{Ti}) (Table 11.1).

The most frequently used methods are: Extended-FAST and Sobol' (Cukier *et al.*, 1973; Schaibly & Shuler, 1973; Sobol', 1993; Saltelli, 2002).

11.2.4.1 Sobol' indices

The core idea behind the Sobol' indices method is the decomposition of the function $f(x)$, which depends on n input factors (x_1, \dots, x_n) , into summands of increasing dimensionality, namely:

$$f(x_1, \dots, x_n) = f_0 + \sum_{i=1}^n f_i(x_i) + \sum_{i=1}^n \sum_{i \neq j}^n f_{ij}(x_i, x_j) + \dots + f_{1,2,\dots,n}(x_1, \dots, x_n) \tag{11.4}$$

The integral of every summand of any of its own variables must be zero and all the summands in Equation 11.4 are orthogonal. Thus, the analysis of variance representation of $f(x)$ is based on the satisfaction of the condition of Equation 11.5.

$$\int f_{i_1, \dots, i_n}(x_{i_1}, \dots, x_{i_n}) dx_k = 0 \quad \text{for } k = i_1, \dots, i_n \tag{11.5}$$

The Sobol' indices for a subset of model factors are defined as:

$$S_{i_1, \dots, i_n} = \frac{D_{i_1, \dots, i_n}}{D} \tag{11.6}$$

where D_{i_1, \dots, i_n} is the partial variance of $f_{i_1, \dots, i_n}(x_{i_1}, \dots, x_{i_n})$ and D is the total variance.

For example, $S_i = D_i/D$ provides the first-order contribution from the i th input parameter to the output variance and $S_{ij} = D_{ij}/D$ is used to compute the second-order contribution from interaction between the i th and j th parameters. Finally, total effect sensitivity indices, which are defined as the sum of all the sensitivity indices as $S_{Ti} = S_i + S_{ij} + \dots + S_{1..i..n}$ quantify the overall effects of one factor on the model output. The required number of simulations is $N^*(n + 2)$; where N is the number of repetitions and n the number of model factors.

11.2.4.2 Extended-FAST

The Extended-FAST method is also based on the variance decomposition. Differently to Sobol' the Extended-FAST evaluates sensitivity independently for each factor using just one simulation because all the terms in a Fourier expansion related to Equation 11.4 are mutually orthogonal.

In the Extended-FAST method S_i , computed according to Equation 11.7, measures how the i th factor contributes to the variance of the model output without taking into account the interactions among factors.

$$S_i = \frac{\text{Var}_{x_i}(E_{x_{-i}}(Y | x_i))}{\text{Var}(Y)} \tag{11.7}$$

where E indicates the expectancy operator and Var the variance operator. According to the notation used by Saltelli *et al.* (2004) the subscripts indicate that the operation is either applied “over the i th factor” X_i , or “over all factors except the i th factor” X_{-i} .

The total effect index S_{Ti} is computed according to Equation 11.8 considering all potential interactions with the i th factor.

$$S_{Ti} = 1 - \frac{\text{Var}_{x_{-i}}(E_{x_i}(Y | x_{-i}))}{\text{Var}(Y)} \quad (11.8)$$

The difference between S_{Ti} and S_i quantifies the interaction among factors (S_{Si}). Variance-based methods allow identification of important factors (high S_i) (factors prioritization).

Only if S_{Ti} is small can the factor be fixed anywhere within its range of uncertainty (factor fixing). Indeed, if the S_i value is small, it does not necessarily mean that the factor may be fixed because a high S_{Ti} value would indicate that the factor is involved in interactions.

11.3 GSA APPLICATIONS FOR WASTEWATER ENGINEERING

In Table 11.2 the main relevant studies on GSA related to the wastewater field are summarized. Further, Table 11.2 contains key elements of relevant GSA applications in other modelling fields.

In the wastewater modelling field the SRC method has often been adopted for: (i) factors prioritization (among others, Corominas & Neumann, 2014; Wagner *et al.*, 2016; Mannina *et al.*, 2017); and (ii) uncertainty analysis (Benedetti *et al.*, 2011; Sin *et al.*, 2011). Sin *et al.* (2011) applied the SRC method in view of uncertainty analysis of a model of a conventional activated sludge system in which three different scenarios were analysed. In particular, the following scenarios were analysed by Sin *et al.* (2011): scenario 1: stoichiometric, kinetic and influent fractionation parameters are uncertain; scenario 2: hydraulics and mass-transfer factors are uncertain; and scenario 3: stoichiometric, kinetic and influent fractionation parameters, hydraulic and mass-transfer related parameters are uncertain. For each scenario, different model inputs (such as bio-kinetic model parameters, influent fractions, mass-transfer parameters and the like), were considered to be either uncertain or known. The study of Sin *et al.* (2011) was aimed at selecting the most important factors that contribute to the uncertainty of wastewater treatment performance criteria (e.g. effluent quality, sludge production and energy consumption). They found a high ability of the SRC method in identifying the main sources of uncertainty and quantifying their impact on process performance criteria. Conversely, Chen *et al.* (2012) found that for more complex wastewater systems such as membrane bio-reactors the SRC method can

Table 11.2 Literature overview on the main GSA applications.

