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EFFECTS OF COLORED NOISE IN SHORT OVERDAMPED JOSEPHSON JUNCTION

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We investigate the transient dynamics of a short overdamped Josephson junction with a periodic driving signal in the presence of colored noise. We analyze noise induced phenomena, specifically resonant activation and noise enhanced stability. We find that the positions both of the minimum of RA and maximum of NES depend on the value of the noise correlation time τ_c. Moreover, in the range where RA is observed, we find a non-monotonic behavior of the mean switching time as a function of τ_c.

Keywords: Josephson junction; colored noise; resonant activation; noise enhanced stability.

1. Introduction

The study of transient dynamics of Josephson junctions (JJs) in the presence of thermal noise is of great importance for the development of rapid single flux quantum (RSFQ)[1] logic. RSFQ technology is mainly based on integrated circuits composed by superconducting quantum interference devices (SQUIDs). The high velocity of processing of RSFQ devices is a very interesting characteristic for their use as logic components of integrated circuits. Moreover JJs are also good candidates as basis components of a solid state quantum computer. In addition, SQUID can be used as a storage element of the single magnetic flux quanta (SFQs), that represent the data bits for logic computation.[2] The commuting time of the JJs is related to the mean life time of the metastable states and to the de-coherence phenomena in quantum computation.[3] In this framework a great attention was payed to the study of the mean life time of metastable states of JJs.[4–6]

Noise induced effects were experimentally observed in underdamped JJs.[7,8] Resonant activation (RA) and noise enhanced stability (NES) phenomena were found in a model of a JJ,[5,6] by considering different values of driving frequency and noise intensity. The curves representing RA and NES showed significant modifications when polarization current changes.

The transient dynamics of long overdamped JJs, under the influence of white noise, was investigated through numerical simulation using the sine-Gordon model.[9] Due to the presence of metastable states, noise delayed decay effects were observed in the range of parameters useful for practical application, depending on the length of the junctions. The results were obtained considering small noise intensity and short junction lengths. In this range of parameters the mean switching time (MST), that is the mean escape time from the metastable state, shows a maximum as a function of the noise intensity. For dimensionless lengths $L > 5$ (where $L = l/\lambda_J$, with l the junction length and λ_J the Josephson penetration length) the noise delayed decay effect disappears. These results can be useful for the design of devices based on Josephson junctions. Short overdamped JJs were studied under the influence of white noise and fluctuating potential by numerical simulation.[10] RA and NES phenomena were observed. In particular it was investigated the NES effect, finding a range of frequency in which the maximum of MST is very prominent. Moreover the presence of NES effect results to be influenced by the variation of the bias current.

In the present work we report the study of the transient dynamics in short JJs. We perform numerical simulation using the Resistively Shunted Junction model, considering short overdamped JJs under the influence of a fluctuating potential. Noise induced effects, such as RA and NES phenomena, in a JJ in the presence of a more realistic noise source possessing a finite correlation time (colored noise) are investigated (see Ref. 11 and references therein).

2. Model

The Langevin equation describing the dynamics of a short overdamped JJ under the influence of colored noise is[12]

$$\frac{d\phi}{dt} = -\omega_c \frac{dU(\phi)}{d\phi} - \omega_c \zeta(t). \tag{1}$$

In Eq. (1), ϕ represents the order parameter, that is the phase difference of the wave functions in the ground state between left and right superconductive sides of the junction. The characteristic frequency of the Josephson junction is $\omega_c = 2eR_N I_c/\hbar$, where e is the electron charge, R_N^{-1} is the normal conductivity, I_c is the critical current and $\hbar = h/2\pi$ with h the Plank constant. $\zeta(t)$ is an Ornstein–Uhlenbeck (OU) process,[13] characterized by a correlation time τ_c, representing a colored noise source. The potential profile $U(\phi)$, is given by

$$U(\phi) = 1 - \cos\phi - i(t)\phi, \tag{2}$$

where $i(t) = i_0 + f(t)$, $i_0 = i_b/I_c$ is the constant dimensionless bias current and $f(t) = A\sin\omega t$ is the driving current with dimensionless amplitude $A = i_s/I_c$ and frequency ω (i_b and i_s represent the bias current and the driving current amplitude respectively).

