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Detector's quantum backaction effects on a mesoscopic conductor and fluctuation-dissipation relation

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When measuring quantum mechanical properties of charge transport in mesoscopic conductors, backaction effects occur. We consider a measurement setup with an elementary quantum circuit, composed of an inductance and a capacitor, as detector of the current flowing in a nearby quantum point contact. A quantum Langevin equation for the detector variable including backaction effects is derived. Differences with the quantum Langevin equation obtained in linear response are pointed out. In this last case, a relation between fluctuations and dissipation is obtained, provided that an effective temperature of the quantum point contact is defined.

1 Introduction

Probing a quantum system implies disturbing its state according to the Heisenberg uncertainty principle. Measurements on a mesoscopic system require quantum detectors, and measurement-induced disturbances result in quantum backaction. Research on quantum electronics has progressed to the point where backaction effects, often near to the limit imposed by the uncertainty relations, are of key relevance to experiments [1–5]. This is the case of nanoelectromechanical systems where quantum electronic conductors have been used as position detection of nanomechanical oscillators [6–11]. Analogous backaction effects occur when measuring quantum mechanical properties of charge transport in mesoscopic conductors. In fact, to perform time-resolved detection of the quantum mechanical current in a quantum transport process, mesoscopic on-chip detectors are required. Some effects of the detector backaction have been addressed already in the literature [12–18] also in connection with qubit measurements [19–29], a relevant issue for quantum networking [30–32].

In the present work we address the quantum backaction effects of a mesoscopic detector on a prototype quantum conductor, a quantum point contact (QPC) consisting of two metallic leads driven out of equilibrium by a static voltage bias which establishes a tunneling current [33–35]. We model the detector as a dissipative quantum LC circuit which is coupled inductively to the QPC [36–42]. In this scheme, the detector is continuously weakly coupled to the mesoscopic conductor. Measurement-induced disturbances on the QPC affect the detector. These are the focus of our work. We derive a Quantum Langevin Equation (QLE) for the charge on the capacitor's plates, corresponding to the x coordinate of the quantum oscillator, accounting for backaction effects. The QLE, derived perturbatively in the LC-QPC coupling, presents non trivial damping and frictional terms in addition to the traditional ones entering the QLE of a dissipative quantum harmonic oscillator [43, 44]. We compare this equation with the QLE for our dissipative detector coupled to the QPC obtained in linear response. In this case the QPC's force noise is not related to the QPC damping kernel via the temperature, as it would be for an equilibrium system [43, 45–47], and

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references therein]. However, similarly to other analyses [15, 48–50], we find that, in linear response, for each given frequency, an effective temperature can be defined. In the present work, the measured system is a non-linear and non-equilibrium system. This places our work in the intriguing and timely research field investigating connections among quantum measurements, fluctuations theorems, and non-equilibrium systems [51–57].

The paper is organized as follows. In the next section we introduce the model. The full Hamiltonian consists of three parts, namely the dissipative resonant circuit described by the Caldeira-Leggett model, the QPC part, and the inductive QPC-detector coupling. In Sec. 3 the Heisenberg equation for the QPC current including backaction contributions is solved and the full QLE is derived. Within linear response theory, we derive a QLE analogous to that found for a classical variable whose average is the expectation value of the operator x [15, 49]. In Sec. 4, a fluctuation-dissipation relation for our system, in linear response regime, is derived provided that an effective temperature related to the QPC is introduced. Finally, in Sec. 5 we draw the conclusions.

2 Model

The Hamiltonian of our measurement setting is the sum of the dissipative LC circuit term, the QPC term, and the LC-QPC interaction term

$$H = H_{\rm LC} + H_{\rm QPC} + H_{\rm int}. \tag{1}$$

The dissipative LC circuit is modeled by a quantum harmonic oscillator of position and momentum operators x and p, respectively, where x is the charge in the capacitor of the LC circuit. The oscillator is linearly interacting with a dissipative environment at finite temperature, which is modeled as a thermal reservoir, or heat bath, of N independent quantum harmonic oscillators of coordinates x_j and momenta p_j. The coupling with the heat bath is not constrained to be small and, to keep the discussion as general as possible, we do not specify a particular spectral density for the oscillators in what follows [44, 58, 59]. The corresponding Hamiltonian is the celebrated Caldeira-Leggett model [60]

$$H_{\rm LC} = \frac{p^2}{2M} + \frac{1}{2}Dx^2 + \frac{1}{2}\sum_j \left[\frac{p_j^2}{m_j} + m_j\omega_j^2\left(x_j - \frac{g_j}{m_j\omega_j^2}x\right)^2\right]. \tag{2}$$

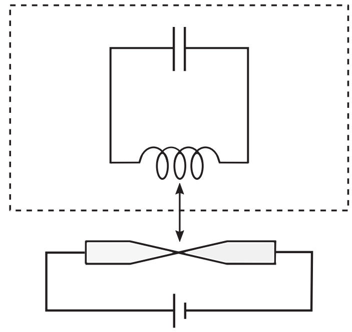

Figure 1 Scheme of the LC oscillator (upper part) coupled to the quantum point contact with external bias $eV = \mu_L - \mu_R$ (lower part). The dashed box indicate the dissipative environment in which the oscillator is embedded.

Identifying the bare oscillator mass M with the inductance L, and the coefficient D with the inverse of the capacitance C yields the resonance frequency of the LC circuit $\Omega = \sqrt{1/LC} = \sqrt{D/M}$.

