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Energy level shifts of a uniformly accelerated atom in the presence of boundary conditions

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Abstract.

We discuss the radiative level shifts of an atom moving with uniform acceleration near an infinite reflecting plate. We first consider the case of a two-level system interacting with a massless scalar field in the vacuum state. The acceleration of the two-level atom is supposed in a direction parallel to the conducting plate. We evaluate the contribution of vacuum fluctuations and of the radiation reaction field to the energy shift of the atomic levels, and discuss their behaviour as a function of the atomic acceleration and of the atom-plate distance. Then, we investigate the more general case of an hydrogen atom accelerating near a perfectly reflecting plate and interacting with the electromagnetic field in the vacuum state.

1. Introduction

A remarkable consequence of Quantum Field Theory (QFT) is that the vacuum state, which is the state of lowest energy of the field, possesses an infinite energy and exhibits fluctuations of the fields, even at zero temperature. These fluctuations are at the origin of many observable effects, such as atomic energy-level shifts, Casimir effect and Casimir-Polder forces. Casimir effect and Casimir-Polder forces are long-range interactions between neutral polarizable bodies (see [1, 2] for a review). These effects have been extensively investigated under very general conditions, for example in the presence of boundary conditions and/or at temperature different from zero [3]. Fluctuations of the atom-wall Casimir-Polder force have been also investigated [4, 5]. Also, both the Casimir effect and the atom-wall Casimir-Polder forces have been recently measured with remarkable accuracy [6–8].

Recently, the properties of quantum vacuum in accelerated frames have been investigated also. In particular, the problem of the interaction of an accelerated neutral atom (or of a charge) with a quantum field has received much attention in recent years. This problem has been investigated in connection with the Unruh effect, according to which accelerated atoms perceive vacuum fluctuations as a thermal bath at temperature $T_U = \hbar a / (2\pi c k_B)$, a being the atomic acceleration [9, 10]. Actually, the problem of the appearance of the vacuum in an accelerated frame is a widely controversial problem [11, 12]. A closely related phenomenon is the dynamical Casimir effect, which concerns with the emission of electromagnetic radiation from a single accelerated mirror in the vacuum. These phenomena clearly show that the quantum

vacuum possesses dynamic properties that are at variance with those of vacuum in classical physics: quantum vacuum can be perturbed and these perturbations leads to observable effects.

Radiative properties of accelerated atoms in vacuum have been extensively investigated from a theoretical point of view until very recently [13–20]. Unfortunately, both the Unruh effect and the dynamical Casimir effect are extremely weak and their detection is very difficult. For example, in order to obtain Unruh radiation corresponding to a temperature of 1 K, an acceleration of the order of 10^{22} cm/s² would be necessary. Experimental schemes have been recently proposed for detecting phenomena related to the Unruh effect, aimed at enhancing the Unruh radiation under very specific circumstances [21].

This paper is devoted to discuss the Casimir-Polder interaction between a uniformly accelerated atom and an infinite conducting plate. We are interested in investigating if thermal effects due to the acceleration of the atom can modify the Casimir-Polder interaction between the atom and the infinite plate. This is indeed expected because, as it is well-known, Casimir-Polder interactions are directly related to vacuum field fluctuations [22]. The interest on this subject is also related to the fact that the static Casimir-Polder interaction between an atom at rest and a wall has been recently measured with good precision [7, 8, 23, 24]. This suggests the possibility of an indirect detection of the Unruh effect by a measurement of the Casimir-Polder interaction between the accelerated atom and the reflecting plate. Also dynamical Casimir-Polder forces have recently attracted much interest in literature [25–29].

In this paper we first consider a neutral two-level atom interacting with a massless scalar field and uniformly accelerated in a direction parallel to an infinite plate. We calculate the energy level shift of the accelerated atom in the presence of the mirror [30]. As it is well-known, the presence of the reflecting plate changes vacuum field fluctuations. The correction to the Lamb-shift of the atom contains terms depending from the atom-mirror distance, yielding the atom-wall Casimir-Polder potential. We identify the contributions of vacuum fluctuations and of radiation reaction to the Casimir-Polder interaction, and discuss their dependence on the acceleration of the atom in the limits of small and large accelerations. The relation with the Unruh effect is then considered, as well as observability of the results obtained.

Then we extend our investigations to the case of an hydrogen atom interacting with the electromagnetic field in the vacuum state and in the presence of a conducting wall. We calculate the energy level shifts of the atom and we compare the results with those obtained in the case of a scalar field.

2. The energy level shift of a uniformly accelerated two-level system interacting with a massless scalar field in the presence of an infinite plate

Consider a two-level system interacting with a massless scalar field in the vacuum state and in the presence of a perfectly reflecting plate located at $z = 0$. The Hamiltonian describing this system in the instantaneous inertial frame of the atom is [14, 15, 30] ($\hbar = c = 1$)

$$H(\tau) = H_A(\tau) + H_F(\tau) + H_{AF}(\tau) \quad (1)$$

where τ is the proper time and

$$H_A(\tau) = \omega_0 S_z(\tau) \quad (2)$$

$$H_F(\tau) = \sum_{\mathbf{k}} \omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \frac{dt}{d\tau} \quad (3)$$

$$H_{AF}(\tau) = \mu S_2(\tau) \phi(x(\tau)) \quad (4)$$

$a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ are the bosonic operators of the scalar field, and μ is the atom-field coupling constant. We have also introduced the pseudospin operators $S_z = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$,

and $S_2 = \frac{i}{2}(S_- - S_+)$, where $S_- = |g\rangle\langle e|$ and $S_+ = |e\rangle\langle g|$ are the atomic lowering and raising operators. Finally, $\phi(\mathbf{x}, t)$ is the scalar field operator,

$$\phi(\mathbf{x}, t) = \sum_{\mathbf{k}} \sqrt{\frac{1}{2V\omega_k}} f(\mathbf{k}, \mathbf{x}) [a_{\mathbf{k}}(t) + a_{\mathbf{k}}^\dagger(t)] \quad (5)$$

where $f(\mathbf{k}, \mathbf{x})$ are the appropriate mode functions taking into account Dirichlet boundary conditions for the field operator. The mode functions satisfy the normalization condition

$$\frac{1}{V} \int d^3\mathbf{x} f(\mathbf{k}, \mathbf{x}) f(\mathbf{k}', \mathbf{x}) = \delta_{\mathbf{k}, \mathbf{k}'} \quad (6)$$

The Hamiltonian $H_F(\tau)$ in (3) governs the evolution of the field in terms of the proper time τ in the instantaneous inertial frame of the atom. It reduces to the usual free-field Hamiltonian in the simple case of an atom at rest, where $dt/d\tau = 1$.

