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Probabilistic Evaluation of the Adaptation Time for Structures under Seismic Loads

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Abstract

In this paper, a probabilistic approach for the evaluation of the adaptation time for elastic perfectly plastic frames is proposed. The considered load history acting on the structure is defined as a suitable combination of quasi-statical loads and seismic actions. The proposed approach utilizes the Monte Carlo method in order to generate a suitable large number of seismic acceleration histories and for each one the related load combination is defined. Furthermore, for each load combination the related adaptation time is determined, if any, as the optimal one for which the structure is able to shakedown under the unamplified applied actions. A known generalized Ceradini's theorem is utilized. The adaptation time values obtained with reference to all the generated seismic acceleration histories for which the shakedown occurs allows us to define the related cumulative conditioned probability function and, therefore, to identify the optimal adaptation time as the one with a probability not lower than a suitably assigned value.

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Keywords: Dynamic shakedown; seismic loading; adaptation time; probabilistic approach;

1. Introduction

An elastic plastic structure, subjected to time-dependent variable loads, exhibits an elastic shakedown behavior if, after a first (transient) phase characterized by a time interval (adaptation time), during which it can suffer some limited amount of plastic deformations, in the subsequent phase no further plastic strains are generated and it has the ability to eventually respond in a purely elastic manner to any subsequent load condition.

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For an assigned structure and a known load history, the main interest within this topic is the determination of the maximum limit load multiplier for which the structure shakes down. Several methods devoted to the reaching of such a multiplier value have been proposed; they are based on the relevant (static or dynamic) shakedown theory and it has been shown that the obtainable value depends on the adaptation time.

In the present paper, a special approach devoted to the search of the optimal value of the adaptation time is proposed. We will assume that the structure be subjected to a combination of a quasi-static load history and a seismic action. For such a load model, the dynamic shakedown problem is based on the Ceradini's theorem [1,2]. The quasi-static load history will be considered as unknown, time-dependent and acting for the entire lifetime of the structure, although defined within an assigned (deterministic) admissibility load domain. The seismic action will be modelled as a random process, acting in a limited time interval and starting at any instant of the lifetime of the structure. In order to probabilistically describe the seismic load history, reference will be made to a Monte Carlo approach [3], which allow us to generate a sufficiently large number of time acceleration histories. For each realization a related load combination is defined. Making reference to the load combinations for which the structure shakes down, an appropriate minimum adaptation time problem is proposed [4]. The solution to the minimum problem, obtained for all the generated samples, provides the adaptation time cumulative conditioned probability function. The optimal value can be chosen as the one with a probability not lower than a suitably assigned value. Some applications related to plane steel frames conclude the paper.

2. Fundamentals and position of the problem

Let us refer to a plane frame constituted by Euler-Bernoulli beams. The material is modelled as elastic perfectly plastic and the limit behavior of the elements is evaluated at the end cross sections, where rigid perfectly plastic hinges are placed, and it is described by an appropriate piece-wise linearized yield surface, function of the axial force *N* and of the bending moment *M*. The actions are represented by nodal loads, by dead loads applied on the beams and by seismic actions. As an example, in Fig. 1a a two spans-three floors frame is represented.

Let us assume that the nodal and the gravity loads, even if they are quasi-static actions, are time dependent too and they permanently act on the structure during its lifetime; on the contrary, the seismic loads act on the structure just for a short time interval randomly situated within the structure's lifetime.

For later use, it is convenient to report the equations governing the purely elastic response of the structure to the quasi-static load history at typical time $t \in \left(0,+\infty\right)$:

$$\boldsymbol{d}_s(t) = \boldsymbol{C}\boldsymbol{u}_s(t), \quad \boldsymbol{Q}_s(t) = \boldsymbol{D}\boldsymbol{d}_s(t) + \boldsymbol{Q}_s^*(t), \quad \boldsymbol{C}^T\boldsymbol{Q}_s(t) = \boldsymbol{F}(t), \tag{1}$$

where $\boldsymbol{d}_s(t)$, $\boldsymbol{Q}_s(t)$ and $\boldsymbol{Q}_s^*(t)$ are displacement, generalized stress and perfectly clamped generalized stress vectors evaluated at the beam ends, respectively, $\boldsymbol{u}_s(t)$ is the structure node displacement vector, $\boldsymbol{C}$ is the compatibility matrix, $\boldsymbol{C}^T$ the equilibrium one and $\boldsymbol{D}$ the block diagonal matrix collecting the beam element stiffness. The solution to problem (1) is given by:

