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## Evolution of grain-size distribution of pumice sands in 1-D compression

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### Abstract

Crushing is one of the micromechanisms that govern the mechanical behaviour of sands at medium-high stresses. It depends on mineralogy, form and strength of single particle, mean stress level, coordination number, time, etc.. It causes changes of grain-size distribution, porosity, number and type of grain contacts, fabric, structure of the material, etc.. Results of an experimental research on the crushing of pumice sands compressed under 1-D conditions to vertical effective stresses  $\sigma'_v$  up to 100 MPa are reported here. They show marked crushing already at  $\sigma'_v$  of about 200 kPa. The evolution of the grain-size distribution can be represented by  $\Delta D_i = h / (K(1 + C \exp(-h \lg \sigma'_v)))$  in which  $\Delta D_i$  is the decrement of the generic characteristic diameter. C, h, K are positive parameters depending on the sand's nature and initial state. This relation properly accounts for the existence of an upper limit to  $\Delta D_i$  (or the existence of a limit grading). It is able also to describe the evolution of the global relative breakage indexes.

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### 1. Introduction

Granular materials are multi-scale and complex systems and their mechanical behaviour depends on the micro, meso and macro scale. The macro response of this type of materials, and in particular the deformation mechanisms, are governed essentially by the relative particle to particle sliding, rolling and breakage (or crushing) of grains, and in

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minor measure of fractures, abrasion and elastic deformations of the single particles. It is well known that grain crushing plays a fundamental role in the mechanical behaviour of particulate materials, being one of the main deformation mechanisms at medium-high stresses. As crushing progresses, for effect of increasing applied stresses, the grain-size distribution undergoes modifications, causing changes in the porosity, grain-to-grain contacts, density, packing density, and in the complex process of generation and fragmentation of particles. Grain crushing influences the permeability, shear strength, deformability, critical state of granular materials. It interests, at “normal” stress level and strain conditions, particles such as weak rock, carbonate sands, expansive clay pellets, pyroclastic soils, and, at high or very high stresses, also materials consisting of very hard particles such as quartz sands. It has relevance in many practical problems such as high earth dams, deep wells and tunnels, driven piles, explosions, impact of projectiles, rock avalanche, “sturzsstroms”, etc.. Breakage of grains has been object of many experimental, theoretical and numerical studies regarding sand type, stress-path, effective stress level, etc. [e.g. 1-9]. However, some aspects are still not fully understood, in particular with reference to the evolution of characteristic diameters, the mean stress level at which the evolution of grading is self-similar or fractal. This paper reports the results of an experimental research, relative to a very high interval of stresses ( $\sigma'_{vmax} = 100$  MPa), on a pumice sand that is particularly crushable even at relatively low mean stress levels.

## 2. Material and test procedure

The tests was performed on pumice sands artificially obtained, from fragmentation, of rock fragments commercially available, of 10-15 mm. The sands used in the experimentation have diameter ranging from 0.18mm to 2 mm, and are almost monogranular, Figure 1. The uniformity coefficient  $C_u$  of tested sands ranges from 1.18 to 1.29 (sands 1-6) and from 1.38 to 1.66 (sands 7-8). The sands 7 and 8 are obtained as mixtures of sands 3, 4 and 5 and sands 4 and 5 respectively. The initial void ratio  $e_0$  ranges from 3.03 to 3.6 for sands 1-6 and from 2.93 to 3.01 for sands 7 and 8. The sand grains have irregular shape, are generally at sharp corners and have a system of intragranular voids. One-dimensional compression tests have been carried out in a specially built oedometer capable of withstanding high stresses up to 120 MPa [4]. The diameter and the height of the specimens were 73mm and 20 mm respectively. The sand has been placed dry in the oedometer and then gently tamped. The vertical load was applied by means of a hydraulic press at constant rate of axial deformation of 0.5mm/min; the tests duration ranging from 5 to 10 min. The tests axial deformation and the duration of tests are such that the influence of creep on grain crushing is negligible. Overall 200 tests have been performed. Tests were performed on dry sands. After testing the specimens were sieved with the aim of studying the evolution of breakage of particles process.

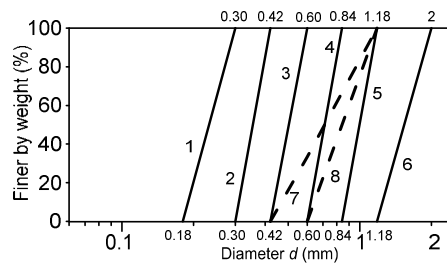


Fig. 1. Initial grading of the sands utilised in the experimentation.