We are interested in evaluating the vacuum field fluctuations and radiation reaction field contributions to the energy level shift of the accelerated atom. As discussed in [14, 15, 30], these quantities can be obtained from an effective Hamiltonian H^{eff} consisting of the sum of two terms

$$H_{vf}^{eff} = \frac{i\mu^2}{2} \int_{\tau_0}^{\tau} d\tau' C^F(x(\tau), x(\tau')) [S_2^f(\tau'), S_2^f(\tau)], \quad (7)$$

$$H_{rr}^{eff} = -\frac{i\mu^2}{2} \int_{\tau_0}^{\tau} d\tau' \chi^F(x(\tau), x(\tau')) \{S_2^f(\tau'), S_2^f(\tau)\}, \quad (8)$$

where

$$C^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | \{\phi^f(x(\tau)), \phi^f(x(\tau'))\} | 0 \rangle \quad (9)$$

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | [\phi^f(x(\tau)), \phi^f(x(\tau'))] | 0 \rangle \quad (10)$$

are the correlation function and the linear susceptibility of the field, respectively, and $[\cdot, \cdot]$ and $\{\cdot, \cdot\}$ denote commutator and anticommutator.

The expectation values of H_{vf}^{eff} and H_{rr}^{eff} on a generic atomic state $|a\rangle$ give the vacuum fluctuations and the radiation reaction contributions to the radiative shift of the atomic level a

$$(\delta E_a)_{vf} = -i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x(\tau), x(\tau')) (\chi^A)_a(\tau, \tau'), \quad (11)$$

$$(\delta E_a)_{rr} = -i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x(\tau), x(\tau')) (C^A)_a(\tau, \tau') \quad (12)$$

where $(C^A)_a(\tau, \tau')$ and $(\chi^A)_a(\tau, \tau')$ are, respectively, the symmetric correlation function and the linear susceptibility of the atom in the state $|a\rangle$

$$\begin{aligned} (C^A)_a(\tau, \tau') &= \frac{1}{2} \langle a | \{S_2^f(\tau), S_2^f(\tau')\} | a \rangle \\ &= \frac{1}{2} \sum_b |\langle a | S_2(0) | b \rangle|^2 (e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')}) \end{aligned} \quad (13)$$

$$\begin{aligned} (\chi^A)_a(\tau, \tau') &= \frac{1}{2} \langle a | [S_2^f(\tau), S_2^f(\tau')] | a \rangle \\ &= \frac{1}{2} \sum_b |\langle a | S_2(0) | b \rangle|^2 (e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')}) \end{aligned} \quad (14)$$

Thus the calculation of the energy level shifts of accelerated atom reduces to the calculation of the statistical functions for the atom and the field.

Let us suppose that the atom is at a distance z_0 from the mirror and that it accelerates along the x direction. In the laboratory frame, its trajectory is described, as a function of the proper time τ , by the equations

$$\begin{aligned} t(\tau) &= \frac{1}{a} \sinh(a\tau), \quad x(\tau) = \frac{1}{a} \cosh(a\tau), \\ y(\tau) &= 0 \quad z(\tau) = z_0 \end{aligned} \quad (15)$$

where a is the proper acceleration. The statistical functions $C^F(x(\tau), x(\tau'))$ and $\chi^F(x(\tau), x(\tau'))$, in the presence of the reflecting plane boundary, can be calculated from the Wightman function, $G(x(\tau), x(\tau'))$ satisfying the Dirichlet boundary conditions on the mirror, $\phi(x) |_{z=0} = 0$,

$$G(x(\tau), x(\tau')) = \langle 0 | \phi(x(\tau)) \phi(x(\tau')) | 0 \rangle \quad (16)$$

This function describes the field correlations at two different points of space-time, $x(\tau)$ and $x(\tau')$. In the presence of a boundary, it consists of the sum of two terms: the empty-space contribution ($G^{(0)}(x(\tau), x(\tau'))$) and a part which depends on the presence of the boundary ($G^{(b)}(x(\tau), x(\tau'))$) [31]

$$\begin{aligned} G(x(\tau), x(\tau')) &= G^{(0)}(x(\tau), x(\tau')) + G^{(b)}(x(\tau), x(\tau')) \\ &= \frac{1}{2\pi^2} \left[\frac{1}{(\Delta(\mathbf{x}))^2 - (\Delta t - i\eta)^2} - \frac{1}{(\bar{\Delta}(\mathbf{x}))^2 - (\Delta t - i\eta)^2} \right], \end{aligned} \quad (17)$$

($\eta \rightarrow 0^+$) where we have introduced the variables $\Delta(\mathbf{x}) = |\mathbf{x}(\tau) - \mathbf{x}(\tau')|$ (the difference between atomic coordinates $\mathbf{x}(\tau)$ taken at two different proper times) and $\bar{\Delta}(\mathbf{x}) = |\mathbf{x}(\tau) - \sigma\mathbf{x}(\tau')|$ where $\sigma\mathbf{x}(\tau')$ is the point corresponding to the reflection of point $\mathbf{x}(\tau')$ on the mirror, with

$$\sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (18)$$

Finally $\Delta t = t(\tau) - t(\tau')$. Substituting (15) into (17), we obtain

$$\begin{aligned} G(x(\tau), x(\tau')) &= -\frac{1}{16\pi^2} \frac{a^2}{\sinh^2 \frac{a(\tau - \tau' - i\eta)}{2}} \\ &+ \frac{1}{16\pi^2} \frac{a^2}{\sinh^2 \frac{a(\tau - \tau' - i\eta)}{2} - z_0^2 a^2}, \end{aligned} \quad (19)$$

