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Mechanical models of amplitude and frequency modulation

L Bellomonte, I Guastella and R M Sperandeo-Mineo

GRIAF (Research Group on Teaching/learning Physics), DI.F.TE.R. (Dipartimento di Fisica e Tecnologie Relative), University of Palermo, Viale delle Scienze, Edificio 18, 90128 Palermo, Italy

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Abstract

This paper presents some mechanical models for amplitude and frequency modulation. The equations governing both modulations are deduced alongside some necessary approximations. Computer simulations of the models are carried out by using available educational software. Amplitude modulation is achieved by using a system of two weakly coupled pendulums, whereas the frequency modulation is obtained by using a pendulum of variable length. Under suitable conditions (small oscillations, appropriate initial conditions, etc) both types of modulation result in significantly accurate and visualized simulations.

1. Introduction

Many topics in the field of physics require formal reasoning, familiarity with mathematical notation, the ability to break problems down into manageable components, manipulation of formulae, and the know-how to extract general principles from specific cases. Many studies [1, 2] have pointed out that the use of visual representations can both accelerate and facilitate learning and that supportive software and visualization tools can further improve this process. Indeed, some scientific visualization techniques, involving suitably prepared images and animations, enable the user to correlate information and to determine cause/effect relationships more readily by taking advantage of the human visual system's capacity to recognize patterns [3].

Moreover, it has been shown that the use of computer tools to model the behaviour of physical systems can bridge the gap between graphic, symbolic and visual representations [4].

Mechanical analogues of electronic circuits and vice versa have been used in physics education for many years [5, 6]. The pedagogical aim is to develop a broader understanding of the phenomenology by highlighting their similarities and differences. To tie similar phenomena together is also the main objective of the new approaches based on modelling and simulation [7, 8].

In this paper, we will present an approach that aims at illustrating some specific aspects of signal modulation through the construction and validation of some mechanical models. It is aimed at introducing the amplitude and frequency modulation concepts in introductory physics courses and/or laboratories.

The techniques used for electric modulation require a basic knowledge of concepts and facts that a typical physics or electrical engineering student does not often have. Moreover, the introduction of such phenomena at a purely technical level barely improves students' understanding, since the electric modulation is produced by effects that cannot be directly observed. This can hinder their conceptual understanding.

For these reasons, our approach involves the formulation of mechanical models which are capable of directly visualizing the main characteristics of amplitude/frequency modulation. These models use simple moving components which help students to understand the concepts of this abstract phenomenon through the visualization of motion and graphical displays of the related time equations.

By analysing these two types of analogue modulation, it is possible to construct some simulated models whose aims are to exploit the basic peculiarities of modulation, determine the numerical values of characteristic mechanical parameters and stimulate an understanding of the relationships with the electronic equivalent.

Section 2 describes the main features of amplitude modulation, in the presence of a sinusoidal modulating signal, and discusses the conditions that are necessary if a mechanical system exhibiting similar kinematic behaviour to the electronic process is to be created. Validation of the model, using appropriate computer simulations, is provided. Section 3 develops an analogous procedure for frequency modulation. The last section draws conclusions about the pedagogical usefulness of the proposed models.

2. A mechanical model for amplitude modulation (AM) with sinusoidal modulating signals

A mechanical model for amplitude modulation, in the simple case of a sinusoidal modulating signal, can be constructed according to the following deductive reasoning.

If $v_c(t) = V_c \cos \omega_c t$ indicates the high frequency carrier and $v_m(t) = V_m \cos \omega_m t$ the modulation term, with $\omega_m \ll \omega_c$, an amplitude modulated signal [9] is defined according to

$$v(t) = (V_c + V_m \cos \omega_m t) \cos \omega_c t. \tag{1}$$

Equation (1) can be considered as the sum of the high frequency carrier $v_c(t)$ and the modulation product

$$v_m(t) \cos \omega_c t = V_m \cos \omega_m t \cos \omega_c t. \tag{2}$$

The modulation product, which contains the information to be transmitted, can be broken up into two harmonic components of equal amplitude, with frequencies $\omega_c - \omega_m$ and $\omega_c + \omega_m$, respectively, constituting the upper and lower side bands of the AM signal. If $i_a = V_m/V_c$ denotes the amplitude modulation index, (1) can then be rewritten in the form

$$v(t) = V_c \left\{ \cos \omega_c t + \frac{i_a}{2} [\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t] \right\}. \tag{3}$$

2.1. A possible mechanical model

A mechanical system capable of representing the modulation product can be created with two simple pendulums of length l and mass m connected by a spiral spring with stiffness constant

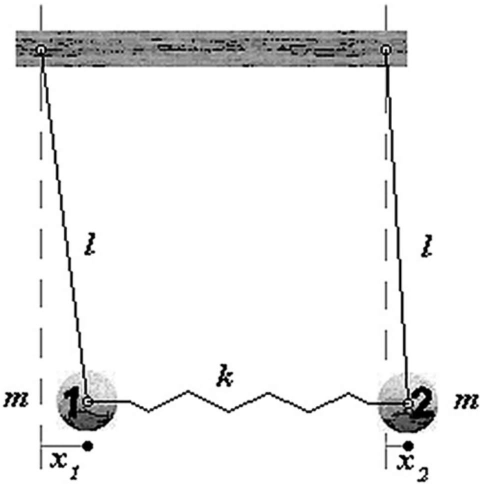


Figure 1. Two pendulums of mass m and length l elastically coupled through a spiral spring of elastic constant k.

k, as shown in figure 1. The spring is unstretched when the pendulums are in the vertical position.

