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OPERATIONAL RISK IN BANK GOVERNANCE AND CONTROL: HOW TO SAVE CAPITAL REQUIREMENT THROUGH A RISK TRANSFER STRATEGY. EVIDENCES FROM A SIMULATED CASE STUDY

E. Scannella*, G. Blandi**

Abstract

Operational risk management in banking has assumed such importance during the last decade. It has become increasingly important to measure, manage, and assess the impact of operational risk in the economics of banking. The purpose of this paper is to demonstrate how an effective operational risk management provides mitigating effects on capital-at-risk in banking. The paper provides evidences that an implementation of an operational risk transfer strategy reduces bank capital requirement. The paper adopts the loss distribution approach, the Monte Carlo simulation, and copula methodologies to estimate the regulatory capital and simulate an operational risk transfer strategy in banking***.

Keywords: Operational Risk, Risk Transfer, Banking, Basel Accord, Risk Management, Financial Regulation

JEL Classification: G01, G2, G18, G21, G24, G28, G32

*Department of Economics, Business, and Statistics, University of Palermo, Viale delle Scienze, 90128 Palermo, Italy
Tel: +39 091 23895305
Fax: +39 1782217366
**Department of Economics, Business, and Statistics, University of Palermo, Viale delle Scienze, 90128 Palermo, Italy
**Although the paper has been written jointly by the two Authors it is possible to identify the contribution of each one as follows. Abstract, and sections 1, 2, 3, 6 have been written by Enzo Scannella. Sections 4 and 5 have been written jointly by Enzo Scannella and Giuseppe Blandi. The data were analysed jointly by the two Authors. All the figures and tables were prepared jointly by the two Authors.

1 Introduction

This paper aims to demonstrate how an effective operational risk management provides mitigating effects on capital-at-risk in banking. The paper provides evidences that an implementation of an operational risk transfer strategy saves capital requirement and reduces the cost of capital in banking. To estimate the regulatory capital the paper adopts an advanced measurement approach, and particularly the loss distribution approach. Such approach is based on a bottom-up methodology. Then, the analysis is conducted on a simulated operational losses database. The estimation of the loss distribution has been carried out using Monte Carlo simulation and copula methodologies.

The analysis is carried out in two parallel steps. In the first one, the operational risk capital requirement is estimated. In the second one, an operational risk transfer policy is implemented through the insurance market. Such policy provides a mitigating impact on the regulatory capital.

The structure of this paper is as follows. Section II introduces operational risk in banking. It aims to frame the specific nature of operational risk. Section III provides a regulatory perspective of the operational risk with reference to the first, second, and third pillar of the New Basel Capital Accord. Section IV analyses the loss modelling process that is based on a separate estimation of the frequency distribution and severity distribution of a single operational event. Section V provides a simulated operational losses database that supports the operational risk transfer strategy in banking. Section VI concludes.

2 An introduction to operational risk in banking

The operational risk is one of the most important risk in the economics of banking. It is defined for the first time by the Basel Committee on Banking Supervision (2001) as "the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events". This definition has been incorporated into the New Bank Capital Accord (Basel Committee on Banking Supervision, 2006). The Basel Committee's definition incorporate the legal risk but excludes the reputational and strategic

risks.

Under the 1988 Capital Accord, there were no capital buffers for such kind of risk. Only with the New Capital Accord in 2006 the Basel Committee recognizes the importance of operational risks, in addition to credit and market risks.

Operational risk management in banking has assumed such importance during the last decades. It has become increasingly important to measure, manage, and assess the impact of operational risk in the economics of banking. Operational risk events may have considerable economic consequences in banking that could compromise the business continuity. As stated by the Basel Committee on Banking Supervision (2001, p. 1) "developing banking practices such as securitization, outsourcing, specialized processing operations and reliance on rapidly evolving technology and complex financial products and strategies suggest that these other risks are increasingly important factors to be reflected in credible capital assessments by both supervisors and banks".

The definition is based on the underlying causes of operational risk. Briefly, the drivers of operational risk are: internal processes, people, information systems, and external events. The operational risk comes from very different causal factors (event risk) and it is inextricably linked to bank activities. From an organization point of view, this kind of risk is pervasive, transversal and similar to a "pure risk". A greater risk is not associated with a higher expected return. The operational risk is not taken in return for an expected reward, like financial risks.

Operational risk is a kind of risk that affects all financial institutions (Santomero and Babbel, 2012). Operational risk is a normal part of banking. There is, however, a trade-off problem in defining an appropriate balance between the benefit of eliminating the risk and the cost of the risk reduction/mitigation (Bessis, 2009). The principles for the management and supervision of operational risk issued by the Basel Committee on Banking Supervision (2011a) recognize that it is essential that banks have a comprehensive risk management process in place that effectively identifies, measures, monitors and controls operational risk exposures, and that is subject to appropriate board and senior management oversight. Sound risk management practices are essential to the prudent operation of banks and the stability of the financial system. A sound risk management process may be divided into four steps. The first one is the identification and understanding of operational risk. The second step is the analysis and the identification of the drivers and principal components of the operational risk in banking. The next step is the measuring of the operational risk, using different models and approaches that are available for different kinds of banks. A bank has to balance between the cost of using a model and the benefits in terms of quality and reliability of risk measures. The final step

is the management of operational risk, in order to reduce/mitigate or eliminate the impact of the operational rate risk in the economics of banking.

Briefly, the introduction of Basel II is important not only because it imposes some standards methodologies for assessing the operational risk capital requirement in banking, but also because it predicts radical changes in the management structures and processes in banking (Birindelli and Ferretti, 2006, 2009; Resti and Sironi, 2007, 2008; Scannella, 2005; Sironi, 2003). A proactive operational risk management, a strong involvement of the top management, a constant auditing activity by the bank Internal Audit, a recurrent review of the operational risk management processes, and well-defined reporting systems and responsibility frameworks at business unit levels are all principles for an effective operational risk management implementation in banking.

3 Operational risk in banking: a regulatory perspective

The New Basel Accord (Basel Committee on Banking Supervision 2006) introduces a capital charge for operational risk in banking. The Basel Committee on Banking Supervision established a minimum regulatory capital charge for operational risk under Pillar 1.

The New Basel Accord presents three methods for calculating operational risk capital charges in a continuum of increasing sophistication and risk sensitivity. The availability of different methodologies aims to ensure correspondence between the complexity of the approaches, and the improvements of risk management practices in banking (Gabbi et al., 2005; Hull, 2012).

The basic approach to measure operational risk is the Basic Indicator Approach. It uses a single indicator as a proxy for the overall operational risk exposure. The bank capital requirement is determined applying a 15% coefficient to the average of the last three years' positive annual gross income. It is an extremely easy approach to implement across banks. A bank's gross income is the only component that is taken into consideration to evaluate the operational risk capital charge.

The second approach is the Standardised Approach. The main difference from the first approach is that a bank's activities are divided into a number of standardised business units and business lines. For each of them it is associated a beta coefficient, which is then multiplied by the last three years' average gross income. The resulting operational risk capital requirement is then obtained as a sum of each business line. Differences in the beta coefficients are linked to the different impact of operational losses on the income capacity of each business lines.

Within each business line, the capital charge is calculated by multiplying a bank's financial indicator

by a "beta factor". It represents a rough proxy for the relationship between the industry's operational risk loss experience for a given business line and the broad financial indicator representing the banks' activity in that business line. The gross income serves as a scale of operational risk exposure within each business line.

The differentiation among business lines represents a step forward in comparison to the basic approach. Nevertheless, the Standardised Approach is also affected by many simplifying assumptions: the existence of a perfect correlation among different loss events, mitigation policies are neglected, extreme events are not caught, and the gross income of each business line is a rough proxy of the bank's risk exposure. In addition, it does not seem to foster the development of appropriate techniques and strategies to face effectively up operational risks in banking.