The second term in Hamiltonian (1) is the QPC part

$$H_{\rm QPC} = \sum_{r\in R} E_r c_r^\dagger c_r + \sum_{l\in L} E_l c_l^\dagger c_l + \hbar\sum_{r,l}\Delta_{r,l}\left(c_l^\dagger c_r + c_r^\dagger c_l\right). \tag{3}$$

This Hamiltonian has free left (L) and right (R) lead parts plus a tunneling term with energy-dependent tunneling frequencies Δ_{rl}. Creation and annihilation operators obey Fermi anticommutation relations.

Finally, the inductive LC-QPC interaction in Hamiltonian (1) couples the oscillator coordinate x with the derivative of the QPC current [36, 40]

$$H_{\rm int} = \alpha x \dot{I}, \tag{4}$$

where α is the inductive coupling strength, with dimension $e^{-2}\omega^{-1}\hbar$ [36, 40].

In what follows we assume that the heat bath is initially in the canonical thermal state at temperature $T_{\rm osc}$ and then the coupling with the LC oscillator is turned on. Similarly, we assume for the leads an initial canonical thermal state with temperature T. At $t = t_0$ the coupling α is switched on and a voltage bias V, which keeps left and right leads at chemical potentials μ_L and μ_R, with $eV = \mu_L - \mu_R > 0$, is applied (see Fig. 1).

3 Quantum Langevin equation for the LC circuit coupled to the QPC

In order to take into account backaction effects we derive an equation for the quantum mechanical evolution of the *whole* system formed by the detector and the measured mesoscopic conductor. An analogous point of view has been taken to address the dynamics of the measurement process in quantum dot systems [61, 62]. The quantum Langevin equation for the LC circuit is derived from the second time derivative for the x operator whose evolution is induced by the full Hamiltonian (1). The Heisenberg equation for x is

$$\dot{x} = \frac{i}{\hbar}[H, x] = \frac{p}{M}. \tag{5}$$

By replacing this equation into $\ddot{x} = \frac{i}{\hbar}[H, \dot{x}]$, we get

$$M\ddot{x} = -Dx + \sum_j g_j \left(x_j - \frac{g_j}{m_j\omega_j^2}x\right) + \frac{i}{\hbar}\alpha\left(x[\dot{I}, p] + i\hbar\dot{I}\right). \tag{6}$$

Further, by replacing the solution of the Heisenberg equations for the coordinates x_j of the bath oscillators into Eq. (6) the quantum Langevin equation for the coordinate of the oscillator coupled to the QPC is obtained ($t_0 = 0$)

$$\begin{aligned} M\ddot{x} + M\int_0^t dt'\gamma(t-t')\dot{x}(t') + Dx - \frac{i}{\hbar}\alpha\left(x[\dot{I}, p] + i\hbar\dot{I}\right) \\ = \xi(t), \end{aligned} \tag{7}$$

where the friction memory kernel reads [44]

$$\gamma(t) = \Theta(t)\frac{1}{M}\sum_j \frac{g_j^2}{m_j\omega_j^2}\cos(\omega_j t). \tag{8}$$

The bath force operator is given by

$$\begin{aligned} \xi(t) = \sum_j g_j\left[x_j(0)\cos(\omega_j t) + \frac{p_j(0)}{m_j\omega_j}\sin(\omega_j t)\right] \\ -M\gamma(t)x(0). \end{aligned} \tag{9}$$

Note that the presence of the *slippage* term, dependent on the initial position of the oscillator, is an artifact due to the choice of a factorized initial condition with the harmonic bath in the thermal equilibrium state. Upon choosing a shifted thermal bath described by the density matrix

$$\begin{aligned} \rho_B = \frac{1}{Z}\exp\Bigg\{-\beta_{\rm osc}\sum_j\Bigg[\frac{p_j^2}{2m_j} \\ +\frac{m_j\omega_j^2}{2}\left(x_j - \frac{g_j}{m_j\omega_j^2}x(0)\right)^2\Bigg]\Bigg\}, \end{aligned} \tag{10}$$

the bath force operator satisfies $\langle\xi(t)\rangle = 0$ [43, 44].

Eq. (7) is the starting point of our analysis. In the following we will derive the QPC current derivative operator including the backaction effect of the meter (LC circuit) on the measured system (QPC).

3.1 Evaluation of the QPC current

The dynamics of the detector, described by the degree of freedom x, depends on the variables of the system to be measured. Here we derive the current operator I and, via its Heisenberg equation, $\dot{I}$. We will distinguish terms describing the current flowing in the QPC in the absence of any coupling with the detector from terms due to backaction effects of the detector on the QPC.

The current I flowing from the left to the right lead of the QPC is given by

$$\begin{aligned} I = \frac{i}{\hbar}[H_{\rm QPC} + H_{\rm int}, Q_R] \\ \equiv I_0 + I_{ba}, \end{aligned} \tag{11}$$

where $Q_R = e\sum_{r\in R} c_r^\dagger c_r$ is the charge on the right lead. Note that I is the sum of two terms: The first is the current which would flow in the QPC in the absence of the detector

$$I_0 = \sum_{r,l} ie\Delta_{rl}\left(c_l^\dagger c_r - c_r^\dagger c_l\right) \equiv \sum_{r,l} I_{0,rl}, \tag{12}$$

and the second is the backaction current

$$I_{ba} = \alpha x\mathscr{I}_{ba}, \qquad \text{where} \qquad \mathscr{I}_{ba} = \frac{i}{\hbar}[\dot{I}, Q_R]. \tag{13}$$

