Elimination of spurious kinematic modes in hybrid equilibrium element

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It is widely recognized that displacement based elements produce solution with poor stress fields, which do not satisfy equilibrium equations. In several fields of computational mechanics, such as cohesive crack propagation and cohesive delamination, stress fields drive all the nonlinear phenomena and very fine meshes have to be employed in order to avoid numerical instabilities. Inter-element equilibrium condition is generally not satisfied and the stress field can abruptly change between adjacent elements, producing numerical instabilities in non linear analysis.

Finite element formulation based on stress fields satisfying locally equilibrium condition are known in literature since 1964 by the pioneering work of de Vebeuke [1,2] and, for its accuracy of solution stress fields, it can be suitable in several fields of computational mechanics, such as: cohesive crack propagation; lower bound limit and shakedown analyses; topology structural optimization.

Equilibrium element can be developed also in hybrid formulation with independent equilibrated stress fields on each element [3,4], producing solution which satisfy strong equilibrium condition throughout the domain with codiffusive traction at each side. Traction equilibrium condition, at sides between adjacent elements and at sides of free boundary, is enforced by use of independent side displacement laws, assumed as lagrangian parameters.

The most relevant problem in the use of hybrid equilibrium element is the presence of Spurious Kinematic Modes (SKMs) [5,6], which are displacement modes with null traction at sides. SKMs can heavily corrupt the elastic solution both in terms of displacement and in terms of stress.

In the present paper, hybrid stress elements are proposed as alternative to standard finite elements in a rigorous mathematical framework and an approach for elimination of spurious kinematic modes is presented. The results of the proposed approach are compared to classical displacement based method by linear elastic analysis of some well known structural problems.

References

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