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The different fates of a low-mass X-ray binary – I. Conservative mass transfer

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ABSTRACT

We study the evolution of a low-mass X-ray binary by coupling a binary stellar evolution code with a general relativistic code that describes the behaviour of the neutron star. We assume the neutron star to be low-magnetized ($B \sim 10^8$ G). In the systems investigated in this paper, our computations show that during the binary evolution, the companion transfers as much as 1 $M_\odot$ to the neutron star, with an accretion rate of $\sim 10^{-9}$ $M_\odot$ yr^{-1}. This is sufficient to keep the inner rim of the accretion disc in contact with the neutron star surface, thus preventing the onset of a propeller phase capable of ejecting a significant fraction of the matter transferred by the companion. In this scenario we find that, for neutron stars governed by equations of state from soft up to moderately stiff, an accretion induced collapse to a black hole is almost unavoidable. The collapse to a black hole can occur either during the accretion phase or after the end of the mass transfer when the neutron star is left in a supramassive sequence. In this last case, the collapse is driven by energy losses of the fast spinning magneto-dipole rotator (pulsar). For extremely supramassive neutron stars, these energy losses cause a spin-up. Consequently, the pulsar will have a much shorter lifetime than that of a canonical, spinning down radio pulsar. This complex behaviour strongly depends on the equation of state for ultradense matter and therefore could be used to constrain the internal structure of the neutron star. In the hypothesis that the r-modes of the neutron star are excited during the accretion process, the gravitational waves emission limits the maximum spin attainable by a neutron star to roughly 2 ms. In this case, if the mass transfer is conservative, the collapse to a black hole during the accretion phase is even more common since the maximum mass achievable before the collapse to a black hole during accretion is smaller due to the limited spin frequency.

Key words: relativity – binaries: close – stars: neutron – pulsars: general – X-rays: binaries.

1 INTRODUCTION

The widely accepted scenario for the formation of millisecond radio pulsars (MSP) is the recycling of an old neutron star (hereafter NS) by a spin-up process. The spin-up is due to the accretion of matter and angular momentum from a Keplerian disc that is fuelled via Roche lobe overflow of a binary late-type companion (see Bhattacharya & van den Heuvel 1991 for a review). If enough mass and angular momentum are transferred, the NS spin attains an equilibrium value that is roughly equal to the keplerian angular frequency at the inner rim of the accretion disc (Ghosh & Lamb 1979). Since the NS has a weak surface magnetic field ($\sim 10^8$ G), the magnetospheric radius (at which the disc pressure is balanced by the magnetic pressure) truncates the accretion disc close by or at the NS surface, and the equilibrium period is expected to be, in most cases, below one millisecond. Typically ~ 0.35 $M_\odot$ are sufficient to reach a spin period of 1 ms (e.g. Burderi et al. 1999). Most donor stars in systems hosting recycled MSPs have certainly lost, during their interacting binary evolution, a mass greater than 0.35 $M_\odot$, since they now appear as white dwarfs of mass 0.15–0.30 $M_\odot$ (e.g. Taam, King & Ritter 2000), the progenitors of which are likely to have been stars of 1.0–2.0 $M_\odot$ (Webbink, Rappaport & Savonije 1983; Burderi, King & Wynn 1996; Tauris & Savonije 1999). Therefore, if the mass transfer is conservative the amount of matter accreted is easily sufficient to spin the star up to periods below 1 ms (Cook, Shapiro & Teukolsky 1994a), or even to produce an accretion induced collapse into a black hole. Once the accretion and spin-up process ends,

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the magnetospheric radius expands beyond the light cylinder radius (where an object corotating with the NS attains the speed of light). This initiates a phase in which the rotational energy of the NS is emitted *via* electromagnetic radiation and the star can be observable as a rapidly rotating radio pulsar. According to this model, low mass X-ray binaries (hereafter LMXBs) are the progenitors of MSPs. Indeed, the discovery of coherent X-ray pulsations in four transient LMXBs, namely SAX J1808.4–3658 with a spin period $P = 2.5$ ms and an orbital period of $P_{\rm orb} = 2$ h (Wijnands & van der Klis 1998), XTE J1751–305 ($P = 2.3$ ms, $P_{\rm orb} = 42$ min, Markwardt et al. 2002), XTE J0929–314 ($P = 5.4$ ms, $P_{\rm orb} = 43$ min, Galloway et al. 2002) and XTE J1807–294 with a spin period $P = 5.2$ ms and an orbital period of $P_{\rm orb} = 40$ min (Markwardt, Juda & Swank 2003) confirmed that NSs in LMXBs can be accelerated to millisecond periods.

Several numerical methods have been developed to solve the Einstein equations for a rotating NS (see Stergioulas 1998 for a review). Stability criteria show that a rapidly rotating NS can support a maximum mass (against gravitational collapse) much higher than the non-rotating mass limit, since the centrifugal force attenuates the effects of the gravitational pull (e.g. Friedman, Ipser & Sorkin 1988). Conversely, if a rotating NS has a mass that exceeds the non-rotating limit (i.e. a supramassive NS), it will be subject to gravitational collapse if it loses enough rotational energy. Numerical simulations of rotating NSs show that, in contrast to the standard behaviour, supramassive NSs spin-up just before collapse, even if they lose energy. This effect is known to be stronger for higher-mass objects (Cook, Shapiro & Teukolsky 1992).

The value of the maximum rotating and non-rotating mass depends on the equation of state (EOS) governing the NS matter. On the other hand, the minimum allowed period for a given mass occurs when gravity is balanced by centrifugal forces at the NS equator (mass shedding limit). Thus the spin period can be used to constrain the mass–radius relation for the NS, i.e. its EOS. In the context of the standard (gravitationally bound) NS models (e.g. Glendenning 2000), several EOSs have been proposed. We usually distinguish different EOSs depending on their stiffness (i.e. the value of $\mathrm{d}p/\mathrm{d}\epsilon$, were p is the fluid pressure and ϵ is the energy density). If the EOS is stiff, the matter is less compressible at high densities, resulting in larger NS radii as compared with a soft EOS, and hence in longer minimum rotational periods. Except for few, very stiff cases, most EOS predict minimum rotational periods well below 1 ms. However, no submillisecond pulsars have been detected up to date: the shortest observed spin period is ∼1.5 ms (Backer et al. 1982), uncomfortably higher than the theoretical predictions.

In an attempt to find an explanation for the apparent clustering of the spin periods of millisecond pulsars around 2 ms, Bildsten (1998) and Andersson (1998) independently suggested that LMXBs emit gravitational waves once they reach a critical spin frequency. Burderi & D'Amico (1997) showed that for non-axisymmetric m-modes, assuming a realistic range of temperatures, the values of the critical spin frequency are remarkably close to the limiting spin frequency determined by the centrifugal limit at the border of the NS. On the other hand, Andersson, Kokkotas & Stergioulas (1999) demonstrated that at a certain spin frequency (much lower than the maximum attainable spin period) an instability to the Rossby waves (r-modes) of the star arises, thus causing emission of gravitational waves. Levin (1999) suggested that the gravitational waves emission causes the onset of a spin-up–spin-down cycle, and not of a steady-state spin equilibrium: in this scenario the NS undergoes a very rapid spin-down (lasting ∼1 yr) due to the rapid heating during the r-mode excitation, and then starts another cycle of accretion driven spin-up. Brown & Ushomirsky (2000) showed that if the NS has a superfluid core the steady-state scenario is not viable because the predicted quiescence luminosity in this case is much higher than the observed one in the X-ray transient Aql X-1. Levin & Ushomirsky (2001) showed that, when keeping into account the presence of the solid crust, the critical spin frequency for the onset of the r-mode varies between ∼600 and ∼200 Hz, depending on the core temperature.