From Eq. (11) the time derivative of the current operator reads $\dot{I} = \dot{I}_0 + \dot{I}_{ba}$, where

$$\dot{I}_0 = \frac{i}{\hbar}[H_{\rm QPC}, I_0] + \frac{i}{\hbar}[H_{\rm int}, I_0] \equiv \dot{I}_0^{(0)} + \delta\dot{I}_0, \tag{14}$$

and

$$\dot{I}_{ba} = \frac{i}{\hbar}[H_{\rm QPC} + H_{\rm LC}, \alpha x\mathscr{I}_{ba}] + \frac{i}{\hbar}[H_{\rm int}, \alpha x\mathscr{I}_{ba}]. \tag{15}$$

Note that, whereas I_0 is of order zero in α, its time derivative $\dot{I}_0$ is the sum of the operators $\dot{I}_0^{(0)}$ (of order zero in α) and $\delta\dot{I}_0$ which includes higher orders in α (see Eq. (14)). In other words, the variation in time of the unperturbed current in the QPC depends on the detector's variables. Both terms, $\dot{I}_{ba}$ and $\delta\dot{I}_0$, represent the backaction effect in the current derivative. We keep these terms separate for comparison with the linear response regime addressed in Section 4.

Up to now everything is exact. To leading order in α the QPC current is given by

$$I \approx I_0 + \alpha x \mathscr{I}_{ba}^{(0)}, \tag{16}$$

where $\mathscr{I}_{ba}^{(0)} = i[\dot{I}_0^{(0)}, Q_R]/\hbar$ (see Eq. (13)). The leading order backaction effect in Eq. (7) is obtained by approximating $\dot{I}$ up to linear order in α. We find

$$\dot{I} \approx \dot{I}_0^{(0)} + \alpha x \left(\dot{\mathscr{I}}_0 + \dot{\mathscr{I}}_{ba}^{(0)}\right) + \alpha \dot{x} \mathscr{I}_{ba}^{(0)}, \tag{17}$$

where we approximated $\delta\dot{I}_0 \approx \alpha x \dot{\mathscr{I}}_0$, with $\dot{\mathscr{I}}_0 \equiv i[\dot{I}_0^{(0)}, I_0]/\hbar$. The terms appearing in Eq. (17) take, in the tunneling limit (second order in Δ) [33], the following form

$$\begin{aligned} \dot{I}_0^{(0)} &= \mathrm{e} \sum_{r,l} \Delta_{rl}\omega_{rl} \left(c_l^\dagger c_r + c_r^\dagger c_l\right) \\ &\quad + 2\mathrm{e} \sum_{r,l,l'} \Delta_{rl}\Delta_{rl'} c_l^\dagger c_{l'} - 2\mathrm{e} \sum_{r,r',l} \Delta_{rl}\Delta_{r'l} c_r^\dagger c_{r'} \end{aligned} \tag{18}$$

$$\begin{aligned} \dot{\mathscr{I}}_0 &= \frac{\mathrm{e}^2}{\hbar} \sum_{r,l} \Delta_{rl} \Bigg[\sum_{l'} \omega_{rl'} \Delta_{rl'} \left(c_l^\dagger c_{l'} + c_{l'}^\dagger c_l\right) \\ &\quad - \sum_{r'} \omega_{r'l} \Delta_{r'l} \left(c_r^\dagger c_{r'} + c_{r'}^\dagger c_r\right) \Bigg] \end{aligned} \tag{19}$$

$$\mathscr{I}_{ba}^{(0)} = \frac{i}{\hbar}\mathrm{e}^2 \sum_{r,l} \Delta_{rl}\omega_{rl} \left(c_l^\dagger c_r - c_r^\dagger c_l\right) = \frac{e}{\hbar} \sum_{rl} \omega_{rl} I_{0,rl} \tag{20}$$

$$\dot{\mathscr{I}}_{ba}^{(0)} = \frac{e^2}{\hbar} \sum_{r,l} \Delta_{rl}\omega_{rl}^2 \left(c_l^\dagger c_r + c_r^\dagger c_l\right) + \dot{\mathscr{I}}_0 \,, \tag{21}$$

where $\omega_\lambda \equiv E_\lambda/\hbar$ with $\lambda = l, r$, $\omega_{rl} \equiv \omega_r - \omega_l$, and $I_{0,rl}$ is given by Eq. (12). We remark that, for energy independent tunneling amplitudes $\Delta_{rl} \equiv \Delta$, and assuming the leads at equal temperatures, the second order approximation is meaningful only in the presence of an applied bias, $\mu_L \neq \mu_R$. This is signalled by the vanishing of the thermal averages of Eqs. (18)–(21) under the above conditions and $V = 0$.

To obtain the explicit solution for $\dot{I}(t)$ we take the time derivative of Eq. (17). We find for each rl component ($\dot{I} = \sum_{rl} \dot{I}_{rl}$)

$$\ddot{I}_{rl} = \ddot{I}_{0,rl}^{(0)} + \alpha x \ddot{\mathscr{I}}_{ba,rl}^{(0)} + \alpha \dot{x} \left(\dot{\mathscr{I}}_{0,rl} + 2\dot{\mathscr{I}}_{ba,rl}^{(0)}\right) + \alpha \ddot{x} \mathscr{I}_{ba,rl}^{(0)}, \tag{22}$$

where we considered that the first non vanishing term in $\ddot{\mathscr{I}}_0$ is $O(\Delta^3)$. The same happens with the last two terms of the expression for $\dot{I}_0^{(0)}$ in Eq. (18). By taking this fact into account and calculating the time derivatives of $\dot{I}_0^{(0)}$ and $\dot{\mathscr{I}}_{ba,rl}^{(0)}$ via the Heisenberg equations we find