$$\boldsymbol{u}_s(t) = \boldsymbol{K}^{(-1)}\boldsymbol{F}^*(t), \quad \boldsymbol{Q}_s(t) = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u}_s(t) + \boldsymbol{Q}_s^*(t) = \boldsymbol{D}\boldsymbol{C}\boldsymbol{K}^{(-1)}\boldsymbol{F}^*(t) + \boldsymbol{Q}_s^*(t), \tag{2}$$

where $\boldsymbol{K} = \boldsymbol{C}^T\boldsymbol{D}\boldsymbol{C}$ is the frame external stiffness square matrix and $\boldsymbol{F}^*(t) = \boldsymbol{F}(t) - \boldsymbol{C}^T\boldsymbol{Q}_s^*(t)$ is the equivalent nodal force vector.

Analogously, the equations governing the purely elastic response of the structure subjected to seismic actions are:

$$\boldsymbol{d}_e(\tau) = \boldsymbol{C}\boldsymbol{u}_e(\tau), \quad \boldsymbol{Q}_e(\tau) = \boldsymbol{D}\boldsymbol{d}_e(\tau) + \boldsymbol{Q}_e^*(\tau), \quad \boldsymbol{M}\ddot{\boldsymbol{u}}_e(\tau) + \boldsymbol{B}\dot{\boldsymbol{u}}_e(\tau) + \boldsymbol{K}\boldsymbol{u}_e(\tau) = -\boldsymbol{M}\boldsymbol{\tau}a_g(\tau), \tag{3}$$

for all $\tau \in \left(0,T_e\right)$, being T_e the length of the seismic time history, $\boldsymbol{\tau}$ the influence vector, $a_g(\tau)$ the horizontal ground acceleration and where $\boldsymbol{d}_e(\tau)$, $\boldsymbol{Q}_e(\tau)$ and $\boldsymbol{u}_e(\tau)$ are displacement, generalized stress vectors evaluated at the beam element ends and structure node displacement vector due to the earthquake, respectively. $\boldsymbol{M}$ and $\boldsymbol{B}$ are lumped mass and damping matrices. As usual, the solution to problem (3) can be obtained by means of a modal analysis, well known procedure here skipped for the sake of brevity.

In order to characterize the special acting load combination, we assume that the quasi-static loads are variable and time-dependent, although defined within a given deterministic load domain, while the ground acceleration is a classical random time-dependent function. For such a load combination, the shakedown theory provides useful tools to predict the structural response. First of all, by virtue of the Ceradinis theorem [4], we can state that: *a necessary and sufficient condition for dynamic shakedown is that there exist a finite time (separation time)* $r \geq 0$, *a free-motion stress field (defined for* $t \geq r$ *) and a time-independent self-stress field such that the sum of these stresses with the purely elastic stress response to the given load history proves to be inside the yield surface at any time,* $t \geq r$. As a consequence of the above stated theorem, we can state that: *if there exists a separation time r, every time of the interval* $J(r) = \{t : t \geq r\}$ *is a separation time. In addition, for a structure subjected to a specified load history, there exist some initial conditions for which the structure adapt within the minimum possible time (minimum adaptation time) and this minimum coincides with the shortest separation time*.

With the aim of probabilistically characterizing the minimum adaptation time t_{mat} , let us firstly suppose that the seismic load history be known, together with its start time t_{ie} and its duration T_e . Furthermore, let us define the plastic potential vector $\boldsymbol{\varphi}_s$ related to the structure subjected just to the quasi-static loads

$$\boldsymbol{\varphi}_s\left(\boldsymbol{P}_s,\boldsymbol{Y}\right) \equiv \boldsymbol{P}_s - \boldsymbol{SY} - \boldsymbol{R}, \tag{4}$$

with $\boldsymbol{S}$ time-independent matrix that transforms the non-negative plastic activation intensities $\boldsymbol{Y}$ into plastic potentials [5,6], $\boldsymbol{R}$ plastic resistance vector and $\boldsymbol{P}_s$ vector that identifies the maximum plastic demand on the rigid perfectly plastic hinges domains:

$$P_s = max_i\ \boldsymbol{P}_{si}, \quad \boldsymbol{P}_{si} = \boldsymbol{N}^T \boldsymbol{Q}_{si}, \quad (i = 1,2,..,n) \tag{5}$$

n being the basic loads which fully define the admissible quasi-static load domain.

