# *The dawning of computational psychoanalysis*

## *A proposal for some first elementary formalization attempts* **[Invited]**

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***Abstract* – In this paper, we wish first to highlight, within the general cultural context, some possible elementary computational psychoanalysis formalizations concerning Matte Blanco's bi-logic components through certain very elementary mathematical tools and notions drawn from theoretical physics and algebra. Afterwards, on the basis of recent work of Giampaolo Sasso (1999; 2005; 2011), relying on the crucial crossroad between neurosciences and psychoanalysis, it will be possible to identify some hints for further formalization attempts turned toward a computational psychoanalysis outlook. Lastly, possible interesting relationships with cognitive informatics are also outlined.**

***Keywords – bi-logic; groupoid; symmetry breaking; double bind; introjective and projective processes; resonance; oscillation; object relationship; consciousness; bundle; denotational mathematics.***

## I. INTRODUCTION

Ignacio Matte Blanco (1908-1995) was an outstanding psychiatrist and psychoanalyst who gave, among other things, notable contributions to the epistemological status of psychoanalysis starting from the Freudian theoretical framework. At the basis of his rigorous formulation of psychoanalytic foundations based on *bi-logic*, he put two main principles, namely the *generalization principle* and the *symmetry principle*. We refer to (Matte Blanco, 1975) for any further deepening of his thought. One of the central points of Matte Blanco thought is then the crucial transition from *symmetric* to *asymmetric* thinking. In this article, we want to make some formal remarks on the symmetry-asymmetry duality of Matte Blanco bi-logic, within the context of theoretical computational psychoanalysis. We think that Matte Blanco bi-logic is the most suitable one to be used for trying to employ a creative dimension from a computational standpoint, if one considers the unconscious as the main source of insight. Indeed, the basic inseparability between the *symmetric logic* and the *asymmetric logic* within the bi-logic context, might turn out to be useful just to this end. In this brief article, in particular, we would like to put into evidence some elementary formalizations which are offered as possible formal tools that may shed light upon the above mentioned link between symmetric and asymmetric logic. This is the chief point upon which we want to focus in the present paper. Within the general cultural context, we shall try to clarify Matte Blanco's intertwining of symmetric and asymmetric logic by means of certain very elementary mathematical tools and notions drawn from theoretical physics and algebra.

## II. THE BASES FOR A FORMALIZATION ATTEMPT

### *A. On Matte Blanco's bi-logic: a brief review*

The central point of Matte Blanco's bi-logic is the inextricable intertwining of *symmetric logic*, which reigns in the unconscious realm and is mainly regulated by the generalization and symmetry principles, and the *asymmetric logic*, which rules conscious thought. We are mainly interested in the passage from the former to the latter, this being that process which, if appropriately formalized, may turn out to be of some usefulness from a computational viewpoint[1]. In pursuing this, we follow a line of contextualization which includes some outstanding figures of the history of culture, amongst whom are Gregory Bateson, Claude Lévi-Strauss, André Weil and Robert R. Bush, as well as some elementary but basic formal tools and notions drawn from theoretical physics and algebra. Herein, we outline some main points of Matte Blanco's thought, following (Figà-Talamanca Dore, 1978).

Matte Blanco's work has been centred around the new and ambitious intention of analysing unconscious logic through rational thought. To do this, he put two main principles at the basis of his framework, the generalization and symmetry principles, which are able to explain the main characteristics of the Freudian unconscious, namely displacement, condensation, absence of time,

[1] In this paper, we mainly stress the passage from symmetric to asymmetric thought. Nevertheless, if one considers the symmetry-asymmetry duality as, in a certain sense, analogous to the Freudian primary-secondary process duality, then it will be very interesting, above all from a computational psychoanalysis standpoint, to look at the general relationships between the elements of these dual pairs. In this regard, for instance, the reverse passage from the asymmetric logic to the symmetric one, presumably is domain of the repression, trauma and dream.

replacement of the external reality with the internal one, and absence of contradiction. Matte Blanco's work allows us to clarify the concept of consciousness. Starting from the paradigm of visual perception, based on the dual relationships between macular (or central) and peripheral vision, the focus on one object is perceivable only through a series of continuous eye pupil oscillations around it and not with a direct, fixed and central focusing on the object itself that will lead to an evanescence of the field of view. Analogously, consciousness, like eye movements, builds up asymmetric relationships around the object which is perceivable only through the latter, allowing to distinguish a thing from another. An object of consciousness is the result of a kind of bridling net of asymmetric relations[2] built up around an emotional nucleus of symmetric relations. Consciousness, therefore, acts in an analytical manner, whereas general emotion, running symmetrically, acts in a global or synthetic manner. Nevertheless, both dimensions, or modes of being, are always inseparable and continuously interacting amongst each other. Psychoanalysis has pointed out the fundamental importance of emotion for the psychic life of every human being. The characters of consciousness may be detected only through introspection, which is an asymmetric phenomenon that concerns the time immediately prior to the introspection act itself. Hence, we may only have a retrospective introspection, because it is not possible to think something and, at the same time, to be aware of thinking. This is mainly due to the basic fact that, into the consciousness, not more than one asymmetric relation per time is presentable. Thus, the typical feature of rational thinking seems to be that of reflecting itself upon consciousness. In this sense, questions on psychological and physical time arise, reaching to touch philosophical issues concerning existentialism and phenomenology.