$$\ddot{I}_{0,rl}^{(0)} + \alpha x \ddot{\mathscr{I}}_{ba,rl}^{(0)} \approx -\omega_{rl}^2 (I_{0,rl}^{(0)} + \alpha x \mathscr{I}_{ba,rl}^{(0)}) \approx -\omega_{rl}^2 I_{rl} \,, \tag{23}$$

where in the last equality we used Eq. (16). Thus Eq. (22) can be cast in the form

$$\ddot{I}_{rl} = -\omega_{rl}^2 I_{rl} + \alpha \dot{x} \left(\dot{\mathscr{I}}_{0,rl} + 2\dot{\mathscr{I}}_{ba,rl}^{(0)}\right) + \alpha \ddot{x} \mathscr{I}_{ba,rl}^{(0)}, \tag{24}$$

which is readily solved by Laplace transform to give the rl component of the QPC current

$$\begin{aligned} I_{rl}(t) &= I_{rl}(0)\cos(\omega_{rl}t) + \frac{1}{\omega_{rl}} \dot{I}_{rl}(0) \sin(\omega_{rl}t) \\ &\quad + \frac{\alpha}{\omega_{rl}} \int_0^t dt' \dot{x}(t') \left(\dot{\mathscr{I}}_{0,rl} + 2\dot{\mathscr{I}}_{ba,rl}^{(0)}\right)(t') \sin[\omega_{rl}(t-t')] \\ &\quad + \frac{\alpha}{\omega_{rl}} \int_0^t dt' \ddot{x}(t') \mathscr{I}_{ba,rl}^{(0)}(t') \sin[\omega_{rl}(t-t')]. \end{aligned} \tag{25}$$

3.2 Quantum Langevin equation including the QPC backaction current

Taking the derivative of Eq. (25) with respect to t, we obtain the rl component of the operator $\dot{I}$ appearing in the QLE (7) ($t > 0$)

$$\begin{aligned} \dot{I}_{rl}(t) &= -\omega_{rl} I_{rl}(0)\sin(\omega_{rl}t) + \dot{I}_{rl}(0)\cos(\omega_{rl}t) \\ &\quad + \alpha \int_0^t dt' \dot{x}(t') \left(\dot{\mathscr{I}}_{0,rl}(t') + 2\dot{\mathscr{I}}_{ba,rl}^{(0)}(t')\right) \cos[\omega_{rl}(t-t')] \\ &\quad + \alpha \int_0^t dt' \ddot{x}(t') \mathscr{I}_{ba,rl}^{(0)}(t') \cos[\omega_{rl}(t-t')]. \end{aligned} \tag{26}$$

This solution, summed over r and l, can be replaced in Eq. (7). By integrating by parts the last term in (26), the QLE in the full Hilbert space, including backaction, takes on the final form ($t > 0$)

$$
\begin{aligned}
&M\ddot{x}(t) + M\int_0^t dt'\gamma(t-t')\dot{x}(t') + Dx(t) \\
&\quad - \alpha^2 x(t)\int_0^t dt' \sum_{rl} X_{rl}(t,t')\cos[\omega_{rl}(t-t')] \\
&\quad + \alpha^2\int_0^t dt'\dot{x}(t')\sum_{rl}\Big\{\Big(\dot{\mathscr{I}}_{0,rl}(t') + \dot{\mathscr{I}}^{(0)}_{ba,rl}(t')\Big) \\
&\quad \times\cos[\omega_{rl}(t-t')] \\
&\quad - \mathscr{I}^{(0)}_{ba,rl}(t')\omega_{rl}\sin[\omega_{rl}(t-t')]\Big\} + \alpha^2\dot{x}(t)\mathscr{I}^{(0)}_{ba}(t) \\
&= \xi(t) + \alpha\sum_{rl}\big(I_{rl}(0)\omega_{rl}\sin(\omega_{rl}t) - \dot{I}_{rl}(0)\cos(\omega_{rl}t)\big) \\
&\quad + \alpha^2\dot{x}(0)\sum_{rl}\mathscr{I}^{(0)}_{ba,rl}(0)\cos(\omega_{rl}t). \qquad (27)
\end{aligned}
$$

The operators X_{rl} in Eq. (27) act in the full QPC-LC detector Hilbert space and read

$$
\begin{aligned}
X_{rl}(t,t') \equiv{}& \frac{i}{\hbar}[\dot{x}(t'),p(t)]\Big(\dot{\mathscr{I}}_{0,rl}(t') + 2\dot{\mathscr{I}}^{(0)}_{ba,rl}(t')\Big) \\
&+ \frac{i}{\hbar}[\ddot{x}(t'),p(t)]\mathscr{I}^{(0)}_{ba,rl}(t'). \qquad (28)
\end{aligned}
$$

Taking the average with respect to the factorized thermal state of the leads, which for lead $\Lambda = R, L$ reads $e^{-\beta(H_\Lambda - \mu_\Lambda N_\Lambda)}/Z_\Lambda$ (where $Z_\Lambda = Tr_\Lambda\{e^{-\beta(H_\Lambda-\mu_\Lambda N_\Lambda)}\}, H_\Lambda = \sum_{\lambda\in\Lambda} E_\lambda c^\dagger_\lambda c_\lambda$, $N_\Lambda = \sum_{\lambda\in\Lambda} c^\dagger_\lambda c_\lambda$), from Eq. (27) we get the following QLE in the Hilbert space of the dissipative oscillator ($t > 0$)

