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To cite this article: F Ciccarello *et al* 2006 *New J. Phys.* **8** 214

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## Entanglement controlled single-electron transmittivity

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*New Journal of Physics* **8** (2006) 214

Received 4 July 2006

Published 27 September 2006

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/8/9/214

**Abstract.** We show that the electron transmittivity of single electrons propagating along a one-dimensional (1D) wire in the presence of two magnetic impurities is affected by the entanglement between the impurity spins. For suitable values of the electron wave vector, there are two maximally entangled spin states which, respectively, make the wire completely transparent whatever the electron spin state or strongly inhibit electron transmission.

The key role that entanglement plays not only in most quantum information processing tasks [1] but also in a broad range of physical processes such as quantum transport [2]–[4] has been considerably clarified over the past few years. In this paper, we illustrate the interplay between entanglement and single-electron transport properties in a one-dimensional (1D) wire in the presence of two scattering magnetic impurities. Such a system is the electron analogue of a Fabry–Perot (FP) interferometer [5], with the two impurities playing the role of two mirrors with a quantum degree of freedom: the spin. This suggests the interesting question of whether

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entanglement between the impurity spins plays any role in modulating the transmittivity. We show that this is indeed the case. In particular, we will show that, under simple suitable circumstances, the maximally entangled Bell states  $|\Psi^+\rangle$  and  $|\Psi^-\rangle$  of the scattering centres spins can either largely inhibit the electron transport or make the wire completely ‘transparent’. Such striking behaviour, which can be intuitively understood in terms of quantum interference, is here quantitatively analysed in terms of constants of motion. Furthermore, we will show that the same scattering mechanism can be used to efficiently create the entanglement between the impurity spins.

Let us assume to inject single conduction electrons into a clean 1D wire—such as a semiconductor quantum wire [6] or a single-wall carbon nanotube [7]—into which two spatially separated, identical spin-1/2 magnetic impurities are embedded (e.g. these could be realized by two quantum dots [8]). Due to the presence of an exchange interaction, the conduction electron undergoes multiple scattering with the impurities before being finally transmitted or reflected. Let us also assume that the electron spin state can be prepared at the input of the wire and measured at its output (this could be achieved through ferromagnetic contacts at the source and drain of the wire [8]). To be more specific, consider a 1D wire along the  $\hat{x}$  direction with the two magnetic impurities, labelled 1 and 2, embedded at  $x = 0$  and  $x = x_0$ , respectively. Assuming that the conduction electrons are injected one at a time (this allows us to neglect many-body effects) and that they can occupy only the lowest subband, the Hamiltonian can be written as

$$H = \frac{p^2}{2m^*} + J\boldsymbol{\sigma} \cdot \mathbf{S}_1\delta(x) + J\boldsymbol{\sigma} \cdot \mathbf{S}_2\delta(x - x_0), \quad (1)$$

where  $p = -i\hbar\nabla$ ,  $m^*$  and  $\boldsymbol{\sigma}$  are the electron momentum operator, effective mass and spin-1/2 operator respectively,  $\mathbf{S}_i$  ( $i = 1, 2$ ) is the spin-1/2 operator of the  $i$ th impurity and  $J$  is the exchange spin–spin coupling constant between the electron and each impurity. All the spin operators are in units of  $\hbar$ . Since the electron-impurity collisions are elastic, the energy eigenvalues are simply  $E = \hbar^2k^2/2m^*$  ( $k > 0$ ) where  $k$  is a good quantum number. As the total spin Hilbert space is 8D and considering left-incident electrons, it turns out that to each value of  $k$  there corresponds an 8-fold degenerate energy level. Let  $\mathbf{S} = \boldsymbol{\sigma} + \mathbf{S}_1 + \mathbf{S}_2$  be the total spin of the system. Since  $\mathbf{S}^2$  and  $S_z$ , with quantum numbers  $s$  and  $m_s$ , respectively, are constants of motion,  $H$  can be block diagonalized, each block corresponding to an eigenspace of fixed  $s$  (for three spins 1/2, the possible values of  $s$  are 1/2, 3/2) and  $m_s = -s, \dots, s$ . Let us rewrite equation (1) in the form