In this paper we consider the full evolution of a LMXB, and try to determine how the results of our modelling of the recycling scenario compare to the observations and which effects peculiar to general relativity are indeed observable. We will also show the differences in the evolution of the system when the r-mode instability is excited during accretion and when it is not excited.

2 EVOLUTION EQUATIONS FOR THE COMPACT OBJECT

A rotating NS is unambiguously defined by the boundary conditions for the integration of the Einstein equations or, equivalently, by suitable pairs of resulting integrated quantities, such as the baryonic mass and the angular momentum or the baryonic mass and the total mass–energy of the star. Therefore, the evolution of the NS is determined by the evolution of such pairs. We consider the evolution of the NS both during the accretion phase and after the accretion has finished: in the former case it is convenient to solve the evolution equations for baryonic mass and angular momentum, since we know the general formula for the torque exerted by the accreting matter on the NS, while in the latter we solve the evolution equations for baryonic mass and mass–energy, since we can evaluate the luminosity of a magnetodipole rotator using Larmor's formula.

2.1 Evolution equations for the accretion phase

According to accretion theories, matter transferred from the companion star to the NS *via* Roche lobe overflow forms an accretion disc around the compact object. The disc is truncated because of one of the following reasons: (i) the interaction with the magnetic field of the NS, which truncates the disc at the magnetospheric radius $r_{\rm m}$; (ii) the presence of the NS surface itself at $R_{\rm NS}$; and (iii) the lack of closed Keplerian orbits for radii smaller than the marginally stable orbit radius, $R_{\rm MSO}$ (at few, depending on the mass and spin of the compact object, Schwarzschild radii from the NS centre).

The magnetospheric radius is defined as the radius at which the pressure of the disc equals the pressure of the magnetic field of the NS. The magnetospheric radius can be written as a fraction ϕ (see Burderi et al. 1998) of the Alfvén radius $R_{\rm A}$ (the radius at which the energy density of the, assumed dipolar, NS magnetic field equals the kinetic energy density of the spherically accreting matter) as

$$\begin{aligned} r_{\rm m} &= \phi R_{\rm A} = 2.45 \times \alpha^{9/35} m_{\rm G}^{1/28} \dot{m}_{\rm B}^{3/70} r_{\rm m}^{-3/28} R_{\rm A} \\ &= 2.244 \times \left(\alpha^{36/5} m_{\rm G} \dot{m}_{\rm B}^{6/5} R_{\rm A}^{28}\right)^{1/31}\ {\rm cm}, \end{aligned} \tag{1}$$

where $\alpha \leqslant 1$ is the Shakura–Sunyaev parametrization of the accretion disc viscosity (for which we will assume a typical value of 0.1), $m_{\rm G}$ is the NS gravitational mass in $\mathrm{M}_\odot$, $\dot{m}_{\rm B}$ is the baryonic accretion rate on to the NS in $10^{-8}\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$, and $R_{\rm A}$ is

$$R_{\rm A} = 1.24 \times 10^6 \mu_{26}^{4/7} m_{\rm G}^{-1/7} \dot{m}_{\rm B}^{-2/7} {\rm cm}, \tag{2}$$

where μ_{26} is the magnetic dipole moment of the NS in units of 10^{26} G cm^3, defined from $B_{\rm s} = \mu/R^3$, where R is the equatorial radius and $B_{\rm s}$ the surface magnetic field of the NS at its equator.

The NS radius, which is always in the order of 10^6 cm, depends both on the mass of the NS and on its angular momentum. In general, smaller radii correspond to larger masses while larger radii correspond to larger angular momenta. Thus a rapidly rotating NS can have a much larger radius than a non-rotating one (the equatorial radius can expand up to 40 per cent, see Cook, Shapiro & Teukolsky 1994b).

The marginally stable orbit is the smallest stable orbit possible for a test particle around an axisymmetric, rotating body of gravitational mass $M_{\rm G}$ and angular momentum J. Its radius, following Bardeen (1970), is:

$$R_{\rm MSO} = R_{\rm g}\left[3 + Z_2 - \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}\right];$$

$$Z_2 = \sqrt{3\left(\frac{a}{R_{\rm g}}\right)^2 + Z_1^2}$$

$$Z_1 = 1 + \left[1 + \left(\frac{a}{R_{\rm g}}\right)^2\right]^{1/3} + \left(1 - \frac{a}{R_{\rm g}}\right)^{1/3}$$

$$a = \frac{J}{M_{\rm G} c}$$

$$R_{\rm g} = \frac{G M_{\rm G}}{c^2},$$

where c is the speed of light and G is the gravitational constant. Therefore, the inner radius of the disc, $R_{\rm D}$, will be:

$$R_{\rm D} = \max\{r_{\rm m}, R_{\rm NS}, R_{\rm MSO}\}. \quad (3)$$

The position of the inner rim of the disc is crucial for the accretion of matter on to the NS. In particular, it is very important in the case $R_{\rm D} = r_{\rm m}$: since at $r_{\rm m}$ the matter is forced by the magnetic field to corotate with the NS, accretion on to a spinning magnetized NS is centrifugally inhibited once $r_{\rm m}$ lies outside the corotation radius $r_{\rm co}$, the radius at which the Keplerian angular frequency of the orbiting matter is equal to the NS angular frequency: $r_{\rm co} = 1.50 \times 10^6 m_{\rm G}^{1/3} P_{-3}^{2/3}$ cm, where P_{-3} is the spin period in milliseconds. Conversely, if the magnetospheric radius is smaller than the corotation radius accretion of matter can proceed undisturbed. However, as $r_{\rm m}$ scales as a negative power of $\dot{m}_{\rm B}$, a decrease in the mass transfer rate, which can occur for instance at the end of the accretion phase, will results in an expansion of the magnetosphere. In this paper we considered NSs with $\mu_{26} \simeq 1$, which is typical of observed MSPs (see, e.g., Lorimer 1994), corresponding to surface magnetic fields of $\sim 10^8$ G. Moreover, in the cases considered here, the accretion rate is always above $\sim 10^{-10}$ $\mathrm{M_{\odot}\ yr^{-1}}$ during the mass transfer phase (see Fig. 1). In this case, equation (1) implies a magnetospheric radius of $\sim 10^6$ cm, which is comparable with the NS radius. Although we considered in our simulations (see below) the possibility of a propeller phase, in these conditions the details of the interaction of the accretion disc matter with the NS magnetosphere during the accretion phase will not greatly affect our results. In fact, the magnetospheric radius only lies outside the corotation radius for a small time ($\lesssim$0.1 per cent of the evolution time) just before of the end of accretion (see Fig. 2 for details). In other words, the spin evolution of a low-magnetized NS is not very different from the spin evolution of a non-magnetized NS, in which no propeller effect is possible since the disc is always truncated near the surface of the NS. We can therefore write the mass evolution equation for $r_{\rm m} < r_{\rm co}$, which is simply given by the condition that the baryonic mass