Let x_1 and x_2 denote the small displacements from the equilibrium position of the two pendulums. The equations of motion of such a system are in first approximation [10]

$$\ddot{x}_1 = -\frac{g}{l}x_1 - \frac{k}{m}(x_1 - x_2) \tag{4a}$$

$$\ddot{x}_2 = -\frac{g}{l}x_2 + \frac{k}{m}(x_1 - x_2). \tag{4b}$$

Assuming $\varepsilon = kl/mg \ll 1$, the normal mode frequencies of the system can be approximately written in the form

$$\omega_S = \left(\frac{g}{l}\right)^{1/2} \tag{5a}$$

$$\omega_F \approx \left(\frac{g}{l}\right)^{1/2}(1+\varepsilon), \tag{5b}$$

where ω_S and ω_F denote the slow and fast frequencies, respectively. Imposing the initial conditions

$$\begin{cases} x_1(0) = x_0 \\ x_2(0) = 0 \end{cases} \quad \text{and} \quad \begin{cases} \dot{x}_1(0) = 0 \\ \dot{x}_2(0) = 0 \end{cases} \tag{6}$$

we finally obtain

$$x_1(t) = \tfrac{1}{2}x_0(\cos\omega_S t + \cos\omega_F t) \approx x_0 \cos\omega_S t \cos\tfrac{1}{2}\varepsilon\omega_S t \tag{7a}$$

$$x_2(t) = \tfrac{1}{2}x_0(\cos\omega_S t - \cos\omega_F t) \approx x_0 \sin\omega_S t \sin\tfrac{1}{2}\varepsilon\omega_S t. \tag{7b}$$

These solutions show that both the pendulums oscillate with frequency ω_S and their oscillation amplitude varies slowly with time, due to the small value of ε.

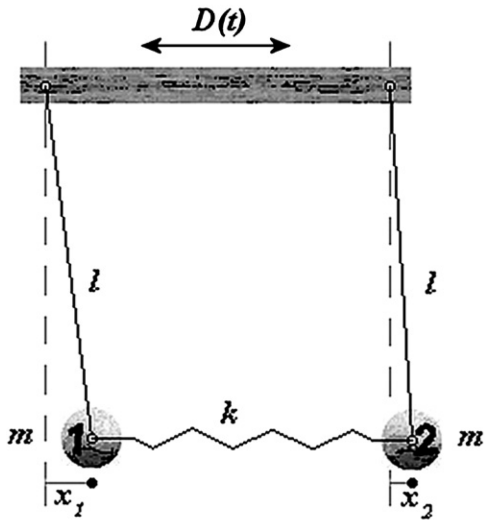


Figure 2. The system of figure 1 with its support oscillating in the horizontal direction.

Setting $\omega_c = \omega_S$ and $\omega_m = \frac{1}{2}\varepsilon\omega_S$, equation (7*a*) becomes analogous to the modulation product (2), since ε is a small quantity. For this reason, it can be deduced that a system of two weakly coupled pendulums may, in amplitude modulation with sinusoidal modulating signals, model the modulation product, or, equivalently, the suppressed carrier amplitude modulation.

In order to introduce an effect which can simulate the contribution of the carrier, the system of figure 1 is modified by applying periodical horizontal motion to the support of the two pendulums, described by the equation $D(t) = D_0 \cos\omega_c t$. This new system makes it possible to visualize a kind of motion whose displacement can be described by (1).

Indeed, upon modifying the system of figure 1 as shown in figure 2, the equations of motion assume the form

$$\ddot{x}_1 = -\frac{g}{l}x_1 - \frac{k}{m}(x_1 - x_2) + D_0\omega_c^2 \cos\omega_c t \tag{8a}$$

$$\ddot{x}_2 = -\frac{g}{l}x_2 + \frac{k}{m}(x_1 - x_2) + D_0\omega_c^2 \cos\omega_c t \tag{8b}$$

with the same normal mode frequencies found in (5*a*) and (5*b*). The general solutions of these equations (8) are

$$x_1(t) = c_{11}\cos\omega_S t + c_{12}\sin\omega_S t + c_{21}\cos\omega_F t + c_{22}\sin\omega_F t + \frac{D_0\omega_c^2}{\omega_S^2 - \omega_c^2}\cos\omega_c t \tag{9a}$$

$$x_2(t) = c_{11}\cos\omega_S t + c_{12}\sin\omega_S t - c_{21}\cos\omega_F t - c_{22}\sin\omega_F t + \frac{D_0\omega_c^2}{\omega_S^2 - \omega_c^2}\cos\omega_c t. \tag{9b}$$