The symmetrical correlation function and the linear susceptibility of the field can be obtained from (19). After some algebra (detailed calculations are given in [30]) we get

$$\begin{aligned} C^F(x(\tau), x(\tau')) &= -\frac{1}{8\pi^2} \left\{ \int_0^\infty d\omega \omega \coth\left(\frac{\pi\omega}{a}\right) \left(e^{i\omega(\tau - \tau')} + e^{-i\omega(\tau - \tau')} \right) \right. \\ &- \frac{1}{2z_0 N^{1/2}} \int_0^\infty d\omega \left(\coth\left(\frac{\pi\omega}{a}\right) \right. \\ &\times \left. \sin\left(\frac{2\omega}{a} \sinh^{-1}(az_0^2)\right) \left(e^{i\omega(\tau - \tau')} + e^{-i\omega(\tau - \tau')} \right) \right) \Bigg\} \end{aligned} \quad (20)$$

and

$$\begin{aligned} \chi^F(x(\tau), x(\tau')) = & -\frac{1}{8\pi^2} \left\{ \int_0^\infty d\omega \omega \left(e^{i\omega(\tau-\tau')} - e^{-i\omega(\tau-\tau')} \right) \right. \\ & - \frac{1}{2z_0 N^{1/2}} \int_0^\infty d\omega \sin \left(\frac{2\omega}{a} \sinh^{-1}(az_0) \right) \\ & \left. \left(e^{i\omega(\tau-\tau')} - e^{-i\omega(\tau-\tau')} \right) \right\} \end{aligned} \quad (21)$$

where $N = 1 + z_0^2 a^2$. Substituting Eqs. (13), (14), (20) and (21) into (11) and (12) and taking the limits $\tau_0 \rightarrow -\infty$, $\tau \rightarrow \infty$, we obtain

$$\begin{aligned} (\delta E_a)_{vf} = & \frac{\mu^2}{8\pi^2} \sum_b |\langle a | S_2(0) | b \rangle|^2 \\ & \times \int_0^\infty d\omega \omega \left(1 - \frac{1}{2z_0 \omega N^{1/2}} \sin \left(\frac{2\omega \sinh^{-1}(z_0 a)}{a} \right) \right) \\ & \times P \left[\frac{1}{\omega + \omega_{ab}} - \frac{1}{\omega - \omega_{ab}} \right] \left(1 + \frac{2}{e^{2\pi\omega/a} - 1} \right) \end{aligned} \quad (22)$$

and

$$\begin{aligned} (\delta E_a)_{rr} = & -\frac{\mu^2}{8\pi^2} \sum_b |\langle a | S_2(0) | b \rangle|^2 \\ & \times \int_0^\infty d\omega \omega \left(1 - \frac{1}{2z_0 \omega N^{1/2}} \sin \left(\frac{2\omega \sinh^{-1}(z_0 a)}{a} \right) \right) \\ & \times P \left[\frac{1}{\omega - \omega_0} + \frac{1}{\omega - \omega_{ab}} \right], \end{aligned} \quad (23)$$

where P denotes the principal part. These expressions give the contributions of vacuum fluctuations and of the radiation reaction field to the energy-level shift of the accelerated atom near the mirror. In contrast to the case of the unbounded space (where the radiation reaction term is not affected by the acceleration of the atom), in the present case, both contributions explicitly depend on the acceleration of the atom, due to the presence of the plate,. In the limit $z_0 \rightarrow \infty$, the function $f(z_0) = 1 - \frac{1}{2z_0 \omega N^{1/2}} \sin \left(\frac{2\omega \sinh^{-1}(z_0 a)}{a} \right)$ tends to 1 and the results obtained for an accelerated atom in unbounded space are recovered [15]. As discussed in [30] Eq.(22) shows that the effect of the uniform acceleration is a thermal-like correction with the Unruh temperature $T_U = \hbar a / 2\pi c k_B$, due to the term containing the $\coth(\pi\omega/a)$ function in the symmetric correlation function.

We now focus our attention on the Casimir-Polder interaction between the two-level atom and the wall. In analogy with the case of an atom at rest near a plate, this quantity can be obtained considering only the z_0 -dependent terms in Eqs. (22) and (23). For a ground-state atom we get

$$E_{CP} = E_{CP}^{(vf)} + E_{CP}^{(rr)} \quad (24)$$

where

$$\begin{aligned} E_{CP}^{(vf)} = & -\frac{\mu^2}{8\pi^2} \frac{1}{8z_0 N^{1/2}} \\ & \int_0^\infty d\omega \sin \left(\frac{2\omega \sinh^{-1}(z_0 a)}{a} \right) \\ & \times P \left[\frac{1}{\omega - \omega_0} - \frac{1}{\omega + \omega_0} \right] \left(1 + \frac{2}{e^{2\pi\omega/a} - 1} \right) \end{aligned} \quad (25)$$

and

$$\begin{aligned}
 E_{CP}^{(rr)} &= \frac{\mu^2}{8\pi^2} \frac{1}{8z_0 N^{1/2}} \\
 &\times \int_0^\infty d\omega \sin\left(\frac{2\omega \sinh^{-1}(z_0 a)}{a}\right) \\
 &\times P\left[\frac{1}{\omega - \omega_0} + \frac{1}{\omega + \omega_0}\right]
 \end{aligned} \tag{26}$$