The OU process of Eq. (1) is represented by the stochastic differential equation[13]

$$d\zeta(t) = -\frac{1}{\tau_c}\zeta(t)dt + \frac{\sqrt{\gamma}}{\tau_c}dW(t) \tag{3}$$

where *W(t)* is the Wiener process and γ is the noise intensity. The correlation function of the OU process is

$$\langle\zeta(t)\zeta'(t)\rangle = \frac{\gamma}{2\tau_c}e^{-\frac{|t-t'|}{\tau_c}}. \tag{4}$$

Equation (1) is a stochastic differential equation, with a time-dependent nonlinear periodic potential, that we analyzed by numerical simulations. We studied the trajectories $\phi(t)$ of the particle along the fluctuating potential. In particular we are interested in noise induced effects influencing the mean escape time from the metastable state. To do this, we considered, as initial condition, the superconductive state corresponding to one of the minima in the potential profile, that is $\phi_0 = \arcsin(i_0)$. Then, we calculated the MST, that is the time spent by the particle to reach the position corresponding to the next maximum of the potential profile. After several simulations, we obtained the behavior of MST for different frequency values of the fluctuating potential. The parameter of simulation are: number of realizations $N_r = 10^4$, time step for integration $\delta t = 10^{-3}$, maximum value for escape time $T_{\max} = 10^3$. The curves obtained show the effects of the correlation on the overall behavior of MST, evidencing RA and NES phenomena.

3. Results

We calculated the trajectory $\phi(t)$ for different values of the characteristic parameters of the junction, such as the bias current i_0 and the noise intensity γ, keeping $\omega_c = 1$. We obtained the curves of the MST and the corresponding standard deviation (SD) vs ω, and the MST vs γ, for different values of τ_c. In Fig. 1, we report the curves of MST vs ω for white noise and colored noise, with different values of the correlation time and noise intensity. We found the presence of the RA phenomenon, in particular we note that the position of the minimum in the curves depends on τ_c. The minimum is shifted towards greater values of ω when τ_c increases and this effect is more evident in correspondence of greater values of the noise intensity (Fig. 1, right). We find a wide range of frequencies ($0.3 < \omega < 0.8$) in which a non-monotonic behavior of MST as a function of the correlation time is present (Fig. 1). In Fig. 2, we report the curves representing MST vs τ_c for $\omega = 0.7$ and noise intensities $\gamma = 0.02$ and $\gamma = 0.5$. The values of MST, in the presence of the white noise, diminishes when the noise intensity grows up. When a colored noise is applied to the system, the curves of Fig. 2 present a maximum depending on the correlation time τ_c. The maximum of MST increases for greater noise intensities. The effect of the colored noise intensity is to enhance MST. Therefore, by changing

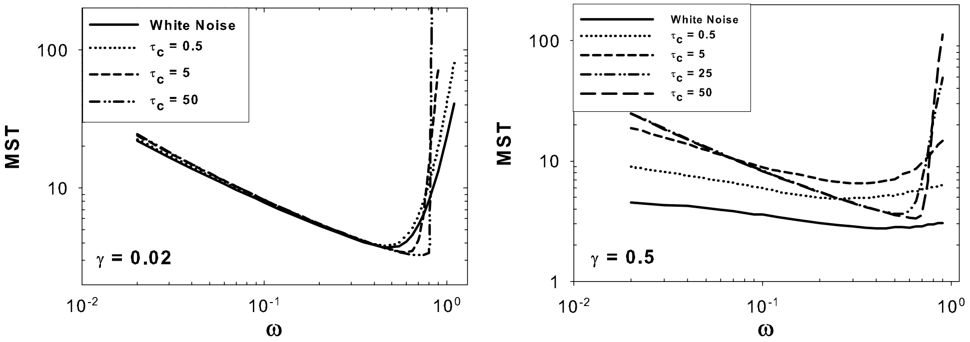

Fig. 1. Left: MST vs ω for white noise and different τ_c, $\gamma = 0.02, i_0 = 0.8$ and A $= 0.7$. Right: MST vs ω for white noise and different τ_c, $\gamma = 0.5$, $i_0 = 0.8$ and A$\,=0.7$.