With an appropriate choice of system parameters

$$c_{11} = c_{21} = i_a V_c/2 \qquad c_{12} = c_{22} = 0 \qquad D_0\omega_c^2\big/\left(\omega_S^2 - \omega_c^2\right) = V_c$$

they reduce to

$$x_1(t) = V_c\left[\cos\omega_c t + \frac{i_a}{2}(\cos\omega_S t + \cos\omega_F t)\right] \tag{10a}$$

$$x_2(t) = V_c\left[\cos\omega_c t + \frac{i_a}{2}(\cos\omega_S t - \cos\omega_F t)\right]. \tag{10b}$$

By setting the carrier and the modulating frequency to $\omega_c = (\omega_F + \omega_S)/2$ and $\omega_m = (\omega_F - \omega_S)/2$ respectively, we finally obtain

$$x_1(t) = V_c(1 + i_a\cos\omega_m t)\cos\omega_c t \tag{11a}$$

$$x_2(t) = V_c(\cos\omega_c t + i_a\sin\omega_c t\sin\omega_m t). \tag{11b}$$

These solutions correspond to the following choice of the initial conditions,

$$\begin{cases} x_1(0) = V_c(1 + i_a) \\ x_2(0) = V_c \end{cases} \quad \text{and} \quad \begin{cases} \dot{x}_1(0) = 0 \\ \dot{x}_2(0) = 0 \end{cases} \tag{12}$$

and make (1) and (11a) identical.

The following equations show the relationships between the mechanical parameters l, k, m, and the values of ω_c, ω_m, V_c, which, together with the modulating amplitude, characterize the amplitude modulated signal

$$\frac{l}{g} = \frac{1}{(\omega_c - \omega_m)^2} \tag{13a}$$

$$\frac{k}{m} = 2\omega_m\omega_c \tag{13b}$$

$$D_0 = \frac{\omega_m}{\omega_c}\left(\frac{\omega_m}{\omega_c} - 2\right)V_c. \tag{13c}$$

2.2. The computer simulation

The model validation was achieved using computer simulations implemented with commercially available educational software [11]. The Fourier analysis was performed using Microsoft Excel and its integrated development environment, Visual Basic for Applications Edition (VBA), to code the routines for the numerical integration of the differential equations of motion (Runge–Kutta method [12]) and the harmonic analysis of signals (FFT algorithm [12]).

The choice of the numerical values for the characteristic parameters of the mechanical system was aimed at producing observable effects in terms of amplitude modulated signals. Figure 3 shows the mechanical system used in our simulation together with the values of the characteristic parameters. It also shows the interface that allows the user to set the initial conditions according to (12). Each mass is $m = 0.1$ kg, the length $l = 0.31$ m and the coupling spring elastic constant $k = 0.79$ N m^{-1}.

The support oscillates horizontally according to the law $D(t) = -1.90 \times 10^{-3}\cos(2\pi t)$.

At time $t = 0$ the two pendulums are at rest in the positions $x_1(0) = 2 \times 10^{-2}$ m and $x_2(0) = 1 \times 10^{-2}$ m, respectively. Although the mechanical system is symmetric, the initial conditions are not and therefore the motions of the two masses are not symmetric either. Figure 4(a) shows that, with this particular choice of system parameters and initial conditions, the time dependence of $x_1(t)$ fits with good approximation an AM signal of the form

$$v(t) = 1 \times 10^{-2}[1 + \cos(2 \times 10^{-1}\pi t)]\cos(2\pi t)$$

with a unitary modulation index. Figure 4(b) shows the Fourier spectrum of $x_1(t)$, which is the classical spectrum of an AM signal.

