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Nonlinear rocking of rigid blocks on flexible foundation: analysis and experiments

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Abstract

Primarily, two models are commonly used to describe rocking of rigid bodies; the Housner model, and the Winkler foundation model. The first deals with the motion of a rigid block rocking about its base corners on a rigid foundation. The second deals with the motion of a rigid block rocking and bouncing on a flexible foundation of distributed linear springs and dashpots (Winkler foundation). These models are two-dimensional and can capture some of the features of the physics of the problem.
Clearly, there are additional aspects of the problem which may be captured by an enhanced nonlinear model for the base-foundation interaction. In this regard, what it is adopted in this paper is the Hunt-Crossley nonlinear impact force model in which the impact/contact force is represented by springs in parallel with nonlinear dampers. In this regard, a proper mathematical formulation is developed and the governing equations of motion are derived taking into account the possibility of uplifting in the case of strong excitation. The analytical study is supplemented by experimental tests conducted in the Laboratory of Experimental Dynamics at the University of Palermo, Italy. In this context, due to their obvious relevance for historical monuments, free-rocking tests are presented for several marble-block geometries on both rigid and flexible foundations. Numerical vis-à-vis experimental data are reported, supporting the usefulness and reliability of the proposed approach.

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Keywords: Rocking motion; Nonlinear contact model; Flexible foundation;

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1. Introduction

The behavior of block-like structures allowed to rock due to base excitation has been a longstanding problem of technical interest and still attracts the attention of a significant number of researchers.

A number of alternative analytical models have been proposed to study the rocking dynamics. However, two models are primarily used to describe the rocking of rigid bodies subjected to ground motion; they are two-dimensional and afford a reasonable representation of the phenomenon.

The first model, hereinafter referred to as the Housner model (HM) [1], deals with the motion of a rigid block rocking about its base corners on a rigid foundation.

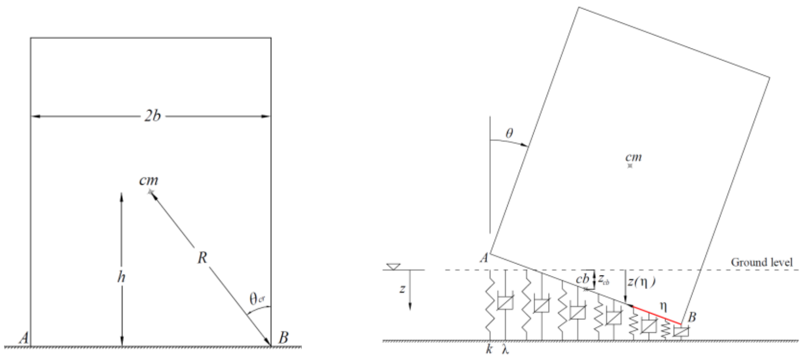
The second model, known as Winkler foundation model (WFM), deals with the motion of a rigid block rocking on a flexible foundation of distributed linear vertical springs and dashpots [2].

Although the rocking phenomenon has been extensively studied, most previous researches have been analytical in nature. Moreover, many experiments on rocking blocks have considered the behavior of rigid blocks on rigid foundations, while the problem of rigid blocks on flexible foundation is less investigated [3].

To further study this complex phenomenon, as well as to take into account the aspects which may arise during the rocking motion of rigid blocks on flexible foundations, in this paper a nonlinear model is used for the base-foundation interaction. Specifically, the Hunt- Crossley nonlinear impact force model [4] is adopted herein; thus, the foundation is treated as a bed of continuously distributed linear tensionless springs in parallel with nonlinear dampers. Note that this model is commonly used in the open literature to represent the nonlinear nature of impact and contact phenomena. The pertinent governing equations of motion are derived taking into account the possibility of uplifting in the case of strong excitation. Further, the analytical study is supplemented by a large number of experiments conducted in the Laboratory of Experimental Dynamics at the University of Palermo, Italy. In this regard, due to their obvious relevance for historical monuments, free-rocking tests are performed for several marble-block geometries on both rigid and flexible foundations. Numerical vis-à-vis experimental results are reported for the proposed model, demonstrating the reliability and accuracy of the proposed formulation.

2. Rocking of a rigid block

Consider a rectangular rigid block with mass m and polar moment of inertia I_{cm} about the center of mass cm, as shown in Fig. 1(a). The variable R is the distance of the base corners from the center of mass, situated at height h above the base of width $2b$. Further, let θ_{cr} be the critical tilt-angle, that is, the maximum angle to which the block can be tilted without overturning under the action of gravity, g, alone.