The third approach is the Advanced Measurement Approach. This approach, in comparison to the previous two, is much more complex and requires qualitative and quantitative standards, in terms of organizational requirements, effective internal control mechanisms and operational risk management techniques. The Advanced Measurement Approach is based on the estimation of a loss frequency and loss severity distribution. The estimation is supported by internal and external historical data. This approach aims to quantify the operational risk exposure, without using any kind of proxy, and differentiate it by business lines. It recognises that the operational risk is the result of two factors: the probability that an event will occur and the consequences of the adverse event. Banks are allowed to implement risk mitigation strategies, use risk transfer mechanisms, and hedge risk exposure with insurance policies (Basel Committee on Banking Supervision, 2003). The Advanced Measurement Approach requires Value at Risk methodologies (Operational VaR) to evaluate the operational unexpected loss, using a 99,9% confidence level and a 1-year time horizon (Basel Committee on Banking Supervision, 2006, 2011b).

In order to implement an Advanced Measurement Approach banks need to categorize operational risks and business lines, according to the Basel Committee-specified event types and business lines. The categorization of operational risk is as follows: internal fraud; external fraud; employment practices and workplace safety; clients, products and business practices; damage to physical assets; business disruption and system failures; execution, delivery and process management. The categorization of business lines is as follows: corporate finance; trading and sales; retail banking; commercial banking; payment and settlement; agency services; asset management; retail brokerage. Banks need to estimate their exposure to each combination of type of risk and business line. Ideally this will lead to 7×8=56 VaR measures that can be combined into an overall VaR measure.

The Advanced Measurement Approach provides incentives for banks to develop measurement methodologies and techniques to internally estimate operational risk and calculate regulatory capital requirements. The Advanced Measurement Approach is the most risk sensitive of the approaches currently being developed for regulatory capital purposes. As market risk capital requirements, the operational risk capital requirements are based on internal models that are developed by banks. These models are subject to qualitative and quantitative standards. They use internal and external loss data (industry loss data).

Nevertheless the above mentioned advantages, the Advanced Measurement Approach is affected by several concerns and criticisms, such as the difficulties to measure the operational risk, the complexity of the calculations, and the non-normal distribution of loss frequency and loss severity (Birindelli and Ferretti, 2006, 2009; Resti and Sironi, 2007, 2008; Tutino, Birindelli and Ferretti, 2011, 2012).

A key issue in the development and implementation of regulatory capital requirements and internal approaches to measure the operational risk is the collection and analysis of loss data, as well as the definition of industry standards to share loss data across banks. Banks need to develop advanced information systems to support an internal measurement approach for operational risk management (Aprile, 2007; Cosma, 2008; Gabbi et al., 2005).

Banks are encouraged to develop sophisticated techniques and practices to manage and monitor their operational risks. The financial regulation issued by the Basel Committee on Banking Supervision aims not only to ensure that banks have adequate capital to support risks (Pillar 1), but also to ensure that banks improve internal control processes, methodologies, and practices to increase the effectiveness of the operational risk management (Pillar 2). Banks have to identify and strengthen policies and strategies that support the assessing, monitoring and controlling/mitigating the operational risk, and establish adequate internal systems for measuring, monitoring, and reporting operational risk exposures. Pillar 2 recognizes that the risk faced by a bank depends on qualitative aspects, such as: organizational structure, internal control systems, and risk management practices.

Supervisors review and evaluate banks' internal capital adequacy assessment and strategies, as well as their ability to monitor and ensure their compliance with regulatory capital requirements. Briefly, capital ratios are not more important than the adequacy and effectiveness of operational risk management practices in banking. The qualitative analysis of the operational risk in banking is put at the center of the Internal capital adequacy assessment process (ICAAP) and the Supervisory review and evaluation process (SREP). There is a strong interlinking

between ICAAP and SREP in banking. It recognizes the relationship that exists between the amount of regulatory capital that a bank has to hold against its operational risk and the strength, soundness, and effectiveness of a bank's risk management and internal control processes. In that view, the ICAAP and SREP are complementary in banking. They are parts of a wider supervisory review process covered by Pillar 2.

To complement the capital requirements and the supervisory review process, the Basel Committee developed a set of risk disclosure requirements (Pillar 3) that aims to remove obstacles that prevent market discipline, and inform the market about a bank's risk exposure. Pillar 3 provides a disclosure framework based on qualitative and quantitative disclosure requirements. Banks are required to disclose: scope and application of Basel regulation; nature of capital held; regulatory capital requirements; risk management objectives, policies, processes and structures; nature of banks' risk exposures. The market discipline of Pillar 3 addresses the issues of transparency in banking.

4 Modelling operational losses in banking

In this section the paper aims to analyse an advanced measurement approach, and particularly the "loss distribution approach", to estimate the regulatory capital requirement in banking. Such approach is based on a bottom-up methodology in which operational loss data coming from internal databases, external consortiums, public data, and scenario

analysis are used to develop an assessment activity at every business process in order to identify and quantify all types of operational risks (Alexander, 2003; Frachot et al., 2001). The loss distribution approach is characterized by the following steps:

- risk class definition: operational risk data are classified in homogeneous categories, in such a way to satisfy the independence and identical distribution hypotheses. The number of classes determines greater or less granularity of the model. The financial regulation requires to test such hypothesis. Collecting data for event type and business line could be considered as a minimal risk class.

- estimation of the severity of operational loss: it identifies a monetary loss caused by an operational event. In order to estimate such severity it is necessary to select a list of possible distribution functions, find out the parameters that best match the observed data to the distribution, and test the distribution functions in order to select the best model.

- estimation of the frequency of operational loss: it is necessary to determine the distribution function that represents the number of observed operational events. The probability distribution should fit the empirical data.

- aggregation of severity and frequency distributions to obtain the aggregate loss distribution. For each risk class, it is necessary to compound severity and frequency into one aggregated loss distribution (Figure 1). It allow to forecast operational losses with a certain degree of confidence.

Figure 1. Aggregated loss distribution
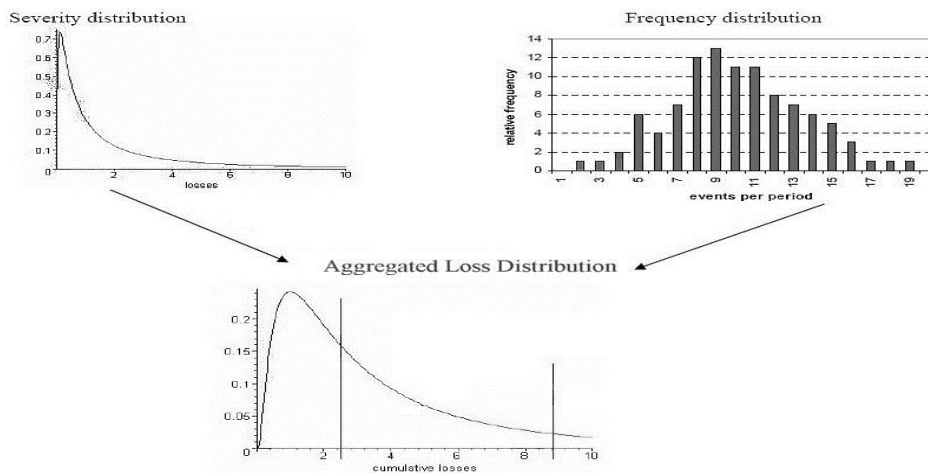
- aggregation of the loss distribution of each risk class to determine the overall annual loss distribution. Adopting a conservative approach (it is based on the assumption of a perfect linear correlation among different risk classes), the operational risk capital requirement is the sum of the Capital-at-Risk for each risk class. However, the Basel Accord allows to use

other aggregating techniques that better take into account the correlation among different risk classes.

- estimation of capital-at-risk: from a regulatory perspective the capital-at-risk is determined as the Value-at-Risk of the overall annual loss distribution with a 99,9% confidence level.

Let us analyse some details of the above

mentioned steps in the following sub-sections.