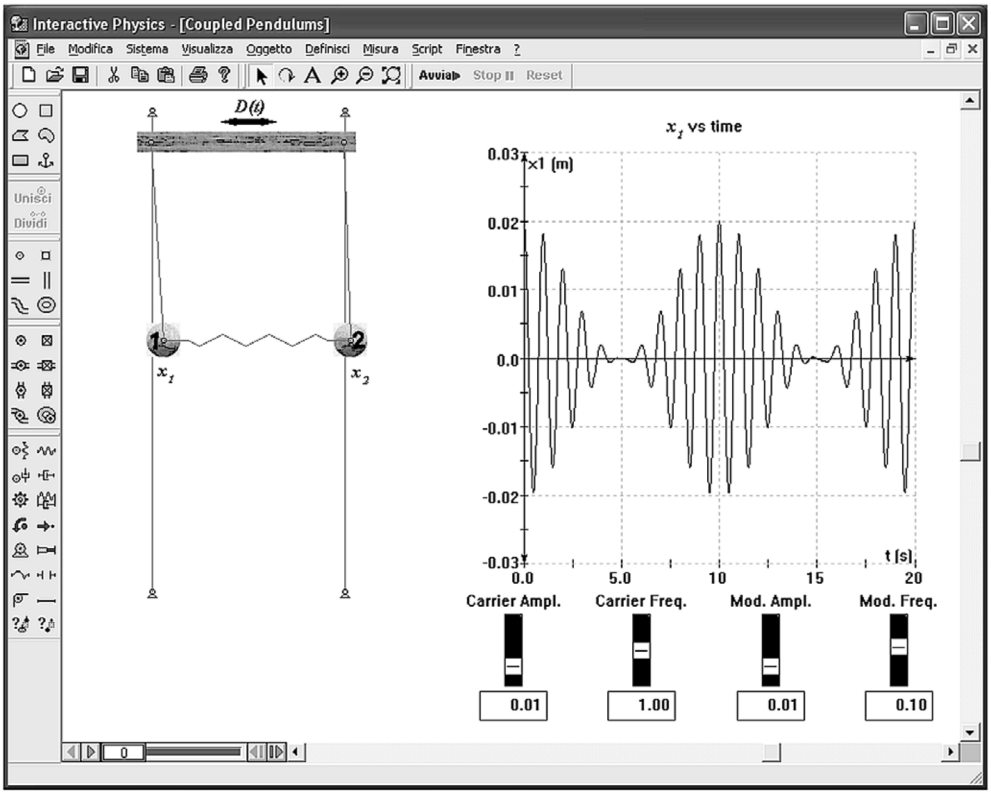


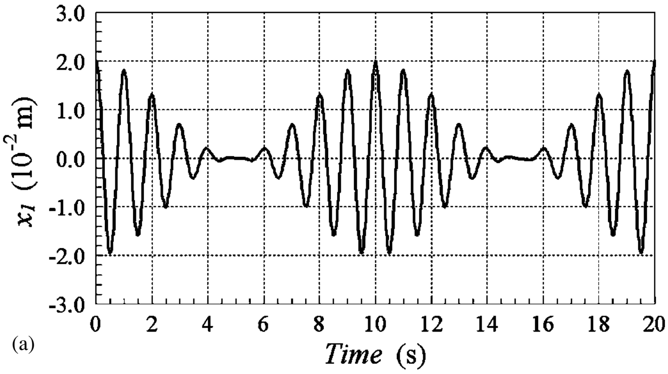
Figure 3. User interface of the simulation, showing the motion of the pendulums and the graphical display of $x_1(t)$. The slider bar controls on the right indicate the values of the carrier amplitude, the carrier frequency, the modulating amplitude and the modulating frequency, respectively. They can be set by the user. In the example shown, the modulation index is 1 and the frequency ratio is 10.

Figure 5(a) shows the temporal behaviour of $x_2(t)$ and figure 5(b) gives its Fourier spectrum.

Although (11*b*) indicates that the analytic form of $x_2(t)$ does not represent an AM signal with a sinusoidal modulation, its spectrum seems to be the same as that of $x_1(t)$. However, a more careful analysis reveals that the phase distribution of the spectral components is not correct. In fact, only the phase spectrum of $x_1(t)$ is symmetric with respect to the carrier phase, as it must be for an AM signal. The matter becomes clearer if we observe from (10*a*) that $x_1(t)$ is a linear superposition of three harmonic components with the same phase, whereas in $x_2(t)$, (10*b*), the higher frequency appears to be out of phase with respect to the lower one. Figure 6 reports the phase distribution of the harmonic components.

Equations (12) show that a change in the initial positions of the two pendulums allows us to settle different values for the carrier amplitude and for the modulation index. The particular choice of initial values for the speed also guarantees the existence of the correct phase relationships amongst the various spectral components.

Equations (13) show that the system parameters are only related to the carrier and modulating frequencies, while the amplitude of the bar's horizontal displacement also depends on the carrier amplitude. As a consequence, an arbitrary choice of parameters, as well as initial



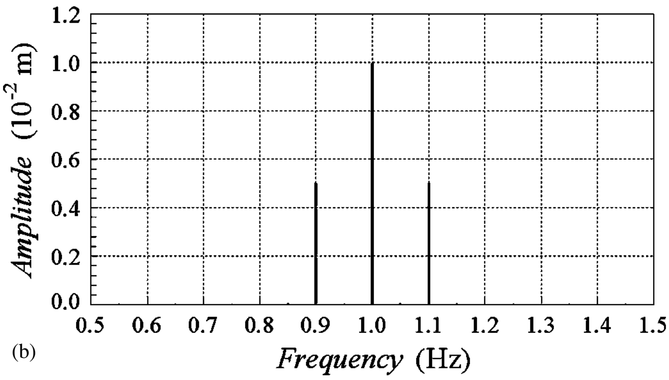


Figure 4. (a) Displacement of pendulum 1 versus time; (b) amplitude spectrum of $x_1(t)$.

conditions, produces a system response which can still be expressed as the sum of three spectral components that do not satisfy the correct amplitude ratio and phase relationships required in an AM signal with a sinusoidal modulation waveform.