Using the relation

$$f\left(\frac{2\omega_0}{a} \sinh^{-1}(z_0 a)\right) = \int_0^\infty d\omega \frac{\sin\left(\frac{2\omega \sinh^{-1}(z_0 a)}{a}\right)}{\omega + \omega_0} \tag{27}$$

the expressions above can be written in the following form

$$\begin{aligned}
 E_{CP}^{(vf)} &= -\frac{\mu^2}{8\pi^2} \frac{1}{8z_0 N^{1/2}} \\
 &\times \left(-2f\left(\frac{2\omega_0}{a} \sinh^{-1}(z_0 a)\right) + \pi \cos\left(\frac{2\omega_0}{a} \sinh^{-1}(z_0 a)\right) \right. \\
 &\left. - \frac{a}{\omega_0} \left(\cos\left(\frac{2\omega_0 \sinh^{-1}(z_0 a)}{a}\right) - 1 \right) \right)
 \end{aligned} \tag{28}$$

and

$$E_{CP}^{(rr)} = \frac{\mu^2}{8\pi^2} \frac{1}{8z_0 N^{1/2}} \cos\left(\frac{2\omega_0}{a} \sinh^{-1}(z_0 a)\right) \tag{29}$$

As expected, the Casimir-Polder interaction depends explicitly on the acceleration of the atom. It is now interesting to investigate the behaviour of the Casimir-Polder interaction as a function of a and z_0 , in the limits $(2\omega_0/a) \sinh^{-1}(z_0 a) \ll 1$ and $(2\omega_0/a) \sinh^{-1}(z_0 a) \gg 1$. These two limits define two different *regions* of the space, $z_0 \ll a^{-1} \sinh(a/2\omega_0)$ and $z_0 \gg a^{-1} \sinh(a/2\omega_0)$, respectively, in analogy with the near- and the far-zone limits of the inertial atom-wall Casimir-Polder interaction. In other words, in the case of accelerated atoms, we can define a new *near-zone* and a new *far-zone* limit for the Casimir-Polder interaction, which depend on the acceleration of the atom too. We now investigate the behavior of the Casimir-Polder interaction in these two regions and in the limits $a < \omega_0$ and $a > \omega_0$. This is equivalent to considering the two cases, $k_B T < \omega_0$ and $k_B T > \omega_0$.

In the limit of acceleration small compared with ω_0 , the two regions defined above coincide with the usual near-zone and far-zone of the stationary Casimir-Polder interaction for an inertial atom. From Eqs. (28) and (29) we obtain

$$\begin{aligned}
 E_{CP}^{(vf)} &= -\frac{\mu^2}{8\pi^2} \frac{1}{8z_0} \\
 &\times (-2f(2\omega_0 z_0) + \pi \cos(2\omega_0 z_0))
 \end{aligned} \tag{30}$$

and

$$E_{CP}^{(rr)} = \frac{\mu^2}{8\pi^2} \frac{1}{8z_0} \pi \cos(2\omega_0 z_0) \tag{31}$$

where we have approximated $2\omega_0/a \sinh^{-1}(z_0 a) \sim 2\omega_0 z_0$. Thus, in the limit of small acceleration we recover, as expected, the well-known stationary atom-wall Casimir-Polder potential,

$$E_{CP} = \frac{\mu^2}{8\pi^2} \frac{\pi}{8z_0} \quad \text{near-zone} \quad (32)$$

$$E_{CP} = \frac{\mu^2}{8\pi^2} \frac{1}{8\omega_0 z_0^2} \quad \text{far-zone.} \quad (33)$$

This shows that, in the case of a scalar field, the Casimir-Polder force between an atom at rest and a plate is repulsive.

We now consider the case $a \geq \omega_0$. For typical values of the atomic transition frequency ($\omega_0 \sim 10^{15} \text{ s}^{-1}$), this corresponds to an acceleration larger than $a \sim 10^{25} \text{ cm/s}^2$. In this limit, $2\omega_0/a \sinh^{-1}(z_0 a) \ll 1$ and from Eqs. (27) and (28) we obtain

$$E_{CP} = E_{CP}^{(vf)} + E_{CP}^{(rr)} \sim \frac{\mu^2}{8\pi^2} \frac{1}{8z_0 \sqrt{1 + (z_0 a)^2}} \times \left(\pi + \frac{a}{\omega_0} \left(\cos \left(\frac{2\omega_0}{a} \sinh^{-1}(z_0 a) \right) - 1 \right) \right) \quad (34)$$

which explicitly depends on the acceleration a . For distances such that $z_0 a \ll 1$, we have

$$E_{CP} = E_{CP}^{(vf)} + E_{CP}^{(rr)} \sim \frac{\mu^2}{8\pi^2} \frac{1}{8z_0} \times \left(\pi + \frac{a}{\omega_0} \left(\cos \left(\frac{2\omega_0}{a} \sinh^{-1}(z_0 a) \right) - 1 \right) \right) \quad (35)$$

In the limit $z_0 a \gg 1$, we obtain

$$E_{CP} \sim \frac{\mu^2}{8\pi^2} \frac{1}{8z_0^2 a} \times \left(\pi + \frac{a}{\omega_0} \left(\cos \left(\frac{2\omega_0}{a} \sinh^{-1}(z_0 a) \right) - 1 \right) \right) \quad (36)$$

which gives the Casimir-Polder interaction between the accelerated atom and the wall for high accelerations. The most striking effect of the acceleration of the atom is the presence of an oscillatory term in the interaction energy, which modulates the interaction as a function of z_0 and a . This oscillatory behaviour is reminiscent of the stationary Casimir-Polder interaction between an excited atom and a mirror, where a spatially oscillating term is also present [32]. This can be explained observing that the limit $a \geq \omega_0$ corresponds to a temperature $T \geq \omega_0/k_B$. In such limit the excitation probability of the atom is nonvanishing and this is reflected in the oscillatory behaviour of the Casimir-Polder interaction. We emphasize that in the *far-zone* limit the Casimir-Polder interaction is essentially due to the vacuum-fluctuation contribution, where a "thermal" term is present, due to the acceleration of the atom. Thus our results show that thermal effects of acceleration may induce observable effects in the far-zone Casimir-Polder interaction between an accelerated atom and a wall, at least in the case of a scalar field. In fig.1, the energy shift E_{CP} is plotted as a function of the atom-plate distance z_0 , for different values of atomic acceleration. A comparison between the different curves shows that the oscillatory behaviour of potential E_{CP} drastically decreases for acceleration smaller than 10^{24} cm/s^2 . Also, depending on the distance z_0 , the Casimir-Polder interaction can be attractive or repulsive, in contrast with the stationary case of a ground-state atom, where the force is always repulsive. Similar features have been recently discussed in the case of dynamical Casimir-Polder interaction

between an atom and a plate [29]. In Fig. 2, we have plotted the quantity $E_{CP}^{a \neq 0} / E_{CP}^{a=0}$, as a function of z_0 . We observe that for given values of z_0 the correction to the potential due to the atomic acceleration is of the same order of the static Casimir-Polder interaction between an atom at rest and a plate.