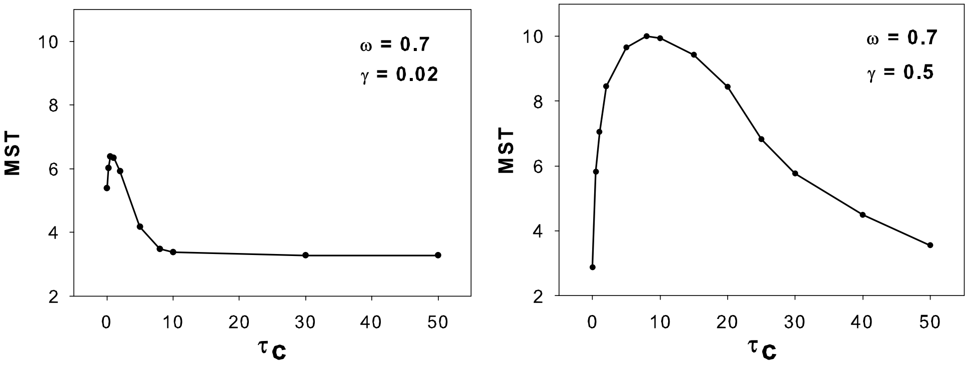

Fig. 2. Left: MST vs τ_c for $\omega = 0.7, \gamma = 0.02, i_0 = 0.8$ and A $= 0.7$. Right: MST vs τ_c for $\omega = 0.7, \gamma = 0.5, i_0 = 0.8$ and A $= 0.7$.

the value of τ_c is possible to control the mean switching time of a JJ device. In particular, for a given noise intensity, we find a correlation time corresponding to a maximum of MST. In Fig. 3, we report the curves for MST and SD as a function of ω. We note a range of frequency ($0.2 < \omega < 0.8$) in which MST and SD present a minimum.[5]

We find also that the RA phenomenon, in the presence of colored noise, presents a scaling effect depending on the values of the correlation time. In Eq. (4) for correlation time greater than the characteristic time scale of the system, $\tau_c \gg |t-t'|$, we obtain $\langle \zeta(t)\zeta(t')\rangle \approx \gamma_{\text{colored}}/2\tau_c$. By comparison with the correlation function of the white noise, $\langle \xi(t)\xi(t')\rangle = \gamma_{\text{white}}\delta(t-t')$, we note that the effective intensity of the colored noise is equivalent, in this approximation, to the intensity of the white noise, scaled by a factor $1/2\tau_c$:

$$\gamma_{\text{white}} = \frac{\gamma_{\text{colored}}}{2\tau_c}. \tag{5}$$

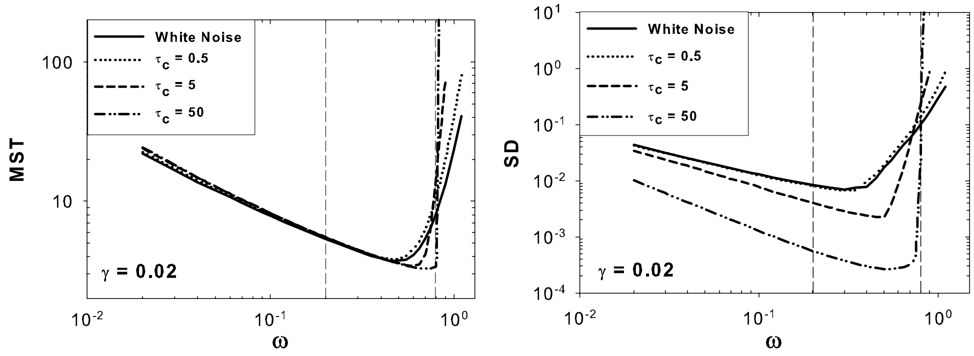

Fig. 3. Left: MST vs ω for different $\tau_c, \gamma = 0.02, i_0 = 0.8$ and A $= 0.7$. Right: Standard deviation (SD) vs ω.