$$H = \frac{p^2}{2m^*} + \frac{J}{2}(\mathbf{S}_{e1}^2 - \frac{3}{2})\delta(x) + \frac{J}{2}(\mathbf{S}_{e2}^2 - \frac{3}{2})\delta(x - x_0), \quad (2)$$

where  $\mathbf{S}_{ei} = \boldsymbol{\sigma} + \mathbf{S}_i$  ( $i = 1, 2$ ) is the total spin of the electron and the  $i$ th impurity. Note that, in general  $\mathbf{S}_{e1}^2$  and  $\mathbf{S}_{e2}^2$  do not commute. Here, we choose as spin space basis the states  $|s_{e2}; s, m_s\rangle$ , common eigenstates of  $\mathbf{S}_{e2}^2$ ,  $\mathbf{S}^2$  and  $S_z$ , to express, for a fixed  $k$ , each of the eight stationary states of the system as an 8D column. To calculate the transmission probability amplitude  $t_{s_{e2}}^{(s'_{e2}, s)}$  that an electron prepared in the incoming state  $|k\rangle|s'_{e2}; s, m_s\rangle$  will be transmitted in a state  $|k\rangle|s_{e2}; s, m_s\rangle$ , we have derived the exact stationary states of the system. To do this, the quantum waveguide theory approach of [9] for an electron scattering with a magnetic impurity has been properly generalized to the case of two impurities. Note that due to the form of  $H$  (see equation (2)) coefficients  $t_{s_{e2}}^{(s'_{e2}, s)}$  do not depend on  $m_s$ .

Let us first consider the subspace  $s = 3/2$ . Since in this subspace  $\mathbf{S}_{e1}^2$  and  $\mathbf{S}_{e2}^2$  commute, the states  $|s_{e2}; s, m_s\rangle = |1; 3/2, m_{3/2}\rangle$  are also eigenstates of  $\mathbf{S}_{e1}^2$  and the effective electron-impurities potential in equation (2) reduces to  $J/4 \delta(x) + J/4 \delta(x - x_0)$ . Note that the two impurities behave as if they were static and the scattering between electron and impurities cannot flip the spins. The four stationary states take therefore the simple form  $|\Psi_{k,1;3/2,m_{3/2}}\rangle = |\phi_k\rangle|1; 3/2, m_{3/2}\rangle$ , where  $|\phi_k\rangle$  describes the electron orbital degrees of freedom and can be easily found imposing suitable boundary conditions at  $x = 0$  and  $x = x_0$ . This allows us to calculate the transmission probability amplitude  $t_1^{(1;3/2)}$  which turns out to depend on the two dimensionless parameters  $kx_0$  and  $\rho(E)J$ , where  $\rho(E) = (\sqrt{2m^*/E})/\pi\hbar$  is the density of states per unit length of the wire [6]. Here, given length limitations, we shall omit its explicit form.

Let us now consider the  $s = 1/2$  subspace. Here,  $\mathbf{S}_{e1}^2$  and  $\mathbf{S}_{e2}^2$  do not commute. This is a signature of the fact that in this space spin-flip can occur. In each of the 2D  $m_{1/2} = -1/2, 1/2$  subspaces, the two stationary states are of the form

$$|\Psi_{k,s'_{e2};1/2,m_{1/2}}\rangle = \sum_{s_{e2}=0,1} |\varphi_{k,s'_{e2},s_{e2}}\rangle |s_{e2}; 1/2, m_{1/2}\rangle, \quad (3)$$

where the index  $s'_{e2} = 0, 1$  indicates that the incident spin state of (3) is  $|s'_{e2}; 1/2, m_{1/2}\rangle$ . In this subspace the 8D column representing each eigenstate of  $H$  has therefore two non-vanishing components. The transmitted part of (3) is given by  $|k\rangle[\sum_{s_{e2}=0,1} t_{s_{e2}}^{(s'_{e2};1/2)} |s_{e2}; 1/2, m_{1/2}\rangle]$  where again the four coefficients  $t_{s_{e2}}^{(s'_{e2};1/2)}$  depend on the two parameters  $kx_0$  and  $\rho(E)J$ . Note that since in the  $s = 1/2$  subspace  $\mathbf{S}_{e2}^2$  is not conserved, unlike the  $s = 3/2$  case,  $t_{s_{e2}}^{(s'_{e2};1/2)} \neq 0$  for  $s_{e2} \neq s'_{e2}$ .