(a) (b)

Fig. 1: Rocking of a rigid block: a) Block geometry; b) Rocking block on nonlinear flexible foundation.

The block is free to rock and bounce on a flexible foundation. For simplicity, the center of the base cb is restricted to vertical relative motion only [2]. Thus, two generalized coordinates are sufficient to specify the configuration of the block relative to the foundation. Specifically, the vertical displacement z_{cb} of the center of the base cb from the undeformed surface of the foundation (positive downward), and the rotation θ of the block from its static equilibrium position (positive clockwise), are chosen as generalized coordinates. Further, assume that the foundation is exposed to a horizontal acceleration, $\ddot{x}_g$, and vertical acceleration, $\ddot{z}_g$. Then, the dynamic equilibrium of forces with respect to the center of the base *cb* yields the equations of motion as

$$\begin{aligned} & m\ddot{z}_{cb} + mh\left(\dot{\theta}^2 \cos\theta + \ddot{\theta}\sin\theta\right) + F_{cb} - mg = m\ddot{z}_g \\ & \left(I_{cm} + mh^2\right)\ddot{\theta} + M_{cb} + mh\left(\ddot{z}_{cb} - g\right)\sin\theta = mh\left(\ddot{x}_g \cos\theta - \ddot{z}_g \sin\theta\right), \end{aligned} \tag{1.a, b}$$

where a dot over a variable denotes differentiation with respect to time; F_{cb} is the resultant vertical contact force; and M_{cb} is the induced moment of the contact force with respect to *cb*.

Equations (1) are the general equations of rocking when no sliding occurs. Clearly, the rocking phenomenon is highly influenced by the kind of foundation considered, which is accounted for in the terms F_{cb} and M_{cb}. For instance, if the case of a rigid foundation is considered, the well-known Housner model (HM) of the rocking motion can be retrieved from Eqs. (1).

Since the transition from rocking about one corner to the other is accompanied by an impact, the associated energy loss, occurring upon impact, must be taken into account. In the HM the impact dynamics is treated by examining the motion immediately before and after the impact, and introducing the so-called "coefficient of restitution" *r*, given as $r = \dot{\theta}(t^+)/\dot{\theta}(t^-)$, where t^+ and t^- are the time instants just after and before the impact, respectively.

For an enhanced treatment of the rocking dissipation mechanism, the foundation can be considered flexible. In this regard, the Winkler foundation model (WFM), involving a flexible foundation of distributed linear springs and dashpots, is widely used in the literature [2, 5]. This model provides a reasonable tool for capturing the energy dissipation associated with the contact/impact forces. In this context, this paper aims at investigating whether additional aspects of the phenomenon can be captured by a nonlinear model for the base-foundation interaction, as detailed in the next section.

3. Proposed Model

As far as the rocking phenomenon is concerned, considering the Hunt-Crossley model [4], the foundation is treated as a bed of continuously distributed and independent parallel spring and nonlinear dampers, with stiffness coefficient k (force units per unit width of base per unit vertical deformation) and damping coefficient λ (force units per unit width of base per unit vertical deformation velocity and per unit vertical deformation) respectively. With these assumptions, the impact/contact force per unit length is given by $F_n(\eta,t) = k\,z(\eta,t) + \lambda\,z(\eta,t)\,\dot{z}(\eta,t)$, where $z(\eta,t)$ is the vertical displacement of the generic point belonging to the base of the block, which is at distance η from the right edge. In this regard, since the block is rigid, $z(\eta,t) = z_{cb} + b\sin\theta - \eta\sin\theta$ (Fig. 1(b)).

Taking into account the characteristic low tensile strength of soils, each spring-dashpot combination in the model is assumed to debond from the block when the springs are about to be in tension. Therefore, two possible different conditions are permitted. Specifically, uplifting condition occurs as soon as either base corners rise above the ground level. On the other hand, no-uplifting condition takes place when the entire block base is under the ground level. Note that this represents an additional relevant difference with respect to the rigid foundation model.