3. A mechanical model of frequency modulation (FM) with sinusoidal modulating signals

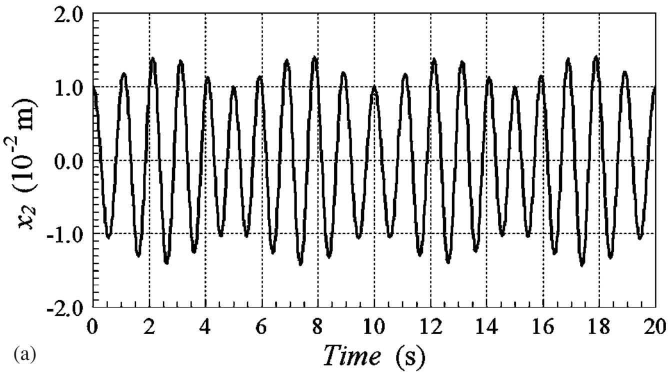
The implementation of a mechanical model of frequency modulation cannot proceed in a similar way to that outlined for the AM. The basic difference is that the FM signal has an extremely large frequency content (in theory infinite, although discrete) even in the simple case of a sinusoidal modulating waveform.

In electronic circuits, if $v_c(t) = V_c \cos \omega_c t$ denotes the high frequency carrier and $v_m(t) = V_m \cos \omega_m t$ is the modulating waveform, the modulated signal [9]

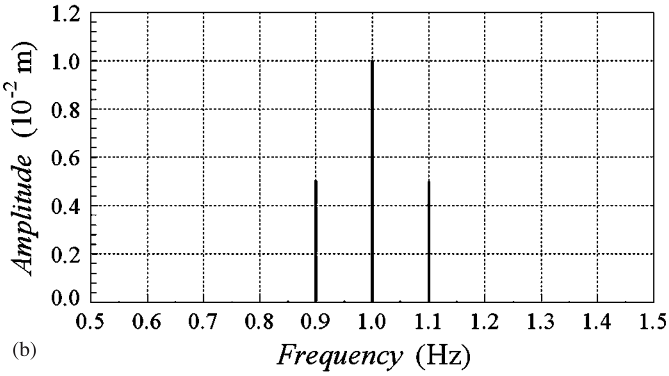
$$v(t) = V_c \cos(\omega_c t + i_f \sin \omega_m t) \tag{14}$$

is produced by varying the frequency ω_c of the carrier $v_c(t)$, according to the low frequency modulating information signal $v_m(t)$. The term $i_f = K_f V_m/\omega_m$ represents the frequency modulation index and K_f denotes the modulator constant whose dimension is such that i_f is dimensionless.

A mechanical system, whose equation of motion is the typical one of frequency modulation, was devised in line with the basic idea of controlling the frequency of a mechanical oscillator by acting on one of its characteristic parameters.



(a)



(b)

Figure 5. (a) Displacement of pendulum 2 versus time; (b) amplitude spectrum of $x_2(t)$.

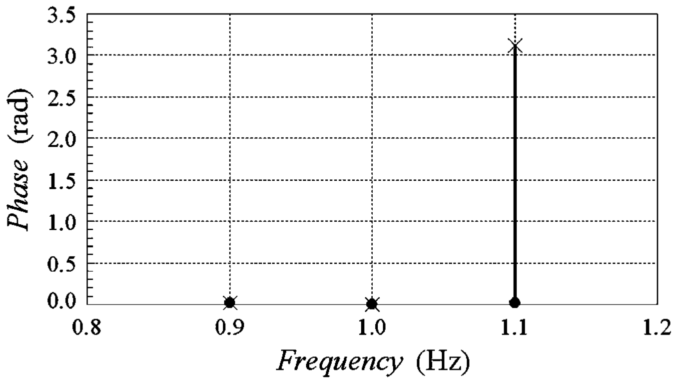


Figure 6. Phases of the significant spectral components of $x_1(t)$ (filled circles) and $x_2(t)$ (crosses).

3.1. A possible mechanical model

Among the various possible mechanical systems, a simple pendulum of mass m and varying length l was chosen. We assume that its length varies with respect to time according to

$$l(t) = l_0 - \Delta l \cos \omega_m t \tag{15}$$

where l_0 is the pendulum rest length, while Δl and ω_m denote the small amplitude and the angular frequency of the length variation, respectively. If θ indicates the instantaneous value of angular displacement, the equation of motion for such a system is given by [10]

$$ml^2\ddot{\theta} + 2ml\dot{l}\dot{\theta} = -mgl\sin\theta. \tag{16}$$

Taking into account (15), in the approximation of small oscillation angles, $\sin\theta \approx \theta$, and also assuming

$$\frac{\Delta l}{l_0} \ll 1 \tag{17}$$

we obtain, to the first-order approximation

$$\ddot{\theta} + 2\frac{\Delta l}{l_0}\omega_m \sin\omega_m t\left(1 + \frac{\Delta l}{l_0}\cos\omega_m t\right)\dot{\theta} = -\frac{g}{l_0}\left(1 + \frac{\Delta l}{l_0}\cos\omega_m t\right)\theta. \tag{18}$$

By imposing the condition

$$\omega_m \ll 1 \tag{19}$$

so that the quantity $\Delta l\omega_m/l_0$ is an infinitesimal of higher order with respect to $\Delta l/l_0$, it is possible to neglect the term which depends on the angular velocity appearing in (18). It follows that, setting $\omega_c = \sqrt{g/l_0}$, (18) can be approximated in the form

$$\ddot{\theta} = -\omega_c^2\left(1 + \frac{\Delta l}{l_0}\cos\omega_m t\right)\theta = -\omega^2(t)\theta \tag{20}$$

where we have set

$$\omega^2(t) = \omega_c^2\left(1 + \frac{\Delta l}{l_0}\cos\omega_m t\right). \tag{21}$$