Method	Convergence Analysis	Number of Factors	Number of Simulation Runs	Threshold Value	Reference	Research Field
SRC	no	26	500	$ \beta_j > 0.1$ important	Sin <i>et al.</i> (2011)	Wastewater
SRC		7	500	$ \beta_j > 0.1$ important		
SRC		33	500	$ \beta_j > 0.1$ important		
SRC	yes	15	600–2250	n.a.	Benedetti <i>et al.</i> (2011)	
Extended-FAST	no	32	n.a.	$S_T > 0.05$ important	Chen <i>et al.</i> (2012)	
Morris Screening	yes	17	90–1260	n.a.	Ruano <i>et al.</i> (2012)	
RSA	no	20	10,000	n.a.	Mannina <i>et al.</i> (2010)	
SRC	no	79	800	$ \beta_j > 0.1$ important	Mannina <i>et al.</i> (2011b); Cosenza <i>et al.</i> (2013a)	
Morris Screening			800	$\mu^* > 0.1$ important		
Extended-FAST			39,500	$S_T > 0.01$ important; $S_{NI} < 0.6$ non-influent		
Extended-FAST	no	79	39,500	$S_T > 0.01$ important; $S_{NI} < 0.1$ non-influent	Cosenza <i>et al.</i> (2014)	
SRC	no	26	500	$ \beta_j > 0.1$ important		
Morris Screening			405	$\mu^* > 0.1$ important	Ramin <i>et al.</i> (2014)	
Sobol'	yes	10	5000	n.a.		
SRC	yes	33	1200	$\beta^* > 0.1$ important	Cogan and Cellam (2017) Mannina <i>et al.</i> (2017)	
Morris Screening	yes	14	300	Factors which lay outside the two lines: $\mu_i = \pm 2SEM_i$	Robles <i>et al.</i> (2014)	
Morris Screening	no	63	–	$\mu^* > 0.1$ important	Valverde-Pérez <i>et al.</i> (2015)	
SRC	no	32	1000	$ \beta_j > 0.1$ important	Wagner <i>et al.</i> (2016)	

(Continued)

Table 11.2 Literature overview on the main GSA applications. (Continued)

Method	Convergence Analysis	Number of Factors	Number of Simulation Runs	Threshold Value	Reference	Research Field
Sobol'	yes	5	36,000	n.a.	Yang (2011)	Relevant applications in other water modelling field
Morris Screening			12,000	n.a.		
Linear regression			3000	n.a.		
RSA			5000	n.a.		
SDP			512	n.a.		
RA	yes	16	3000	n.a.	Gan <i>et al.</i> (2014)	
Morris Screening			2800	n.a.		
MARS			3000	n.a.		
FAST			2777	n.a.		
McKay-1			3000	n.a.		
McKay-2			2890	n.a.		
Sobol'			3000	n.a.		
Sobol'	yes	5	420,000	n.a.	Sarrazin <i>et al.</i> (2016)	
		13	600,000	$S_{T_i} < -0.0084$ non-influent		
		50	520,000	$S_{T_i} < -0.004$ non-influent		
		5	78,000	n.a.		
Morris Screening		13	560,000	$\mu^* < 0.026$ non-influent		
		50	102,000	$\mu^* < 0.027$ non-influent		
		5	10,000	n.a.		
RSA		13	20,000	n.a.		

OAT	no	50	30,000	n.a.	Sun <i>et al.</i> (2012)
RSA		6	85	n.a.	
Morris Screening			10,000	n.a.	
SRC	no	42	10,000	n.a.	Reder <i>et al.</i> (2017)
PCC			300	n.a.	
Spearman					
RSA	yes	6	30,000	n.a.	Dotto <i>et al.</i> (2012)
RSA	no	17	10,000	n.a.	Mannina and Viviani (2010)
RSA	no	28	10,000	n.a.	Freni <i>et al.</i> (2009)
SRC	no	17	1000	$ \beta_j > 0.1$ important	Mannina <i>et al.</i> (2012)
Morris Screening			540	$\mu^* > 0.1$ important	
Extended-FAST			8500	$S_i > 0.01$ important;	
				$S_{Ti} - S_j < 0.1$ non-influent	
SRC	no	19	500	$\beta_j^2 > 0.05$ important	Corominas and Neumann (2014)
Morris Screening			200	$\mu^* < 0.1$ non-influent	
SRC			4000	$ \beta_j > 0.1$ important	
Morris Screening	yes	17	36,000	$\mu^* > 0.1$ important	Vanrolleghem <i>et al.</i> (2015)
Extended-FAST			85,000	$S_i > 0.01$ important;	
				$S_{Ti} - S_j < 0.1$ non-influent	