$$
\begin{aligned}
&M\ddot{x}(t) + M\int_0^t dt'\gamma(t-t')\dot{x}(t') + Dx(t) \\
&\quad - \alpha^2 x(t)\int_0^t dt' \sum_{rl} \langle X_{rl}(t,t')\rangle\cos[\omega_{rl}(t-t')] \\
&\quad + \alpha^2\int_0^t dt'\dot{x}(t')\sum_{rl}\Big\{\langle\dot{\mathscr{I}}_{0,rl}(t') + \dot{\mathscr{I}}^{(0)}_{ba,rl}(t')\rangle \\
&\quad \times\cos[\omega_{rl}(t-t')] \\
&\quad - \langle\mathscr{I}^{(0)}_{ba,rl}(t')\rangle\omega_{rl}\sin[\omega_{rl}(t-t')]\Big\} + \alpha^2\dot{x}(t)\langle\mathscr{I}^{(0)}_{ba}(t)\rangle \\
&= \xi(t) + \alpha\sum_{rl}\langle I_{rl}(0)\omega_{rl}\sin(\omega_{rl}t) - \dot{I}_{rl}(0)\cos(\omega_{rl}t)\rangle\,, \qquad (29)
\end{aligned}
$$

where we considered that $\langle\mathscr{I}^{(0)}_{ba,rl}(0)\rangle = 0$ (see Appendix A). Eq. (29) is the central result of this work. The detector, considered as an open quantum system in contact with a heat bath including backaction effects on the measured system, obeys a generalized QLE. It is a non-linear equation due to the presence of detector's variables entering $\langle X_{rl}(t,t')\rangle$, in the second line of Eq. (29). We interpret terms in the third and fourth lines as a QPC contribution to the friction memory kernel. In the second member of Eq. (29) we find a stochastic force contribution from the QPC.

3.3 Quantum Langevin Equation in linear response

In this section we derive the Quantum Langevin equation for the LC detector in linear response regime. To this end we identify $\dot{I}$ in the interaction Hamiltonian Eq.(4) with the unperturbed current in the QPC, $\dot{I}^{(0)}_0$, that is

$$
H_{\rm int} \approx \alpha x \dot{I}^{(0)}_0\,. \qquad (30)
$$

Under these conditions, the effect on the meter of the current flowing in the QPC can be obtained following the same procedure used to solve the Heisenberg equations for the harmonic oscillators, namely by solving

$$
\dddot{I}^{(0)}_{0,rl} = -\omega^2_{rl}\dot{I}^{(0)}_{0,rl} - \omega^2_{rl}\alpha x\dot{\mathscr{I}}_{0,rl}\,. \qquad (31)
$$

Its solution is formally similar to that for a heath bath oscillator driven by the coordinate of the particle, namely

$$
\begin{aligned}
\dot{I}^{(0)}_{0,rl}(t) ={}& -\omega_{rl}I_{0,rl}(0)\sin(\omega_{rl}t) + \dot{I}^{(0)}_{0,rl}(0)\cos(\omega_{rl}t) \\
&+ \alpha\int_0^t dt'\dot{x}(t')\dot{\mathscr{I}}_{0,rl}(t')\cos[\omega_{rl}(t-t')] \qquad (32) \\
&- \alpha x(t)\dot{\mathscr{I}}_{0,rl}(t) + \alpha x(0)\dot{\mathscr{I}}_{0,rl}(0)\cos(\omega_{rl}t),
\end{aligned}
$$

where we disregarded terms from the derivative of $\dot{\mathscr{I}}_0$, to order Δ^2 (see Appendix B for an outline of the derivation). Inserting this solution into Eq. (7) and taking the average with respect to the thermal state of the QPC, we end up with the following QLE in the Hilbert space of the dissipative oscillator

$$
\begin{aligned}
&M\ddot{x}(t) + M\int_0^t dt'\gamma(t-t')\dot{x}(t') + \big(D - D_{\rm QPC} - \langle D(t)\rangle\big)\,x(t) \\
&\quad + \alpha^2\int_0^t dt'\dot{x}(t')\sum_{rl}\langle\dot{\mathscr{I}}_{0,rl}(t')\rangle\cos[\omega_{rl}(t-t')] \qquad (33) \\
&= \xi(t) + \langle\xi_{\rm QPC}(t)\rangle - \alpha^2 x(0)\sum_{rl}\langle\dot{\mathscr{I}}_{0,rl}(0)\rangle\cos(\omega_{rl}t).
\end{aligned}
$$

Here $D_{\rm QPC}$ describes a renormalization of the LC potential. In fact, by including time dependent terms to order Δ^2 in Eq.(19) (see Appendix A), it follows that

$$D_{\text{QPC}} \equiv 2\alpha^2 \sum_{rl} \langle \dot{\mathscr{I}}_{0,rl}(t)\rangle = \alpha^2 \frac{4e^2}{\hbar} \sum_{rl} \Delta^2_{rl}\omega_{rl}\left[f_L(\omega_l) - f_R(\omega_r)\right] , \quad (34)$$

where $f_\Lambda(\omega) = \{\exp[\beta(\hbar\omega - \mu_\Lambda)] + 1\}^{-1}$ is the Fermi function for the Λ-lead. Also in this case the QLE is of non-linear form due to the additional contribution

$$D(t) = \alpha^2 \frac{i}{\hbar}\Big\{[x(0), p(t)] \sum_{rl} \dot{\mathscr{I}}_{0,rl}(0)\cos(\omega_{rl}t) + \int_0^t dt'[\dot{x}(t'), p(t)] \sum_{rl} \dot{\mathscr{I}}_{0,rl}(t')\cos[\omega_{rl}(t-t')]\Big\}. \quad (35)$$