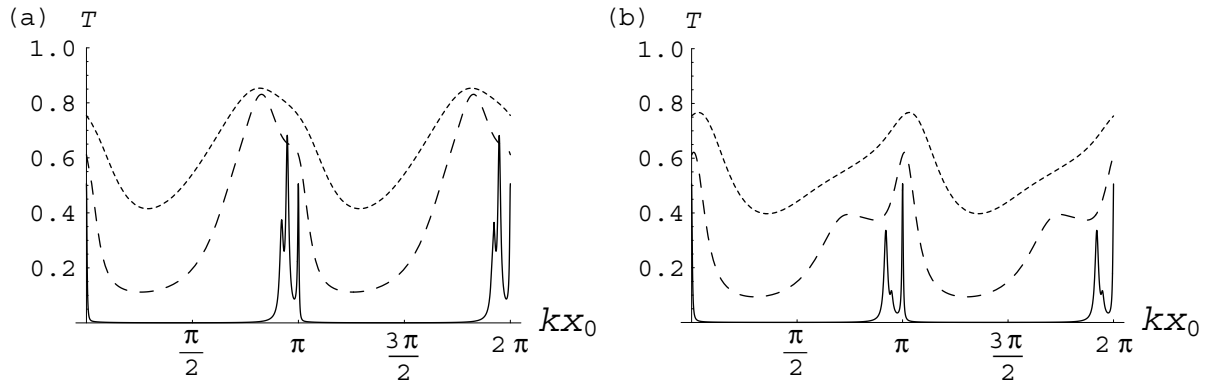
The knowledge of all the exact transmission amplitudes  $t_{s_{e2}}^{(s'_{e2};s)}$  allows us to determine, at all orders in the coupling constant  $J$ , how an incident wave  $|k\rangle|\chi\rangle$ , where  $|\chi\rangle$  is an arbitrary overall spin state, is transmitted after scattering. The state  $|k\rangle|\chi\rangle$  is the incident part of the stationary state

$$|\Psi_{k,\chi}\rangle = \sum_{s'_{e2},s,m_s} \langle s'_{e2}; s, m_s | \chi \rangle |\Psi_{k,s'_{e2};s,m_s}\rangle \quad (4)$$

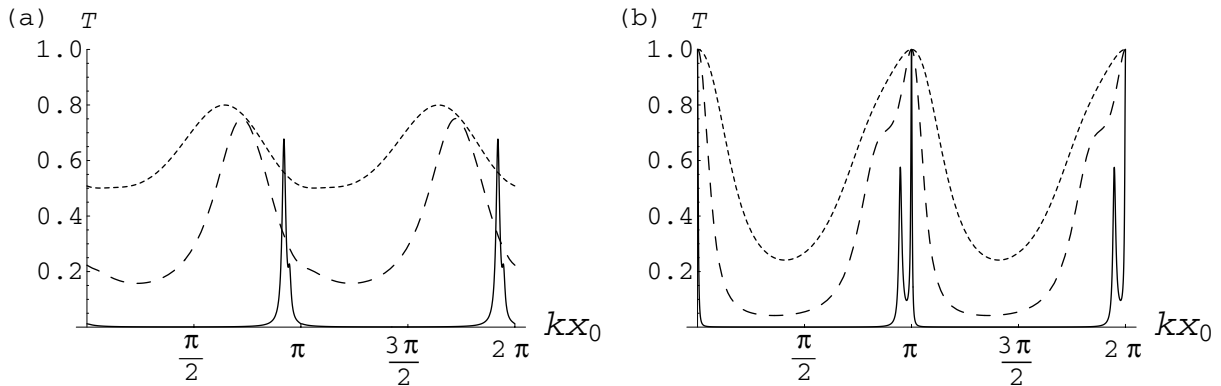
where  $s'_{e2} = 1$  for  $s = 3/2$ , while  $s'_{e2} = 0, 1$  for  $s = 1/2$ . The overall electron transmittivity  $T$  is obtained by expressing the transmitted part of each stationary state  $|\Psi_{k,s'_{e2};s,m_s}\rangle$  in terms of  $t_{s_{e2}}^{(s'_{e2};s)}$ , rearranging (4) as a linear expansion in the basis  $|s_{e2}; s, m_s\rangle$  and then summing the squared modules of the coefficients of the expansion. We are now able to investigate how electron transmission depends on the state in which the two impurities are prepared. Let us start with the following family of impurity spins states