Moreover, we define the new plastic potential $\boldsymbol{\varphi}_e$ related to the combination of the quasi-static loads with the seismic action and the free vibrations due to the (unknown) initial conditions $\boldsymbol{x}_a(0)$, $\dot{\boldsymbol{x}}_a(0)$:

$$\boldsymbol{\varphi}_e\left(\boldsymbol{P}_s,\boldsymbol{P}_{fv}(\tau),\boldsymbol{P}_e(r+\tau),\boldsymbol{Y}\right) \equiv \boldsymbol{P}_s + \boldsymbol{P}_{fv}(\tau) + \boldsymbol{P}_e(r+\tau) - \boldsymbol{SY} - \boldsymbol{R}, \quad \tau \in (r,T_e) \tag{6}$$

where $\boldsymbol{P}_{fv}(\tau)$ and $\boldsymbol{P}_e(r+\tau)$ are the plastic demand analogous to $\boldsymbol{P}_s$ but related to the free vibration history and to the seismic load history, respectively.

Let us now generate a large number of time acceleration histories by means of a Monte Carlo method: the minimum adaptation time for the typical k^{th} time acceleration history $a_g^k(\tau)$ can be determined by solving the following problem:

$$t_{mat}^k = \min_{(r,\boldsymbol{x}_a(0),\boldsymbol{x}_a(0),Y)} r \tag{7a}$$

subject to:

$$\boldsymbol{\varphi}_s\left(\boldsymbol{P}_s,\boldsymbol{Y}\right) \leq \boldsymbol{0} \tag{7b}$$

$$\boldsymbol{\varphi}_e\left(\boldsymbol{P}_s,\boldsymbol{P}_{fv}(\tau),\boldsymbol{P}_e(r+\tau),\boldsymbol{Y}\right) \leq \boldsymbol{0}, \quad \tau \in (r,T_e) \tag{7c}$$

$$\boldsymbol{Y} \geq \boldsymbol{0} \tag{7d}$$

The obtained large number *m* of minimum adaptation times t_{mat}^k (k=1,2,…,m) are random and they allow us to define the corresponding cumulative probability function. The related optimal value can be chosen as the one for which the probability for the shakedown of the structure is not lower than a suitably fixed high value.

3. Results and Discussions

As a case study, the simple three-floor plane steel frame shown in Fig. 1a is considered. The geometry of the frame is fully described by the span lengths (here assumed as L_1=600 cm, L_2=400 cm) and by the inter-storey height (H=400 cm). The material is assumed having an elastic perfectly plastic constitutive behavior and it is therefore completely described by the yield stress σ_y=235 Mpa and the Young's modulus E=210 Gpa. All the elements of the considered frame have box cross sections (Fig. 1b) with width b=200 mm, height h=300 mm and constant thickness t=6 mm. For the chosen cross sections, a particular convex yielding domain of the typical plastic hinge, posed in terms of interaction between the bending moment M and the axial stress N, is defined (Fig 1c). For the dynamic analyses, a floor-wise lumped mass model is adopted. In particular, the mass of each floor $m_f = q_0 \left(L_1 + L_2 \right) / g$ has been concentrated just in the intermediate node of the two spans, where g is the gravitational acceleration, $q_j = q_0 = 30$ kN/m (j=1,2,3) is the dead loads and (L_1+ L_2) is the total length of each floor.

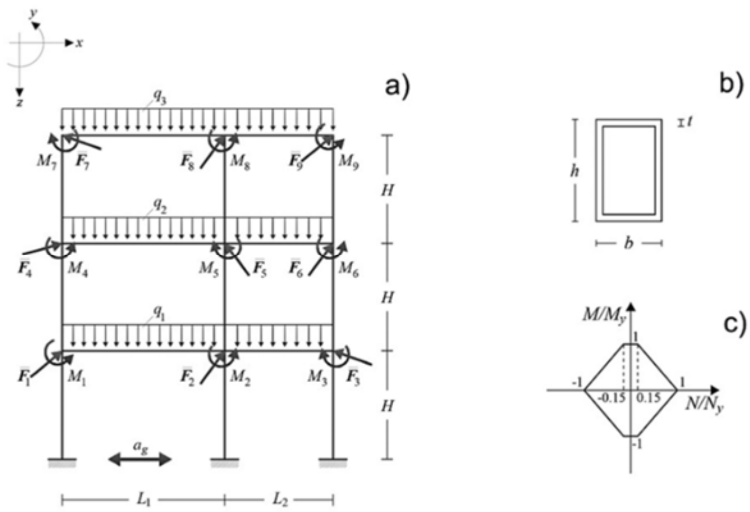

Fig. 1. (a) flexural frame: geometry and load condition; (b) typical box cross section; (c) rigid plastic domain of the typical plastic hinge.