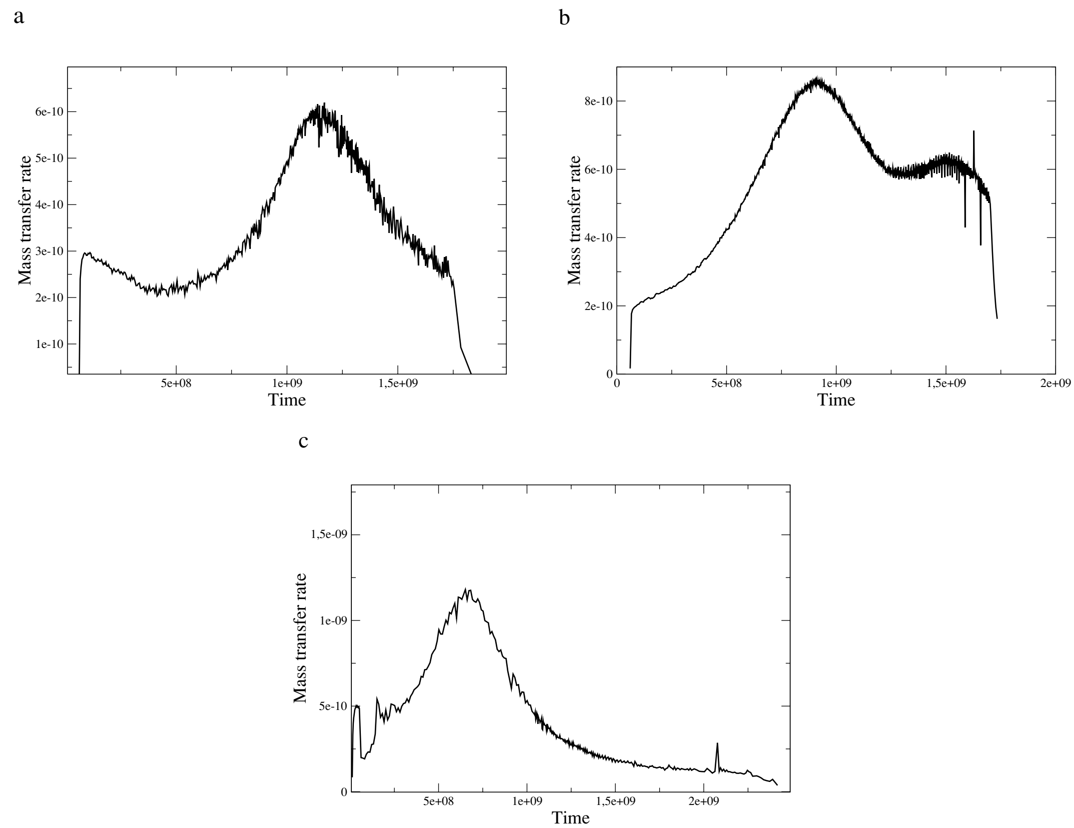

Figure 1. Baryonic mass accretion rates (in $\mathrm{M_{\odot}\ yr^{-1}}$) as a function of time (in yr) for systems consisting of an NS with initial gravitational mass of 1.4 $\mathrm{M_{\odot}}$ and: (a) a population II (low metallicity) star of 0.85 $\mathrm{M_{\odot}}$ transferring approximately 0.65 $\mathrm{M_{\odot}}$; (b) a 1.15-$\mathrm{M_{\odot}}$ population I star transferring approximately 0.91 $\mathrm{M_{\odot}}$; (c) a 1.199-$\mathrm{M_{\odot}}$ population I star transferring approximately 0.99 $\mathrm{M_{\odot}}$.

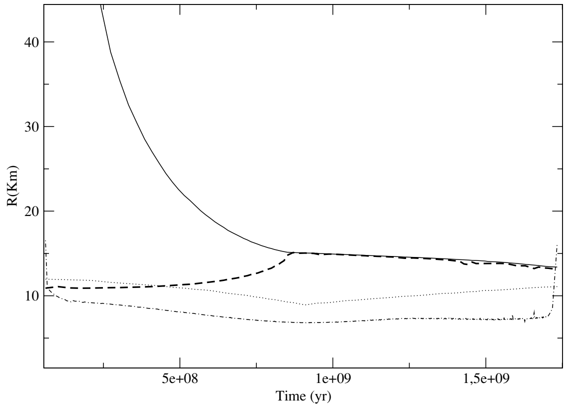

Figure 2. Time evolution of the relevant radii for the accretion process on to the neutron star in the binary system of Fig. 1(b). The EOS governing the ultradense matter is EOS FPS. The solid line is the corotation radius, the thick dashed line is NS radius, the dot dashed line is the magnetospheric radius, the dotted line is the radius of the marginally stable orbit. Note that the magnetospheric radius exceeds the corotation radius only at the end of accretion, when the mass transfer rate decreases, and that for the first part of the accretion process the last stable orbit is outside the NS. The magnetospheric radius data are smoothed in order to avoid the disturbing visual effects due to the numerical instabilities of $\dot{M}_{\rm B}$.

per unit time accreted on to the NS is equal to the mass lost by the companion per unit time due to Roche lobe overflow, $\dot{M}_{\rm RL}$, i.e.

$$\dot{M}_{\rm B} = \dot{M}_{\rm RL}. \quad (4)$$

Lamb, Pethick & Pines (1973) wrote a general equation describing the flow of angular momentum into the stellar magnetosphere including the material stress at the inner edge of the disc as well as magnetic and viscous stresses in a disc partially threaded by the NS magnetic field. In particular, they assumed that the threading occurs in a transition region near the inner rim of the disc where the magnetic field of the NS couples to the accretion disc. However, they noted that, for slow rotators, magnetic and viscous stresses can be neglected with respect to the contribution of the material stress. Ghosh, Lamb & Pethick (1977) showed that, in the case of a disc rotating in the same sense of the star, if the transition region where the magnetic field couples to the disc is small, these additional contributions are still negligible even for rapidly rotating NS. In the present discussion we assume therefore that the torque exerted on the NS is only due to the contribution of the material stress at the inner edge of the disc (Pringle & Rees 1972). We will study the evolution of high-magnetized NSs with heavily threaded accretion discs in a future paper. Note, however that Wang (1996) showed that threading effects can modify the maximum achievable period (i.e. the equilibrium period) of the NS only by a few per cent with respect to the unthreaded case.

The angular momentum per unit baryonic mass of a particle orbiting around a rotating, axisymmetric object at a distance R is (see Bardeen 1970)

$$j(R) = (GM_{\rm G})^{1/2}\left[R^2 - 2(J/M_{\rm G}c)(RGM_{\rm G}/c^2)^{1/2} + (J/M_{\rm G}c)^2\right] \times \left\{R^{3/4}\left[R^{3/2} - 3(GM_{\rm G}/c^2)R^{1/2} + 2(J/M_{\rm G}c)(GM_{\rm G}/c^2)^{1/2}\right]^{1/2}\right\}^{-1}. \quad (5)$$

Thus we can write the equation for angular momentum evolution of the NS simply as

$$\dot{J} = j(R_{\rm D})\dot{M}_{\rm B}, \qquad J < J_{\rm max}(M_{\rm B})$$
$$\dot{J} = \frac{{\rm d}J_{\rm max}}{{\rm d}M_{\rm B}}\dot{M}_{\rm B}, \qquad J = J_{\rm max}(M_{\rm B}). \quad (6)$$

The first equation does not apply when the NS is at the mass shedding, i.e. the regime in which the matter at the border of the star has the Keplerian velocity at that radius; for each value of baryonic mass, the mass shedding regime is made individual by the corresponding maximum angular momentum of the star, $J_{\rm max}(M_{\rm B})$. At mass shedding, the matter of the disc should dissipate angular momentum to accrete on to the star. If the disc matter has an angular momentum that will make the star exceed the mass shedding limit, it will just not be able to accrete since it will not be gravitationally bound to the star, until viscous forces drive the excess of angular momentum to the outer zones of the disc (where it will be given back to the companion through tidal forces) allowing matter to accrete on to the star. In this situation, the second equation holds. Integrating it we obtain $J_f - J_i = J_{{\rm max},f} - J_{{\rm max},i}$, but since $J_i = J_{{\rm max},i}$ we have $J_f = J_{{\rm max},f}$: the accreted matter will give the NS only the angular momentum that keeps it at mass shedding, and the star will continue to move along the maximum rotation line.