$$|\Psi(\vartheta, \varphi)\rangle = \cos \vartheta |\uparrow\downarrow\rangle + e^{i\varphi} \sin \vartheta |\downarrow\uparrow\rangle, \quad (5)$$

with  $\vartheta \in [0, 2\pi]$  and  $\varphi \in [0, \pi]$ . This family includes both maximally entangled and product states. The electron transmittivity  $T$  when the injected electron spin state is  $|\uparrow\rangle$  with the impurities prepared in the product states  $|\uparrow\downarrow\rangle$  and  $|\downarrow\uparrow\rangle$  is plotted in figures 1(a) and (b), respectively. A behaviour similar to a FP interferometer with partially silvered mirrors, with equally spaced maxima of transmittivity, is exhibited. In figure 1(a), principal maxima occur around a value of  $kx_0 \neq n\pi$  which tends to  $kx_0 = n\pi$  for increasing values of  $\rho(E)J$ , while in figure 1(b) they occur at  $kx_0 = n\pi$ . As  $\rho(E)J$  is increased, maxima get lower and lower and sharpen. Remarkably,

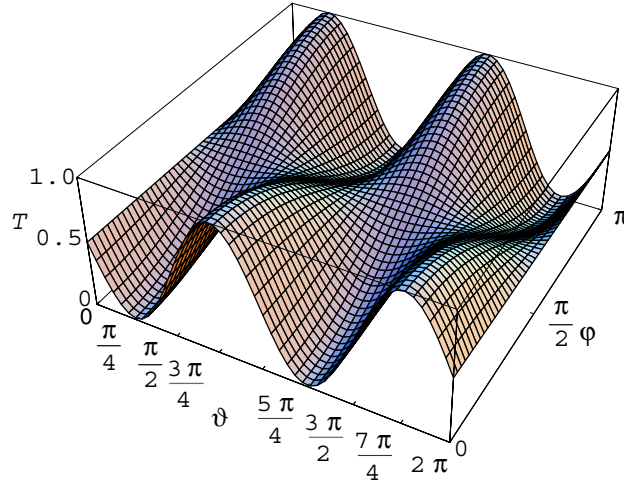


**Figure 1.** Electron transmittivity  $T$  as a function of  $kx_0$  when the electron is injected in the state  $|\uparrow\rangle$  with the impurities prepared in the state  $|\uparrow\downarrow\rangle$  (a) and  $|\downarrow\uparrow\rangle$  (b). Dotted, dashed and solid lines stand for  $\rho(E)J = 1, 2, 10$ , respectively.



**Figure 2.** Electron transmittivity  $T$  as a function of  $kx_0$  when the electron is injected in the state  $|\uparrow\rangle$  with the impurities prepared in the state  $|\Psi^+\rangle$  (a) and  $|\Psi^-\rangle$  (b). Dotted, dashed and solid lines stand for  $\rho(E)J = 1, 2, 10$ , respectively.

