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Can we learn from “outside”? A dialogue with a Chinese teacher: the “two basics” as a meaningful approach to mathematics teaching

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Abstract: Since many years Confucian heritage students (Chinese ones in particular), acquire leading positions in numerous international scientific programmes and display excellent performance in international assessments as PISA or TIMMS (OECD, 2013). To understand the “reasons” of this excellence we tried to explore some aspects of the cultural background of teaching practices and classroom life in those countries. With this aim a Chinese teacher was interviewed; we asked him about principles, values and beliefs and their impact on teaching/learning Math in classroom. The paper discusses what emerged from this dialogue and in particular from the idea of the “Two Basics” mathematics teaching approach, typical for the Chinese educational context. Furthermore, this work tries to underline (in a implicit or explicit way) similarities and differences between East and West didactical approaches and to define a sort of integration of these in order to improve a better mathematics education for all students.

Résumé: Depuis de nombreuses années, les élèves de l'héritage confucéen (les chinois en particulier) atteignent des positions de leader dans de nombreux programmes scientifiques internationaux et obtiennent d'excellentes performances dans les évaluations internationales PISA ou TIMMS (OECD, 2013).

Pour comprendre les «raisons» de cette excellence, nous avons essayé d'explorer certaines caractéristiques du contexte culturel des pratiques d'enseignement et de la vie scolaire dans ces pays.

Avec cet objectif, nous avons interviewé un professeur chinois; nous lui avons demandé à propos des principes, des valeurs et des croyances et du leur impact sur l'enseignement /apprentissage des mathématiques en classe. L'article traite de ce qui a ressorti de ce dialogue et en particulier de l'idée des "Deux Principes", l'approche à l'enseignement des mathématiques typique du contexte éducatif chinois.

En outre, ce travail veut mettre en évidence (de manière implicite ou explicite) similitudes et différences entre les approches didactiques de l'Est et de l'Ouest, et veut définir une sorte d'intégration entre eux dans le but de parvenir à un meilleur enseignement des mathématiques pour tous les élèves.

Cultural aspects related to Chinese mathematics educational context: an overview of possible values and principles

A general overview of the cultural aspects that could be the linked to education in China is related to the social status of school and the value that School as institution has for teachers, students and families. In this area of remarkable economic development, with the connected social changes, school in fact is seen as an instrument of social promotion. Many expectations of families and students are pinned on the result in English and mathematics. So students study very seriously till late. In the Confucian view school has to be difficult. Teachers must assign “many” homework and require great commitments. The authority of teachers is not discussed and the confidence of

students is granted.

According to Zhang et alii (2004), the didactic aspects related to this and the implication on the way of teaching/learning in classroom could be summarised in five points:

- *Rituals*: in Chinese, Korean and Japanese school, lessons follow a repetitive pattern that always includes welcoming the teacher, summary and review of past lessons, various other activities depending on the grade level, and a final greeting. This habitual structure is heartening for students and makes the lesson a separate space in their lives, where they can concentrate and where specific rules apply.

- *Progressivity*: a good part of the lesson is devoted to summary and review of past lessons. Then you can add some new contents. Knowledge is built with steady rhythm, step by step, with great consistency.

- *Rote learning strategies and significance*: memorization, repetition, repetitive exercises are fundamental in teaching practices in these countries, but it seems that students develop considerable metacognitive skills, which appears like a paradox. In *Chéng chú tōng Bian běnmò* (乘除通变本末 “Full Explanation on multiplication and division”, 1274) the Chinese mathematician *Yang Hui* (杨辉 1238-1298) wrote a list of important skills and knowledge for students, indicating timetables and ways to learn: 260 days in all. According to him, it takes a day to figure out a method (an algorithm or a procedure) and you have to work on it for two months to seize its meaning and applications without forgetting. But, for example, once you understand the method of addition (加法 *jiāfǎ*), studying subtraction (减 *jiǎn*) you can reverse all the exercises made by adding the differences to subtrahends. This gives the method its "origins" and reduces the time of study only 5 days. So understanding significance is the target.