2.2 Evolution equations at the end of the accretion phase

The NS is thought to switch on as a radio pulsar when the inner edge of the disc lies outside the light cylinder radius (i.e. the radius at which a particle corotating with the star will have velocity c), $r_{lc} = c/\omega_{\rm NS}$ – where $\omega_{\rm NS}$ is the angular velocity of the NS: this certainly happens when the accretion stops and thus the disc disappears. The emission mechanisms for a radio pulsar are believed to be rotating magnetic dipole radiation and magnetospheric currents associated with the emission of relativistic particles, both depending on the angle i between the NS magnetic moment μ and the spin axis (Goldreich & Julian 1969). These two emission mechanisms compensate in such a way that the total energy emitted is nearly independent of i (Bhattacharya & van den Heuvel 1991). The energy loss per unit time will be $\dot{E} = -(2/3c^3)\mu^2\omega_{\rm NS}^4$. It is then convenient to describe NS evolution in terms of baryonic mass and total energy rather than in terms of baryonic mass and angular momentum as we did for the accretion phase. We can write the evolution equations as

$$\dot{M}_{\rm B} = 0$$
$$\dot{M}_{\rm G} = -\frac{2}{3c^5}\mu^2\omega_{\rm NS}^4. \quad (7)$$

Depending on the different conditions, we use either equations (4) and (6) or equations (7) to compute the spin evolution of the NS.

2.3 Excitation of r-modes

Since our main goal is to qualitatively compare the evolution of systems in which the gravitational waves damping is not present and systems in which it is present, we will not go into the details of the theory of r-modes excitation. We will simply assume that, if the r-mode mechanism is present, any star attaining a period of 2 ms (corresponding to a frequency of 500 Hz) undergoes a rapid spin-down to a period of 5 ms due to gravitational waves emission following the scenario proposed by Levin (1999). After this rapid spin-down, the NS starts its accretion driven spin-up process again.

2.4 Numerical methods of integration

During accretion phases we coupled through equations (4) and (6) a detailed description of the binary evolution of the system obtained with the stellar evolution code with a fully relativistic calculation of NS physical properties given its EOS, its baryonic mass and its angular momentum.

The evolution of the binary system is followed self-consistently including the full computation of the structure of the secondary star, by means of the ATON1.2 stellar evolution code (D'Antona, Mazzitelli & Ritter 1989). The equations of stellar structure are numerically solved by a full Newton–Raphson integration from the centre up to the basis of the stellar atmosphere. The numerical inputs are described in Mazzitelli (1989). The secondary star is considered to be the mass losing component of the binary system, and its mass-loss rate is computed following the formulation by Ritter (1988), as an explicit exponential function of the distance of the stellar radius to the Roche lobe, in units of the pressure scaleheight. The evolution of the binary parameters can be followed by considering several possible cases for the transfer of mass, and for the loss of mass and associated angular momentum from the system. The orbital evolution also includes losses of orbital angular momentum through magnetic braking, in the Verbunt & Zwaan (1981) formulation, in which the braking parameter is set to $f = 1$ and through gravitational radiation.

The relativistic computations for the compact object are implemented using a slightly modified version of RNS (rotating neutron stars) public domain code by Stergioulas & Morsink (1999). The RNS routines provide a numerical solution of Einstein equations for a rotating axisymmetric body, integrating the equations *via* the Komatsu–Eriguchi–Hachisu method (Komatsu et al. 1989; see Stergioulas & Friedman 1995 for a comparison with other integration methods). The main problem of this approach is that we have evolution equations for the baryonic mass and the angular momentum of the compact object, while the boundary conditions for the solution of Einstein equations are the central energy density and the equatorial expansion of the star. To solve this problem, we used a grid of relativistic equilibrium NS models integrating the Einstein equations for a wide range of initial conditions, spanning all allowed values for stable configurations. This was obtained modifying the code to build a complete grid of relativistic equilibrium configurations with the necessary numerical precision. To this we added a stability control routine to exclude unstable configurations.

We obtain the accretion rate $\dot{M}_{\rm B}$ and a corresponding time interval (in which $\dot{M}_{\rm B}$ remains unchanged) from the stellar evolution code. Then we integrate the differential equations (4) and (6) using a finite-differences method. For each integration time-step the accreted baryonic mass is $\Delta M_{\rm B} = \dot{M}_{\rm B}(t)\Delta t$ and the accreted angular momentum is $\Delta J = j[R_{\rm D}(t)]\dot{M}_{\rm B}(t)\Delta t$. For the nth time-step of the evolution, we search on the grid the single equilibrium configuration with the updated values of $M_{\rm B}(t_n) = M_{\rm B}(t_{n-1}) + \Delta M_{\rm B}$ and $J(t_n) = J(t_{n-1}) + \Delta J$; in this way we obtain the corresponding values of gravitational mass, radius, spin frequency and momentum of inertia of the NS. We fine-tuned the time-step so that $\Delta M_{\rm B}$, ΔJ are always comparable with the distance of two neighbour points in the grid.

Obviously the integration over a grid may introduce numerical uncertainties in our results: in particular, we are implicitly assuming that the spin frequency, the gravitational mass and the radius of the NS remain unchanged between two points on the grid. Therefore, we introduce an error in the evaluation of these quantities that is equal to half the distance between two points on the grid, which is always well below 0.5 per cent in our simulations. This also yields an uncertainty in evaluating, step by step, the value of the accretion radius given by equation (3) and the angular momentum of the matter at the inner rim of the disc, which is always in the order of a few per cent.

During the pulsar phase, we integrated equations (7) in a similar way using the same grid of relativistic equilibrium NS models. To avoid the grid imprecisions we made a cubic spline interpolation on the values of the function $\omega_{\rm NS}(M_{\rm G})$ for the integration of the equation set (7).

3 RESULTS

We consider three simple examples that show different possible fates for an LMXB, depending on the characteristics of the companion star. In all cases we start our evolution with a slowly rotating ($P \sim 1$ s) NS with a gravitational mass of 1.4 $M_\odot$, which is a typical value for a newborn NS (Thorsett & Chakrabarty 1999). The EOS adopted to describe the ultradense matter is that proposed by Friedman, Pandharipande & Smith (FPS, see, e.g., Lorenz, Ravenhall & Pethick 1993 for a recent discussion), which has been widely used in the literature and has average values of stiffness and physical parameters when compared with other EOS (see Arnett & Bowers 1977 for a catalogue). For our simulations we consider the maximum mass configuration equal to the maximum rotation configuration (although this is not precisely true, see Stergioulas & Friedman 1995, these configurations differ from each other for less than 0.1 per cent in our case), and we have $M_{\rm max} = 2.123$ $M_\odot$, $\omega_{\rm max}/2\pi = 1882$ Hz. Finally, as already mentioned, we assume that the NS has a magnetic dipole moment of 10^{26} G cm^3.