in both cases the electron and impurities spin state is changed after the scattering and the electron undergoes a loss of coherence, since we always have  $T < 1$  [10]–[12]. A similar behaviour with decoherence occurs when the two impurity spins are prepared in the maximally entangled state  $|\Psi^+\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$  (see figure 2(a)). Again the transmitted spin state differs from the incident one and, in particular, when  $kx_0 = n\pi$ , it turns out to be a linear combination of  $|\uparrow\rangle|\Psi^+\rangle$  and  $|\downarrow\rangle|\uparrow\uparrow\rangle$ . A striking behaviour, however, appears when the impurity spins are prepared in the maximally entangled state  $|\Psi^-\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ : as shown in figure 2(b), the wire becomes ‘transparent’ for  $kx_0 = n\pi$ . In other words perfect transmittivity  $T = 1$  is reached at  $kx_0 = n\pi$  regardless of the value of  $\rho(E)J$ , with peaks getting narrower for increasing values of  $\rho(E)J$ . Furthermore, under the resonance condition  $kx_0 = n\pi$ , the spin state  $|\uparrow\rangle|\Psi^-\rangle$  is transmitted unchanged. Note that this occurs even if  $|\uparrow\rangle|\Psi^-\rangle$  belongs to the  $s = 1/2$  subspace where spin-flip is allowed. Using the explicit form of  $t_{s_e2, s}^{(s_e2, s)}$  it can be proved that for  $kx_0 = n\pi$  the only spin state which is transmitted unchanged is  $(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|\Psi^-\rangle$ , with arbitrary complex values of  $\alpha$  and  $\beta$ . Thus the wire becomes transparent regardless of the electron spin state. This effect,



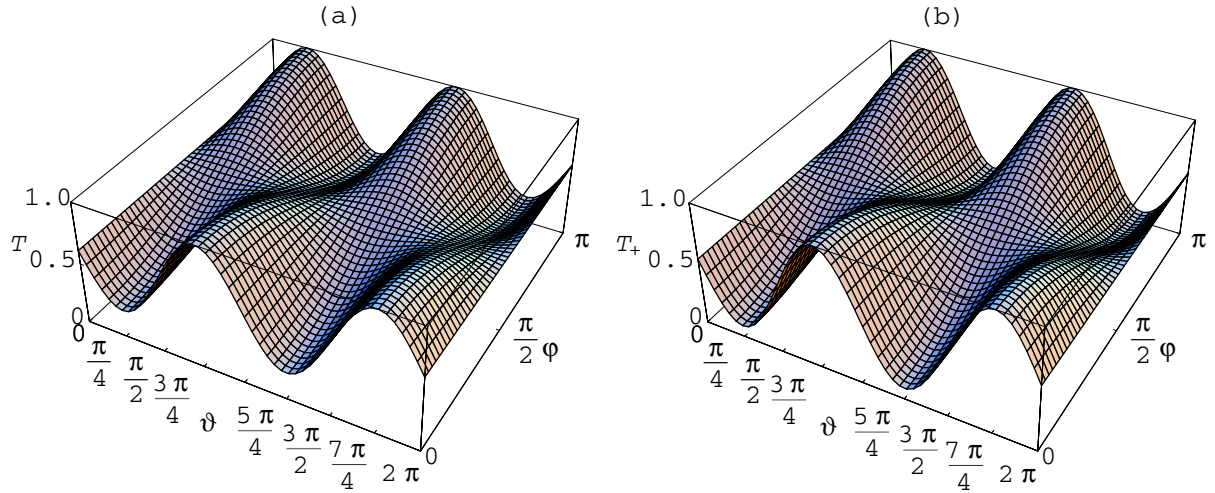
**Figure 3.** Electron transmittivity  $T$  at  $kx_0 = n\pi$  and  $\rho(E)J = 10$  when the electron is injected in an arbitrary spin state  $(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$  with the impurities prepared in the state  $\cos\vartheta|\uparrow\downarrow\rangle + e^{i\varphi}\sin\vartheta|\downarrow\uparrow\rangle$ .

clearly due to constructive quantum interference, can be quantitatively analysed in terms of Hamiltonian symmetries. Denoting by  $\delta_k(x)$  and  $\delta_k(x - x_0)$ , the effective representations of  $\delta(x)$  and  $\delta(x - x_0)$ , respectively, in a subspace of fixed energy  $E = \hbar^2 k^2 / 2m^*$ , it can be easily proved that  $\delta_k(x) = \delta_k(x - x_0)$  for  $kx_0 = n\pi$ . When this occurs the non-kinetic part  $V$  of  $H$  in equation (1) assumes the effective representation

$$V = J\boldsymbol{\sigma} \cdot \mathbf{S}_{12}\delta_k(x) = \frac{J}{2}(\mathbf{S}^2 - \boldsymbol{\sigma}^2 - \mathbf{S}_{12}^2)\delta_k(x), \quad (6)$$