Under condition (17), $\omega^2(t)$ results in a real function of time and, in a first approximation, we obtain

$$\omega(t) = \omega_c\left(1 + \frac{1}{2}\frac{\Delta l}{l_0}\cos\omega_m t\right). \tag{22}$$

Equation (20) is a particular kind of Mathieu equation. The Floquet theorem [13] shows that, with a particular choice of parameters, this equation leads to a periodic solution[1]. Nevertheless, for our purpose we do not need to go into this matter in depth. However, a heuristic reasoning may be followed so as to search for a periodic solution of (20) which has the form

$$\theta(t) = \theta_0\cos\varphi(t). \tag{23}$$

By substituting (23) into (20), we obtain

$$-\dot{\varphi}^2(t)\theta_0\cos\varphi(t) - \ddot{\varphi}(t)\theta_0\sin\varphi(t) = -\omega^2(t)\theta_0\cos\varphi(t)$$

from which we deduce that (23) may be a solution of (20) if

$$\dot{\varphi}(t) = \omega(t) \tag{24}$$

and

$$|\ddot{\varphi}(t)\theta_0\sin\varphi(t)| \ll 1. \tag{25}$$

[1] Mathieu's equation is a differential equation, occurring very commonly in physics. It is written in the form $y'' + f(x)y = 0$, where $f(x)$ is a periodic function of the independent variable x. This happens when x is an angle ($x \equiv x + 2\pi n$) or the time in an oscillation of period T ($x \equiv x + T$). The Floquet theorem states that there exists a particular solution $y(x)$ such that, when x is incremented by $2\pi n$, the solution itself is multiplied by a constant k. If $|k| = 1$ (i.e. $k = \mathrm{e}^{\mathrm{j}2\pi n}$), the solution is periodic.

Integrating (24) with respect to time, and also neglecting a trivial phase factor, the result is

$$\varphi(t) = \omega_c t + \frac{1}{2}\frac{\Delta l}{l_0}\frac{\omega_c}{\omega_m}\sin\omega_m t. \tag{26}$$

By substituting (26) into (23), we find

$$\theta(t) = \theta_0 \cos\left(\omega_c t + \frac{1}{2}\frac{\Delta l}{l_0}\frac{\omega_c}{\omega_m}\sin\omega_m t\right). \tag{27}$$

Equation (27) was obtained by choosing the following initial conditions:

$$\begin{cases} \theta(0) = \theta_0 \\ \dot{\theta}(0) = 0 \end{cases} \tag{28}$$

If, finally, we set

$$\theta_0 = V_c \tag{29}$$

and

$$i_f = \frac{1}{2}\frac{\Delta l}{l_0}\frac{\omega_c}{\omega_m} \tag{30}$$

we can rewrite (27) in the form

$$\theta(t) = V_c \cos(\omega_c t + i_f \sin\omega_m t) \tag{31}$$

which is identical to (14).

By differentiating (26) twice with respect to time, (25) becomes

$$\left|\frac{1}{2}\frac{\Delta l}{l_0} V_c \omega_m \omega_c\right| \ll 1. \tag{32}$$

The definition of i_f, given in (30), indicates that, in order to have a frequency modulation index which is of the same order as unity, the ratio $\Delta l/l_0$ must be of the same order of magnitude as the ratio ω_m/ω_c.

The following relations highlight the connection between the characteristic parameters l_0, Δl, of the mechanical system and ω_c, V_m, K_f, of the electronic modulating system, which, together with the modulating frequency and the carrier amplitude, characterize the frequency modulated signal

$$\frac{l_0}{g} = \frac{1}{\omega_c^2} \tag{33a}$$

$$\frac{\Delta l}{g} = 2\frac{K_f V_m}{\omega_c^3}. \tag{33b}$$

3.2. The computer simulation

The model validation was achieved through computer simulations carried out using specific educational software [11] and the above-mentioned VBA routine for harmonic analysis.

Figure 7 shows the mechanical system used in the simulation with a description of the characteristic parameters. It also shows the interface that allows the user to set the initial conditions easily, according to (28) and (29). Here, we refer to a pendulum with varying length $l(t) = 1 + 0.1\cos(5 \times 10^{-2}\pi t)$. The numerical value of the modulator constant is set equal to $K_f = 1.57$.