- *Relational approach*: in Chinese or Japanese textbooks whenever you find a rule, you find nearby the reverse one; whenever you learn an operation, you learn the reverse one; whenever you see an identity, like in polynomial algebra, you find the same written in reverse form. So you learn together addition and subtraction, polynomial algebra and factorization, and so on... That means that you are not just learning some object and procedure, but their mathematical relations too.

- *Visualization*: schemas, pictures, drawings, blackboard sketch, magnets, stickers, page layouts, graphic arrangement in books, animations and screens, all are strongly used by eastern teachers to represent, organize, and explain mathematical properties and objects at school. Visualization tradition is still alive. Students remember images that mean concepts, not especially words.

In particular, about Mathematics, a cultivated person in China has to know many mathematics and a clever person can do mental calculations quickly and accurately. Mathematics as a subject has big space in school timetable.

If we look at the history of the Discipline, Chinese mathematics has been so important that one of foundational myths of the country is related to magic squares (幻方 *huàn fāng*, the myth is the *Luo river magic square* 洛书 *luò shū*). Mathematics is up to now one of the traditional *Six Arts* (六艺 *liù yì*) required in young lords education and played a strategic role in imperial bureaucrats curriculum and examinations (Nicosia, 2010).

Due to historical reasons, in many Eastern countries, including Japan, Korea, Singapore, and China (Mainland China, Taiwan, and Hong Kong), and even Russia, mathematics educators emphasize the importance of foundational training more than is usually seen in the West.

In all the Confucian heritages the didactical consequence of this approach is the principle of the “two basics”, that is the most typical teaching framework in all these eastern countries (Zhang, 2004).

In China the “two basics” principle (basic knowledge and basic skills) in mathematics teaching is a broad and loose concept without a strict definition. According to Zhang (2004), its general meaning is that, between “solid foundation” and “individual development and creativity”, although both are important, foundation is the more important. As a Chinese proverb declares: “although a tower is beautiful its groundwork is more important”.

With the aim to better understand this important principle hidden in Chinese teaching

framework, and its possible link to mathematical competences acquired by Chinese students (as evidenced in international competitions), we met Lai in Guangzhou last year and we interviewed him. With his help we tried to define a general framework of cultural background of teaching practices and classroom life in China. Furthermore, according to his point of view, we tried to define a possible link between these cultural situations, these social values and the excellent training that students showed in international assessments as PISA or TIMMS (OECD, 2013).

The “two basics”: solid foundation and individual development. A dialogue with a Chinese teacher

Lai is a Chinese teacher; he lives/teaches Mathematics in a Upper Secondary School in Guangzhou (廣州) that it is the biggest city in the south of China in the region of Guangdong. He’s teaching since 1978 and he is considering an expert Professor in his school. We met him last year in May, we spent 3 month with him observing his typical way of teaching and its impact in class. After each lesson we discussed with him exploring what happened in class and why.

This paper discusses what emerged from an informal dialogue with him about social and cultural aspects related to the Chinese school and their impact of the teaching/learning phase in mathematics. This dialogue was registered after the first Lai Math lesson at his Upper Secondary School.

In particular we report what we discussed and what Lai put in evidence about the “two basics” principle and its value for the Chinese teaching approaches and its characteristic.

Researcher: *What do you mean with Two basics?*

Lai: *Two basics means “Basic Knowledge” and “Basic Skills”. According to the Chinese prospective, skills can be developed into knowledge and knowledge can induce skills. So ... knowing and practicing. In fact, skills and knowledge overlap at some examples. You have to understand that there are two kinds of knowledge, one is direct and it is possible to acquire it through direct exploration and investigation. The second kind of knowledge is indirect knowledge. It came from imitation, repetition, memorization ... We have two basics that are the slabs, these slabs connected together by ability. The possible success of problem solving in Math but also in the other subject depends on whether such slabs are connected.*