4.1 Estimation of severity distribution

In order to select all possible distributions that best fit observed data it is necessary to analyze the features of an operational loss database. Most empirical distribution show a positive skewness and high kurtosis. It happens because generally an operational loss database is characterized by many low intensity losses and few high intensity losses or extreme losses. In this regard, the most commonly used distribution is the Log-normal, although there exist a quite list of distributions that can be used in modelling the severity of operational losses like Weibull, Exponential, Gamma etc. (Dahen and Dionne, 2010;

Klugman et al., 2012). The main issue is related to the fact that they tend to underestimate the losses in the right tail of the distribution. In order to avoid such problem, it is common practice to split up the monetary distribution impact in two parts. One for the body and one for the tail. They are then aggregated through a mixture function to obtain a single model. Such a new distribution will result more reliable in taking into account the impact of rare events. Crucial to this process is the threshold "*u*", which will be used as discriminatory element to separate the body from the tail of the distribution. Therefore, the severity distribution function of a single internal operational loss is a mixture between two different distributions (Figure 2).

Figure 2. Mixture function
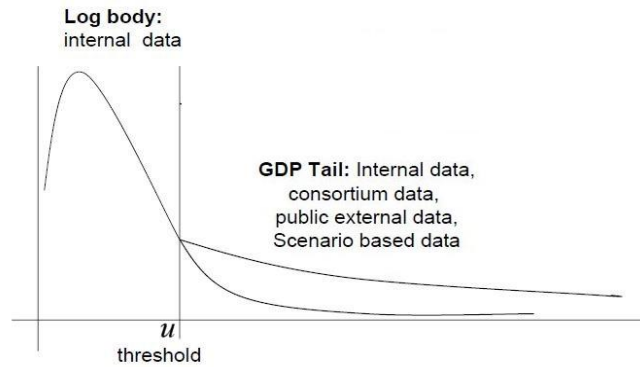
In addition, a bank may not keep recording of very low impact losses. In this case the body distribution must be modified in such a way to take into account the truncation effect of data below a threshold *T*. With this regard it is necessary to modify the body distribution function, and introducing the left truncated conditional probability distribution. The most widely used approaches to estimate parameters are the followings: a system of equations equal to the number of parameters (the popular methods are: moments, percentile matching, probability weighted moments) and an optimization process. As far as tail parameters estimation concerns, Extreme Value Theory (EVT) provides a theoretical framework for

studying rare event by focusing on the tails of the probability distributions. Two different approaches are used to estimate the tail distribution when dealing with EVT: Block Maxima and Peaks Over Threshold methodology (Abbate et al., 2009; Cruz, 2002; Da Costa Lewis, 2004).

4.2 Estimation of frequency distribution

The estimation of frequency distribution of operational loss implies the representation of the pattern of observed operational events through a discrete random variable. The most commonly used distribution is the Poisson (Table 1).

Table 1. Poisson distribution

Distributions		Parameters
Poisson	$p(x) = \lambda^x \dfrac{\exp(-\lambda)}{x!}$	$(\lambda > 0)$

When a bank decides not to account operational losses below a certain threshold *T*, it is necessary to estimate the parameter λ taking into account the truncation effect:

$$\hat{\lambda} = \frac{\hat{\lambda}_s}{1 - F_b(T; \theta)}$$

Where $\hat{\lambda}_s$ is the estimated parameter from the loss events database. After having computed the

parameter, it is possible to estimate the parameter for the body and the tail of the distribution.

4.3 Aggregation of loss distribution

Having separately assessed both frequency and severity distributions, it is now necessary to combine them into one aggregate loss distribution. The most used aggregation methodology is Monte Carlo simulation. However, before moving on the simulation itself we need to satisfy the independence hypothesis between frequency and severity in order to adopt that methodology. The aggregate loss distribution comes from a convolution process between the previous estimated frequency and severity distributions. As well as in the previous stage we need to carry out the Monte Carlo simulation separately for the body and tail. After that, through a convolution process we can join them to estimate the aggregate loss distribution.

 The Monte Carlo simulation involves different steps for the body and tail of the loss distribution: sampling the number of annual losses, generating as many uniform random variables as demanded by the frequency, using such variables as probabilities to find out the quantile in the chosen severity distribution function. After repeating several times and sorting out the loss data from the smallest to the largest, the aggregate loss distribution is obtained. Finally, the aggregate loss distribution for the body and the tail are summed to obtain the annual aggregate loss distribution.

4.4 Aggregation of risk classes and estimation of capital-at-risk

The most conservative approach requires to estimate the total capital as the sum of the capital-at-risk of each risk class. This approach assumes a perfect linear correlation hypothesis among different risk classes. To remove such limitation and estimate the overall annual loss distribution may be used an aggregating technique based on a Copula methodology.

 A Copula distribution function is obtained by starting from marginal distributions and dependence structure. The main dependence measures between random variables are the followings: the Pearson linear correlation, the rank correlation coefficients, and tail dependence. The main Copula functions exploited for their technical prescriptions in the operational risk framework are the Archimedean Copula and the Elliptical Copula functions (Chernobai et al., 2007; McNeil et al., 2005). Once identified the most suitable copula that represents the operational loss multivariate distribution, then it is possible to determine the capital requirement using Value-at-Risk measurements.

5 A simulated operational risk transfer strategy

This section of the paper aims to demonstrate how an operational risk transfer policy based on insurance contracts can mitigate the impact on the bank regulatory capital. The analysis is based on simulated data instead of empirical ones because of the high sensitivity and confidentiality of banks' operational loss databases. The simulation is conducted on the Loss Data Collection Exercise that has been carried out by the Operational Risk Subgroup of the Standards Implementation Group (SIGOR). The analysis is performed using the statistical language R. The analysis is carried out in two parallel steps. In the first one, after a comprehensive description of the datasets, the operational risk capital requirement is estimated. In the second one, the mitigating impact on the regulatory capital is the result of a transferring operational risk policy.

 In order to carry out the analysis, two aspects are crucial. Firstly, the definition of operational risk class in order to satisfy the hypotheses of independence and identical distribution. Secondly, the number of risk classes to be considered in the analysis. As regards the number of risk classes, the low amount of data stored within the database would not provide full robustness of the statistical results. Thus, it would be not possible to use as risk class the minimum one – i.e. intersection of business line and event type – because almost each bank lies at the initial stage in the use of such methodology. In addition it will be used the breakdown by event type rather than business line, since the former provides a direct insight into transferring techniques, and therefore the effects on regulatory capital (Cruz, 2002; Davis, 2006).

 In particular, the data on which the analysis will be performed are extracted from a simulated operational losses database. Such database will represent the internal operational database that a bank may hold. Moreover, in order to get closer to the reality it is assumed that a bank does not keep recording of losses under 2,000 euro. The features of such database are summarized in Table 2 and Table 3.

5.1 Modelling the severity distribution

Earlier we explained the reason why we need to spilt up the severity distribution in two parts. One for the body and the other for the tail. In addition, we need to take into consideration the truncation effect since we are assuming a bank does not keep recording for losses under 2,000 euro. Over the course of this paper we present the analysis exclusively for the Event Type 1, as we can easily extend the same conclusions to the other risk classes. Firstly, for each risk class we need to justify the use of Extreme value theory and demonstrate that loss data satisfies the hypothesis of independence and identical distribution (Abbate et al., 2009; Klugman et al., 2012).