Where: RSA = Regionalized Sensitivity Analysis; RA = Regression analysis; McKay-1 = McKay main effect analysis; McKay-2 = McKay two-way interaction analysis; MARS = Multivariate adaptive regression splines screening; SDP = State-Dependent Parameter; OAT = One-at-A-Time; PCC = Pearson correlation coefficients; SRC = Standardized Regression Coefficient; Extended-FAST = Extended Fourier Amplitude Sensitivity Testing; n.a. not available.

lead to erroneous results, due to non-linearity. Studying a similar system, Cosenza *et al.* (2013a) found that the SRC method provided similar results to the Extended-FAST method in terms of factors prioritization despite the presence of non-linearity. This result has peculiar interest. Indeed, if the modeller knows to deal with a non-linear model and is interested only in factor prioritization he/she can apply the less computationally-demanding method (SRC) without creating errors. Indeed, Mannina *et al.* (2017) have recently successfully applied the SRC method for the factors prioritization of a complex ASM-based model including greenhouse gases as state variable, despite the average R^2 model output being equal to only 0.5. Conversely, Cosenza *et al.* (2013a) found a low similarity in results obtained by applying the Morris screening and Extended-FAST methods, mainly due to a convergence problem.

Despite the fact that the core aim of the Morris screening method is to provide information in terms of factors fixing, it has often been applied in the wastewater field with the aim of factors prioritization thanks to the qualitative knowledge on the factor interactions provided by σ .

Few variance-based method applications are found in the wastewater modelling field mainly due to their higher computational cost compared to other GSA methods. Applications include Cosenza *et al.* (2014) who found significant interactions among the model factors of the ASM2d–SMP model (Cosenza *et al.*, 2013b). Contrary to previous GSA studies for ASMs, Cosenza *et al.* (2014) found the relationship between variables and factors to be non-linear and non-additive.

Some GSA applications derived from other water modelling fields merit being cited here for their peculiarities, which could be of useful interest for future applications in the wastewater field.

Vanrolleghem *et al.* (2015) have applied three GSA methods (SRC, Extended-FAST and Morris screening) to an urban drainage stormwater quality–quantity model. They discussed the issue of sensitivity convergence as further explored in Section 11.5. Vanrolleghem *et al.* (2015) found a considerable deviation of the results of Morris screening from that of the other methods, demonstrating that for their model the best method was the Extended-FAST. However, they underlined the need for using multiple GSA methods and multiple objectives in view of increasing the robustness of the results as previously performed by Corominas and Neumann (2014).

Neumann (2012) compared five SA methods (derivative-based local sensitivity analysis, Morris Screening, SRC, Extended-FAST and an entropy-based method) applied to micropollutants removal in a drinking water model. The author found the same parameter ranking results for the different methods. However, for chemicals leading to high non-linearity, the approximation of first-order effect indices using the local or regression-based methods was poor, leading to different classifications.

Reder *et al.* (2017) have recently applied GSA to a water quality model aimed at evaluating the river water quality in Africa. Specifically, they applied the lowest computationally demanding GSA methods: SRC, Pearson correlation coefficients (PCC) and the Spearman rank correlation coefficient. Despite obtaining reasonable

results using a sampling size larger than that recommended in the literature, they pointed out the need to investigate and compare different methodologies. This latter statement embraces all fields of GSA applications since no best GSA method exists and different methods and sampling strategies have to be applied to obtain reasonable results (Pianosi *et al.*, 2016).

Recently Sarrazin *et al.* (2016) presented a research paper focusing on key issues such as the adequate sample size, convergence and the choice of the screening threshold. Sarrazin *et al.* (2016) applied three GSA methods to three hydrological models of increasing complexity (with 5, 13 and 50 model factors). Further, as discussed below, they also proposed a method for a quantitative assessment of the convergence of the sensitivity analysis. They demonstrated the importance of assessing convergence analysis with respect to the objective of the modeller, for example to obtain stable sensitivity indices, a ranking or a screening.

11.4 NUMERICAL SETTINGS

One of the crucial aspects in the GSA applications is the sample size to be adopted in view of obtaining reliable results. Some relevant GSA applications have demonstrated that the sample size suggested in Saltelli *et al.* (2008) can be too low and inadequate to obtain reliable results (Vanrolleghem *et al.*, 2015; Sarrazin *et al.*, 2016).

As recently demonstrated by Sarrazin *et al.* (2016), no relationship exists between the number of model factors and the sample size required to reach convergence. The number of model simulations to be run for carrying out the sensitivity analysis depends on the choice of the sampling repetitions which is therefore crucial. For complex models (large model factors) the sample size can be the bottleneck of the methodology (long calculation time for running the requested model runs).

The sample sizes suggested in literature are often insufficient; therefore, a convergence analysis would be required to ensure that stable results are obtained given the limited sample size. However, a dilemma exists: complex models are likely to be computationally expensive and therefore require sophisticated (again expensive) GSA methods making a convergence analysis unfeasible. In Section 11.4.3 a convergence pursuing numerical method is proposed to obtain reliable results at minimal computational cost.

11.4.1 Open issues

A generally accepted common GSA terminology, permitting ease of comparison between the methods, is still lacking (Mannina *et al.*, 2012; Vanrolleghem *et al.*, 2015).