In particular, we consider three typical examples of binary evolution in which the accretion rate is low enough to remain conservative, both above and below the bifurcation period, i.e. the period below which the orbital evolution, during the accretion phase, proceeds towards shorter binary periods. The bifurcation period is $P_{\rm orb,bif} \simeq 18$ h for a binary composed of a 1.4-$M_\odot$ NS plus a 1.0-$M_\odot$ secondary (Podsiadlowski, Rappaport & Pfahl 2002). In the cases we considered, the secondary is one of the following.

(i) A population II donor with an initial mass of 0.85 $M_\odot$ and an orbital period above the orbital bifurcation period; it loses 0.646 $M_\odot$ during the accretion phase. The initial binary period is 14.3 h and it steadily increases to ∼124 h at the end of the accretion phase, which lasts $\sim 1.7 \times 10^9$ yr (see Fig. 1a), leaving a helium white dwarf remnant. Note that the orbital bifurcation period for a system composed of a 1.4-$M_\odot$ NS plus a 0.85-$M_\odot$ companion is smaller than 14.3 h, because the bifurcation period depends strongly on the companion mass. In this case, Roche lobe overflow is driven by the nuclear evolution of the companion.

(ii) A population I donor (initially below the bifurcation period) with an initial mass of 1.15 $M_\odot$, which loses 0.91 $M_\odot$ during the accretion phase. The orbital period evolves from 10.6 to 3.5 h. In this case the companion star overflows its Roche lobe due to angular momentum losses caused magnetic braking. The accretion phase ends when the companion becomes fully convective, according to the classical scenario for the evolution of cataclysmic variables in the period gap (Verbunt & Zwaan 1981). The accretion phase lasts $\sim 1.7 \times 10^9$ yr, see Fig. 1(b).

(iii) A population I donor (initially above the bifurcation period) with an initial mass of 1.199 $M_\odot$; it loses 0.99 $M_\odot$ during the accretion phase. The period evolves from 19 to 92 h, leaving again a helium white dwarf remnant. The accretion phase lasts $\sim 2.2 \times 10^9$ yr (see Fig. 1c). In this case, Roche lobe overflow is driven by the nuclear evolution of the secondary star.

In these evolutions the mass transfer rate is smaller than the Eddington limit. Therefore, it is reasonable to assume that the mass transfer is conservative. Moreover, the matter flow from the

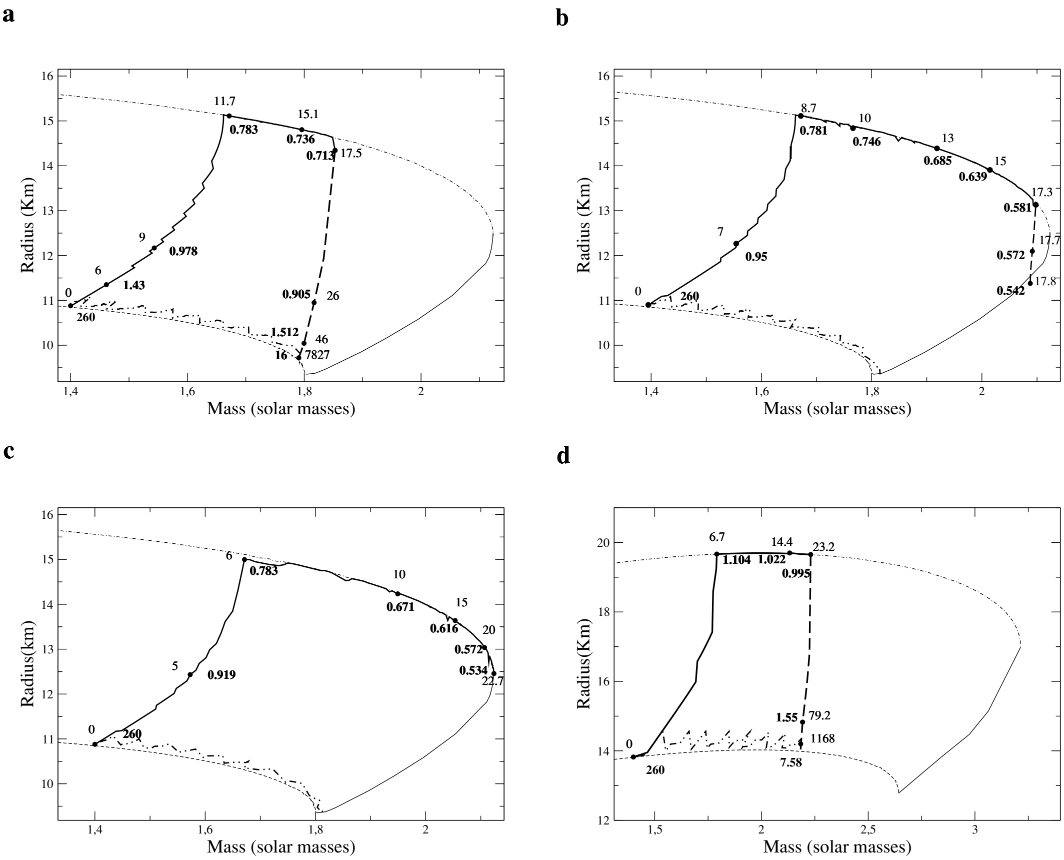

Figure 3. Evolution of the NS in the binary systems of Fig. 1 in the gravitational mass–radius plane. The mass is in solar masses and the radius is in kilometres. The numbered dots indicate evolutionary stages with system age since the start of the accretion in units of 10^8 yr (external numbers) and the spin period in milliseconds (internal, bold numbers). These numbers refer to simulations in the absence of gravitational waves emission. The thin dot dashed line is the maximum rotation limit, the dashed thin line indicates the stable non-rotating configuration and the thin solid line indicates the stability limit to gravitational collapse. The thick solid line marks the evolution during the accretion phase in absence of gravitational waves emission, the thick dot-dot-dashed line marks the evolution during the accretion phase, with gravitational waves driven by the r-modes instability cycle, while the thick dashed line marks the pulsar phase. In case (i) the pulsar evolution is shown until the star reaches a spin period comparable to that of the fastest pulsar observed, PSR B1937+21, which is 1.56 ms and then slowly brakes to 16 ms in $\sim 8 \times 10^{11}$ yr. In case (ii) pulsar evolution, after a relatively short radio-pulsar phase lasting 5×10^7 yr, brings the star to gravitational collapse as it crosses the stability limit. In case (iii) there is no pulsar phase since the accretion of matter brings the star directly to gravitational collapse. Case (iv) features the same companion star as case (iii), but the NS is governed by the ultrastiff EOS N. The evolution is similar to case (i). Little jumps along the evolution curve are due to the resolution of the grid of relativistic values we used.

companion is not subject to large fluctuations. The NS mass and spin evolution in more complicated cases, such as those observed in X-ray transients (see Campana et al. 1998 for a review) or those discussed by Burderi et al. (2001), are not considered here and will be discussed in a following paper. We considered both the case in which the accretion process does not excite the r-mode instability, and the case in which the spin evolution is influenced by the gravitational waves emission.