Table 2. Loss frequencies by event type

Code Event Type	Event Type	Operational Events within one year
Et1	Internal Fraud	1374
Et2	External Fraud	8564
Et3	Employment. Practices. & Workplace. Safety	5714
Et4	Clients Products & Business Practices	5915
Et5	Damage to Physical Assets	383
Et6	Business Disruption & System Failures	642
Et7	Execution, Delivery & Process Management	9970
	All	**32562**

Table 3. Loss severity by event type

Event Type	Minimum Loss	Maximum Loss	Median Loss	Mean Loss	Standard Deviation	Skewness	Kurtosis
Et1	2,058	398,6143	33,228	85,065	186,318	9.8	159
Et2	2,006	4,089,191	23,861	5,2347	109,477	11.8	282
Et3	2,000	3,424,911	25,816	71,738	159,956	7.8	97
Et4	2,021	343,170,300	293,599	1,548,775	7,400,414	24	887
Et5	2,023	375,254	12,177	25,469	37,946	4.38	27
Et6	2,119	765,084	10,564	24,754	50,163	8,28	96
Et7	2,094	42,775,230	146,408	416,194	1,070,166	14	381

Figure 3. Event type 1 - Box plot (left) and log-scale box plot (right)
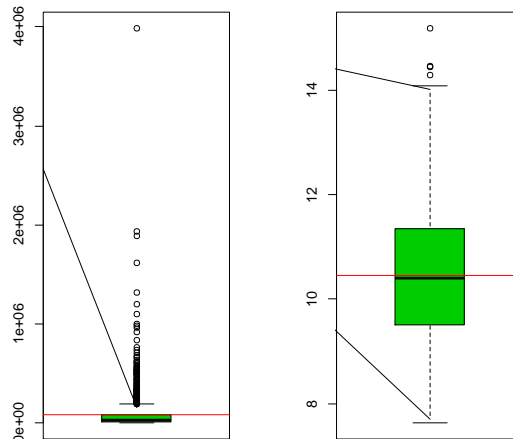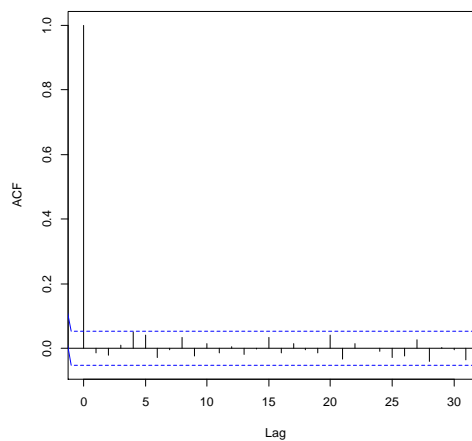

Figure 4. Autocorrelation plot

From the box-plot (Figure 3) we can immediately observe that data are showing a highly skewed to the right such that to justify the use of Extreme value theory. In order to check the independence and identical distribution hypothesis we have to look at the autocorrelation plot and to the Box-Jenkins test.

Assuming that the sample data is sorted by accounting date, the autocorrelation plot (Figure 4) shows that the independency hypothesis is satisfied since the autocorrelation values are within the 95% confidence level. Whereas, Box-Jenkins test returns a statistic value X-squared = 0.277 with consequent p-value=0.5987. Therefore, we can apply the methodology that has been introduced so far.

Now, it is possible to move onto the parameters estimation for the body distribution. In order to explain the procedure and make it simple, we choice to fit our simulated internal data to two theoretical distributions, the left-truncated log-normal distribution and the left-truncated Weibull distribution. Before moving on the parameters estimation itself, we need to identify the threshold over which the tail has to be estimated through extreme value theory. In particular we decided to recur on Peaks Over Threshold methodology, in order to find out the generalized Pareto distribution which explains the tail behavior. As mentioned earlier a primary tool is the Sample mean excess plot.

Figure 5. Sample mean excess plot for different level of T
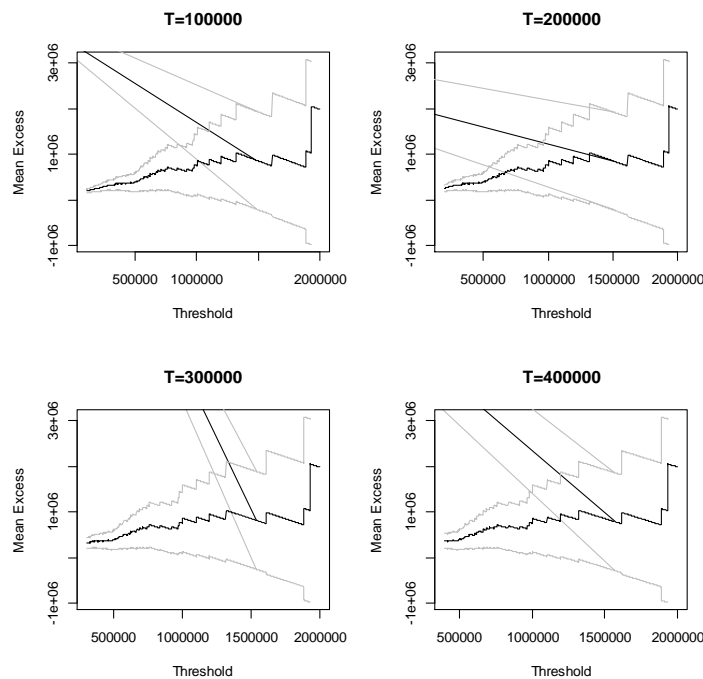
The four graphs pictured in Figure 5 represent the sample mean excess function for different threshold levels. If the Generalized Pareto distribution is a good fit to the tail, the plot should become approximately linear. Our purpose here is to pick the largest threshold beyond which the plot starts to become linear. Indeed, if the threshold is chosen too high, then there are not enough exceedance over the threshold to obtain good estimators of the extreme value parameters, and consequently, the variances of the estimators are high. Conversely, if the threshold is too low, the Generalized Pareto distribution may not be a good fit to the excesses over the threshold and there will be bias in the estimations. Other useful tools are the parameter stability plots (Figure 6), which help us to pick a right threshold. They should become stable above the right threshold (Horbenko et al., 2011).

In our example a threshold of T = 400,000 seems good enough both in terms of linearity of the sample mean excess and stability of the parameter estimates.

Once we have identified the right threshold we can estimate the parameters from the left-truncated log-normal distribution. So, firstly we need to find out the log-likelihood for a left-truncated log-normal distribution. In general we define the likelihood of a particular model the following expression:

$$L(x; \boldsymbol{\theta}) = \prod_{i=1}^{n} f(x_i; \boldsymbol{\theta})$$

Where the maximum likelihood estimate is:

$$\hat{\boldsymbol{\theta}} = argmax \, L(x; \boldsymbol{\theta}); \, \boldsymbol{\theta} \in \Theta 0$$

Figure 6. Parameter stability plots
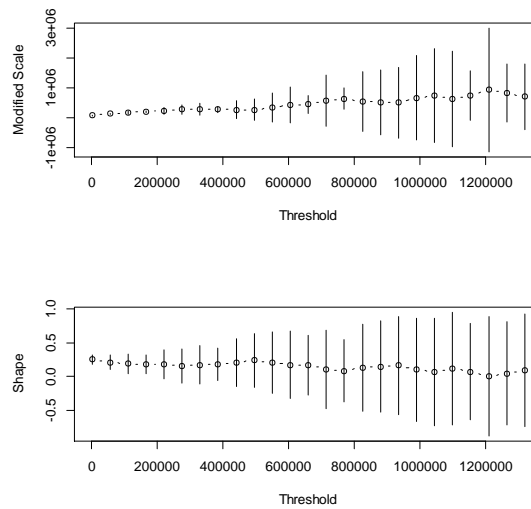
Actually, it is more convenient to deal with the log-likelihood, specified as:

$$l(x; \boldsymbol{\theta}) = \sum_{i=1}^{n} \log f(x_i; \boldsymbol{\theta})$$

Maximizing the log-likelihood first requires taking the partial derivatives with respect to the parameters and setting them equal to zero:

$$\frac{\vartheta \, l(x; \boldsymbol{\theta})}{\vartheta \theta} = 0$$

Sometimes it is not possible to get an explicit expression, in those cases we recurr to numerical optimization methods. Comig back to the issue regarding to left-truncated distributions we need to set up the likelihhod function of a conditional density function, as in the following expression:

$$L(x; \theta) = \prod_{i=1}^{n} \frac{f_b(x; \theta)}{1 - F_b(T; \theta)}$$

With corresponding log-likelihhod function:

$$l(x; \boldsymbol{\theta}) = \sum_{i=1}^{n} \log \frac{f_b(x; \boldsymbol{\theta})}{1 - F_b(T; \boldsymbol{\theta})} = \sum_{i=1}^{n} \log f_b(x; \boldsymbol{\theta}) - n \log\left(1 - F_b(T; \boldsymbol{\theta})\right)$$