On the other side, because of the mentioned time independence of $\langle \dot{\mathscr{I}}_{0,rl}(t)\rangle$ used in Eq.(34), the dissipative term in Eq. (33) is convolutive and allows to cast the QLE in linear response in the form

$$M\ddot{x}(t) + M\int_0^t dt'\gamma(t-t')\dot{x}(t') + \big(D - D_{\text{QPC}} - \langle D(t)\rangle\big)\, x(t) + M\int_0^t dt'\gamma_{\text{QPC}}(t-t')\dot{x}(t') = \xi(t) + \langle \xi_{\text{QPC}}(t)\rangle - M\gamma_{\text{QPC}}(t)x(0), \quad (36)$$

where the QPC memory damping kernel $\gamma_{\text{QPC}}(t)$ is given by

$$\gamma_{\text{QPC}}(t) \equiv \frac{\alpha^2}{M}\Theta(t)\sum_{rl}\langle \dot{\mathscr{I}}_{0,rl}(0)\rangle \cos(\omega_{rl}t) = \alpha^2 \frac{2\mathrm{e}^2}{M\hbar}\Theta(t)\sum_{rl}\Delta^2_{rl}\omega_{rl}\left[f_L(\omega_l) - f_R(\omega_r)\right]\cos(\omega_{rl}t) \quad (37)$$

and the stochastic force $\xi_{\text{QPC}}(t)$ by

$$\xi_{\text{QPC}}(t) \equiv \alpha\sum_{rl}\left(\omega_{rl}I_{0,rl}(0)\sin(\omega_{rl}t) - \dot{I}^{(0)}_{0,rl}(0)\cos(\omega_{rl}t)\right). \quad (38)$$

Notice that this stochastic force contribution is the same as the one entering the QLE (29). Therefore it is not related to backaction effects. On passing, we observe that the slippage term dependent on $x(0)$ of the RHS of Eq. (36) is analogous to that of Eq. (9).

4 Fluctuation-dissipation relation and effective temperature

The QLE (36) for the detector in linear response includes, in addition to damping and fluctuating force due to the equilibrium bath of harmonic oscillators, analogous contributions due the QPC, similarly to [15, 49]. Considering that $\langle\xi(t)\rangle = 0$, the correlation function of the full force $\xi(t) + \xi_{\text{QPC}}(t)$ has no cross terms of mixed origin (heat bath and QPC). Since the QPC is a non-equilibrium system, we cannot expect that the spectrum of the QPC stochastic force and the corresponding dissipative term are related by the standard equilibrium relation holding for the thermal bath

$$\bar{S}_\xi(\omega) = \hbar\omega\coth\left(\frac{\hbar\omega}{2K_BT}\right)\tilde{\gamma}'_\xi(\omega)\,, \quad (39)$$

where $\bar{S}_\xi(\omega)$ is the symmetrized quantum noise spectral density of $\xi(t)$. Neverthless, an indication of the asymmetry of the QPC's quantum noise can be obtained by defining, for any given frequency, an effective temperature [15, 49], $T_{\text{eff}}(\omega)$, via the relation

$$\bar{S}_{\xi_{\text{QPC}}}(\omega) = M\hbar\omega\coth\left(\frac{\hbar\omega}{2K_BT_{\text{eff}}(\omega)}\right)\tilde{\gamma}'_{\text{QPC}}(\omega)\,. \quad (40)$$

The QPC damping kernel in the continuum limit reads

$$\gamma_{\text{QPC}}(t) = \Theta(t)\frac{2\alpha^2}{M\hbar}e^2\int d\omega'\int d\omega''\omega''\rho_L(\omega')\rho_R(\omega'+\omega'') \times\Delta^2(\omega'')\left[f_L(\omega') - f_R(\omega'+\omega'')\right]\cos(\omega''t)\,. \quad (41)$$

The real part of the Fourier transform of $\gamma_{\text{QPC}}(t)$ is

$$\tilde{\gamma}'_{\text{QPC}}(\omega) = \pi\omega\frac{\alpha^2}{M\hbar}\mathrm{e}^2\Delta^2(\omega)\int d\omega'\rho_L(\omega') \times\Big\{\left[\rho_R(\omega'-\omega)f_R(\omega'-\omega) - \rho_R(\omega'+\omega)f_R(\omega'+\omega)\right] + f_L(\omega')\left[\rho_R(\omega'+\omega) - \rho_R(\omega'-\omega)\right]\Big\}. \quad (42)$$

In the above expressions $\Delta(\omega)$ is the continuum limit of Δ_{rl}, and $\rho_\Lambda(\omega)$ denotes the density of states in the lead Λ. By using the expression for the QPC force operator given in Eq. (38), we can calculate the symmetrised noise

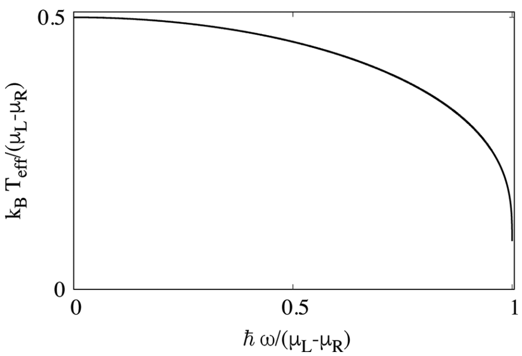

Figure 2 Behavior of the effective temperature defined by Eq. (46) for small frequencies $\omega < (\mu_L - \mu_R)/\hbar$. It is a result of different excitation and relaxation rates of the oscillator caused by the shot noise in the QPC for zero temperature of the leads.