In Fig. 3 we plot the evolution of these systems in the gravitational mass–radius plane. The area of this plane that a stable NS equilibrium configuration can span is limited from below by the sequence of stable non-rotating equilibrium configurations, which are those with minimum radius for a given gravitational mass, and from above by the mass-shedding sequence – that is the sequence of equilibrium configurations for which the NS angular velocity equals the Keplerian angular velocity at the NS surface radius. These two stability lines are connected by a third curve, the secular stability curve, which limits from the right the region in which equilibrium configurations are stable against gravitational collapse.

The system evolution without gravitational waves emission are indicated by the thick solid and dashed lines in Fig. 3, where the numbered dots give the corresponding age and spin period. We see that the first phase of accretion (thick solid lines) is similar in all cases: a rapid spin-up brings the star to the mass-shedding limit. Thus the star remains on the mass shedding sequence in the M–R diagram; it will detach from it only when the accretion stops and the NS switches on as a radio pulsar (thick dashed lines).

When we consider the spin-down due to r-modes excitation, all evolutionary tracks (dot-dot-dashed lines in Fig. 3) are even more similar: after a brief spin-up phase, the NS attains the critical spin period of 2 ms. After that, the spin-up–spin-down cycle begins and it ends either in a direct collapse to a black hole or in a radio pulsar phase. In both hypotheses, the amount of mass accreted before the pulsar lights up will determine the fate of the system.

In Fig. 4 we show the possible behaviours of NS angular velocity as a function of gravitational mass for given values of baryonic mass. From equation (7) we see that during the pulsar phase the spin evolution of the star will only depend on the baryonic mass

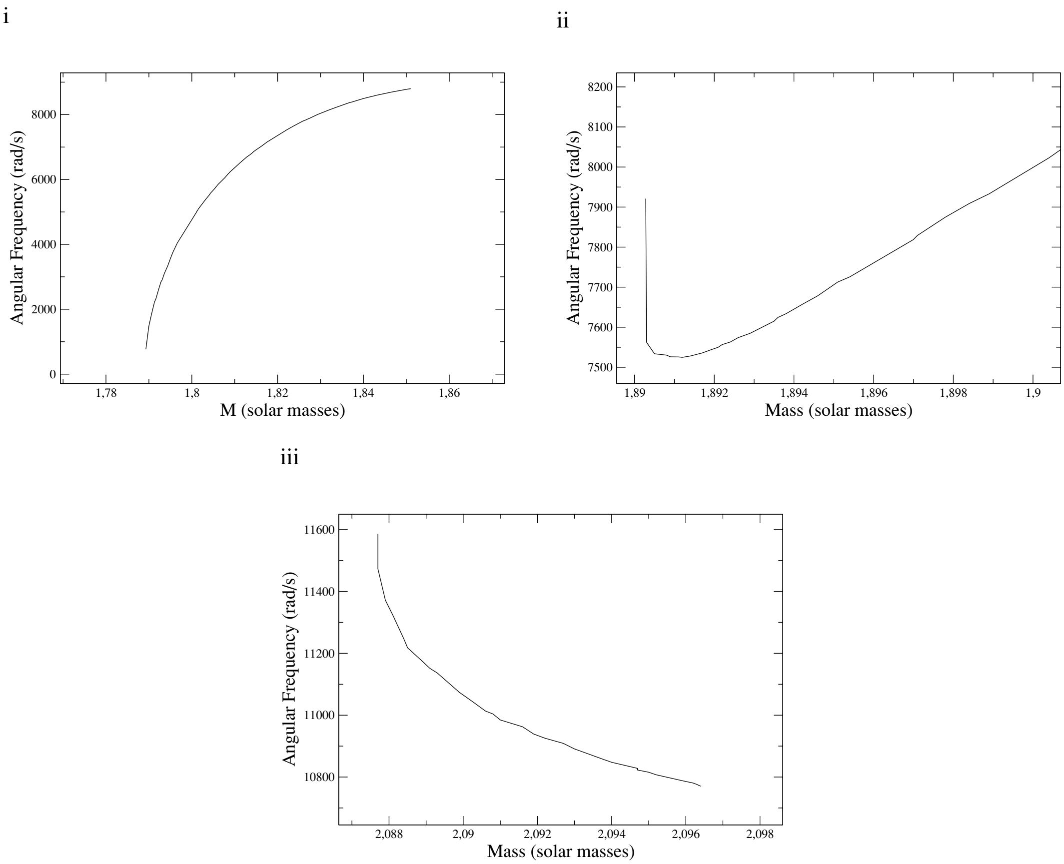

Figure 4. Different evolutionary tracks for a pulsar in the $M_{\rm G} - \omega$ plane depending on its baryonic mass. (a) NS with a baryonic mass equal to that at the end of accretion in the evolutionary case (i). The baryonic mass is low enough so that the rotating star has a stable non-rotating counterpart and the derivative $(\partial\omega/\partial M_{\rm G})_{M_{\rm B}}$ is always positive. (b) NS in the supramassive sequence. In this case, $(\partial\omega/\partial M_{\rm G})_{M_{\rm B}} \rightarrow -\infty$ near the onset of the secular instability, as noted by various authors, but is still positive away from it. The spin-up phase for a pulsar of this mass, assuming a magnetic field of $\sim 10^8$ G, lasts $\sim 10^6$ yr. (c) NS in the extremely supramassive regime, with a baryonic mass equal to that at the end of accretion in evolutionary case (ii). In this case, the mass is so high that $(\partial\omega/\partial M_{\rm G})_{M_{\rm B}} < 0$ for any stable configuration. The spin-up phase, as can be seen from our simulations, lasts $\sim 5 \times 10^7$ yr.

of the star and on the sign of the derivative $(\partial\omega/\partial M_{\rm G})_{M_{\rm B}}$. If there is a non-rotating stable configuration for the given baryonic mass, then $(\partial\omega/\partial M_{\rm G})_{M_{\rm B}} > 0$ always and the pulsar will spin-down until it stops. If such a configuration does not exists for the given baryonic mass, two different evolutionary tracks are possible: either the pulsar spins down until it comes close to the stability limit to gravitational collapse, where it spins up rapidly (in fact $(\partial\omega/\partial M_{\rm G})_{M_{\rm B}} \rightarrow -\infty$ at the instability), or it spins up until it collapses, with no spin-down phase (i.e. $(\partial\omega/\partial M_{\rm G})_{M_{\rm B}} < 0$ always). We say that in the former case the pulsar lights up in the supramassive regime, while in the latter it lights up in the extremely supramassive regime. The spin-up phase, which occurs in correspondence to a loss of rotational energy, is caused by the fact that, near the onset of the instability and in the extremely supramassive regime, a little loss of energy (gravitational mass) corresponds to a rapid decrease of the NS radius. Since the momentum of inertia of the NS strongly depends on NS radius R and on its compactness $GM_{\rm G}/Rc^2$ ($I \sim 0.21\, M_{\rm G}R^2 / (1 - 2\, GM_{\rm G}/Rc^2)$, see Ravenhall & Pethick 1994), in these regimes the loss of rotational energy is achieved by means of a reduction of the momentum of inertia rather than via a spin-down of the NS, which, instead, will spin-up to partially compensate the reduction of the momentum of inertia.