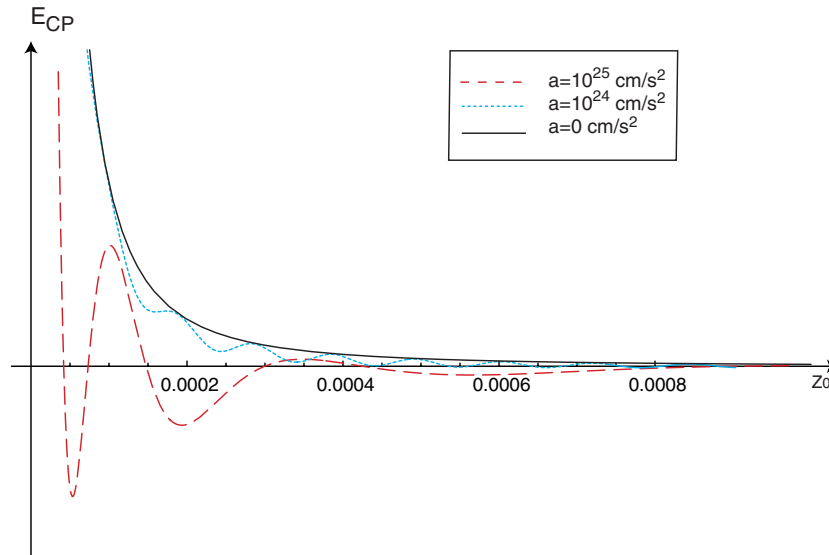


Figure 1. Casimir-Polder interaction as a function of the atom-plate distance z_0 and for different values of the atomic acceleration. Red, cyan and black (long-dashed-dot, dashed and continuous lines, respectively) represent the interaction for $a = 10^{25} \text{ cm/s}^2$, $a = 10^{24} \text{ cm/s}^2$, $a = 0 \text{ cm/s}^2$, respectively.

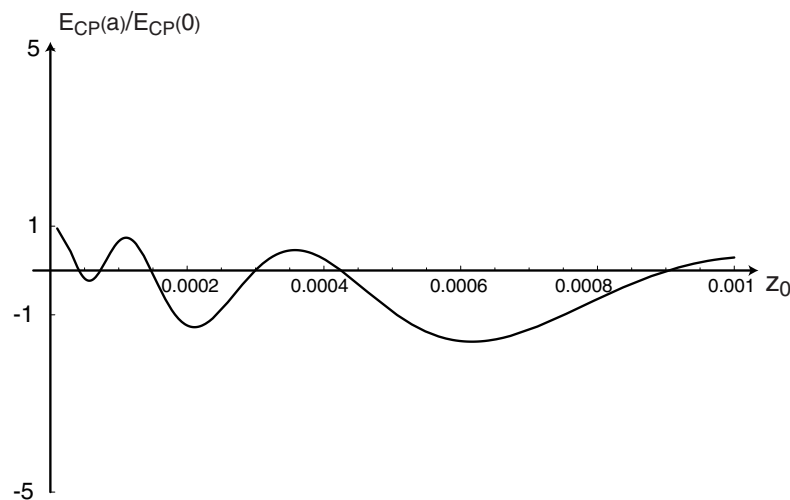


Figure 2. Ratio between the atom-plate Casimir-Polder interaction with accelerated atom ($a = 10^{25} \text{ cm/s}^2$) and the atom-plate Casimir-Polder interaction with the atom at rest.

We conclude comparing our results with the Casimir-Polder interaction between an atom

at rest and a plate immersed in a thermal bath. As it is well-known [33], the atom-wall Casimir-Polder interaction at temperature T is proportional to the temperature of the bath. A comparison of the results in [33] with Eqs. (34)-(36), immediately shows that the Casimir-Polder interaction between the accelerated atom and the plate is qualitatively different from the static counterpart at the Unruh temperature T_U , because of the nontrivial dependence from the acceleration a in (35) and (36). This consideration indicates that, in general, uniformly accelerated atoms behave differently from static ones in a thermal bath at the Unruh temperature. In a different context, this aspect has been discussed in [19, 20]

3. Energy shifts of an accelerated hydrogen atom interacting with the electromagnetic field in the presence of a conducting plate

In this section we extend our investigation to the case of the electromagnetic field. Recently, the radiative level shifts of an accelerated atom interacting with the electromagnetic field in the vacuum state have been considered [16] and significant differences compared to the case of scalar field have been found. This motivates us to consider the case of a uniformly accelerated hydrogen atom interacting with the electromagnetic field in the presence of a perfectly reflecting plate.

Let us consider a uniformly accelerated hydrogen atom interacting with the electromagnetic field in the vacuum state near a perfectly conducting plate. The Hamiltonian that describes the time evolution of this system with respect to the proper time τ , in the multipolar coupling scheme, is [16]

$$H(\tau) = H_A(\tau) + H_F(\tau) + H_{AF}(\tau) \quad (37)$$

where

$$H_A(\tau) = \sum_m \omega_m \sigma_{mm}(\tau) \quad (38)$$

$$H_F(\tau) = \sum_{\mathbf{k}j} \omega_k a_{\mathbf{k}j}^\dagger a_{\mathbf{k}j} \frac{dt}{d\tau} \quad (39)$$

$$H_{AF} = -e \mathbf{r}(\tau) \cdot \mathbf{E}(x(\tau)) = -e \sum_{\ell m} \mathbf{r}_{\ell m} \cdot \mathbf{E}(x(\tau)) \sigma_{\ell m}(\tau) \quad (40)$$

where $\sigma_{\ell m} = |\ell\rangle\langle m|$ and $e\mathbf{r} = \sum_{\ell m} \mu_{\ell m} \sigma_{\ell m}$ is the atomic electric dipole moment.