Firstly, we proceed with fitting our data sample with the left-truncated log-normal distribution. As regards to the left-truncated log-normal, we cannot obtain an explicit expression for the MLE estimate. In such a case we decide to perform the estimate recurring to the Nelder-Mead numerical optimization method in order to find out the two parameters characterizing the distribution. We report the R code to process the MLE estimates:

ltlnorm<-function(x,meanlog,sdlog)

dlnorm(x, meanlog, sdlog) / plnorm(2000, meanlog, sdlog,lower.tail=FALSE)

fitdistr(x,ltlnorm,start=list(meanlog=5,sdlog=2))

The resulting estimates are:

$$\hat{\mu} = 10.38 \quad \hat{\sigma} = 1.38$$

In order to check how well our model fits a set of observations, we perform both graphical and quantitative tests. Firstly, we report the Q-Q plot relative to the body of the severity (Figure 7).

As the picture shows the model seems to fit well the observations especially for values under the threshold *u* (green line), while for values above *u* the model seems to lose adaptabilty to the data. This is not a problem since those observations will be processed and modeled in the tail of the distribution. Regarding to the quantitative tests, we report the well studied Kolmogorov-Smirnov test and Anderson-Darling test for left-truncated data (Table 4).

All the p-values are sufficiently high. So, the null hypothesis is not rejected. The same analysis has been carried out assuming as theoretical distribution a left-truncated Weibull distribution.

The resulting MLE estimates are:

$$\hat{\alpha} = 7.33 \quad \hat{\beta} = 10.85$$

We report the Q-Q plot resulting from these estimates (Figure 8).

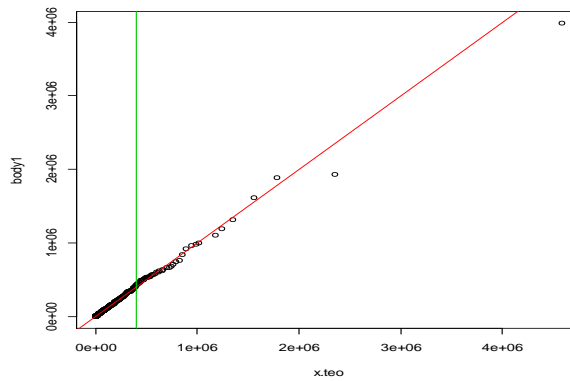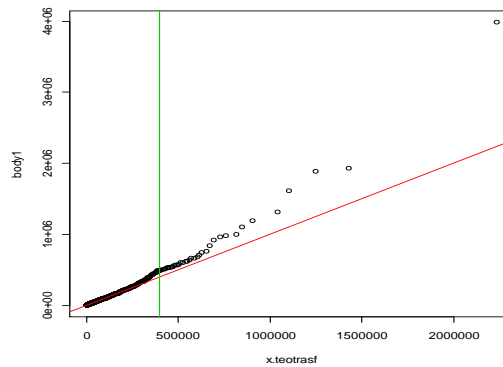**Figure 7.** Q-Q plot for left truncated Log-Normal distribution

Table 4. Goodness test results between the left-truncated log-normal distribution and the sample data

Test	Statistics	p-value
Kolmogorov-Smirnov	0.7335	0.185
Anderson-Darling	2.0298	0.618
Anderson-Darling up	71.9778	0.346

Figure 8. Q-Q plot for left truncated Weibull distribution

As showed by the picture, the model seems to fit well the observations especially for values under the threshold u (green line), while for values above u the model seems to lose adaptabilty to the data, as well as the previous case. Regarding to the quantitative tests, again we report the Kolmogorov-Smirnov test and Anderson-Darling test for left-truncated data (Table 5).

Table 5. Goodness test results between the left-truncated Weibull distribution and the sample data

Test	Statistics	p-value
Kolmogorov-Smirnov	1.13	0.0099
Anderson-Darling	9.52	0.0222
Anderson-Darling up	3397.245	0.009259

All the quantitative tests do not reject the null hypothesis. Therefore the body of the severity distribution relating to Event type 1 will be modeled as a log-normal distribution. Once identified the body distribution we can move forward to the estimation of the tail distribution. Relating to the parameter estimates, we apply again maximum likelihood estimation. The resulting scale and shape parameters are:

$$\hat{\sigma} = 368,400 \quad \hat{\varepsilon} = 0.181$$

We report the Q-Q plot for the tail distribution (Figure 9).

Figure 9. Q-Q plot for generalized Pareto distribution
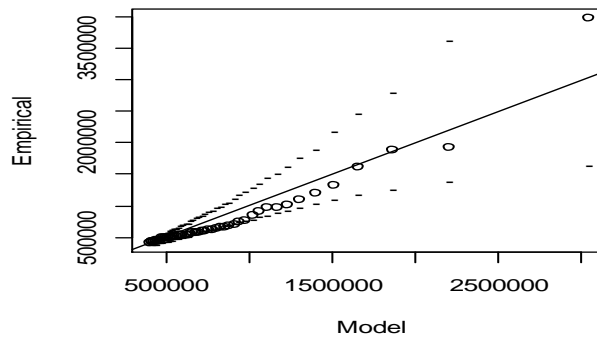
As the picture shows the model seems to fit well the observations for values over the threshold u.

Now for any event type we report the model and the corresponding parameter estimates (Table 6).

Table 6. Body and tail severity distributions parameter estimates

Event Type	Model Body	$\hat{\mu}$	$\hat{\sigma}$	Boundary threshold (u)	Model tail	$\hat{\sigma}(scale)$	$\hat{\varepsilon}(shape)$
Et1	Lognormal	10.38	1.38	400,000	GPD	368,400	0.181
Et2	Lognormal	10.04	1.24	500,000	GPD	385,600	0.111
Et3	Lognormal	10.05	1.48	600,000	GPD	439,600	0.084
Et4	Lognormal	12,58	1.80	5,000,000	GPD	1,130,000	0.279
Et5	Lognormal	9.38	1.18	50,000	GPD	50,030	0.104
Et6	Lognormal	8.96	1,41	100,000	GPD	115,600	0.183
Et7	Lognormal	11.87	1.46	5,000,000	GPD	4,114,000	0.1054

5.2 Modelling the frequency distribution

The Poisson distribution represents the frequency distribution of an operational event. The only parameter λ is estimated through the method of moments (Horbenko et al., 2011). We only use the annual frequency of occurrence for operational losses higher than the threshold T = 2,000 euro.

$$\hat{\lambda}sample = \frac{\sum_{i=1}^{365} n_i}{365} = \frac{1374}{365} = 3.76$$

$$P_{body}(400000) = 0.9659312$$

$$\hat{\lambda}_{body} = \hat{\lambda}\left[P_{body}(400000) - P_{body}(2000)\right] = 3.85(0.9659 - 0.02201) = 3.63$$

$$\hat{\lambda}_{tail} = \hat{\lambda}\left[1 - P_{body}(400000)\right] = 3.85(1 - 0.9659) = 0.13$$

This value does not take into account the effect of operational events causing a loss below a threshold T. Thus, in order to deal with a more reliable estimate we have to consider the truncation effect:

$$\hat{\lambda} = \frac{\hat{\lambda}sample}{P_{body}(loss>2000)} = \frac{3.76}{1-P(2000)} = \frac{3.76}{0.9779876} = 3.85$$

Starting from this estimation, we can define the frequency of loss for the body and the frequency of occurrence for the tail, as it follows:

$$P_{body}(2000) = 0.02201243$$

Now for any event type we report the corresponding parameter estimates (Table 7).