On this basis, the impact/contact force F_{cb} and the moment M_{cb} in Eqs. (1) can be then calculated as:

for No-uplifting

$$\begin{aligned} & F_{cb} = \int_0^{2b} F_n(\eta,t)\,d\eta = 2bk\,z_{cb} + 2b\lambda\,z_{cb}\dot{z}_{cb} + \frac{2}{3}b^3\lambda\dot{\theta}\sin\theta\cos\theta \\ & M_{cb} = \int_0^{2b} F_n(\eta,t)\left[(b-\eta)\cos\theta\right]d\eta = \frac{2}{3}b^3 k\sin\theta\cos\theta + \frac{2}{3}b^3\lambda\dot{z}_{cb}\sin\theta\cos\theta + \frac{2}{3}b^3\lambda z_{cb}\dot{\theta}\cos^2\theta \end{aligned} \tag{2.a, b}$$

and for uplifting, i.e. when $z_{cb} - b\sin|\theta| < 0$

$$\begin{aligned}
F_{cb} &= \int_{\eta_I}^{\eta_U} F_n(\eta,t)\,d\eta = \frac{1}{2}b^2 k\sin\theta\,\mathrm{sgn}\,\theta + k\,z_{cb}\left(b+\frac{1}{2}\frac{z_{cb}}{\sin\theta}\mathrm{sgn}\,\theta\right)+\frac{1}{2}b^2\lambda\,\dot z_{cb}\sin\theta\,\mathrm{sgn}\,\theta+\lambda z_{cb}\dot z_{cb}\left(b+\frac{1}{2}\frac{z_{cb}}{\sin\theta}\mathrm{sgn}\,\theta\right)\\
&\quad+\frac{1}{3}b^3\lambda\dot\theta\cos\theta\sin\theta+\frac{1}{2}\lambda z_{cb}\dot\theta\cos\theta\left(b^2-\frac{1}{3}\frac{z_{cb}^2}{\sin^2\theta}\right)\mathrm{sgn}\,\theta\\
M_{cb} &= \int_{\eta_I}^{\eta_U} F_n(\eta,t)\left[(b-\eta)\cos\theta\right]d\eta = \frac{b^3}{3}k\cos\theta\sin\theta+\frac{1}{2}k\,z_{cb}\cos\theta\left(b^2-\frac{1}{3}\frac{z_{cb}^2}{\sin^2\theta}\right)\mathrm{sgn}\,\theta+\frac{b^3}{3}\lambda\dot z_{cb}\cos\theta\sin\theta\\
&\quad+\frac{1}{2}\lambda z_{cb}\dot z_{cb}\cos\theta\left(b^2-\frac{1}{3}\frac{z_{cb}^2}{\sin^2\theta}\right)\mathrm{sgn}\,\theta+\frac{1}{4}b^4\lambda\dot\theta\sin\theta\cos^2\theta\,\mathrm{sgn}\,\theta+\frac{1}{3}\lambda z_{cb}\dot\theta\cos^2\theta\left(b^3+\frac{1}{4}\frac{z_{cb}^3}{\sin^3\theta}\mathrm{sgn}\,\theta\right),
\end{aligned}\tag{3.a, b}$$

where $\mathrm{sgn}(\bullet)$ denotes the signum function, yielding the sign of its argument, and the integration limits η_I and η_U in Eq. (3) depend on the different configuration of the block. Specifically, for $\theta<0$ $\eta_I = z_B/\sin\theta$ and $\eta_U = 2b$, while for $\theta>0$ $\eta_I = 0$ and $\eta_U = z_B/\sin\theta$, where $z_B = z_{cb} + b\sin\theta$.

Note that the case of WFM [2] is a restrictive case of the proposed one, with the contact force of the form $F_n(\eta,t) = \bar{k}\,z(\eta,t) + c\,\dot z(\eta,t)$, where the damping coefficient c has units of force per unit width per unit vertical deformation velocity; and $\bar{k}$ is the stiffness coefficient. Thus, taking into account Eqs. (2)-(3), the expressions of F_{cb} and M_{cb} for the WFM can be obtained, leading exactly to the equations of motion reported in [2, 5].

4. Experimental investigation

4.1. Experimental set-up and data acquisition

To assess the usefulness of the preceding formalism, a series of free-rocking experimental tests has been conducted in the Laboratory of Experimental Dynamics at the University of Palermo. Mainly the experimental investigation dealt with the evaluation of the influence of both block geometries and foundation materials on the rocking response. To this aim, three configurations with three different marble blocks heights were considered (Tab. 1) and two different foundation materials were used. Specifically, a marble slab was adopted for simulating rigid foundation condition, and a mat of viscoelastic materials, labeled Aerstop CN20 (currently employed as anti-vibration material), was used to simulate flexible foundation conditions.

For each block on every foundation free-rocking was triggered by releasing the block from an initial rotation angle α, close to the corresponding critical tilt-angle θ_{cr}. In Tab. 1 the various values of α for each configuration are reported.