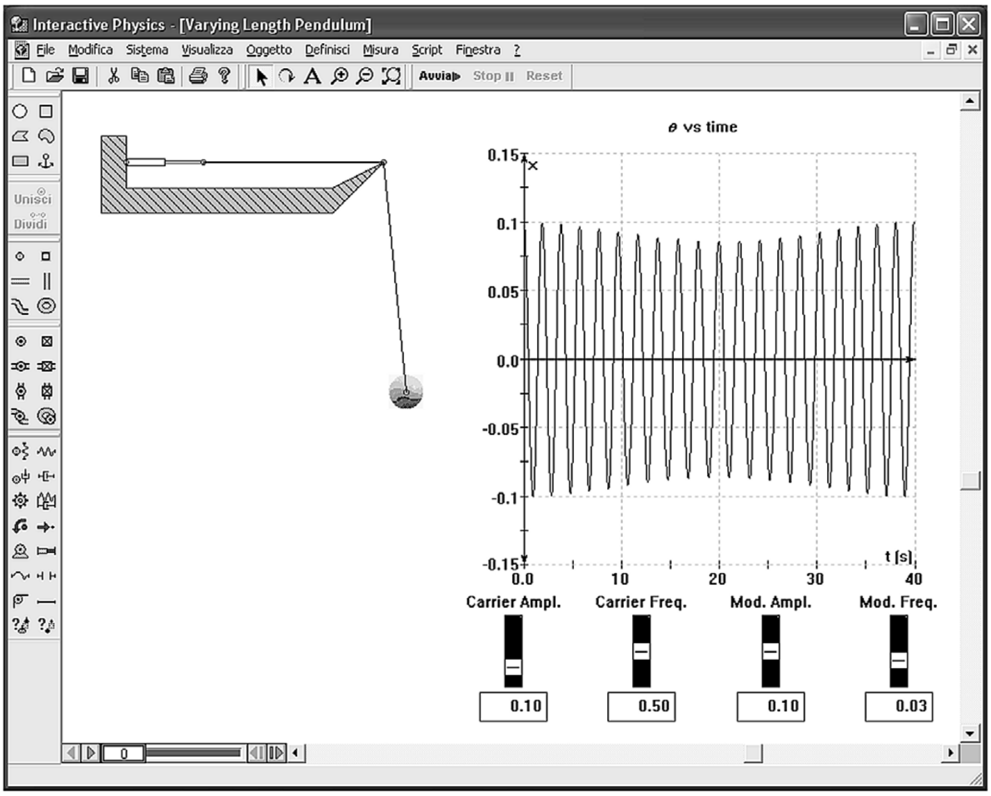


Figure 7. User interface of the simulation showing the motion of the pendulum with the varying length device and the graphical display of $\theta(t)$. The slider bar controls on the right indicate the values of the carrier amplitude, the carrier frequency, the modulating amplitude and the modulating frequency, respectively. They can be set by the user. In the example shown, the modulation index is 1 and the frequency ratio is 20.

At time $t = 0$, the pendulum is at rest in the angular position $\theta(0) = 0.1$ rad. Figure 8(a) shows that with this particular choice of the system parameters and initial conditions, the time dependence of $\theta(t)$ fits with a good degree of approximation a FM signal of the form

$$v(t) = 10^{-1}\cos[\pi t + \sin(5 \times 10^{-2}\,\pi t)]$$

with unitary modulation index.

The Fourier spectrum of the angular position $\theta(t)$ is shown in figure 8(b) where the crosses refer to the spectrum of the theoretical FM signal $v(t)$ and the horizontal lines indicate the values of the simulated one.

The differences between the two waveforms decrease as the conditions given in (17), (19) and (32) are better satisfied. Nevertheless, the particular choice of the parameters is due to a compromise between the necessity of satisfying the above-mentioned conditions and that of limiting the number of carrier oscillations during a complete period of the modulating signal, so as to achieve a better visualization of the time response. Condition (17), in particular, limits the frequency deviation. As a result, and differently to what happens in the AM case, here the effects of the modulation are difficult to visualize in the time domain, but they become particularly evident in the frequency domain.

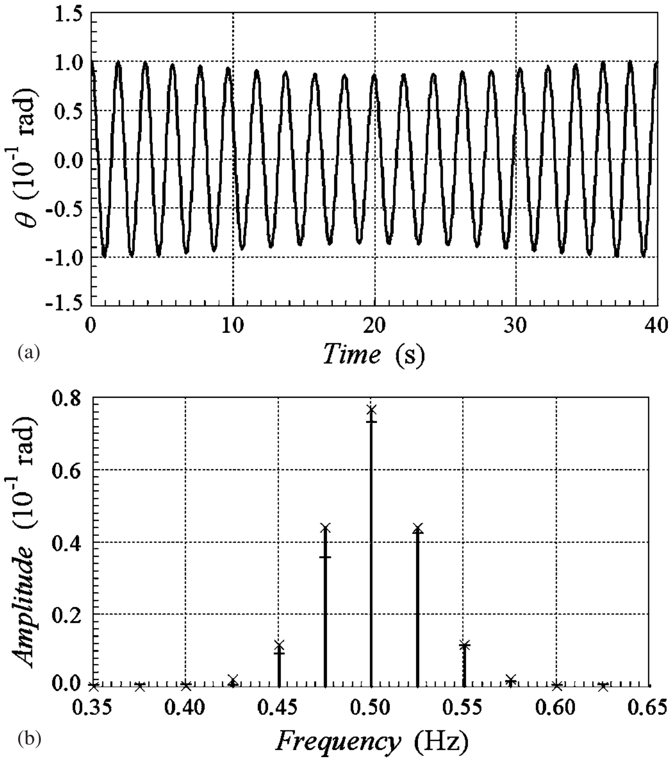


Figure 8. (a) Angular pendulum displacement versus time; (b) amplitude spectrum of $\theta(t)$. The crosses refer to the spectrum of the theoretical FM signal $v(t)$ and the horizontal lines indicate the values of the simulated one.