In case (i), in the absence of r-mode excitation, the pulsar lights up, with a period of 0.71 ms, when the star has a gravitational mass larger than the maximum non-rotating mass i.e. $M_{\rm stat} = 1.803\ \mathrm{M}_{\odot}$ (see Fig. 4i). As the pulsar loses energy due to dipole radiation according to equation (7), it leaves the mass-shedding sequence and returns in the normal gravitational mass range, below $M_{\rm stat}$. Thus the pulsar will slow down (see Fig. 3a), without collapsing, until it almost stops on a very long time-scale (it reaches 16 ms in $\sim 8 \times 10^{11}$ yr). If the r-modes are excited during the accretion process, the star lights up as a pulsar with the same baryonic mass (since the same amount of mass has been accreted), but with a different gravitational mass, $1.792\ \mathrm{M}_{\odot}$ instead of $1.805\ \mathrm{M}_{\odot}$: the larger rotational period of the newborn pulsar, 3.31 ms, with respect to the period of 0.71 ms of the evolution in absence of gravitational waves, means that there is not enough energy to increase the gravitational mass above the maximum non-rotating mass. In this case, the pulsar slows down on a similar long time-scale, reaching 16 ms in $\sim 7 \times 10^{11}$ yr.

In case (ii), if r-modes are not excited the star becomes a pulsar when it is in the extreme supramassive regime (see Fig. 3b) and thus equations (7) imply that the NS is spinning up rather than spinning down. As it loses energy, however, the star begins to shrink and heads towards the secular instability limit, which brings the NS to the gravitational collapse. Its radio pulsar phase, characterized by periods well below 1 ms (its initial spin period is 0.581 ms, its period when it crosses the stability line is 0.542 ms) and by the unusual sign of the period derivative, lasts $\sim 5 \times 10^7$ yr. Such a short lifetime, if compared with typical lifetimes for spinning down pulsars, is due to the positive feedback we obtain in this case for the second of equations (7): gravitational mass loss causes a spin-up, which in turn will make the term on the right of this equation bigger, causing a further mass loss. If the r-modes are excited no such behaviour arises, since spin of the NS is kept above 2 ms, the maximum sustainable mass is smaller than that for an NS rotating at mass shedding (the maximum sustainable gravitational mass for an NS spinning at a period of 2 ms is 1.817 $M_{\odot}$). Thus the star collapses to a black hole $\sim 5 \times 10^8$ yr before the end of the accretion process.

In case (iii), the star never lights up as a pulsar, as too much matter is accreted on the star and the maximum mass limit is exceeded. Thus the NS will directly collapse to a black hole (see Fig. 3c).

All these behaviours are strongly dependent on the EOS adopted to describe the ultradense matter. As a comparison we studied the evolution of a system consisting of the same companion star as in case (iii) and a 1.4-$M_{\odot}$ NS with an ultrastiff EOS (EOS N by Walecka & Serot, see Arnett & Bowers 1977), which has a maximum non-rotating mass of 2.634 $M_{\odot}$. The result of this evolution is different, as we show in Fig. 3(d); in this case (hereafter case iv) no spinning up pulsar shows up, nor any accretion induced collapse happens. Instead, we end up with a spinning down submillisecond pulsar with a very long lifetime, comparable with that we obtained in the first case. If the r-modes are excited (see again Fig. 3d) the minimum attainable period is limited to 2 ms, and the pulsar lights up with a period of 2.27 ms. In a following paper, we will discuss in full detail the effects of the EOS of the ultradense matter on the evolution of LMXBs.

4 CONCLUSIONS

We have shown that, depending on the characteristics of the system, especially on the amount of mass accreted on to the NS, on the EOS adopted to describe the NS matter, and on the excitability of the r-modes, LMXBs can have quite different fates: they can light up as a spinning down radio pulsars, they can directly collapse to a black hole during the accretion phase or if, at the end of the accretion phase, the NS is left in the extreme supramassive regime, it will light up as an exotic, spinning up submillisecond radio pulsar with a relatively short lifetime.

It is then evident that, in the hypothesis of a conservative mass transfer from the companion on to a low-magnetized NS and in the absence of r-modes excitation, the accretion process, if the amount of mass accreted is not enough to collapse into a black hole, will end with a very fast spinning object, as it has been suggested before (Cook et al. 1994a). If the NS becomes then detectable as a radio pulsar, it will have a spin period well below one millisecond. In fact, our simulations show (cases i and iv) that we obtain submillisecond pulsars with long lifetimes (in the former case the pulsar lifetime before reaching a period as long as that one of the fastest millisecond pulsar known to date, PSR B1937+21, is $\sim 3 \times 10^9$ yr, while in the latter it becomes $\sim 5 \times 10^9$ yr).

If r-modes are excited by accretion, pulsars are constrained to spin slower than a critical frequency, and this could explain why no NS spinning at submillisecond periods has been observed to date. However, in this situation any binary system in which enough mass is transferred from the companion to the NS will collapse to a black hole, without lighting up as a pulsar. Thus pulsar formation could be much less favoured than in other cases.

It is likely that millisecond pulsar systems such as those observed to date (i.e. systems with $P > 1$ ms) originate from different binary evolution scenarios, in which some critical mechanism has prevented the accretion process to continue until a mass as large as a significant fraction of a solar mass has been transferred. It is probable that the magnetic field of the NS has values much higher than the value we chose (at least at the beginning of the accretion), so that the inner edge of the disc could be outside the corotation radius for at least part of the evolution and magnetic torques could play an important role in the spin evolution of the NS. Moreover, systems in which the mass transfer rate has large fluctuations will light up as pulsars before the end of the accretion process, losing a large amount of mass in a so-called radio ejection phase as proposed by Burderi et al. (2001). We will investigate the evolution of such systems in a future paper.

Although there are selection effects that could have prevented the discovery of submillisecond pulsars (Burderi et al. 2001), if a self-limiting mechanism such as the r-mode instability does not operate, submillisecond radio pulsars should exist and should be detectable in the future. On the other hand we have shown (case iii) that if the NS matter is governed by a moderately stiff EOS such as FPS (i.e. with a maximum non-rotating mass of $\leqslant 1.9$ $M_{\odot}$), the mass transfer can end in an accretion induced collapse to a black hole if as much as 1 $M_{\odot}$ is accreted. Although not much in known on the range of progenitor masses for the present-day population of LMXB in the Galaxy, a recent study by Pfahl, Rappaport & Podsialowski (2003) argues that a great fraction of observed LMXBs may have descended from intermediate mass X-ray binaries, that is form systems with initial donor mass $\gtrsim 1.5$ $M_{\odot}$. If this is true, and if we assume that in such a system, we have an equal probability that the companion transfers any amount of mass between 0.5 and 2 $M_{\odot}$ on to a 1.4-$M_{\odot}$ NS, for NS governed by EOS N we have a 100 per cent probability of obtaining a submillisecond pulsar, while for NS governed by EOS FPS we have only a 20 per cent probability of obtaining a spinning down millisecond pulsar, a 9 per cent probability of the formation of a spinning up submillisecond pulsar (doomed to gravitational collapse) and a 71 per cent probability of direct collapse into a black hole during the accretion phase. Therefore, if the EOS governing NS matter is soft, a conservative mass transfer is more likely to end with a direct accretion-induced collapse to a black hole than with the formation of a submillisecond radio pulsar, and thus submillisecond pulsars could be hard to detect because of their low formation probability.