Table 1. Marble blocks configuration parameters.

	Configuration #1	Configuration #2	Configuration #3
2h	0.42 m	0.28 m	0.14 m
2b	0.07 m	0.07 m	0.07 m
h/b	6	4	2
m	8.67 kg	5.84 kg	2.98 kg
α	6°	10°	20°

As far as the data acquisition systems is concerned, blocks displacements were recorded through a laser sensor, model Micro-Epsilon optoNCDT. Laser voltage signals were acquired and digitalized by means of a National Instruments NI 4497 PXI Acquisition Board provided inside the chassis of a National Instruments PXIe model 1082 (see Fig. 2(c)). Finally the signals were processed and converted to rotation time-histories *θ(t)* in LabView and MATLAB environments. Note that this conversion was possible since neither three-dimensional rotations around the vertical axes nor sliding effects along the horizontal directions were observed during the experimental tests. Further, due to the laser contactless sensor no external disturbances to the free-rocking motion were introduced during the experiments.

Free-rocking experimental results of the three configurations on the two different base materials are shown in Fig. 2. As seen in this figure, free-rocking responses are strongly influenced by the foundation material. Specifically, Fig. 2(a) shows the behavior of the slender marble block with $(h/b=6)$. In this case it appears that the marble foundation leads to much longer free-rocking time-histories, while Aerstop CN20 foundation dissipates vibrations just after few seconds. A similar behavior is also shown in Fig. 2(b) for the marble block with $(h/b=4)$. On the other hand, a different response feature is shown in Fig. 2(c), where the free-rocking experimental result of the squat block with $(h/b=2)$ is reported. In this case, the Aerstop CN20 foundation yields longer free-rocking response with higher amplitudes than the one obtained with the marble foundation.

Thus, it can be argued that rocking behavior cannot be presumptuously treated if viscoelastic materials are used, for instance for vibration isolation of art objects. Counterintuitive responses may in fact be observed for some combinations of particular geometries and material foundations, as in Fig. 2(c). This feature points out the need for further investigations on the theoretical models of rocking, as detailed in the next section.

4.2. Experimental results vis-à-vis numerical simulations

To examine whether additional aspects of the rocking behavior can be captured by introducing the nonlinear flexible foundation model, comparisons among experimental data with the theoretical models previously introduced have been performed. Specifically, model parameters of the HM, WFM and nonlinear flexible foundation (proposed model) have been identified minimizing the mean square error between numerical and experimental data. Therefore, a numerical optimization procedure has been implemented to find the parameters which yield the smallest mean square error for each model. In this regard, the identified parameters are reported in Tab. 2, while Fig. 3 shows the experimental free-rocking responses for Configuration #2 vis-à-vis pertinent numerical results of the three different theoretical models. Clearly, a good agreement is achieved both for the proposed and the Winkler foundation models. Similar results have been also obtained for the other cases (Configurations #1 and #3), here not reported for brevity sake.

Further, to obtain a deeper understanding of the discrepancy between the experimental data and the numerical results of the various models, a marker labeled ε_θ has been introduced as $\varepsilon_\theta = \int_0^{t_f} \left[\theta_{th}(t) - \theta_{ex}(t)\right]^2 dt \Big/ \int_0^{t_f} \theta_{ex}(t)^2\, dt$, where t_f is the final time instant; the subscripts *th* and *ex* stand for numerical and experimentally measured, respectively. The values of ε_θ for each theoretical model and different foundation materials are reported in Tab. 3 for Configuration #2.

These findings show that both the proposed and the Winkler foundation models can predict the rocking responses of a rigid block on rigid and flexible foundations. However, with very flexible foundation (Aerstop CN20) an enhanced nonlinear model, as the one proposed, may be adopted to capture additional features of the rocking behavior.