4. Conclusion

This paper outlines the implementation of two models of mechanical modulation. It aims at providing a simpler way of teaching the main features of modulation in signal transmission at an introductory level and at helping students understand the phenomenon in greater depth. The simulations were validated on small groups of physics and engineering undergraduates. They attended laboratory sessions dedicated to the subject in hand. Through informal interviews with the students the authors carried out an assessment of the material's pedagogical usefulness. The students showed a positive attitude towards this didactic approach and a real interest in the concepts, models and approximations lying behind the process of modulation. From their answers, the authors have been able to infer both the students' level of understanding of the main concepts and their ability to differentiate the effects of the modulating frequency on the carrier. Moreover, it has become evident that the visualization of these effects plays an important role in the process of understanding modulation.

The students, who had already studied amplitude modulation in electronics courses, were aware of the fact that this process is substantially achieved by means of nonlinear devices. Consequently, their first approach was to think that the mechanical equivalent could be obtained through the introduction of nonlinear terms in the equations of motion. Indeed, this fact cannot be applied in mechanical modulation since nonlinearity produces higher order harmonics. The use of two weakly coupled pendulums allows the experimenter to obtain amplitude modulation, since the coupling produces effects similar to those caused by the nonlinearity in electronic circuits.

In frequency modulation, as opposed to amplitude modulation, a mechanical device has been proposed as an equivalent to the electronic circuit. Electronic frequency modulation is usually achieved through capacity variation (e.g., using a varicap device) in an oscillating circuit. The mechanical frequency modulation, obtained by changing the length of a pendulum, has similar effects.

Moreover, it was emphasized that in electric modulation as well as in mechanical modulation, unwanted combination frequencies or harmonics occur. They are eliminated, in electronics, by the use of appropriate filters. The same task is accomplished in mechanical modulation by setting suitable initial conditions or imposing restrictions on the dynamics of the system (i.e. weak coupling between the pendulums) or on the pendulums' motion (i.e. low amplitude oscillations). This aspect has been highlighted during laboratory courses so as to make the students aware of the fact that some treatment of modulated output is necessary if an acceptable quality of the modulated signal is to be obtained, since both electronic and mechanical modulations are approximate processes.

In conclusion, the mechanical systems here depicted, by visualizing cause–effect relationships, supply an effective support for the conceptual understanding of the widely used process of modulation.

Appendix. Modulation

Modulation is the process of producing a wave some characteristic of which varies as a function of the instantaneous value of another wave called the modulating wave. The modulating wave is usually called the signal, the wave to be modulated is denominated the carrier, Usually, the modulation frequency is considerably lower than the carrier frequency. The ultimate purpose is to alter some characteristic parameter of the carrier wave in a nonlinear manner to carry information.

There are at least two reasons for transmitting information at a relatively high frequency level: (1) transmission by radiation is practicable at high frequencies. In fact, efficient radiation and reception of electromagnetic waves require the use of antennas and circuits tuned to the frequencies of the waves. The antennas required at audio frequencies would be impractical because of their great lengths, and they would not respond equally well to all frequencies in the audio range or TV range. (2) It is possible to transmit a number of messages simultaneously without interference if the frequency level is different for each message.

Since the sounds of the human voice and of musical instruments or the images of a TV picture involve a large number of frequencies produced simultaneously, the form of the modulating wave is rarely sinusoidal. To simplify theoretical analyses, however, each modulating frequency can be treated separately. Consider a wave (the carrier), which may be represented analytically by the expression

$$e(t) = A\cos(\omega t + \varphi)$$

where t is the time, ω the angular frequency and φ the phase. If either A, ω, or φ is varied according to some nonlinear function of the instantaneous value of a modulating wave, then this expression will represent the modulated wave. Although, it is possible to produce a wave in which all three parameters vary simultaneously, in each of the modulating methods that are important practically, only one of these parameters is varied.

In amplitude modulation, the amplitude A is varied in accordance with the modulating wave, while ω and φ remain constant. In frequency modulation, the frequency ω is varied, and both A and φ remain constant. In phase modulation, the phase φ is varied, while A and ω remain constant. Phase modulation is not of much practical importance in itself and it is formally analogous to frequency modulation.

Frequency modulation has two main advantages with respect to amplitude modulation: (1) the frequency modulated signal is less affected by noise than the amplitude modulated since the most common sources of noise are due to spurious signals producing amplitude variation, they have no effect on a frequency modulated wave. (2) The characteristic of the amplifiers used in signal transmission/reception is nonlinear. This fact introduces distortions, which depend on the amplitude of the modulating signal. They may be relevant and variable in amplitude modulation, but they can be small and constant in frequency modulation.

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