We wish to evaluate the energy level shift of the atom accelerating along the x direction parallel to the plate. Our approach generalizes the method used by Takagi [34] to the case in which boundary conditions are present. The trajectory of the atom is described by equations (15) of the previous section.

In order to calculate the statistical functions of the field, we consider the Wightman's function for the photon field in the presence of a boundary condition, which can be expressed as a sum of two terms

$$\begin{aligned} \langle M0 | A_\alpha(x) A_\beta(x') | 0_M \rangle &= \langle M0 | A_\alpha(x) A_\beta(x') | 0_M \rangle_0 + \langle M0 | A_\alpha(x) A_\beta(x') | 0_M \rangle_b \\ &= \eta_{\alpha\beta} G^{(0)}(x, x') + (\eta_{\alpha\beta} - 2n_\alpha n_\beta) G^{(b)}(x, x') \end{aligned} \quad (41)$$

where $A_\alpha(x)$ is the electromagnetic 4-vector potential, $G^{(0)}(x, x')$ and $G^{(b)}(x, x')$ are the Wightman's functions of the scalar field discussed in the previous section. Also, $x = (t, \mathbf{x})$, $|0_M\rangle$ is the Minkowski vacuum, $\eta_{\alpha\beta} = \text{diag}(-1, 1, 1, 1)$ and $n_\alpha = (0, 0, 0, 1)$. We now focus our attention only on the boundary-dependent term, which is relevant for the calculation of the Lamb shift of the accelerated atom in the presence of a reflecting plate. The Wightman function

for the fields (in the laboratory frame) follows from differentiating Eq.(41),

$$\begin{aligned}
 G_{\alpha\beta\gamma\delta}^{(b)}(x, x') &= \langle_M 0 | F_{\alpha\beta}(x) F_{\gamma\delta}(x) | 0_M \rangle_b \\
 &= (\eta_{\beta\delta} \partial_\alpha \partial'_\gamma - \eta_{\beta\gamma} \partial_\alpha \partial'_\delta - \eta_{\alpha\delta} \partial_\beta \partial'_\gamma + \eta_{\alpha\gamma} \partial_\beta \partial'_\delta) \\
 &\quad - 2(n_\beta n_\delta \partial_\alpha \partial'_\gamma - n_\beta n_\gamma \partial_\alpha \partial'_\delta - n_\alpha n_\delta \partial_\beta \partial'_\gamma + n_\alpha n_\gamma \partial_\beta \partial'_\delta) \\
 &\quad \times G^{(b)}(x, x')
 \end{aligned} \tag{42}$$

where $F_{\alpha\beta}(x) = \partial_\alpha A_\beta - \partial_\beta A_\alpha$.

We are now interested in calculating the two-point correlation function of the electric field in the reference frame co-moving with the atom. The electric field $\hat{\mathbf{E}}(\tau)$ observed at proper time τ by the atom accelerating along the x direction (at a fixed distance z_0 from the plate) is related by a Lorentz transformation to the electromagnetic field in the laboratory frame

$$\hat{E}_i = \hat{F}_{i0} = (\Lambda_\tau)_i^\alpha (\Lambda_\tau)_0^\beta F_{\alpha\beta} \tag{43}$$

where \hat{E} indicates the electric field in the instantaneously inertial frame, and

$$\Lambda = \begin{pmatrix} \cosh(a\tau) & -\sinh(a\tau) & 0 & 0 \\ -\sinh(a\tau) & \cosh(a\tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{44}$$

After some algebra we obtain

$$\begin{aligned}
 g^{(b)}(\tau - \tau') &= \langle_M 0 | \hat{\mathbf{E}}_\ell(\tau) \hat{\mathbf{E}}_m(\tau') | 0_M \rangle_b \\
 &= \frac{a^4}{16\pi^2} \left[\delta_{\ell m} \left(\sinh^2(a(\tau - \tau')/2) + a^2 z_0^2 \right) - 2n_\ell n_m \sinh^2(a(\tau - \tau')/2) \right] \\
 &\quad \times \frac{1}{\left[\sinh^2(a(\tau - \tau')/2) + a^2 z_0^2 \right]^3} - \frac{a^4}{16\pi^2} \left[2a^2 z_0^2 (\delta_{\ell m} - k_\ell k_m) + 2az_0 (n_\ell k_m + k_\ell n_m) \right] \\
 &\quad \times \frac{\sinh^2(a(\tau - \tau')/2)}{\left[\sinh^2(a(\tau - \tau')/2) + a^2 z_0^2 \right]^3}
 \end{aligned} \tag{45}$$

where $k_i = (1, 0, 0)$. This expression reduces to the two-point field correlation function obtained for an atom at rest near a reflecting plate in the limit of $a \rightarrow 0$.

From relation (45) we can obtain the symmetrical and anti-symmetrical correlation function. In analogy with the case of the scalar field, we may express the field correlation function and the field susceptibility as integrals over frequencies. After some algebra, we obtain

$$\begin{aligned}
 C_{\ell m}^F(x(\tau), x(\tau')) &= \frac{1}{2} \langle_M 0 | \{E_\ell(x(\tau)), E_m(x(\tau'))\} | 0_M \rangle \\
 &= -\frac{1}{8\pi^2} \frac{1}{(2z_0)^3 N} \int_0^\infty \left[\sigma_{\ell n} f_{nm}(\omega; z_0, a) + 2a^2 z_0^2 \sigma_{\ell n} g_{nm}(\omega; z_0, a) + az_0 h_{\ell m}(\omega; z_0, a) \right] \\
 &\quad \times \coth\left(\frac{\pi\omega}{a}\right) \left(e^{i\omega(\tau-\tau')} + e^{-i\omega(\tau-\tau')} \right)
 \end{aligned} \tag{46}$$