Researcher: *What do you mean with connection?*

Lai: *Connection of pieces means connection of knowledge. The quality of foundation depends on number of reinforcement and also the quality of the concrete activities ... so we can say that ... the slabs are joined together.*

Trying to better explain his idea, he had drawn the Model of learning cycles in the “two basics” as follow:

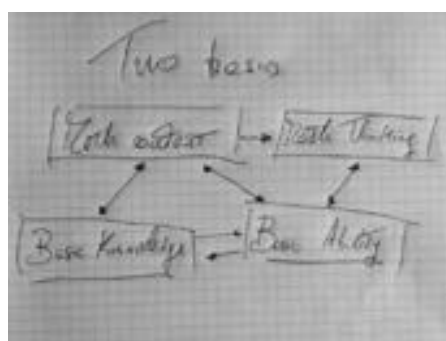


Fig.1 Model of learning cycles in the “two basics” by Lai

After that Lai explains the drawing:

Lai: *If we think to a mathematical content, we can summarise the process in this way. I hope to be clear for you. The first step is imitation, with teacher’s guidance.*

Researcher: *Imitation? What do you mean?*

Lai: *Imitation means students need to observe and internalize, personalize. Ok?*

Researcher: *Yes but in which way?*

Lai: *it depends on teacher and students ... The second step in fact is the intervention by teacher. In this phase we have criticism, correction of concepts, strong examples for the concept, and a brief summary of knowledge learned in a trunk and practice.*

Research: *But how to avoid having practice become mechanical repetition?*

Lai: *because ... yes ... only in this way we can have a real understanding by students. The two basics provide the following principles that are the base of a significant learning:*

1. *memory leads to recognition and become intuition,*
2. *good speed of operation provides grounds for efficient thinking,*
3. *using deduction and reasoning to sustain precise logical thinking,*
4. *rising of standard through variation of problems and learning process.*

Lai: *The third step is then abstraction process by students, which is a kind of internalization and self monitoring. By internalization, students with the help of teachers can connect different knowledge, and deduce new knowledge.*

Researcher: *And what about the internalization and self monitoring? How can facilitate it?*

Lai: *it depends on the teacher. On the examples discussed with students, the time allocated to this activities ... It is difficult, I tried to summarize. There are two levels of teaching. The first one is using daily lives context aimed to capture the interest of students. The second level is the learning process through abstraction. Teachers teach mathematics based on the process of “correspondence, variation, induction and deduction”.*

After a brief pause linked to the difficulties of the dialogue Lai argues:

Lai: *Correspondence means to map the concept of mathematical problems to another problem with variation⁷; it also involves relations of expression, structure, meaning ... We will discuss more about it. It is complex. Tomorrow I will show you some practices in classroom. These practices refer to the “routine” daily practice commonly accepted by Chinese teachers.*

The model of learning cycles in the “two basics” applied to variation problems: an example form Primary School

To better discusses his idea of variation and trying to generalize this Chinese practice Lai shws a mathematics book (published in Japan) for Primary grade and in particular discusses the exercise N.1 (pages 62 and 63) reported below:

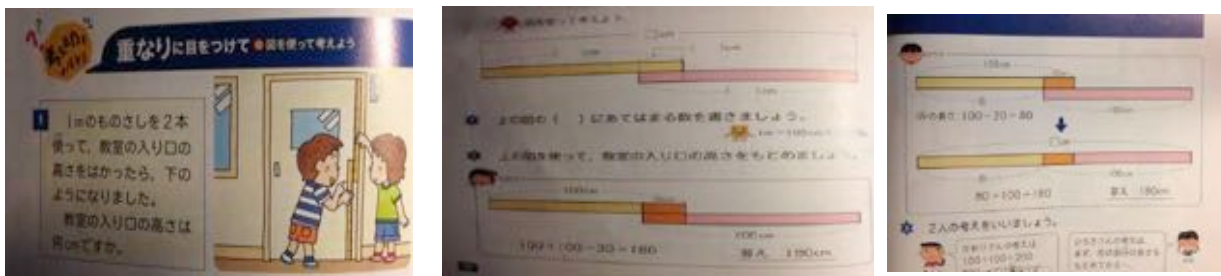


Fig.3 Use of variation problems in Primary textbook (pages 62 and 63)