spectrum of the QPC force operator $\bar{S}_{\xi_{\rm QPC}}(\omega)$

$$\begin{aligned}\bar{S}_{\xi_{\rm QPC}}(\omega) &= \int_{-\infty}^{+\infty} dt \langle \xi_{\rm QPC}(t)\xi_{\rm QPC}(0)\rangle \cos(\omega t)\\ &= \omega^2\pi\alpha^2 {\rm e}^2\Delta^2(\omega)\int d\omega' \rho_L(\omega')\\ &\quad\times\Big\{\rho_R(\omega'-\omega)\big[f_L(\omega')\left(1-f_R(\omega'-\omega)\right)\\ &\quad+f_R(\omega'-\omega)\left(1-f_L(\omega')\right)\big]\\ &\quad+\rho_R(\omega'+\omega)\big[f_L(\omega')\left(1-f_R(\omega'+\omega)\right)\\ &\quad+f_R(\omega'+\omega)\left(1-f_L(\omega')\right)\big]\Big\},\end{aligned} \tag{43}$$

where we neglected the squared average of the operator $\dot{I}_0^{(0)}$, which is of order Δ^4.

In the limit $T \to 0$, assuming a constant density of states $\rho_\Lambda = \Gamma$ around $\omega < (\mu_L - \mu_R)/\hbar$, Eq. (43) yields

$$\bar{S}_{\xi_{\rm QPC}}(\omega) \approx 2\pi\omega^2\alpha^2\Delta^2(\omega)\Gamma^2(\mu_L-\mu_R)/\hbar. \tag{44}$$

Under the same conditions, the real part of the damping kernel in Fourier space reads

$$\tilde{\gamma}'_{\rm QPC}(\omega) \approx \frac{2\pi}{M\hbar}\omega^2 {\rm e}^2\alpha^2\Delta^2(\omega)\Gamma^2. \tag{45}$$

The effective temperature $T_{\rm eff}(\omega)$, resulting of the non-equilibrium fluctuations that arise during the evolution of the *entire* system follows from Eq. (40) and is given by

$$\coth\left(\frac{\hbar\omega}{2K_B T_{\rm eff}(\omega)}\right) \approx \frac{\mu_L-\mu_R}{\hbar\omega}. \tag{46}$$

The frequency dependence of the effective temperature resulting from Eq. (46) is reported in Fig. 2. Under stationary conditions, $\omega \to 0$, $T_{\rm eff}(\omega)$ reduces to

$$T_{\rm eff}(\omega\to 0) \approx \frac{\mu_L-\mu_R}{2k_B}. \tag{47}$$

Analogously to single electron transistor and tunnel junction detectors [48, 63, 64], the effective temperature at zero frequency is proportional to the applied source-drain voltage drop V, that is $K_B T_{\rm eff} \approx {\rm e}V$. Physically, the finite effective temperature entering the QLE is a result of different excitation and relaxation rates of the oscillator caused by the shot noise in the QPC when the leads are at zero temperature. For a tunnel junction it has been shown [48] that the effective oscillator temperature is responsible for a quadratic term in the I-V characteristic. An analogous backaction effect can be expected in our case, but it is beyond the scope of the present paper.

In concluding this section we note that, by considering the full QLE (29) with backaction terms, additional contributions to the memory kernel are involved which display nontrivial time dependencies, while the QPC force operator is left untouched. Therefore, an analogous effective temperature can not be defined when considering backaction effects in our non-equilibrium and non-linear system.

5 Conclusions

In the present work we addressed the quantum backaction effects of a mesoscopic detector on the tunneling current in a QPC, a prototype quantum conductor. The detector has been modelled as a dissipative quantum LC circuit inductively coupled to the QPC as in Refs. [36, 40]. In those articles no backaction effect was included, whereas dissipation in the resonant circuit measuring finite-frequancy current moments was the subject of Ref. [40]. Measurement-induced disturbances on the QPC are originated by the continuos and weak meter-QPC coupling and we found that they enter both the backaction current *and* its derivative. These backaction effects, treated in lowest order in the coupling strength, enter the non-linear QLE for the dissipative resonator, Eq. (29), which is the main result of this work. Backaction gives rise to non-trivial damping and frictional terms in the QLE. We also derived the QLE in linear response. In this case, the QPC's force noise can be related to the damping kernel by a frequency-dependent effective temperature. Interestingly, the same QPC stochastic force enters the QLE in linear response and QLE including backaction effects. However, due to the more involved damping contributions originated by backaction, the stochastic force noise and damping kernel can not be related via a similar relation. A further step of our work, currently in progress, consists in evalating the role of measurement induced disturbances (backaction) on measurable quantities, like the second current cumulant both under stationary conditions and at finite frequencies, extending the analysis of Ref. [40].

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A Time evolution of the QPC operators

The Heisenberg equation for the QPC operator c_l, to zero order in α, is

$$\dot{c}_l = -i\omega_l c_l - i\sum_{r'} \Delta_{r'l} c_{r'}. \tag{A.1}$$

Eq. (A.1) has solution

$$\begin{aligned} c_l(t) &= c_l(0)e^{-i\omega_l t} - i\sum_{r'} \Delta_{r'l} \int_0^t dt' e^{-i\omega_l(t-t')} c_{r'}(t') \\ &\simeq c_l(0)e^{-i\omega_l t} - i\sum_{r'} \Delta_{r'l} c_{r'}(0) \int_0^t dt' e^{-i\omega_l(t-t')} e^{-i\omega_{r'} t'}, \end{aligned} \tag{A.2}$$

where, in passing to the second line, we replaced the similar solution for $c_r(t)$ taken to zero order in Δ.