Table 7. Body and tail frequency distributions parameter estimates

Event Type	Model	$\hat{\lambda}_{body}$	$\hat{\lambda}_{tail}$
Et1	Poisson	3.85	0.13
Et2	Poisson	23.29	0.16
Et3	Poisson	15.34	0.24
Et4	Poisson	15.25	0.95
Et5	Poisson	0.91	0.13
Et6	Poisson	1.69	0.08
Et7	Poisson	27.09	0.22

5.3 Severity and frequency convolution

After estimating the severity and frequency distribution, the next step involves their aggregation in order to perform the aggregate loss distribution. Such aggregating procedure is based on a convolution process. It is a mathematical operation through which two functions return a third function (Da Costa Lewis, 2004; Frachot et al., 2001; Shevchenko, 2010). Assuming that the two distributions satisfy the independence hypothesis necessary to exploit the convolution mechanism, it is therefore possible to implement the algorithm based on Monte Carlo simulation. As mentioned, such algorithm provides subsequent sampling from the frequency and severity distribution to return the aggregate loss distribution. Once again we report the analysis only for Event type 1. In details, we run the simulation 10^6 times to obtain a suitable distribution.

Let $i = 1\ldots\ldots N$, the Monte Carlo simulation methodology involves the following steps:

a. Sampling the number of annual losses for the body:

$$m_i \sim Pois(\hat{\lambda}_{1b} = 3.63)$$

b. Generating as many uniform random variables as demanded by the frequency.

c. Those variables will be used as the probability (*p*) that we use to find out the quantile in the chosen body severity distribution function.

$$b_{i,r} \sim F_{1b}(x; (\hat{\mu} = 10.38; \hat{\sigma} = 1.38/x \geq 2000))$$

With $r = 1, \ldots\ldots, m_i$

d. Repeating several times the process to obtain the aggregate loss distribution for the body.

In the same way we proceed to get the aggregate loss distribution for the tail:

a. Sampling the number of annual losses for the tail:

$$n_i \sim Pois(\hat{\lambda}_{1t} = 0.13)$$

b. Generating as many uniform random variables as demanded by the frequency.

c. Those variables will be used as probabilities (*p*) that we use to find out the quantile in the chosen tail severity distribution function:

$$t_{i,r} \sim F_{1t}(x; (\hat{\sigma} = 368,400; \hat{\varepsilon} = 0.181))$$

With $r = 1, \ldots\ldots, n_i$

d. Repeating several times the process to obtain the aggregate loss distribution for the tail.

Lastly, in order to obtain the annual aggregate loss distribution for a certain risk class we compute it, as it follows:

$$G_i = \sum_{r=1}^{m_i} b_{i,r} + \sum_{r=1}^{n_i} t_{i,r} + c$$

Where *c* is the mean of the operational loss data empirical distribution below a certain threshold *T*.

Once produced seven aggregate simulated loss distributions, we can as well as extract the expected loss and the Value-at-Risk, with a 99,9% confidence level, and assuming to use the conservative approach, compute the capital at risk for the bank as a whole (Table 8).

Table 8. Value-at-risk

Code Event Type	Expected Loss €	Unexpected Loss €	Value-at-Risk 99,9% €
Et1	361,728	4,379,121	4,740,849
Et2	1,233,710	3,675,625	4,909,335
Et3	1,186,874	6,996,498	8,183,372
Et4	24,360,690	297,000,914	321,361,604
Et5	29,027	489,534	518,561
Et6	47,414	1,001,130	1,048,545
Et7	12,442,230	52,922,301	65,364,531
Capital-at-Risk	**€ 39,661,674**	**€ 366,465,123**	**€ 406,126,797**

In the graphs below (Figure 10) we report the histograms of the square root transformations of the seven aggregate annual loss distributions. These graphs show the existence of a strong lack of homogeneity among different event types, pointing out that each event type is characterized by different risk drivers. Heterogeneity, which is explained by the different impact that each individual operational event has in terms of operational loss. Moreover, we can immediately observe, as well as which types of events have a greater weight in determining the capital at risk, which ones represent major concerns for the operational risk manager and the bank as a whole. In particular, the graphs show that the event type 4 represents the major bank's source of operating losses, given that approximately 18% of operational events explains more than 79% of regulatory capital.

Figure 10. Histograms for the sqrt transformation of the seven event type distribution
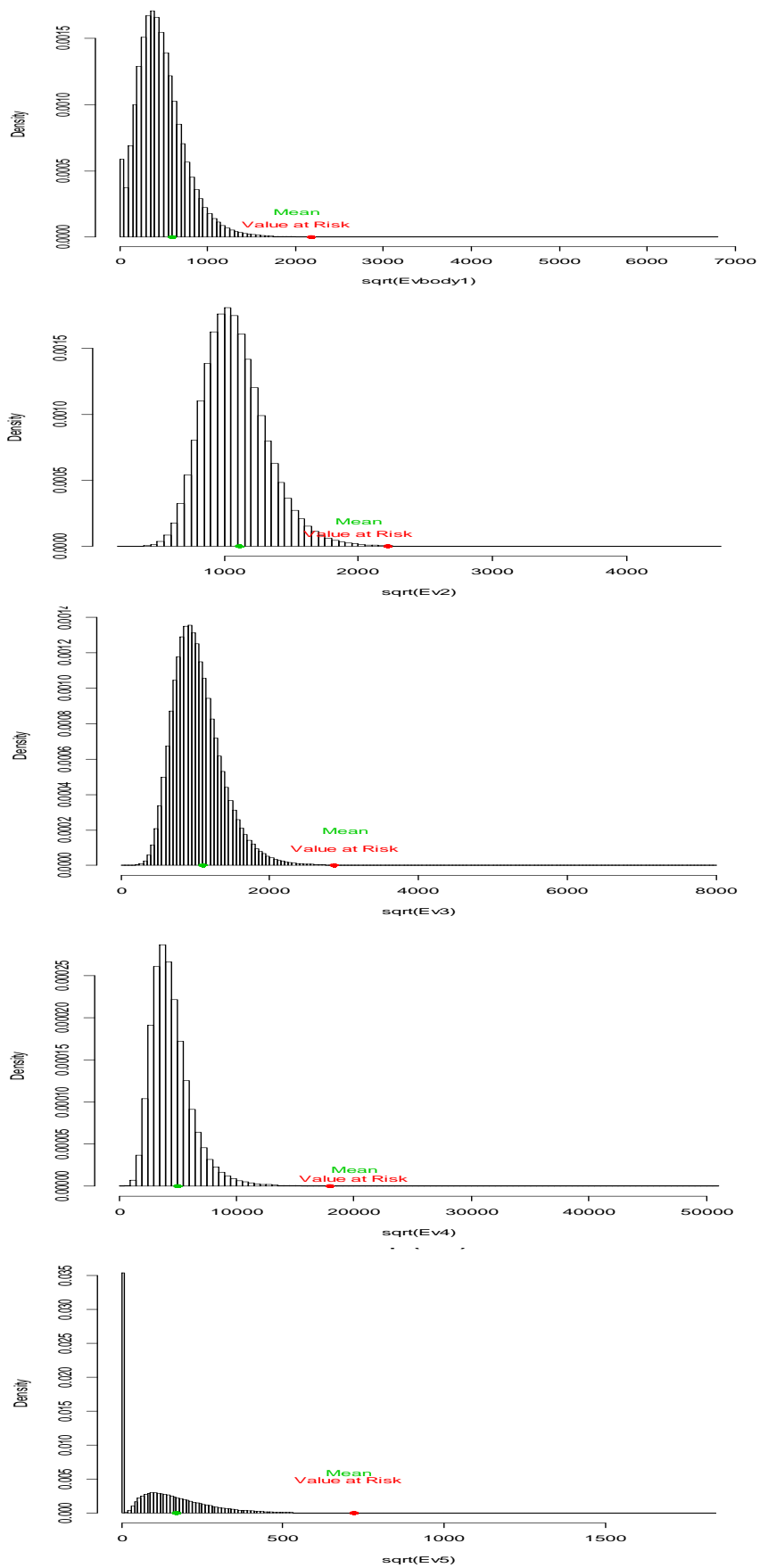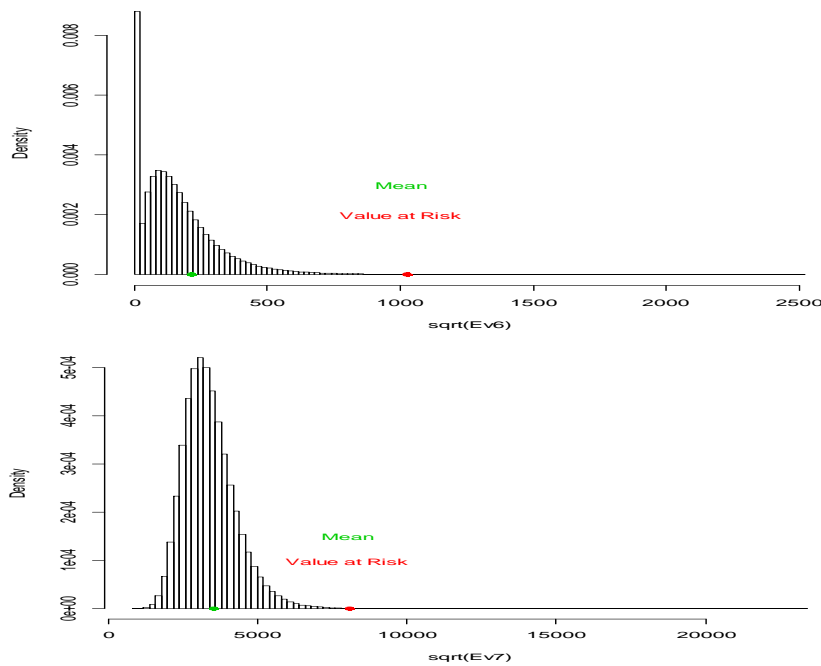
Figure 10. Histograms for the sqrt transformation of the seven event type distribution (continued)