## 3. Experimental results

Figure 2 shows some photos (obtained by using the optical microscope at different magnifications) after sieving, of some particles of the material of different sizes of a specimen subjected to  $\sigma'_v = 15$  MPa. There are sand particles of the same sizes of the initial sand ( $0.42 < d < 0.60$ mm); this means that not all the grains are crushed at the end of the test. The deformation process consequent to the load application produces particles of various sizes, all with very sharp corners, including those with very small sizes ( $d \leq 0.074$ mm). These observations demonstrate that also the smaller particles are still potentially crushable, having a small coordination number, C.N. (number of contacts with its neighbor

grains, which strongly depends on the type of corners of the single particles; C.N. normally decreases with the increase of the particle asperities and sharpness of the edges).

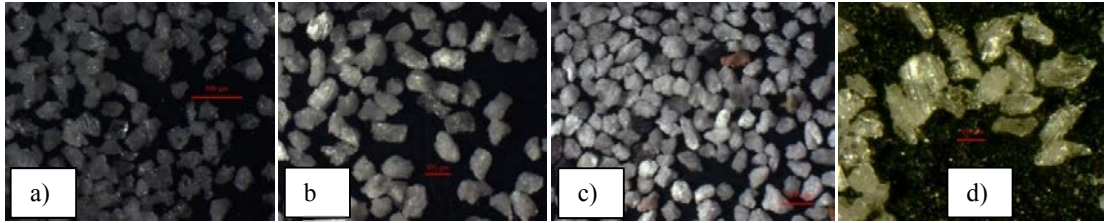


Fig. 2. Grains of sample subjected to  $\sigma'_v = 15$  MPa, at the end of the test and after sieving. a) grains with  $0.42 < d < 0.60$  mm (survived particles). Generated grains: b)  $0.30 < d < 0.42$  mm; c)  $0.125 < d < 0.18$  mm, d) particles with  $d < 0.074$  mm. All grains have very sharp corners.

#### 4. Evolution of grain size distribution

Figure 3 shows some results of the evolution of the grading in function of  $\sigma'_v$ . This figure also reports the fractal ultimate curves ( $F_u$ ) of the grain size distribution as predicted assuming the self-similar condition for the material in correspondence to the very high stress level reached in some tests. The sands undergo very high grading modifications as a consequence of the applied  $\sigma'_v$ . The changes in the grading induced by  $\sigma'_v$  depend on many and interdependent factors such as initial void ratio  $e_0$ , form, size, angularity and strength of individual soil particle, eventual presence of grain cracks, granulometric sorting and related coefficient of uniformity, distribution of grain-to-grain contacts, variability within the sample of the coordination number, N.C., loading rate and creep. It is not surprising, therefore, that the quantitative description of the breakage process in deterministic as well as in probabilistic micromechanical terms is extremely difficult. Nevertheless, the experimental results of the present research permit to identify some general macroscopic trend in the evolution of the granulometric distribution.

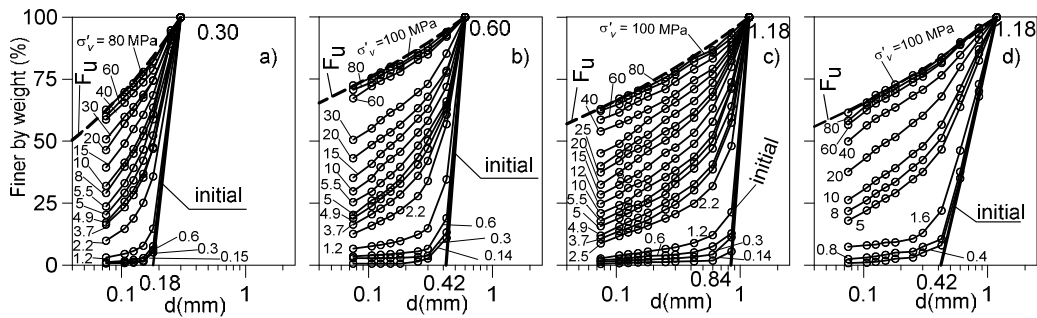


Fig. 3. Modifications of grading as a function of the maximum applied vertical stress  $\sigma'_v$  for sands 1, 3, 5 and 7 of Fig. 1.