According to Lai, the typical approach proposed by the text in this exercise is significant to “facilitate” the students strategy solution of the proposed problematic in order to work on their

⁷ In this paper we don't discuss the typical Chinese variation approach to mathematical problem solving. Some interesting didactical considerations on this subject are in Bartolini Bussi, Sun, Ramploud, (2013); Spagnolo, Di Paola (2010); Di Paola, Ramploud (2013).

memorization procedure leads to recognition and intuition; to favour their efficient and logical thinking and to generalize through variation of problems and learning process.

Lai: *In general in a mathematics book it is easy to note how the use of schemes (as the ones proposed in this exercise) is totally linked to a didactics focused on the research of solving methods, rather than on the presentation of isolated contents. This happen in many western book. Do you agree?*

Researcher: *What do you mean?*

Lai: *If you look for example to these pages (he refers to 1 pages 62 and 63) it is clear ... when observing the associated schemes ... the operation of addition and subtraction and the strong interdependence between them is clear, they are proposed in the same context, their use is evident for students. This approach is typical in Chinese curriculum. I think it is not typical in your country. The operation requested to students are linked to the idea of variation as I said before. All Chinese students are able to solve this problem, all are able to read it with different “lent” and to move inside the mathematics calculation requested by the interdependence between the operation of addition and subtraction together. The representation help also students to conceptualize this strongly relation and to generalize it.*

...

Researcher: *And what about the model of “two basics”? Why you shows me this exercise?*

Lai: *Do you remember what I said on the “two basics” model? This is an example of connection of pieces of knowledge. I shows it because this is one of the mail point of the Chinese curriculum, an example of the philosophical idea of two basics to practical activities in classes trough the use of variation.*

...

Lai is right, if we look at a typical Italian textbook (we can generalize to the western textbook), the approach is different. The idea of variation is distant from the western culture, it is instead a key point of the curriculum of China and all the Confucian area for all the grades. It is possible to find some example just from the pre-school grades as showed below.



Fig.3 Use of variation problem in pre-school Chinese textbook

Conclusion

Can we learn from “outside”? Can we *contaminate* (Ramproud, Di Paola, 2013) our typical didactical approaches with something “different” in order to enrich it? We think yes. The tables below tries to summarize what emerged from the dialogue with Lai and in particular discusses some possible key aspects stressed by Lai on the typology of mathematical activities that are typically propose in Chinese classroom and the teaching/learning time related to their implementation according to the two basics approach at School.

Mathematical activities	<ul style="list-style-type: none"> • Activities mainly individual or in a big group, • Learning by imitation: “small steps” teaching instead of free discover, • Great importance to mental calculation and memorization: “practice makes perfect”, • Few importance to proofs, • Strong importance to the relationship between comprehension and manipulation, • Emphasis on the teaching of problem solution schemes.
Time	<ul style="list-style-type: none"> • Students are required to adapt to the pace of progress setted for most students by the teacher, • Teachers present the main mathematics contents as quickly as possible to avoid students spending too much time on winding paths.

Table 1. Typical didactical approaches in China

Is it possible to integrate in our western didactical practices some different activities and teaching proposals coming out from abroad (for example from China) in a sort of continuous dialogue from distances and cultures crossing? We think yes.

The different cultural background in teaching and learning hidden in the Easter context can, according to us, help us to better understand our own background and, on the other side, be pertinent, for example, to better understand the excellent performance in international assessments of Confucian area.

To study similarities and differences between our western typical teaching approaches and the Chinese one could represent for teacher and researchers only the starting point to adapt possible new “multicultural approaches” in own classes.

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