“Sensitivity indices” provide a measure of the importance of a model factor in determining variation in the model output. When applying sensitivity analysis, one of the most relevant parameters that has to be employed is the so-called “cut-off threshold”. The cut-off threshold is a subjective number which allows classification of model factors (i.e. factors classified as being important or non-influential). Therefore,

the “cut-off threshold value” of the sensitivity indices for establishing whether a model factor is important or not has a key role in applying GSA (see below Section 11.4.2).

There is missing terminology for comparing results in view of classification (establishing an order of importance of model factors) and for comparing cut-off threshold values for sensitivity indices obtained with different methods.

Another issue surrounding GSA, for which no consensus still exists, is the convergence of the results obtained for different numbers of simulations. The lack of uniformity in the definition of convergence makes it difficult to compare the results obtained for models of different complexities when using different GSA methods (Sarrazin *et al.*, 2016).

In Figure 11.3, a first attempt to standardize the GSA terminology for comparability of results among three classes of methods (i.e. SRC, Morris screening and Extended-FAST) is provided (Vanrolleghem *et al.*, 2015).

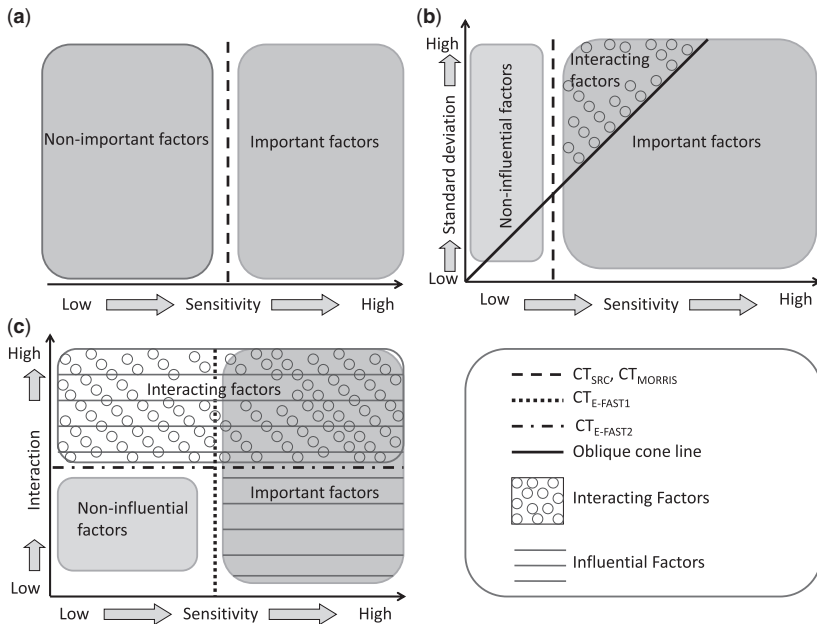


Figure 11.3 Schematic overview of the terminology used for classifying SRC (a), mean (b), first-order effect (c) according to different GSA methods: SRC (a), Morris screening (b) and Extended-FAST (c) (Vanrolleghem *et al.*, 2015).

On the basis of a cut-off threshold (CT) for the sensitivity indices it is possible to classify factors according to their importance (Vanrolleghem *et al.*, 2015). As detailed below, each method provides different sensitivity indices which provide the entity of the importance of each model factor.

The SRC method allows calculation of the standardized regression coefficient (SRC) or slope (β_i) (obtained with respect to the i th model factor and the model output standard deviation – see Section 11.2.2.1) of the multivariate linear regression model. A CT threshold (CT_{SRC} for β_i) has to be defined in view of classifying model factors in terms of importance. Once CT_{SRC} is defined, on the basis of its value, factors can be classified in the following way (Figure 11.3a):

- *important factors*: if SRC coefficient $> CT_{\text{SRC}}$;
- *non-important factors*: if SRC coefficient $< CT_{\text{SRC}}$.

In the case of linear models the non-important factors can be fixed anywhere in their space. In the case of non-linear models, non-important factors could be involved in interactions with other factors thus they cannot be fixed (see below).

In the case of the Morris screening method, two indices are calculated. Specifically, the absolute mean (μ^*) and the standard deviation of the elementary effect (difference with and without factor variation) computed at different points of the input space. These values provide information about the importance of model factors. A CT threshold (CT_{MORRIS} for μ^*) has to be established in view of classifying model factors in terms of importance.

By means of the Morris screening method three different types of factors can be distinguished with respect to the mean and the standard deviation of the elementary effect function (Figure 11.3b):

- *important factors*: if absolute mean sensitivity $> CT_{\text{MORRIS}}$;
- *interacting factors*: if absolute mean sensitivity $> CT_{\text{MORRIS}}$ and the standard deviation is above a specified cone line;
- *non-influential factors*: if absolute mean sensitivity $< CT_{\text{MORRIS}}$.