##### 4.1. Evolution of characteristic diameters

Figure 4 shows the variations of the absolute values of the decrement  $\Delta D_i$  of the characteristic diameters  $D_i$  (such as  $D_{10}$ ,  $D_{15}$ ,  $D_{25}$ ,  $D_{50}$ ) with of  $\sigma'_v$ .  $D_i$  is the diameter corresponding to the percentage  $i$  finer by weight. The values of  $D_i$  always decrease with the increasing of  $\sigma'_v$ . However, experimental results show that there is an upper limit,  $\Delta D_{i,max}$ , to  $\Delta D_i$ .  $\Delta D_{i,max}$  is lower than the initial value of  $D_i$ .  $\Delta D_i$  can be related to  $\sigma'_v$  according to the following function:

$$\Delta D_i = \frac{1}{k} \frac{h}{(1 + C \exp(-h \log \sigma'_v))} \tag{1}$$

where  $K, h, C$  are positive parameters depending on nature and the initial properties of the sand, and can be found by using curve fitting methods. Other functions may be used to fit experimental data too. The parameter  $h$  is related to the propensity to crushing of sand particles, the parameter  $K$  accounts for the resistance to the changes of  $D_i$ , that is higher the higher the number of particles for volume unit and, consequently, the smaller are the dimensions of particles. The physical meaning of the parameter  $C$  is not as yet clear. Function (1) plots as a logistic or Verhulst curve on the plane  $\Delta D_i - \log \sigma'_v$ ; it has two horizontal asymptotes ( $\Delta D_i=0$  and  $\Delta D_{i,max}=h/K$ ). The existence of an upper bound  $\Delta D_{i,max} < D_i$  implies that at very high stresses the particles of the sand become so tiny or so mutually coordinated that further breakage becomes negligible as predicted by Kendall [10]. The relation (1) well fits other experimental data relative to other materials such as carbonate sand, quartz sand, angular glass, both in 1-D compression as in triaxial compression [11].

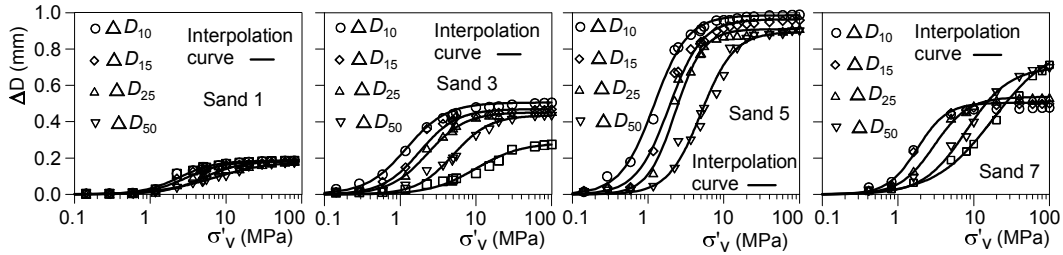


Fig. 4. Evolution of characteristic diameters with vertical effective stress  $\sigma'_v$  for sands 1, 3, 5 and 7 of Fig. 1.

#### 4.2. Evolution of breakage indexes

Several measures have been proposed in literature for quantifying the amount of grain crushing, among which [2] and [12] that take the global changes of grading into account. Figure 5 reports the trend of the Hardin’s [2] and Einav’s [12] relative breakage indexes in function of  $\sigma'_v$  for sands 1,3,5 and 7. The Hardin’s relative breakage index  $B_r$ , defined as the ratio between the total breakage  $B_t$  (i.e. the area on the semilogarithmic plot bounded by the initial grading curve, the grain-size curve pertaining to the given applied stress and the vertical line drawn at the abscissa  $d=0.074\text{mm}$ ) and the breakage potential  $B_p$  (i.e. the area bounded by the initial grading curve, the horizontal line through the 100% point of the initial grading curve, and the vertical line at  $d=0.074\text{mm}$ ). The Hardin’s relative breakage index  $B_r$  ranging from 0 (no crushing) to 1 (all particles have crushed and their diameter is smaller than  $0.074\text{mm}$ ). Einav [12], on the basis of the observation that the grain size composition of a compacted particulate material should be limited by an ultimate grading, attained under extremely large stress level or under very high shear strains, has proposed the following new relative breakage index  $B_r$ :

$$B_r = \frac{B_p}{B_p} = \frac{\int_{d_m}^{d_M} (F(d) - F_0(d)) d (\log(d))}{\int_{d_m}^{d_M} (F_u(d) - F_0(d)) d (\log(d))} \tag{2}$$