where  $\mathbf{S}_{12} = \mathbf{S}_1 + \mathbf{S}_2$ . This means that, for  $kx_0 = n\pi$ ,  $\mathbf{S}_{12}^2$  (with quantum number  $s_{12}$ ) becomes a constant of motion. This is physically reasonable since the condition  $kx_0 = n\pi$  implies that the electron is found at  $x = 0$  and  $x = x_0$  with equal probability and, as a consequence, the two impurities are equally coupled to the electron spin. Furthermore,  $V$  turns out to vanish for  $s = 1/2$  and  $s_{12} = 0$ . This is the case for the initial state  $(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|\Psi^-\rangle$  as this is an eigenstate of  $\mathbf{S}^2$  and  $\mathbf{S}_{12}^2$  with quantum numbers  $s = 1/2$  and  $s_{12} = 0$ , respectively. Therefore, when this state is prepared and  $kx_0 = n\pi$ , no spin-flip occurs and the wire becomes transparent. This is not the case for the state  $|\uparrow\rangle|\Psi^+\rangle$  belonging to the degenerate 2D eigenspace of  $\mathbf{S}_{12}^2$  and  $S_z$  with quantum numbers  $s_{12} = 1$  and  $m = 1/2$ , respectively. As a consequence, when  $kx_0 = n\pi$ , the transmitted spin state will result in a linear combination of  $|\uparrow\rangle|\Psi^+\rangle$  and  $|\downarrow\rangle|\uparrow\uparrow\rangle$ , implying spin-flip and decoherence. To further illustrate these results, we have plotted the transmittivity  $T$  when the electron is injected in an arbitrary spin state  $(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$  with the impurities prepared in a state (5) as a function of  $\vartheta$  and  $\varphi$ , for  $kx_0 = n\pi$  and  $\rho(E)J = 10$  (see figure 3). Note how the electron transmission depends crucially on the relative phase  $\varphi$  between the impurity spin states  $|\uparrow\downarrow\rangle$  and  $|\downarrow\uparrow\rangle$ . The maximum value of  $T$  occurs when the impurities are prepared in the singlet state  $|\Psi^-\rangle$ , while its minima occur for the triplet state  $|\Psi^+\rangle$ . As we have discussed,  $T < 1$  (it gets smaller and smaller for increasing values of  $\rho(E)J$ ) for  $|\Psi^+\rangle$  due to decoherence effects. Since the set of states (5) is spanned by  $|\Psi^-\rangle$  and  $|\Psi^+\rangle$ , the transmittivity for a generic state (5) will have intermediate values between the value of  $T$  for  $|\Psi^+\rangle$  and 1.





**Figure 4.** Electron transmittivity  $T$  (a) and conditional electron transmittivity  $T_+$  (b) at  $kx_0 = n\pi$  and  $\rho(E)J = 2$  when the electron is injected in the state  $|\uparrow\rangle$  with the impurities prepared in the state  $\cos \vartheta |\uparrow\downarrow\rangle + e^{i\varphi} \sin \vartheta |\downarrow\uparrow\rangle$ .

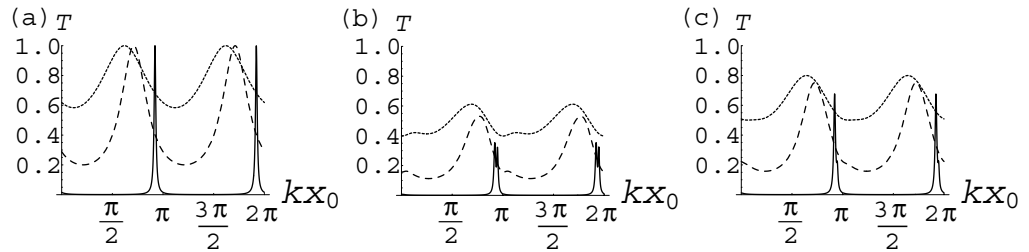
The most remarkable result emerging from the above discussion is that, within the set of initial impurity spins states (5), maximally entangled states  $|\Psi^-\rangle$  and  $|\Psi^+\rangle$  have the relevant property to maximize or minimize electron transmission. We have chosen  $\rho(E)J = 10$  to better highlight this behaviour, but this happens for any value of  $\rho(E)J$ . This result suggests the appealing possibility to use the relative phase  $\varphi$  as a control parameter to modulate the electron transmission in a 1D wire.