In particular, the Morris screening method as modified by Campolongo *et al.* (2007) basically defines a cone whose edges are set by a CT_{MORRIS} and an oblique line called the core line, corresponding to two times the standard error of the sensitivity index (Figure 11.3b) (quantitative characteristics are given below).

In the case of the Extended-FAST method, the i th factor contribution to the variance of the model output with and without taking into account the interactions among factors (S_{T_i} and S_i , respectively) are considered as sensitivity indices. The difference between S_{T_i} and S_i represents the contribution of the i th factor in terms of interaction. A CT for each of the indices ($CT_{\text{E-FAST1}}$ and $CT_{\text{E-FAST2}}$ for S_i and $S_{T_i} - S_i$, respectively) has to be established in view of classifying model factors in terms of importance.

The Extended-FAST distinguishes three classes of factors on the basis of two CTs ($CT_{\text{E-FAST1}}$ and $CT_{\text{E-FAST2}}$) (Figure 11.3c):

- *important factors*: if $S_i > CT_{\text{E-FAST1}}$;
- *interacting factors*: if $S_{T_i} - S_i > CT_{\text{E-FAST2}}$;
- *non-influential factors*: if $S_i < CT_{\text{E-FAST1}}$ and $S_{T_i} - S_i < CT_{\text{E-FAST2}}$.

Non-influential factors which can be identified by both the Morris screening as well as the Extended-FAST method can be fixed anywhere within their variation range without changing the model output variance. With respect to this, it is important to stress once more that non-important does not imply non-influential.

11.4.2 Cut-off criteria for factors classification

In order to classify factors as being important or non-influential or interacting, cut-off thresholds of the sensitivity indices values (CTs) are required. The choice of the cut-off thresholds is arbitrary and depends on the objective of the modeller and on the model structure. For example, in the case where the modeller is interested only in the factors that contribute significantly to the model variability, a high cut-off threshold is selected. A general criterion to select the cut-off thresholds is that the CT value has to be effective in view of differentiation of model factors (e.g. as important or non-influential or interacting). Therefore, a value that classifies all involved factors as either important or non-influential is not suitable. Indeed, this latter case would make the GSA application un-useful, especially when the modellers' aim is to select the factors that need to be calibrated.

In the following, CT-values typically found in literature are reported (see also Table 11.2).

A CT value equal to 0.1 is suggested to select important factors, for the SRC ($|\beta_i|$) and Morris screening (μ^*) methods (CT_{SRC} and CT_{MORRIS} , respectively). This latter value is equivalent to a CT value of 0.01 for S_i obtained in the Extended-FAST method ($CT_{\text{E-FAST1}}$) (since for linear models $S_i = \beta_i^2$) (Sin *et al.*, 2011; Neumann, 2012; Cosenza *et al.*, 2013a; Vanrolleghem *et al.*, 2015).

For the Extended-FAST method a CT of 0.1 ($CT_{\text{E-FAST2}}$) was chosen by Vanrolleghem *et al.* (2015) for the threshold value of the interaction (i.e. $S_{T_i} - S_i$). Neumann (2012) chose a CT of 0.05 for S_{T_i} and μ^* in order to identify non-influential factors (factors with S_{T_i} (or μ^*) < 0.05 were considered to be non-influential).

Sarrazin *et al.* (2016) have proposed the method presented below to validate the screening results after applying GSA methods, which could provide support to the modeller to investigate the adequacy of the chosen screening threshold.

11.4.3 GSA applications dealing with convergence analysis

Convergence analysis represents another crucial aspect when dealing with GSA. The high time demand required for convergence analysis is often contrasting with the need of a modeller to achieve results as fast as possible. Therefore, very few quantitative criteria exist for implementing convergence analysis in the wastewater field.

Benedetti *et al.* (2011) proposed a method to reduce the computational cost of Monte Carlo based GSA methods. The authors used two criteria (the model

output variability and the stability of the set of factors classified as important as the number of iterations increases) to select the minimum number of simulations to be performed. They found that depending on the analysed variable the results of the convergence analysis vary, highlighting that the achievement of convergence is strongly dependent on the model output considered.

Ruano *et al.* (2012) investigated the convergence of the Morris screening for a wastewater treatment plant model. They proposed a criterion (the position factor) for establishing the achievement of convergence. By increasing the number of replicates (r) of the OAT sampling from 5 to 70, Ruano *et al.* (2012) analysed the average variation of the sum of the rank of the model factors (position factor). Ruano *et al.* (2012) found that the optimal number of replicates was 60–70, which is considerably higher than recommended by Saltelli *et al.* (2008) (namely 4–10 as reported in Table 11.1). The work of Ruano *et al.* (2012) confirmed that the modeller must take care in employing “default” numerical settings proposed in literature especially in the case of complex models.

For the sake of completeness, some quantitative criteria adopted in other water modelling fields are reported below. These criteria could represent the starting point for future convergence analysis applications in the wastewater field.