By substituting Eq. (A.2) and the analogous expression for $c_r(t)$ (and their Hermitian conjugates) into Eq. (20) we get, to order Δ^2,

$$\begin{aligned} \mathscr{I}^{(0)}_{ba}(t) = \frac{i}{\hbar}e^2 \sum_{rl} \Delta_{rl}\omega_{rl} \Bigg\{ & c_l^\dagger(0)c_r(0)e^{-i\omega_{rl}t} - h.c. \\ & - \sum_{l'(\neq r)} \Delta_{rl'} \left[c_l^\dagger(0)c_{l'}(0)\frac{e^{-i\omega_{l'l}t} - e^{-i\omega_{rl}t}}{\omega_{rl'}} - h.c. \right] \\ & + \sum_{r'(\neq l)} \Delta_{r'l} \left[c_{r'}^\dagger(0)c_r(0)\frac{e^{-i\omega_{rr'}t} - e^{-i\omega_{rl}t}}{\omega_{r'l}} - h.c. \right] \Bigg\}. \end{aligned} \tag{A.3}$$

Similarly, by replacing the solutions for $c^\dagger(t)$ and $c(t)$ into Eq. (21), the time derivative of the operator $\mathscr{I}^{(0)}_{ba}$, to order Δ^2, reads

$$\begin{aligned} \dot{\mathscr{I}}^{(0)}_{ba}(t) = \frac{e^2}{\hbar} \sum_{rl} \Delta_{rl}\omega_{rl}^2 \Bigg\{ & c_l^\dagger(0)c_r(0)e^{-i\omega_{rl}t} + h.c. \\ & - \sum_{l'(\neq r)} \Delta_{rl'} \left[c_l^\dagger(0)c_{l'}(0)\frac{e^{-i\omega_{l'l}t} - e^{-i\omega_{rl}t}}{\omega_{rl'}} + h.c. \right] \\ & + \sum_{r'(\neq l)} \Delta_{r'l} \left[c_{r'}^\dagger(0)c_r(0)\frac{e^{-i\omega_{rr'}t} - e^{-i\omega_{rl}t}}{\omega_{r'l}} + h.c. \right] \Bigg\} \\ + \frac{e^2}{\hbar} \sum_{r,l} \Delta_{rl}\omega_{rl} \Bigg\{ & \sum_{l'} \Delta_{rl'} \left[c_l^\dagger(0)c_{l'}(0)e^{-i\omega_{l'l}t} + h.c. \right] \\ & - \sum_{r'} \Delta_{r'l} \left[c_r^\dagger(0)c_{r'}(0)e^{-i\omega_{rr'}t} + h.c. \right] \Bigg\}. \end{aligned} \tag{A.4}$$

The average value with respect to the equilibrium QPC thermal state selects the terms with $l' = l$ and $r' = r$ in Eq. (A.4). As a result we have

$$\begin{aligned} \langle \dot{\mathscr{I}}^{(0)}_{ba}(t) \rangle &= \frac{2}{\hbar} e^2 \sum_{rl} \Delta_{rl}^2 \omega_{rl} \left[f_L(\omega_l) - f_R(\omega_r) \right] \cos(\omega_{rl} t) \\ &= \sum_{rl} \langle \dot{\mathscr{I}}_{0,rl}(t) \rangle \cos(\omega_{rl} t), \end{aligned} \tag{A.5}$$

where, in passing to the second line we used the explicit expression for $\dot{\mathscr{I}}_0$ given in Eq. (19). Note that, up to second order in Δ, this average value is constant in time. This is easily seen by replacing in Eq. (19) the solutions for the QPC creation and annihilation operators to zero order in Δ given by Eq. (A.2).

B Time derivative of the QPC current in linear response

Here we outline the derivation of Eq. (32), namely the solution for the time derivative of the current operator in linear response. Eq. (31) is obtained by taking twice the time time derivative of $\dot{I}^{(0)}_{0,rl}$, the rl component of Eq. (18). This is done by means of the Heisenberg equation $\dot{A} = i/\hbar[H_{\rm QPC} + H_{\rm int}, A]$, where the interaction Hamiltonian features $\dot{I}^{(0)}_0$ itself (see Eq. (30)). We get

$$\ddot{I}^{(0)}_{0,rl} = ie\Delta_{r,l}(-\omega_{rl}^2)\left(c_l^\dagger c_r - c_r^\dagger c_l \right). \tag{B.1}$$

The time derivative $\dddot{I}^{(0)}_{0,rl}$ of the above operator is obtained via the commutator $i/\hbar[H_{\rm QPC} + H_{\rm int}, \ddot{I}^{(0)}_{0,rl}]$ which yields Eq. (31). This equation can be formally solved, for example by Laplace transform, and has solution

$$\begin{aligned} \dot{I}^{(0)}_{0,rl}(t) =& \frac{\ddot{I}^{(0)}_{0,rl}(0)}{\omega_{rl}} \sin(\omega_{rl} t) + \dot{I}^{(0)}_{0,rl}(0)\cos(\omega_{rl} t) \\ & - \alpha\omega_{rl}\int_0^t dt' x(t') \dot{\mathscr{I}}_{0,rl}(t') \sin[\omega_{rl}(t-t')]. \end{aligned} \tag{B.2}$$

Now, by comparing Eq. (B.1) with Eq. (12) one finds the relation $\ddot{I}^{(0)}_{0,rl}(0)/\omega_{rl} = -\omega_{rl} I_{0,rl}(0)$. Using this relation for the first term (RHS) of Eq. (B.2), integrating by parts the third term (RHS), and neglecting the time derivative of $\dot{\mathscr{I}}_{0,rl}$, Eq. (B.2) can be cast in the form of Eq. (32).

Key words. Quantum backaction, mesoscopic conductor, quantum Langevin equation, fluctuation-dissipation relation.

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