5.4 The overall annual loss distribution

The sum of 7 value-at-risk measures that has been calculated on the annual loss distributions to determine the bank's capital at risk involves the implementation of an extremely protective policy in terms of regulatory capital. In fact, we are implicitly assuming that there is a perfect correlation among operational loss distributions. If we remove such hypothesis it will be necessary to carry on with the overall annual loss distribution. Consequently, we can estimate the value-at-risk with a 99,9% confidence level. Such solution leads to a more appropriate estimation of the capital-at-risk in banking.

In this regard, we carry out the analysis using the elliptical copula family, since it allows to take into consideration the real dependency structure among different event types and to mark the role of the dependence structure in the proximity of extreme values.

$$
R = \begin{pmatrix}
 & Et1 & Et2 & Et3 & Et4 & Et5 & Et6 & Et7 \\
Et1 & 1 & 0.00022 & -0.00019 & -0.0016 & 0.00051 & -0.00048 & -0.00087 \\
Et2 & & 1 & -0.00005 & 0.00076 & -0.00053 & -0.00188 & -0.00119 \\
Et3 & & & 1 & 0.00089 & 0.00039 & 0.00118 & -0.00128 \\
Et4 & & & & 1 & -0.00154 & -0.00025 & 0.00054 \\
Et5 & & & & & 1 & -0.00040 & 0.00061 \\
Et6 & & & & & & 1 & -0.00020 \\
Et7 & & & & & & & 1
\end{pmatrix}
$$

We estimate the overall annual loss distribution with the Monte Carlo simulation method. Then we compare the results of Gaussian copula with those of t-Student copula. For the latter, it is necessary to estimate the parameter *v*, through the use of the maximum likelihood estimator.

The algorithm for both distributions has been carried out for a number of simulations equal to 10^6. In the following figures, we report the square root transformation of the histograms of the overall annual loss distribution in three cases: the existence of a perfect positive correlation among all risk classes, the Gaussian copula, and the t-Student copula (Figure 11).

The results are summarized in the following Table 9.

The above mentioned results confirm two important conclusions. The first one refers to a substantial saving capital requirement by implementing an aggregation mechanism, which takes into account the correlation among different risk classes. We obtain a lower regulatory capital when we use the Gaussian or t-Student copula in comparison to the sum of single value-at-risk for each annual loss distribution. The second one refers to the tail dependence of the t-Student copula. Such peculiarity brings to a higher regulatory capital of the t-Student copula in comparison to the Gaussian copula one.

Figure 11. Histograms for the sqrt transformation overall annual loss distribution
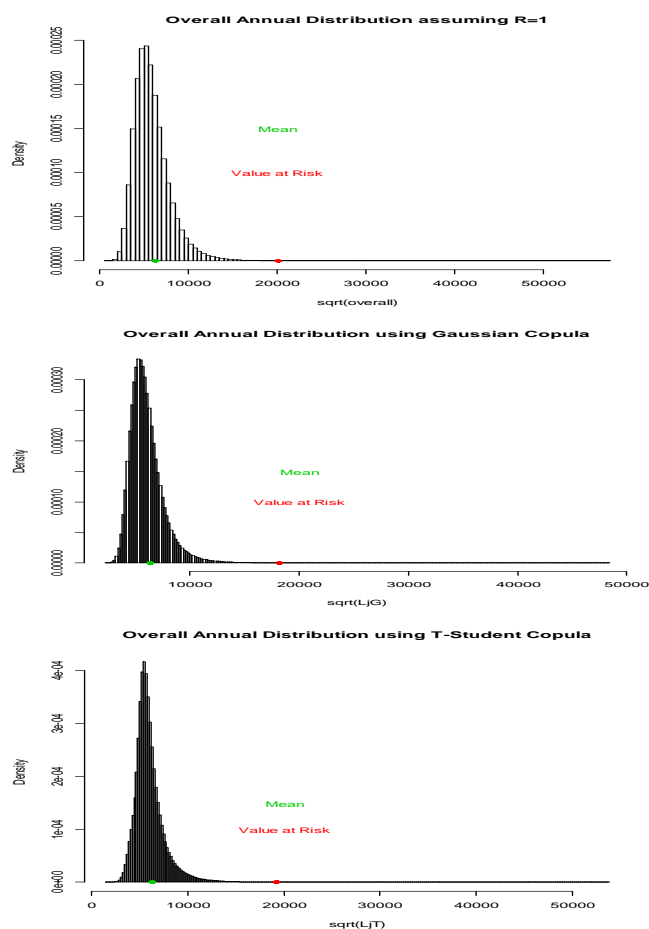

Table 9. Value-at-risk using different aggregating mechanisms

Aggregating Mechanism	Expected Loss €	Unexpected Loss €	Value-at-Risk 99,9% €
R=1	39,661,674	366,465,123	406,126,797
Gaussian Copula	39,666,040	290,961,758	330,627,798
t-Student Copula	39,641,997	328,155,528	367,797,525

5.5 Hedging against operational risk

This section of the paper aims to illustrate how an effective operational risk transfer strategy can result in saving regulatory capital. In order to analyse the risk mitigating strategy, let us suppose a bank decides to negotiate 5 insurance contracts that aim to attenuate the negative impact of operational risk. In particular, the structure of each insurance contract has been set to avoid moral hazard problems. For this purpose, insurance contracts fix deductibles (amounts of money subtracted from the value of a loss, which is not covered by insurance) and policy limits, in such a way to not encourage bank's opportunistic behavior. With reference to the insurance pricing, we consider the pure risk premium. The Table 10 shows some details of 5 insurance contracts.