Recently, Sarrazin *et al.* (2016) have defined quantitative criteria to assess convergence of sensitivity indices, ranking and screening. They propose to assess the maximum width of the confidence intervals of the sensitivity index ($Stat_{indices}$) as a summary statistic to assess converge (Equation 11.9).

$$Stat_{indices} = \max(S_i^{ub} - S_i^{lb}), \quad i = 1, \dots, n \tag{11.9}$$

where S_i^{ub} and S_i^{lb} are the upper and lower bounds of the sensitivity index of i th input factor, respectively; n represents the number of the input factors. The convergence is achieved when the $Stat_{indices}$ value is close to zero. Regarding the convergence in terms of ranking, Sarrazin *et al.* (2016) have proposed a weighted rank correlation coefficient ($\rho_{s,j,k}$) (Equation 11.10).

$$\rho_{s,j,k} = \sum_{i=1}^n \left| R_i^j - R_i^k \right| \frac{\max_{j,k} (S_i^j, S_i^k)^2}{\sum_{i=1}^M \max_{j,k} (S_i^j, S_i^k)^2} \tag{11.10}$$

where S_i^j and S_i^k relate to the i th input factor evaluated at the j th and k th resample of the model factors, respectively; while, R_i^j and R_i^k are the related rank positions. Sarrazin *et al.* (2016) suggested that the convergence can be considered reached when the 95% quantile value of the $\rho_{s,j,k}$ distribution (obtained over all possible resamples) is equal to one. Regarding the convergence of the input factor screening, Sarrazin *et al.* (2016) have proposed as a summary statistic ($Stat_{screening}$) the maximum width of the 95% confidence intervals across the sub-set (X_0) of the input factors that can be considered “low-sensitivity” (having sensitivity index value lower than a fixed threshold) (Equation 11.11).

$$Stat_{\text{screening}} = \max(S_i^{ub} - S_i^{lb}), x_i \in X_0 \quad (11.11)$$

The screening convergence can be considered reached when $Stat_{\text{screening}}$ is close to zero.

In the urban wastewater field Vanrolleghem *et al.* (2015) proposed a quantitative method for testing the convergence achievement for SRC, Morris screening and Extended-FAST methods. The core idea of the method, is to evaluate the variability of the sensitivity indices (SC) as the number of simulations increases. In particular, for each model output j , the S_{SCj} is computed as the sum of the sensitivity SC of all factors normalized with respect to the number of factors (n).

$$S_{SCj} = \frac{\sum_{i=1}^n SC_{i,j}}{n} \quad (11.12)$$

The variability of S_{SCj} is analysed for increasing numbers of Monte Carlo simulations (n_{MC}). More specifically, for the j th model output the variability of S_{SCj} is expressed as the percentage of change of S_{SCj} from n_{MCi-1} to n_{MCi} (Equation 11.13).

$$\text{Variability} = \left[\frac{\left(\sum_{i=1}^n SC_{i,j} \right)_{n_{MCi-1}} - \left(\sum_{i=1}^n SC_{i,j} \right)_{n_{MCi}}}{n} \right] \cdot 100 \quad (11.13)$$

According to this method, the convergence can be considered achieved if Variability (Eq. 13) stays within a selected precision threshold.

In Figure 11.4 an example of a convergence analysis for two different types of models (simplified and complex) is reported (Vanrolleghem *et al.*, 2015). The results are related to the application of SRC, Morris screening and Extended-FAST methods. A variability threshold of the results equal to $\pm 3\%$ is considered for evaluating convergence (Figure 11.4) of the two models. The simple model is a rainfall-runoff urban drainage model (Mannina & Viviani, 2010) with five model factors. In Figure 11.4a–c two model outputs are reported (i.e. O1 and O2). The complex model is able to simulate processes related to quantity and quality inside sewer systems and includes 17 model factors. For the sake of conciseness only two model outputs of the complex model are considered in Figure 11.4d–f (i.e. O3 and O4).

From Figure 11.4 one may observe that the sensitivity index of a simple model's outputs with the simple method achieves convergence at a lower number of simulations than with the complex methods. For example, for SRC convergence is achieved with 1000 simulations (Figure 11.4a). Conversely, for the complex model, convergence is achieved at 4000 simulations (Figure 11.4d). Moreover, by analysing Figure 11.4 one may observe that, especially for the complex model, sensitivity indices of the two model outputs do not achieve convergence simultaneously.

