




MATHEMATICS  
EDUCATION

*How to solve it?*



PROCEEDINGS  
OF THE 40<sup>TH</sup> CONFERENCE OF THE  
INTERNATIONAL GROUP FOR THE  
PSYCHOLOGY OF MATHEMATICS EDUCATION

EDITORS: CSABA CSÍKOS • ATTILA RAUSCH • JUDIT SZITÁNYI

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## WELCOME TO PME 40

We are delighted to welcome you to the 40<sup>th</sup> Annual Conference of the International Group for the Psychology of Mathematics Education, being held in Szeged, Hungary. PME40 is being hosted by the University of Szeged, and the theme of the conference is *Mathematics Education: How to solve it*. This title reminds all participants that 70 years ago the Hungarian Pólya György (George Pólya) published his seminal book entitled "How to solve it?". This book was used by generations of mathematics teachers as their inspiring source of teaching ideas. Besides commemorating Pólya's oeuvre, the title evokes the everlasting debate on the role of mathematical problem solving in fostering children's thinking. We invite all participants to contribute actively to the discourse and analysis of ideas. We also encourage all of you to foster a welcoming and stimulating atmosphere at the conference, that all participants may feel included as members of the PME community. We extend a special welcome to those attending their first PME conference. Our hope is that the conference will provide a chance to attain some pinnacles and to establish some fruitful connections.

It is the second time Hungary is hosting a PME conference. PME 12 was held in Veszprém, and several presenters of that conference are still active members of the PME community. Another special welcome is due to them!

Do extra-terrestrial beings exist? – the Nobel Prize winning Italian physicist, Enrico Fermi, was once asked by his disciples in California. Of course, Fermi answered – they are already here among us, they are called Hungarians... A headline article published in *Nature* in 2000 ("Genius Loci") claimed that the 20th century was made in Budapest. The article goes on to enumerate all the amazing contributions to progress by Hungarian scientists early in the century. Recognizing and developing mathematical talent has long been and is still a central issue in the Hungarian educational system. However, international assessment projects (TIMSS, PISA) in the last decades warned us that our mathematics education should be reformed according to the principle of evidence-based educational policy. We hope that our educational system will benefit from hosting such a highly prestigious scientific conference.

The Local Organizing Committee has 13 members from different universities, thus making the occasion a national endeavor. The University of Szeged is proud of hosting such a highly prestigious event. Szeged is most famous for its culture, including the University which is among the 500 best universities of the world. The name of the town is also closely intertwined with sport events: the Canoe Sprint World Championships were hosted in Szeged three times. Moreover, the town is a gastronomical and spa event itself worth being discovered.

The Program Committee and the Local Organizing Committee want to express our thanks for the support we have received from members of the PME community, including previous conference organizers and Bettina Rösken-Winter, PME's administrative manager.

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## PROBLEM SOLVING AND ARGUMENTATIVE SKILLS IN MULTICULTURAL ITALIAN CLASSROOM

Valeria Di Martino

University of Turin – Department of Philosophy and Science Education

In the last decades the population in Italian classrooms has become increasing more multicultural. Even if the law guarantees to all students the access to school, the gap between Italian students and migrant ones is becoming more and more evident in the results of national surveys in Italian Language and Mathematics and in the high dropout rates. In wide spectrum of Italian research studies, to date, a great deal of attention has been focussed on integration processes and on the subject of interculturalism. Relatively little, however, has been dedicated to specifically didactic and disciplinary matters.

A preliminary investigation was carried out to highlight the main difficulties faced in mathematics by these students, particularly related to problem solving and argumentation. The research aimed therefore to the development and testing of an educational intervention that, taking into account cultural and linguistic differences, aims at strengthening the aforementioned skills. It was tested in an experimental research design with a control group. The sample comprises 453 fourth and fifth graders. The intervention took place once a week, for twenty sessions of two hours. All activities comprising the intervention aim to: reinforce problem solving by developing cognitive processes (memory, comprehension, reasoning, creativity and critical thinking) (Coggi, 2015); encourage comprehension, representation, categorization, planning, monitoring, identification of multiple solution strategies; develop argumentative skills, both as a strategy for communicating knowledge and also for the purposes of self-clarification (Boero et al. 2008). For this purpose, both quantitative and qualitative analysis was conducted (pre-post-test specifically constructed, video recordings, interviews with teachers). Results showed that argumentation in multicultural classes can become the moment in which it's possible to share different reasoning and solving strategies, in order to constitute an enrichment for all, Italians and immigrants.

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## HIERARCHICAL AND NON-HIERARCHICAL CLUSTERING METHODS TO ANALYSE AN OPEN-ENDED QUESTIONNAIRE ON ALGEBRAIC THINKING

Benedetto Di Paola\*, Onofrio Rosario Battaglia\*\*, Claudio Fazio\*\*

\* Department of Mathematics and Computer Science, University of Palermo, Italy  
\*\* Department of Physics and Chemistry, University of Palermo, Italy

In recent years, some papers have tried to develop detailed models of the reasoning competences of the student populations tested, or to subdivide a sample of students into intellectually similar subgroups, by using quantitative or qualitative analysis methods. It is worth noting that research papers using quantitative analysis methods to study student responses to open-ended questionnaire can be found in Science and Physics education (Springuel et al., 2007), but the same cannot be said for research in Mathematics education. In this paper we focus on the application of hierarchical and non-hierarchical clustering methods referred to *dendrograms* and *k-means* approaches (Everitt, et al., 2011), trying to make sense to answers given by 118 Tenth Grade Italian students to six open-ended questions on algebraic thinking. In particular we discuss the results on the study of typical students behaviour in tackling the algebraic resolution of word problems and, at the same time, at understanding how the student semantically and syntactically control questions containing symbolic algebraic expressions (Radford & Puig, 2007).

The two methods (K-means and dendrograms) both allowed us to partition and characterize our student sample, without making any a priori assumptions and giving as output student's behaviour interesting for the researcher in Education. The first method identified 3 groups of students, the second one 5. The results we found are largely coherent with the ones already reported in the literature obtained by means of qualitative methods. For this reason, we can consider the use of both hierarchical and non-hierarchical clustering a valid tool to complement the use of qualitative analysis to study a large number of students with respect to the way they give answer to the questionnaire.

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