and

$$\begin{aligned}
 \chi_{\ell m}^F(x(\tau), x(\tau')) &= i \langle_M 0 | \{E_\ell(x(\tau)), E_m(x(\tau'))\} | 0_M \rangle \\
 &= -i \frac{1}{8\pi^2} \frac{1}{(2z_0)^3 N} \int_0^\infty d\omega \left\{ -2\sigma_{\ell n} f_{nm}(\omega; z_0, a) - 2a^2 z_0^2 \sigma_{\ell n} g_{nm}(\omega; z_0, a) - 2az_0 h_{\ell m}(\omega; z_0, a) \right\} \\
 &\quad \times \left(e^{i\omega(\tau-\tau')} - e^{-i\omega(\tau-\tau')} \right)
 \end{aligned} \tag{47}$$

where

$$f_{nm}(\omega; z_0, a) = \frac{1}{\sqrt{N}} \left\{ (\delta_{nm} - n_n n_m) (2\omega z_0)^2 \sin(2\omega/a \sinh^{-1}(z_0 a)) \right. \\ \left. + (\delta_{nm} - 3n_n n_m) \left[\frac{2z_0 \omega}{\sqrt{N}} \cos(2\omega/a \sinh^{-1}(z_0 a)) - \frac{1}{N} \sin(2\omega/a \sinh^{-1}(z_0 a)) \right] \right\}; \quad (48)$$

$$g_{nm}(\omega; z_0, a) = \frac{1}{N} \left\{ (2\delta_{nm} - 3n_n n_m) (2\omega z_0) \cos(2\omega/a \sinh^{-1}(z_0 a)) - \frac{1}{\sqrt{N^3}} \right. \\ \left. \times [(\delta_{nm} - 4n_n n_m) - 2a^2 z_0^2 (\delta_{nm} - 2n_n n_m)] \sin(2\omega/a \sinh^{-1}(z_0 a)) \right\}; \quad (49)$$

$$h_{\ell m}(\omega; z_0, a) = a z_0 (a z_0 (\delta_{\ell m} - k_\ell k_m) + n_\ell k_m + k_\ell n_m) \\ \times \left[(2\omega z_0)^2 \sin(2\omega/a \sinh^{-1}(z_0 a)) - \frac{2\omega z_0}{N} (1 - 2a^2 z_0^2) \cos(2\omega/a \sinh^{-1}(z_0 a)) \right. \\ \left. + \frac{1 + 4a^2 z_0^2}{\sqrt{N^3}} \sin(2\omega/a \sinh^{-1}(z_0 a)) \right] \quad (50)$$

We are now ready to calculate the energy level shifts of the accelerated atom in the presence of an infinite reflecting plate. The symmetric and antisymmetric correlation functions for the accelerated hydrogen atom are

$$C_{\ell m}^A(\tau, \tau') = \frac{1}{2} \langle a | \{ \mathbf{r}_\ell(\tau), \mathbf{r}_m(\tau') \} | a \rangle \\ = \frac{1}{2} \sum_b \left[\langle a | r_\ell(0) | b \rangle \langle b | r_m(0) | a \rangle e^{i\omega_{ab}(\tau - \tau')} + \langle a | r_m(0) | b \rangle \langle b | r_\ell(0) | a \rangle e^{-i\omega_{ab}(\tau - \tau')} \right] \quad (51)$$

$$\chi_{\ell m}^A(\tau, \tau') = i \langle a | [\mathbf{r}_\ell(\tau), \mathbf{r}_m(\tau')] | a \rangle \\ = i \sum_b \left[\langle a | r_\ell(0) | b \rangle \langle b | r_m(0) | a \rangle e^{i\omega_{ab}(\tau - \tau')} - \langle a | r_m(0) | b \rangle \langle b | r_\ell(0) | a \rangle e^{-i\omega_{ab}(\tau - \tau')} \right] \quad (52)$$

Using Eqs.(46),(47),(51) and (52) in (11) and (12), we obtain

$$(\delta E_a)_{(b)}^{VF} = -\frac{1}{4\pi^2} \sum_b \left(\mu_\ell^{ab} \mu_m^{ba} \right) \frac{1}{2N(2z_0)^3} \int_0^\infty d\omega \\ \times \left(\sigma_{\ell n} f_{nm}(\omega; z_0, a) + 2a^2 z_0^2 \sigma_{\ell n} g_{nm}(\omega; z_0, a) + z_0 a h_{\ell m}(\omega; z_0, a) \right) \\ \times \coth(\pi\omega/a) P \left(\frac{1}{\omega + \omega_{ab}} - \frac{1}{\omega - \omega_{ab}} \right) \quad (53)$$

and

$$(\delta E_a)_{(b)}^{RR} = \frac{1}{4\pi^2} \sum_b \left(\mu_\ell^{ab} \mu_m^{ba} \right) \frac{1}{2N(2z_0)^3} \int_0^\infty d\omega \\ \times \left(\sigma_{\ell n} f_{nm}(\omega; z_0, a) + 2a^2 z_0^2 \sigma_{\ell n} g_{nm}(\omega; z_0, a) + z_0 a h_{\ell m}(\omega; z_0, a) \right) P \left(\frac{1}{\omega + \omega_{ab}} + \frac{1}{\omega - \omega_{ab}} \right) \quad (54)$$

Equations (53) and (54) give the correction to the energy level shifts of the accelerated atom due to the presence of boundary.