in which  $B_p$  is the area between the original and the current grain size distribution;  $B_t$  is the total area between the initial and the ultimate fractal grain size composition.  $F_0(d)$  and  $F_u(d)$  represent the initial grading (before grain crushing) and the ultimate fractal distribution respectively;  $F(d)$  is the current grading;  $d_M$  and  $d_m$  are the maximum and minimum grain size of the material. This definition differs from Hardin’s one and was supported by several experimental and theoretical studies [e.g. 14-16]. As shown in Figure 5, these indexes have similar trend. The grain crushing process is very intense for  $0 < \sigma'_v < 8-10$  MPa; for  $8-10 < \sigma'_v < 25$  MPa it undergoes a reduction. For  $\sigma'_v$  ranging from 25 to 40 MPa breakage reduces its intensity further, and for  $\sigma'_v > 40$  MPa it tends to disappear. In this last part of stress interval the Einav’s  $B_r$  reaches the unity in all cases, while the Hardin’s  $B_r$  reaches values ranging from 0.65 to 0.72. The fitting of data was made with a relation similar to the relation (1) with  $B_r$  instead of  $\Delta D_i$ . The fact that the values of the Einav’s  $B_r$ , in all cases are very close to one for  $\sigma'_v > 40$  MPa, and hence for a very large range of applied stress ( $40 < \sigma'_v < 100$  MPa), implies that for the tested pumice sands the ultimate fractal grading has

been reached. For a fractal (or self-similar) grading evolution is valid the next relation [12]:

$$F(d) = \frac{d^{3-\alpha} - d_m^{3-\alpha}}{d_M^{3-\alpha} - d_m^{3-\alpha}} \quad (3)$$

where  $d$  is the generic diameter,  $F(d)$  is the percent in weight of material with a diameter smaller than  $d$ ,  $d_M$  is the maximum diameter,  $d_m$  is the minimum diameter and  $\alpha$  is the fractal dimension. The distribution (3) has a limit per  $\alpha=3$ . If  $d_m=0$  (hypothesis that a comminution limit does not exist) it becomes the next relation (4), that is Turcotte's relation [13], for which there is no limit for  $\alpha$ .

$$F(d) = \left(\frac{d}{d_M}\right)^{3-\alpha} \quad (4)$$

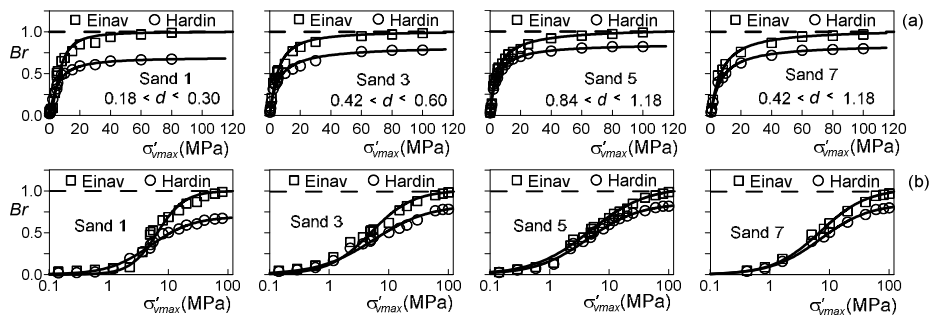


Fig. 5. Evolution of Hardin's and Einav's  $Br$  with  $\sigma'_v$  for sands 1, 3, 5 and 7.  $\sigma'_v$  axis: (a) natural scale; (b) log scale (b).

For sands 1, 3, 5 and 7 the trend of the coefficient  $\alpha$  that fitted the experimental data are plotted in Fig. 6; even these interpolations have been made with a relation similar to the (1) (or 3) with  $\alpha$  instead of  $\Delta D_i$  (or  $B_r$ ). The initial values (uncrushed sands) of  $\alpha$  are negative. For  $0.3 < \sigma'_v < 2-3$  MPa the values of  $\alpha$  ranging from about zero to about 1.5-1.7; for  $5 < \sigma'_v < 10$  MPa,  $\alpha$  ranging from 1.5-1.7 to 2.2-2.3 and for  $\sigma'_v > 10$  MPa  $\alpha$  varies from 2.5 to 2.85. The ultimate values of  $\alpha$  ranging from 2.55 (sand 1) to 2.85 for the other sands. These values are the fractal dimension of pumice sands and are very similar to other fractal dimensions reported in literature relative to other materials e.g. [13-15]. The results in Figure 6 show that, at least for the pumice sands, the evolution of grading is far from fractal (or self-similar) for  $\sigma'_v < 2-3$  MPa. In this stress range the breakage is principally governed by the dimension of particles, and in minor measure by the C.N., according to McDowell et al. [15]. For  $2-3 < \sigma'_v < 8-10$  MPa the particle sizes and the C.N. are equally important in determining the probability of grain crushing. For  $\sigma'_v$  greater than about 10 MPa ( $\alpha > 2.2-2.3$ ) the grading evolves as a self-similar (or fractal) grain size distribution, with probability of splitting of particles essentially governed by the C.N.. In this range of  $\sigma'_v$  the probability of splitting of largest particles is very low, as a consequence of the corresponding state of stress (relative to the single particle) that tends to uniform compression. Figure 7 shows, for the sand 3, the evolution of some granulometric fractions with  $\sigma'_v$ . The fraction with  $0.84 < d < 1.18$  mm (that coincide with the initial grading of the sand) is not completely eliminated (there are particles that survived) also for very high values of applied  $\sigma'_v$ . The particles with  $d > 0.074$  mm always increase with the vertical applied stress  $\sigma'_v$ , but with a rate which is reduced with  $\sigma'_v$  when  $\sigma'_v$  is greater than about 15-20 MPa. The other generated fractions (Fig. 7b-d) initially increase with  $\sigma'_v$ , they reach a peak value at  $\sigma'_v$  that reduces with the increasing of the mean diameter of the considered fraction, and then they reduce with  $\sigma'_v$ . This data clearly confirms that the evolution of grading is essentially fractal only for  $\sigma'_v$  greater than about 8-10 MPa.