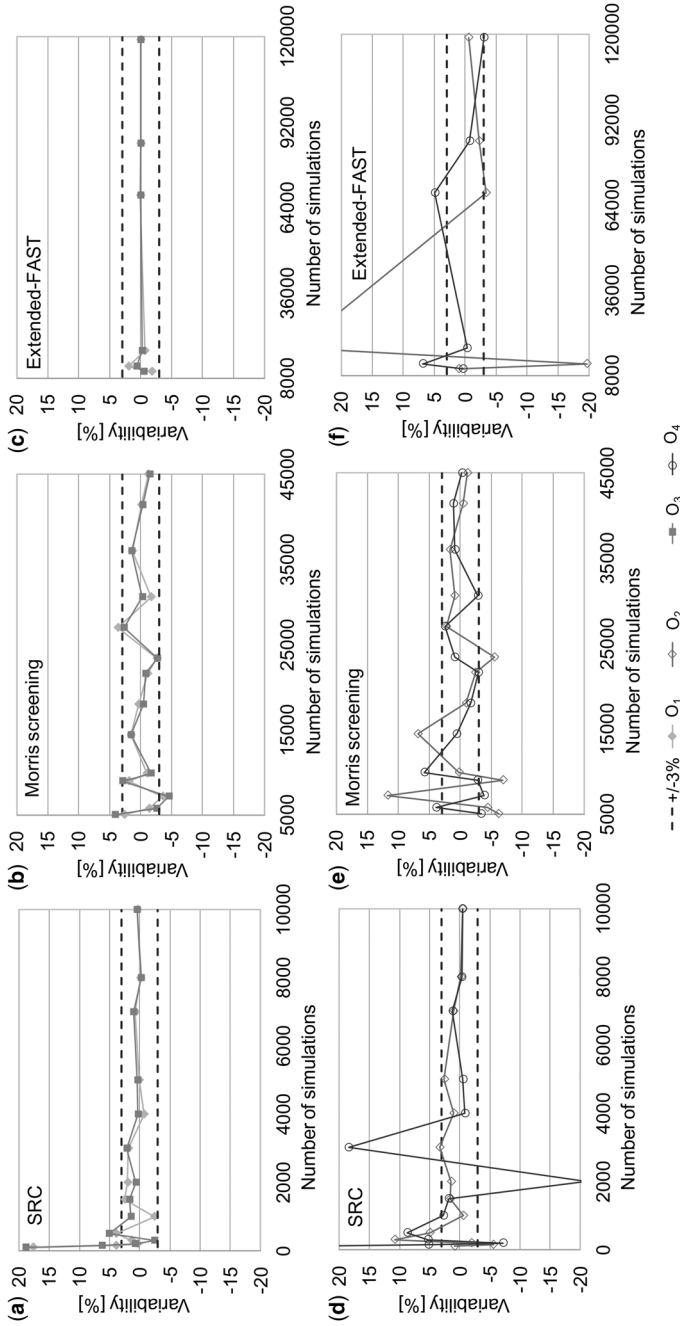


Figure 11.4 Quantitative convergence analysis with increasing sample size (expressed as number of simulations) versus variability (according to Equation 11.7) for different mathematical models and different GSA methods: simple model (a–c) and complex model (d–f) (modified from Vanrolleghem *et al.*, 2015).

11.5 USING MULTIPLE GSA METHODS

11.5.1 Comparison studies

The specific key features of each method compared to the others can be analysed by applying different methods to the same model. Cosenza *et al.* (2013a) have compared the SRC, Morris screening and Extended-FAST methods for a membrane bioreactor wastewater treatment model. They showed a poor similarity between Morris screening and Extended-FAST results in terms of classification into influential/non-influential factors. In terms of identifying important factors Mannina *et al.* (2012) obtained a good agreement applying SRC, Morris screening and Extended-FAST methods to a simple model. In the hydrological field, slight differences of factors ranking were obtained by Yang (2011), for the application of regionalized sensitivity analysis and non-parametric smoothing methods to the HYMOD model. For the regionalized sensitivity analysis method, the different results were debit to the dependence on the choice of the adopted filtering criterion to separate parameter sets into behavioural and non-behavioural. For the non-parametric smoothing method despite the convergence achievement Yang (2011) debited the different results to the low number of simulations performed compared to the other methods.

The results of the comparison studies obtained by Mannina *et al.* (2012) (simple model) and Cosenza *et al.* (2013a) (complex model) are reported in Figure 11.5. More specifically, the Venn diagram of Figure 11.5 shows the important factors selected by applying SRC, Morris screening and Extended-FAST methods to a simple (Figure 11.5a) and a complex model (Figure 11.5b).

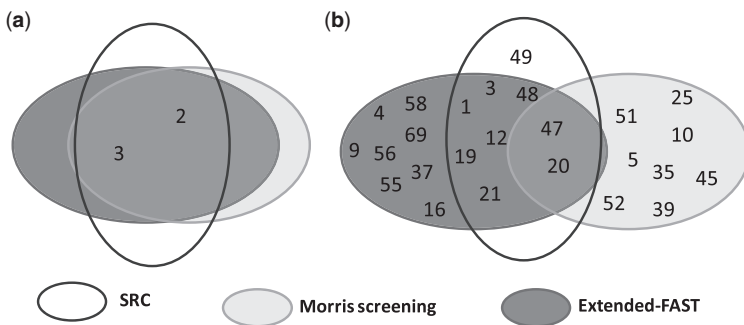


Figure 11.5 Venn diagram related to the comparison of important factors obtained by applying SRC, Morris screening and Extended-FAST methods for a simple model (a) and a complex model (b); numbers represent the factor order index; adapted from Mannina *et al.* (2012).

By analysing Figure 11.5, one may observe that for the case of the simple model the three methods provide the same results in terms of important factors (Figure 11.5a). Conversely, for the complex model a poor similarity is shown in Figure 11.5b.