On the other hand, if the r-modes are excited, the spin period will remain well above one millisecond for all of the evolution. However, making the same assumptions as before, the probability of forming a pulsar drops to 10 per cent for EOS FPS, while for EOS N we still have a very high probability, $\sim$90 per cent. Therefore, this scenario, in which the mass transfer is conservative but the spin frequency is limited by the emission of gravitational waves, implies that the EOS is stiff in order to have a high probability of formation of millisecond pulsars.

Therefore we should predict that the EOS of NSs is very stiff in order to explain the observational evidence (MSP are formed), if gravitational waves are indeed emitted due to r-modes excitation, while we should predict that the EOS of NSs is soft if they are not emitted, so that submillisecond pulsars are very uncommon, as the observations seem to indicate. If future observations will allow one to constrain the stiffness of the NS EOS on an observational basis, this will give an indication on whether r-modes are indeed excited in LMXBs or not.

We have shown that, if r-modes are not excited in LMXBs, the accretion process can leave us with an extremely supramassive NS, which will spin-up during all of its life as a radio pulsar (case ii). It is evident that, being the critical baryonic mass for getting to the

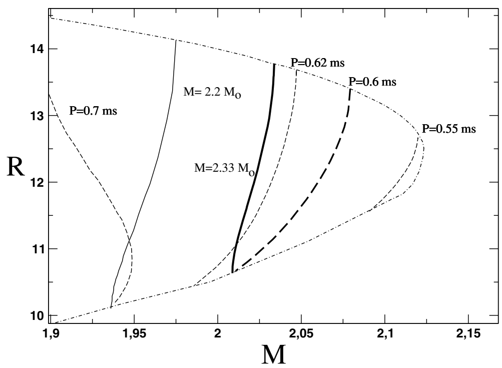

Figure 5. Sequences of NSs with the same spin period are plotted as dashed lines in the gravitational mass–radius plane, while sequences of NSs with the same baryonic mass–area plotted as solid lines. We consider NSs governed by EOS FPS. The dashed-dotted line limits stable configurations. We plot with a thick solid line the sequence of NS with baryonic mass equal to the critical mass, $M_{\rm crit} = 2.33$ M$_\odot$. Any star with $M_{\rm B} \geqslant M_{\rm crit}$ is extremely supramassive, i.e. it spins up under magnetodipole radiation. It is interesting to note that while sequences of constant baryonic mass always have the same shape in the gravitational mass–radius plane, bending from left to right with increasing radii, sequences of constant spin frequency have completely different topologies below the critical mass and above it (see, for example, the sequence with $P = 0.7$ ms and the one with $P = 0.6$ ms). During the radio pulsar phase the star moves along a sequence of constant baryonic mass, decreasing its gravitational mass. It moves therefore from top right of the figure to the bottom left. This implies that the pulsar spins down as long as constant spin frequency sequences bend from top left to bottom right (as the one with $P = 0.7$ ms), and that it spins up if the constant spin frequency sequences it crosses bend from bottom left to top right in the plane (as the one with $P = 0.6$ ms does). As shown in the figure, any stable NS attaining a period $P \leqslant P_{\rm crit} = 0.6$ ms (i.e. any star who lies on the right of the thick dashed line) has $M_{\rm B} \geqslant M_{\rm crit}$. Thus any NS governed by EOS FPS attaining a period $\leqslant$0.6 ms will spin-up once it becomes a pulsar.

extremely supramassive regime $M_{\rm crit}$ an EOS-dependent feature, in principle, the observation an accelerating (or braking) submillisecond pulsar can allow one to exclude several EOS on an observational basis.

To clarify how such an effect can help to constrain the EOS of ultradense matter we need to introduce the new concept of a critical spin period $P_{\rm crit}$ that, together with the minimum period ($P_{\rm min} = 2\,\pi\,/\,\omega_{\rm max}$), is peculiar to each EOS. $P_{\rm crit}$ is the period below which the EOS allows only extremely supramassive stable configurations. In Fig. 5 we show sequences of equilibrium configurations with constant spin period, together with the critical baryonic mass sequence. It is evident from the figure that $P_{\rm crit}$ is equal to the minimum allowed period to avoid gravitational collapse if the star has $M_{\rm B} = M_{\rm crit}$. In fact, any constant period sequence $P < P_{\rm crit}$ will only include stars of baryonic mass greater than the critical one. Thus any NS with $P < P_{\rm crit}$ will accelerate as a consequence of energy loss due to magnetic dipole radiation. Being $P_{\rm crit}$ EOS-dependent, the detection of a submillisecond radio pulsar and the determination of the sign of its period derivative will allow one to effectively constrain the equation of state governing ultradense matter.

Thus the detection of a submillisecond radio pulsar can impose two constraints on the EOS of the NS:

(i) the spin period must be larger than the minimum allowed period, i.e. the spin period of the maximum rotation configuration, $P_{\rm min}$;

(ii) if the period is shorter than $P_{\rm crit}$, the radio pulsar must spin-up rather than spin-down.

Both $P_{\rm min}$ and $P_{\rm crit}$ are EOS dependent and are longer for stiffer EOSs. In fact, the detection of a submillisecond radio pulsar with spin period $P_{\rm obs}$ undergoing a spin-up will rule out all the stiff EOSs with $P_{\rm min} > P_{\rm obs}$. On the other hand the detection of a spinning down submillisecond radio pulsar, with spin period $P_{\rm obs}$, will allow us to rule out all the stiff EOSs with $P_{\rm crit} > P_{\rm obs}$, because they cannot explain a spinning down radio pulsar with such a short spin period. In this case the limit is more stringent because $P_{\rm crit} > P_{\rm min}$! As an example, suppose that a spinning down radio pulsar with a period of 0.713 ms (such as the one we obtain in case i) will be detected: this will allow us to rule out EOS N, since although the minimum period for this EOS is 0.69 ms, any radio pulsar governed by EOS N with such a low spin period will spin-up, being for EOS N $M_{\rm crit} = 3.63$ M$_\odot$ and $P_{\rm crit} = 0.74$ ms.

In summary, in this paper we presented the first results obtained with a new code that allows one to study in details the binary system evolution and the spin evolution of the NS, on the basis of fully relativistic calculations. We used this code to study the evolution of systems with conservative mass transfers and confirmed that the large amount of matter that is transferred on to the NS will spin it up to periods well below one millisecond, unless the emission of gravitational waves dissipates the excess of angular momentum. However, in this last case the amount of mass accreted on to the NS is easily big enough to cause a direct collapse to a black hole. Therefore, we concluded that presumably the recycled systems that give origin to the MSP observed to date should have origin from systems with a highly non-conservative mass transfer. We showed that if the EOS of ultradense matter is not very stiff the direct collapse to a black hole is very likely to happen even if the r-modes are not excited. This could explain the lack of any observation of submillisecond radio pulsars even without invoking gravitational waves emission. As a last remark, since we showed that there is the possibility of obtaining from binary evolution some unusual, accelerating submillisecond radio pulsars, we introduced the new concept of the critical spin period $P_{\rm crit}$, peculiar to each EOS, which can allow one to effectively constrain the EOS of NS matter if a radio pulsar with a period below one millisecond will be observed in the future.

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