## 5. Conclusions

Results presented in the paper show that the grain-size distribution of pumice sands in 1-D compression undergo appreciable changes at stress  $\sigma'_v$  as low as about 0.2 MPa. It is shown that the reduction  $\Delta D_i$  of the generic

characteristic diameter  $D_i$  increases with the applied stress  $\sigma'_v$  according to the relation  $\Delta D_i = h/(K(1+C \exp(-h \lg \sigma'_v)))$ , where  $C$ ,  $h$ ,  $K$  are parameters depending on the initial properties and state of the sand. This Verhulst type function properly accounts for the existence of an upper bound to  $\Delta D_i$  and then of a limit grading. It well describes other literature geotechnical data relative to other sand types and for different stress-path. It is also able to describe very well also the evolution of the global breakage indexes (Hardin's and Einav's  $B_r$ ). For the pumice sands the evolution of grading is essentially fractal for  $\sigma'_v > 8-10$  MPa. For lower values of  $\sigma'_v$  the evolution of grain size distribution is not fractal or self-similar, with probability of fragmentation of grains governed essentially from the particle sizes for  $\sigma'_v < 2-3$  MPa, and influenced in equal measure by the particle sizes and the C.N. for  $3 < \sigma'_v < 8$  MPa. The ultimate values of the coefficient  $\alpha$  ( $\alpha=2.5-2.85$ ) represent the fractal dimension of the tested pumice sands. The variation of the coefficients  $B_r$  and  $\alpha$ , the fractal dimension, and the not fractal evolution of grading for  $\sigma'_v$  lower than about 8-10 MPa must be taken into account in the implementation of constitutive models that consider within them the possibility of grain crushing at least for materials like the pumice sands that present a double voids system and are very crushable also at relatively low stress levels.

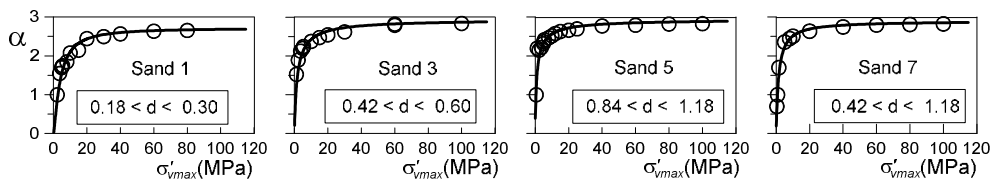


Fig. 6. Evolution of the coefficient  $\alpha$  of equation 7 with  $\sigma'_v$  for the sands 1, 3, 5 and 7 by Fig. 1.

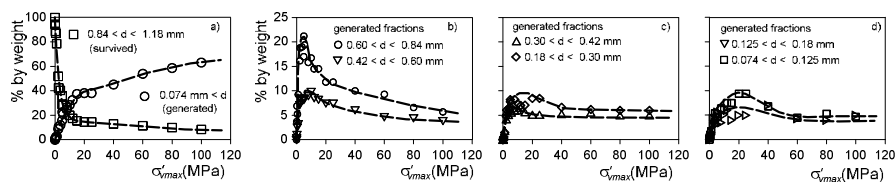


Fig. 7. Evolution of survived and generated fractions with  $\sigma'_v$ , for the initial sand 3. Scale of vertical axis for a) is different from b,c and d.

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