The findings deduced from one GSA method application are often affected by the subjectivity of the modeller (e.g. cutting threshold for selecting important factors; number of runs; etc.) during the analysis, thus limiting the transferability of the knowledge to other models.

Despite the huge efforts aimed at understanding the peculiarities of each GSA method “*no ideal GSA method exists for all case studies*” (Pianosi *et al.*, 2016).

11.5.2 Sequential use

The Morris screening method is often suggested as a first step to discard non-influential factors and then pursue the GSA by applying a more elaborate method with only the selected potentially influential factors. However, care needs to be taken to assess the convergence for the Morris screening (among others, Cosenza *et al.*, 2013a).

In case of computationally demanding complex models, a general recommendation is that of applying a screening method to discard non-influential factors before applying more demanding methods such as Extended-FAST. Indeed, this is a general recommendation in GSA literature (Saltelli *et al.*, 2008). However, to ensure that this procedure works one must ascertain that no factors are eliminated wrongly in the screening. Examples of such approaches are reported in Sun *et al.* (2012) who suggested using, in the case of a model with a large number of factors, a two-step procedure including first a factors screening step (by using a local method) followed by a GSA step to be applied only to the important factors identified during the first step. For complex models, Gan *et al.* (2014) suggested first using approximate methods (e.g. SRC or local methods) for a rough factor screening and then applying more demanding methods such as Extended-FAST or Sobol’s indices.

The problem of the high computational burden can be managed by adopting the meta-model approach, typical of the hydrological field (Song *et al.*, 2015). The core idea behind this approach is the adoption of statistical or experimental design methods as surrogate models (emulators) to assess the relationship between model factors and state variables. However, despite the fact that this approach has often been used in the hydrological field several issues still remain, for example the truthfulness of the meta-models with respect to the real process that has to be described.

11.5.3 Complementary use

The choice of GSA method to apply is not always clear a priori due to unknown model behaviour and the unknown relationship between model outputs and factors. Only by applying different methods simultaneously, can a robust assessment of the impact of model factors on the model outputs and the model’s degree of non-linearity be obtained (Neumann, 2012; Corominas & Neumann, 2014).

With this in mind, the complementary use of GSA methods can provide support to the modeller. The complementary use of GSA methods can be performed with two aims:

- (1) *Increase robustness of analysis*: if the modeller uses four GSA methods and three are providing similar results and one is not then he/she has to check what is going on;
- (2) *Studying multiple objectives*: since not all methods provide information for all objectives, the modeller has to apply complementary methods.

Regarding the multiple objectives, Corominas and Neumann (2014) have applied the SRC and the Morris screening methods to an urban wastewater system model in order to understand how the operational variables of the system (including WWTP) influence the receiving water quality. Specifically, two GSA methods have been applied to identify important operational variables (i.e. SRC) as well as variables that do not influence (i.e. Morris screening) river water quality. The authors have confirmed the recurring statement in the field of water modelling that no ideal GSA method exists and that the combination of several methods can improve the results.

However, despite the advantage of the complementary use of different GSA methods several questions remain to be addressed, such as: Is this really an effective practice for overcoming the computational burden? A way to provide an answer is to test how the results of the sensitivity analysis change after performing a complementary use of GSA methods.

11.6 SUMMARY AND OUTLOOK

This chapter has outlined the application and issues surrounding the use of GSA in the wastewater modelling field. Sometimes applications or criteria from other water modelling fields have been discussed in view of providing the reader with a general overview on GSA. The following summary points can be deduced from this chapter:

- (1) The adoption of GSA methods for WWTP modelling has several advantages with respect to local methods. One of the most significant advantages is the capability of GSA methods to take into account interactions among factors. This capability makes the selection of important model factors more precise than local methods. Consequently, the model predictions are more accurate. Moreover, the possibility of identifying the interactions/relationships among the model factors may also reduce the amount of model factors (stoichiometric, kinetic or fractionation) that need to be assessed during the experiments or allow a modeller to better interpret the measured data.

- (2) Since the WWTP models often contain tens of model parameters, the sequential use of several GSA methods represents a way for reducing the high computational demand typical of these methods. Therefore, the first adoption of a screening method to discard non-influential factor before applying more demanding methods (such as Extended-FAST) is suggested.
- (3) No single GSA method is ideal for all WWTP models. Therefore, the adoption of multiple methods is suggested in view of selecting the best method to be adopted on the basis of the model structure (linear or non-linear, monotonic or non-monotonic). However, a systematic approach to verify the effectiveness of running multiple GSA methods is also required.
- (4) Some aspects related to GSA still require further investigations in view of favouring the adoption of GSA in the WWTP modelling field and of improving results of future applications: (i) setting-up of quantitative criteria to assess convergence of GSA results; (ii) establishing a methodology to support modellers for the identification of an adequate screening threshold to be adopted; and (iii) adopting a standardized nomenclature/terminology related to convergence and sensitivity thresholds facilitating comparability among methods.

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