The total energy level shift is obtained by summing the contributions of vacuum fluctuations and of the radiation reaction field

$$\delta E_a = (\delta E_a)^{VF} + (\delta E_a)^{RR} \quad (55)$$

Equations (53) and (54) have several interesting properties. First, while in the case of the unbounded space only the contribution of vacuum fluctuations depends on the acceleration, in the presence of a boundary both vacuum fluctuations and radiation reaction depend on the

atomic acceleration, in analogy with the case of the scalar field discussed in the previous section. In particular, in the limit of small acceleration, they reduce to the usual Lamb-shift for an atom at rest near a reflecting plate. Secondly, when z_0 goes to infinity the boundary-dependent terms vanish and we obtain the results for the Lamb-shift of an accelerated atom in the unbounded space [16]

$$(\delta E_a)^{VF} = \frac{1}{12\pi^2} \sum_b |\langle a | \mu | b \rangle|^2 \int_0^\infty d\omega \omega^3 (1 + \frac{a^2}{\omega^2}) \coth(\pi\omega/a) P(\frac{1}{\omega + \omega_{ab}} - \frac{1}{\omega - \omega_{ab}}) \quad (56)$$

and

$$(\delta E_a)^{RR} = -\frac{1}{12\pi^2} \sum_b |\langle a | \mu | b \rangle|^2 \int_0^\infty d\omega \omega^3 P(\frac{1}{\omega + \omega_{ab}} + \frac{1}{\omega - \omega_{ab}}) \quad (57)$$

Finally, the effect of the acceleration is not simply a *thermal* correction given by a term $\coth(\pi\omega/a)$. In fact, the functions $f_{\ell n}(\omega; z_0, a)$, $g_{\ell n}(\omega; z_0, a)$ and $h_{\ell m}(\omega; z_0, a)$ depend explicitly on the atomic acceleration. Thus, in analogy with the scalar field case, also in the case of electromagnetic field we find a non-thermal correction to the vacuum fluctuation contribution, due to the presence of the boundary. This point will be discussed in detail in a future publication.

4. Conclusions

In this paper we have discussed the energy level shifts of a uniformly accelerated two-level system interacting with a massless scalar field in the presence of an infinite plate with Dirichlet boundary conditions. In particular, we have considered the contributions of vacuum fluctuations and of radiation reaction to the Casimir-Polder interaction energy and we have discussed their dependence on the acceleration of the two-level atom, in the limits of *near-* and *far-zone*. We have shown that the atom-wall Casimir-Polder interaction in the limit of small accelerations coincides with the stationary atom-wall Casimir-Polder potential. In contrast, for high atomic accelerations the Casimir-Polder interaction depends explicitly on the acceleration of the atom and exhibits an oscillatory behaviour in space. Therefore it appears that effects due to the acceleration of the atom may become evident in the atom-wall Casimir-Polder interaction. Finally, we have discussed the case of an hydrogen atom moving with constant acceleration near a perfectly conducting plate and interacting with the electromagnetic field in the vacuum state. The results obtained have been compared with those of the scalar field case.

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References

- [1] Compagno G, Passante R, Persico F, *Atom-Field Interactions and Dressed Atoms*, Cambridge University Press, Cambridge (1995)
- [2] Salam A, Int. Rev. Phys. Chem. **27**, 405 (2008)
- [3] Passante R, Spagnolo S, Phys. Rev. A **76**, 042112 (2007)
- [4] Wu C-H, Chung-I Kuo, Ford L-H, Phys. Rev. A **65**, 062102 (2002)
- [5] Messina R, Passante R, Phys. Rev. A **76**, 032107 (2007)
- [6] Bressi G, Carugno G, Onofrio R, Ruoso G, Phys. Rev. Lett. **88**, 041804 (2002)
- [7] Sukenik C I, Boshier M G, Cho D, Sandoghdar V, Hinds E A, Phys. Rev. Lett. **70**, 560 (1993)
- [8] Druzhinina V and DeKieviet M, Phys. Rev. Lett. **91**, 193202 (2003)

- [9] Davies P C W, J. Phys. A: Math. Gen. **8**, 609 (1975)
- [10] Unruh W G, Phys. Rev. D **14**, 870 (1976)
- [11] Jaekel M T and Reynaud S, Quant. Semicl. Opt. **7**, 499 (1995)
- [12] Hu B L, Roura A, Phys. Rev. Lett. **93**, 129301 (2004)
- [13] Boyer T H, Phys. Rev. D **21**, 2137 (1980)
- [14] Audretsch J, Muller R, Phys. Rev. A **50**, 1755 (1994)
- [15] Audretsch J, Muller R, Phys. Rev. A **52**, 629 (1995)
- [16] Passante R, Phys. Rev. A **57**, 1590 (1998)
- [17] Yu H and Lu S, Phys. Rev. D **72**, 064022 (2005)
- [18] Yu H and Zhu Z, Phys. Rev. D **74**, 044032 (2006)
- [19] Zhang J and Yu H, Phys. Rev. D **75**, 104014 (2007)
- [20] Obadia N, Phys. Rev. D **76**, 045013 (2007)
- [21] Belyanin A, Kocharovskiy V V, Capasso F, Fry E, Zubairy M S and Scully M O, Phys. Rev. A **74**, 023807 (2006)
- [22] Power E A and Thirunamachandran T, Phys. Rev. A **48**, 4761 (1993)
- [23] Obrecht J M, Wild R J, Antezza M, Pitaevskii L P, Stringari S and Cornell E A, Phys. Rev. Lett. **98**, 063201 (2007)
- [24] Bordag M, Mohideen U, Mostepanenko V M, Phys. Rep. **353**, 1 (2001)
- [25] Shresta S, Hu B L, Phillips N G, Phys. Rev. A **68**, 062101 (2003)
- [26] Rizzuto R, Passante R and Persico F, Phys. Rev. A **70**, 012107 (2004)
- [27] Per-Olof Westlund and Wennerstrom H, Phys. Rev. A **71**, 062106 (2005)
- [28] Rizzuto L, Passante R and Persico F, Phys. Rev. Lett. **98**, 240404 (2007)
- [29] Vasile R and Passante R, Phys. Rev. A **78**, 032108 (2008)
- [30] Rizzuto L, Phys. Rev. A **76**, 062114 (2007)
- [31] Birrel N D, Davies P C W, *Quantum Field in curved space*, Cambridge Monographs on Mathematical Physics, Cambridge 1982
- [32] Messina R, Passante R, Rizzuto L, Spagnolo S and Vasile R, J. Phys. A: Math. Theor., **41**, 164031 (2008)
- [33] Boyer T H, Phys. Rev. A **11**, 1650 (1975)
- [34] Takagi S, Progr. Theor. Phys. Suppl. **88**, 1 (1986)