Table 10. Insurance contracts: some details

Code Event Type	Premium €	Deductible €	Policy Limit €
Et-1	46,189	800,000	1,700,00
Et-2	8,930	2,000,000	3,800,000
Et-3	22,725	3,000,000	5,800,00
Et-4	1,680,030	55,000,000	95,000,00
Et-7	237,234	20,000,000	40,000,00
	€1,995,108		

In order to show how insurance contracts work, it is necessary to observe the following graph, where we report the Event type 1 annual loss distribution in the presence of an insurance contract that has the above mentioned characteristics (Figure 12).

Figure 12. Event type 1 - Insurance coverage effect
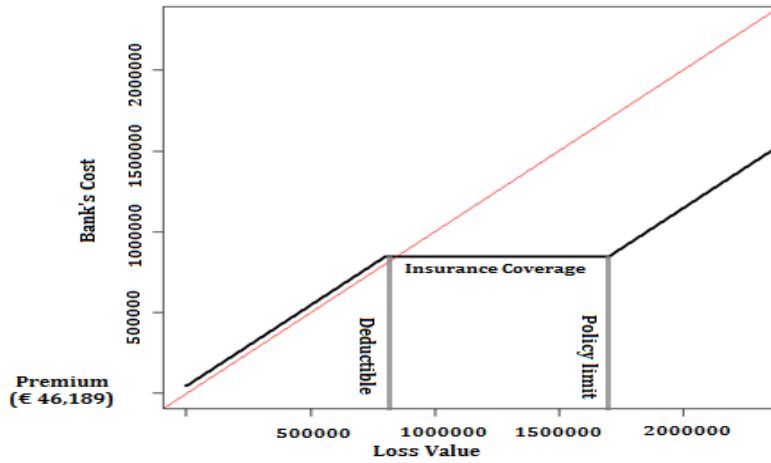
The insurance contract transforms the distribution in a step function that is characterized by the followings:

$$\begin{cases} P + ET_1(x), & x \leq D_1 \\ D_1, & D_1 \leq x \leq D_2 \\ ET_1(x) - D_1, & x > D_2 \end{cases}$$

with ET_1 the loss value, and D_1 and D_2 the value of deductible and policy limit respectively.

Briefly, the insurance contract modifies the monetary impact coming from an operational event. Thus, we need to modify the severity distribution (Banks, 2004; Committee of European Banking Supervisors, 2009; Cruz, 2002). Conversely, if insurance contracts have not any effects on the occurrence of an operational event we will not need to modify the previously estimated frequency distribution.

The measurement of the new capital requirement takes into account the mitigating effect of insurance contracts. The estimation of the overall annual loss distribution has been carried out using Monte Carlo simulation and copula methodologies. The following pictures report the histograms of the overall annual loss distribution, respectively for the Gaussian and t-Student copula aggregating mechanisms (Figure 13, 14).

Figure 13. Histograms for the sqrt transformation overall annual loss distribution with insurance contracts (using Gaussian Copula)
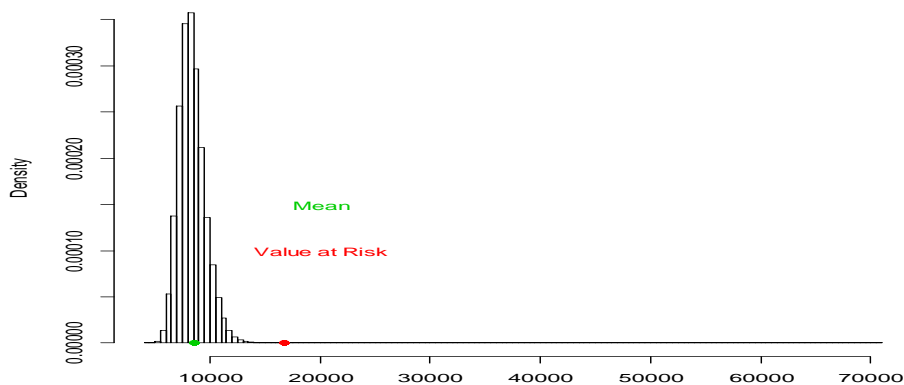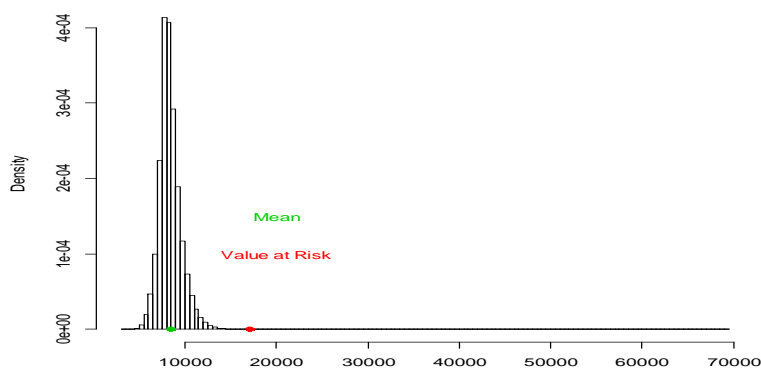
Figure 14. Histograms for the sqrt transformation overall annual loss distribution
with insurance contracts (using t-Student Copula)

Finally, in order to summarize the results of an operational risk transfer strategy, the Tables 11 and 12 show the estimation of value-at-risk using the different aggregation mechanisms (under insurance contracts) and a comparison between regulatory capital with insurance contracts and regulatory capital without insurance contracts.

Table 11. Value-at-risk under insurance contracts (using different aggregation mechanisms)

Aggregating Mechanism	Expected Loss €	Unexpected Loss €	Value-at-Risk 99,9% €
Gaussian Copula	72,218,751	206,547,581	278,766,332
t-Student Copula	72,172,803	218,604,831	290,777,634

Table 12. A comparison of capital requirements with and without insurance contracts

Aggregating Mechanism	Var 99% without insurance contracts	Var 99,9% with insurance contracts	% Saving on Capital Requirement
Gaussian	330,627,798	278,766,332	16%
t-Student	367,797,525	290,777,634	21%

The operational risk transfer strategy involves a regulatory capital saving to an extent of 16% when we use a Gaussian copula and 21% when we use a t-Student copula. Although the capital adequacy regulation imposes a maximum level (20%) of the mitigating effect on the capital requirement (Basel Committee on Banking Supervision, 2006, 2010). Therefore, with the adoption of t-Student copula methodology the regulatory capital should not be less than the 80% of the value-at-risk without insurance contracts.

6 Conclusion

Banking industry has made significant progress over the past years in understanding, measuring and managing operational risk. Banking authorities have been pressuring banks to adopt a proactive operational risk management. In addition they have imposed protective measures based on the provision of a minimum level of regulatory capital to absorb risk operational losses.

The paper has been designed to demonstrate how an effective operational risk management provides a regulatory capital saving, and a resulting reduction of bank capital costs. A simulated operational losses database supported the operational risk transfer strategy. The estimation of the overall annual loss distribution has been carried out using Monte Carlo simulation and copula methodologies. The operational risk transfer strategy involves a regulatory capital saving to an extent of 16% when we use a Gaussian copula and 21% when we use a t-Student copula.

In addition, it is important to note several aspects that need further developments. Firstly, nowadays only a limited number of banks are using an advanced methodology to estimate the regulatory capital. Nevertheless, it is only through the use of advanced measurement tools that it is possible to implement an effective hedging strategy. Therefore, in a perspective of proactive risk management, it is necessary to stimulate the adoption of AMA methodologies. Sources of incentives may arise from the standardization of the methodologies to estimate the regulatory capital, and the increasing accessibility of AMA methodologies to smaller banks.

Finally, in order to avoid that the operational risk transfer market becomes a possible source of financial instability in the banking industry it is necessary to develop transparent procedures and policies, to ensure

that a certain amount of operational risk still stays within the responsibilities of the bank management.

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