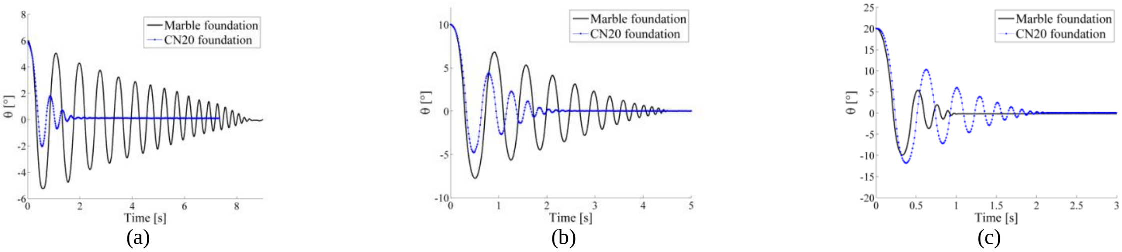
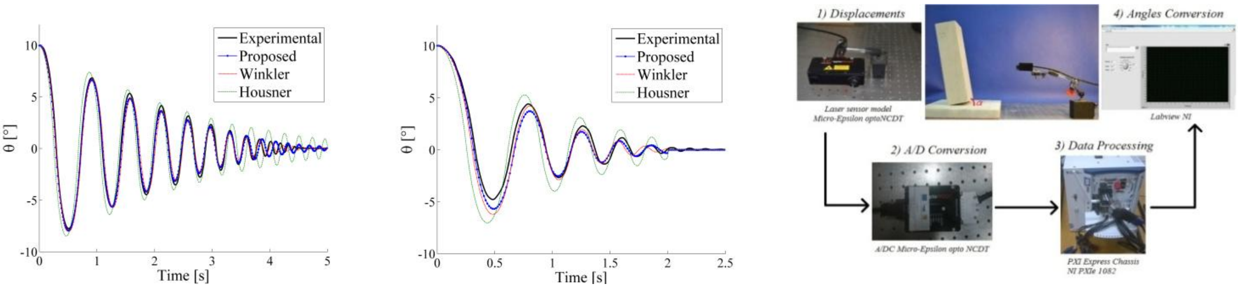


Fig. 2: Effect of the foundation material on the free-rocking responses: a) Configuration #1; b) Configuration #2; c) Configuration #3. Continuous black line – Marble foundation; Blue dashed line – Aerstop CN20 foundation.

Table 2. Values of the identified parameters.

	Marble foundation	Aerstop CN20 foundation
Housner's model	$r=0.958$	$r=0.895$
Winkler foundation model	$\bar{k}=6.77\cdot10^6$; $c=2.17\cdot10^4$	$\bar{k}=4.50\cdot10^6$; $c=4.05\cdot10^4$
Proposed model	$k=6.88\cdot10^6$; $\lambda=1.30\cdot10^8$	$k=2.86\cdot10^6$; $\lambda=8.16\cdot10^7$

Further, similar analyses can be performed in terms of the coefficient of restitution. Specifically, the coefficient of restitution of the i-*th* impact, can be readily computed from the experimental and the numerical data. The results show that the proposed theoretical model always yields a better prediction of the coefficient of restitution for all the foundation materials, herein omitted due to text-space limitations.

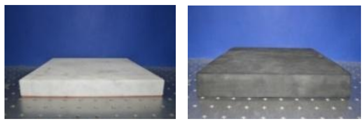


(a) (b) (c)

Fig. 3: Comparison of experimental vis-à-vis numerical results in time domain for Configuration #2 and test set-up: a) Marble foundation; b) Aerstop CN20 foundation; c) test set-up.

Table 3. Values of ε_θ for Configuration #2 and various foundation materials.

	Marble foundation	Aerstop CN20 foundation
Housner's model	15.25%	21.66%
Winkler foundation model	2.45%	4.58%
Proposed model	1.40%	2.87%



5. Concluding Remarks

A study on the rocking response of a rigid block resting on a nonlinear flexible foundation has been made. To account for the additional features of the rocking behavior on very flexible foundations, a novel nonlinear model has been proposed for the base-foundation interaction, based on the Hunt-Crossley nonlinear impact force model. Specifically, the foundation has been treated as a bed of continuously distributed vertical springs in parallel with nonlinear dampers. The governing equations of motion have been derived taking into account the possibility of uplifting in the case of strong base excitation. It has been shown that the governing equations reduce to the classical equations of motion of the Winkler foundation model for an appropriate choice of the relevant parameters.

Further, an extensive experimental study has been conducted to validate the proposed model. In this regard, free-rocking tests have been carried out for three marble-block geometries on both rigid and flexible foundations. The data have pointed out the strong influence of the foundation materials on the rocking dynamics. Further, it has been shown that the combination of particular geometries and material foundations may lead to complex, and somewhat counterintuitive, responses. Furthermore, numerical vis-à-vis experimental results have been reported for the proposed model in the time domain. The results have shown the reliability and accuracy of the proposed model for all the tests performed. Finally, comparisons with the Housner model and the Winkler foundation model, commonly used in the literature, have shown that the Winkler foundation model, as well, is able to predict free-rocking responses. Nevertheless, for very flexible foundations, the proposed model can, solely, capture additional aspects of the rocking dynamics.

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