For this task to be correctly performed, it is required that the state  $|\Psi^+\rangle$  in which the impurities must be prepared to inhibit electron transmission, can be protected from spin-flip events. This can be achieved if the electron is injected in a fixed spin state, let us say  $|\uparrow\rangle$ , and analysed in the same state when transmitted. Let us denote by  $T_+$  the conditional probability that the electron is transmitted in the state  $|\uparrow\rangle$ . In figures 4(a) and (b), we have plotted  $T$  and  $T_+$ , respectively, for an initial impurity spins state (5) with the electron injected in the state  $|\uparrow\rangle$  and for  $kx_0 = n\pi$  and  $\rho(E)J = 2$ . Note how the filtering can be used to efficiently reduce the electron transmission with the impurities prepared in the state  $|\Psi^+\rangle$ . We found that  $T_+ \simeq T$  for high values of  $\rho(E)J$  as in figure 3 for  $\rho(E)J = 10$ . Thus in these cases, no spin-filtering is required.

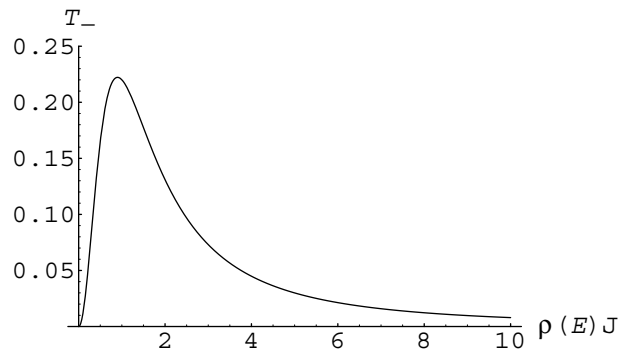
The above features are however not present for all sets of maximally entangled states. Let us for instance consider the family of initial impurity spins states

$$|\phi(\vartheta, \varphi)\rangle = \cos \vartheta |\uparrow\uparrow\rangle + e^{i\varphi} \sin \vartheta |\downarrow\downarrow\rangle, \quad (7)$$

with  $\vartheta \in [0, 2\pi]$  and  $\varphi \in [0, \pi]$ . Our calculations show that, in this case, the two impurities behave as if they were prepared in a statistical mixture of  $|\uparrow\uparrow\rangle$  and  $|\downarrow\downarrow\rangle$  with weights  $\cos^2 \vartheta$  and  $\sin^2 \vartheta$ , respectively. This is illustrated in figures 5(a)–(c) for the cases  $\vartheta = \pi/4$ ,  $\vartheta = 0$ ,  $\vartheta = \pi/2$ , respectively, with arbitrary  $\varphi$  and the electron injected spin state being  $|\uparrow\rangle$ . The phase  $\varphi$  thus plays no role and no interesting interference effect occurs. The reason for this is that  $|\uparrow\rangle|\uparrow\uparrow\rangle$  and  $|\uparrow\rangle|\downarrow\downarrow\rangle$  are eigenstates of the constant of motion  $S_z$  with different quantum numbers  $m = 3/2$  and  $m = -1/2$ , respectively and therefore, unlike the set of states (5), no quantum interference



**Figure 5.** Electron transmittivity  $T$  as a function of  $kx_0$  when the electron is injected in the state  $|\uparrow\rangle$  with the impurities in the initial state  $|\uparrow\uparrow\rangle$  (a),  $|\downarrow\downarrow\rangle$  (b) and  $(|\uparrow\uparrow\rangle + e^{i\varphi}|\downarrow\downarrow\rangle)/\sqrt{2}$  for arbitrary  $\varphi$  (c). Dotted, dashed and solid lines stand for  $\rho(E)J = 1, 2, 10$ , respectively.