Therefore, just the horizontal displacement of the intermediate node of each floor has been chosen as dynamically significant degree of freedom. Furthermore, it is assumed that the structure is subjected to perfect cyclic quasi-static horizontal loads applied on all the nodes (wind effect) described by the vector $\boldsymbol{F}_{cw}^T = \left| 25.05\ 29.32\ 33.18 \right|$ (kN), where the typical component F_{cwj} is the resultant horizontal force at the j^{th} floor.

The generic k^{th} ground acceleration history $a_g^k(\tau)$ affecting the structure has been generated as a filtered uniformly modulated zero mean Gaussian process. The filter set is the one proposed by Clough and Penzien and the site parameters are those suggested by the same authors for stiff soil, whereas the white noise intensity it is chosen as $S_0 = 0.0029$ m^2·sec^{-3} (see e.g. [7]). Assuming a time length of 30 secs for the generic seismic acceleration, the Iwan-Hou modulating function has been tuned in such a way that the time instant where the maximum is attained is equal to 8 sec. As one can see, the minimum adaptation time search problem for the typical k^{th} time acceleration history $a_g^k(\tau)$ described by problem (7) is a strongly non-linear one involving unbounded vector variables $\boldsymbol{x}_a(0)$ and $\dot{\boldsymbol{x}}_a(0)$, and positive plastic activation intensity variables $\boldsymbol{Y}$. In order to efficiently solve this problem, the arbitrary free vibration contribution is dropped. In this way the founded solution will be an upper bound to the real one but in any case it provides an essential information for engineering problems. Furthermore, a special heuristic algorithm (not reported here for the sake of brevity) based on an outer-inner loop procedure has been coded to find the approximate solution. In estimating the failure probability of the system, a total of 1,000 simulations have been used in running the Monte Carlo simulation algorithm. For each realization of time acceleration history $a_g^k(\tau)$, the minimum adaptation time search problem (7) has been solved, therefore yielding a realization of the probabilistic minimum adaptation time distribution T_{mat} (indicated here with the capital letter T in order to distinguish it from its deterministic counterpart).

Fig. 2 shows the distribution of T_{mat}. This distribution provides invaluable information on the sensitivity of the probabilities associated with the time necessary to the adaptation. In particular, making reference to Fig. 2, the lifetime conditioned probability of the structure to adapt with a time lower than the maximum peak of the forcing function (8 sec) is of 6.2%. It should be noted that in calculating these values the external loads are not amplified; therefore, the stochastic excitation rigorously retains the spectral characteristics of the loading model.

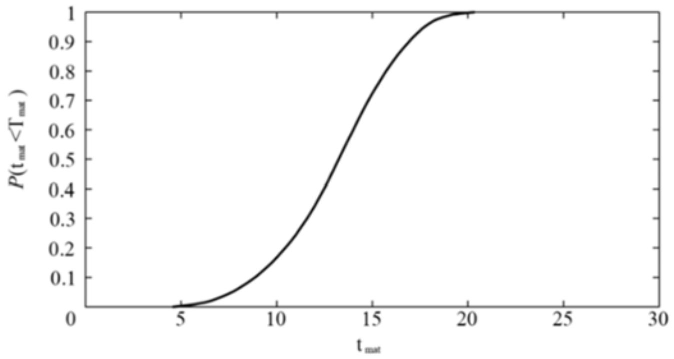

Fig. 2 Distribution of minimum adaptation times.

4. Conclusions

In the present paper, a special approach devoted to the search of the optimal value of the adaptation time for elastic perfectly plastic structures subjected to suitable combinations of quasi-static and seismic loads has been proposed. The cited actions are all considered as unknown and time dependent, in such a way that no distinctions between quasi-static and dynamic shakedown need to be made. In particular, the structure is thought as subjected to an unknown time-dependent history of quasi-static loads, defined within an assigned admissibility load domain and acting for the entire lifetime of the structure and, simultaneously, to a typically random time-dependent seismic load history. The proposed approach is devoted to the simple case of elastic perfectly plastic plane frame and it has been based on the Ceradini's dynamic shakedown theorem. As usual, the quasi-static loads have been described by means of deterministic values while, due to the natural random features of the seismic load, the latter has been treated by a probabilistic approach, basing on a Monte Carlo method. The numerical application related to a three-floor plane steel frame confirms the theoretical expectations and it provide very meaningful results on the structure sensitivity and safety.

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