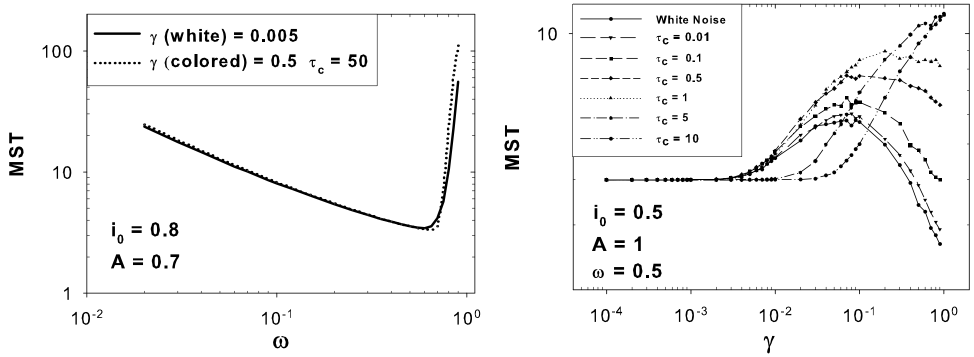

Fig. 4. Left: MST vs ω with $i_0 = 0.8, \mathrm{A} = 0.7$: white noise (line) and colored noise with $\tau_c = 50$ (dots). Right: MST vs γ for white noise and colored noise with different value of $\tau_c, \omega = 0.5$, $\mathrm{A} = 1, i_0 = 0.5$.

To verify this result we considered the effect on the system of a white noise with intensity $\gamma_{\text{white}} = 0.005$ and a colored noise with intensity $\gamma_{\text{colored}} = 0.5$, setting $\tau_c = 50$. In Fig. 4, on the left, we report the curves of MST vs ω calculated for these two values of noise intensity. We see that for MST less than 50, there is a good agreement between the curves. In this conditions the time evolution is faster than the correlation time and the system is not affected by the correlation.

We investigated the NES phenomenon by studying the curves of MST as a function of the noise intensity, for different values of the correlation time (Fig. 4, right). The NES phenomenon had been already found in short JJs in the presence of white noise.[10] Here we found a range of values of the correlation time, $0.01 < \tau_c < 1$, in which the curves present a non-monotonic behavior. For $\tau_c = 5, 10$, the non-monotonic behavior disappears. As already noted in Fig. 1, we observe the presence of a non-monotonic behavior of MST as a function of τ_c. One of the main results of this work concerns the optimum range of the system parameter values in which the MST and its SD are minimized. Specifically, for $0.3 < \omega < 0.7$, $\gamma \approx 0.02$ and $\tau_c > 5$, the MST and its SD take the lower values and the enhancement of MST, due to the NES effect, is vanishing (see Figs. 3 and 4-right). As a consequence the efficiency of JJ device, as a logic component, in this parameter range increases.

4. Conclusion

We studied the transient dynamics of Josephson junctions under the influence of colored noise and driving potential oscillating with frequency ω. We found the presence of noise induced effects such as resonant activation and noise enhanced stability phenomena. RA is influenced by the presence of colored noise: varying the correlation time τ_c affects the position of the minimum of MST. In particular, we noted a non-monotonic behavior of MST as a function of τ_c. We found a range in which both MST and the corresponding SD present a minimum, as a function of ω, for different values of the correlation time. Moreover we found a scaling effect of the colored noise when large values of the correlation time are considered. Finally we studied the curves of the mean switching time as a function of the noise intensity, finding that the correlation time affects the noise enhanced stability phenomenon.

Acknowledgments

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