**Figure 6.** Conditional electron transmittivity  $T_-$  at  $kx_0 = n\pi$  as a function of  $\rho(E)J$  when the electron is injected in the state  $|\uparrow\rangle$  with the impurities prepared in the state  $|\downarrow\downarrow\rangle$ .

effects are possible. Finally, note that while in the cases of figures 5(b) and (c) a loss of electron coherence is exhibited similarly to the cases of figures 1(a), (b) and 2(a), a coherent behaviour completely analogous to a FP interferometer with partially silvered mirrors [5] is observed when the impurities are prepared in the state  $|\uparrow\uparrow\rangle$  with the electron injected in the state  $|\uparrow\rangle$  (see figure 5(a)). Indeed, the spin state  $|\uparrow\rangle|\uparrow\uparrow\rangle$  belongs to the non degenerate eigenspace  $s = 3/2$ ,  $m = 3/2$  where spin-flip does not occur and the impurities behave as they were static. However, we note that at variance with the case of figure 2(b),  $T = 1$  for values of  $kx_0$  which depend on  $\rho(E)J$  and only if the electron spin is initially aligned with the spins of the impurities.

To conclude we show how the scattering itself can be used to generate the desired entanglement between the impurity spins. To modulate the electron transmission, we are interested in generating either a  $|\Psi^+\rangle$  or a  $|\Psi^-\rangle$  state. Such states can be easily transformed into each other by simply introducing a relative phase shift through a local field. A  $|\Psi^+\rangle$  state can be generated by injecting an electron in the state  $|\uparrow\rangle$  with the two impurities prepared in the state  $|\downarrow\downarrow\rangle$ , in the spirit of [13]. When  $kx_0 = n\pi$ , due to conservation of  $S_{12}^2$  and  $S_z$  the transmitted spin state will be a linear combination of  $|\uparrow\rangle|\downarrow\downarrow\rangle$  and  $|\downarrow\rangle|\Psi^+\rangle$ . An output filter selecting only transmitted electrons in the state  $|\downarrow\rangle$  can thus be used to project the impurities into the state  $|\Psi^+\rangle$ . The conditional probability  $T_-$  for this event is plotted in figure 6 as a function of  $\rho(E)J$ . A probability larger than 20% can be reached with  $\rho(E)J \simeq 1$ .



To estimate the feasibility of the device here proposed, let us assume an electron effective mass of  $0.067 m_0$  (as in GaAs quantum wires) and the impurities to be two quantum dots each one of size 1 nm. As a consequence, the maximum electron energy allowing to assume a contact electron-dot potential—as in equation (1)—is around 2 meV. In this case, for  $\rho(E)J \simeq 1$  we obtain  $J \simeq 1 \text{ eV}\text{\AA}$  which appears to be a reasonable value.

Finally, we would like to point out that the above main result showed in figure 3 opens the possibility of a new maximally entangled states detection scheme. Indeed, electron transmission can be used as a probe to detect maximally entangled singlet and triplet states of two localized spins within the family (5). In particular, it should be clear from the above discussion that in the case of  $|\Psi^-\rangle$ , this would be a quantum non-demolition (QND) detection scheme.

In summary, in this paper, we have considered a 1D wire into which single electrons are injected. Such electrons undergo multiple scattering with two spin-1/2 magnetic impurities embedded at a fixed distance before leaving the wire. We have derived the exact stationary states of the system thus obtaining all the necessary transmission probability amplitudes to describe electron transport. We have shown that for suitable electron wave vectors (independent on the electron-impurity coupling constant) perfect transmittivity without spin-flip takes place provided the impurity spins are prepared in the singlet maximally entangled state. In this regime, singlet and triplet entangled states of the localized spins are found to maximize and minimize, respectively, electron transmission. This suggests the appealing idea to use entanglement as a tool to control electron transmission through a wire.

## Acknowledgments

Helpful discussions with V K Dugaev are gratefully acknowledged. YO and VRV thank the support from Fundação para a Ciência e a Tecnologia (Portugal), namely through programs POCTI/POCI and projects POCI/MAT/55796/2004 QuantLog and POCTI-SFA-2-91, partially funded by FEDER (EU).

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