A pure sensation is something having an elusive and fleeting character, which can be caught by consciousness in an extremely brief time before it is inextricably harnessed into some implicit or explicit proposition (asymmetric relation). At a given instant of time, in the consciousness field there may be present only one phenomenon, the other ones going out from the field. Matte Blanco points out the primary fact according to which any human psychic act is roughly an inseparable pair made both by an emotional nucleus and by a surrounding rational thought component, with a variable ratio of reciprocal combination. The emotional component is made by symmetric relations, whereas the rational thought component is made by asymmetric relations. These two components are inseparable from each other, and in continuous reciprocal interaction. Analysing some types of emotions, like falling in love, fear and discouragement, it emerges that the main aspect of the symmetric relations characterizing emotion, is that each emotion refers not only to the intentional concrete object but to the wider and whole class to which such an object belongs, that is to say, to the equivalence class of which such an object is one of its representative elements. Indeed, when one, for instance, loves a person, this latter is seen as someone who goes on beyond herself or himself, personifying the attraction and representing all the attractive persons (maximization – see below). Therefore, three main features of the symmetric relations involved in a (strong) emotional status, are identifiable, namely: *i)* the generalization of all the properties of the intentioned object; *ii)* the maximization of the properties of this object; *iii)* the transposition of this object to every other one who may be represented by it. All this recalls in mind the basic features of an algebraic equivalence relation. Then, the rational thought will build up a covering net made by asymmetric relations around such an emotional nucleus, made by symmetric relations, which constitutes the first and unavoidable apperceptive capture moment of the intentional cathexis object. The main consciousness activity is essentially analytic because it basically subdivides such a globally intentioned object (by emotion) into its constitutive parts, to give rise then to those asymmetric relations which will characterize the consciousness grasp of such an object[3]. Symmetric and asymmetric modes of being are inseparable from each other because an utterly symmetric mode is a typical feature of psychotic or loss of consciousness states, whereas a complete asymmetric mode is also impossible since it would entail a total absence of any cathexis object, which is impossible because of the intentional nature of human desire. As we will see later, we shall try to formalize in an elementary way this last basic process of consciousness putting it into analogy with a formal process consisting of a kind of truly elementary (inversion) symmetry breaking of equivalence relations (of symmetry thought) into asymmetric ones (asymmetry thought). These very simple considerations may also be laid out within a wider and ambitious research program of a sort of *psychoanalytic physics* centred around the general relationships between primary and secondary psychic process, and whose early origins may be retraced in the Freudian *Project for a Scientific Psychology*[4] (of 1895).

### *B. From theoretical physics: on symmetry breaking*

Symmetry arguments are very important in natural sciences. These have also played a certain role in linguistics, starting from Noam Chomsky's formalization attempts up until recent linguistic invariant theory (Guay & Hepburn, 2009). Following almost verbatim (Brading & Castellani, 2013), the symmetry of the initial state of a given situation implies the complete equivalence between the existing possible alternatives (for example, the left bundle of hay with respect to the right one of Buridan's ass argument). If the alternatives are completely equivalent, then there is not a sufficient reason for choosing between them, the initial

[2] A mental image, which often is prior to a definition, is also a set of relationships, hence an asymmetric thought outcome.

[3] This situation is paradigmatically similar to the essence of the so-called *inverse scattering* according to which a given non-directly observable physical object is indirectly detectable only through the set of asymmetric relations emerging from it (like those forming the scattered field emerging from the scattered object). This resembles a metaphoric explanation of what scattering is, due to P.T. Matthews (1974, Chapter 10), according to which the essence of the scattering methods is like to determine the form of an invisible statue (i.e., the mathematical form of the scattering potential) from the angular distribution and intensity of the emerging rays due to a watering of it.

[4] For instance, when, in Chapter 1, Section 3 of this work, Freud talks about *contact-barriers*, we may identify a certain conceptual analogy with the *potential wells* of elementary quantum mechanics. From a historical viewpoint, the positivistic thought (above all, physics) of the 19th-century greatly influenced Freud: in this regard, see (Hall, 1999).

situation remaining unchanged if also the Leibnizean *principle of sufficient reason* (in short, PSR) cannot be applied to settle the question. Arguments of this kind – that is, arguments leading to definite conclusions on the basis of an initial symmetry of the situation plus PSR – have been used in science since antiquity. The form they most frequently take is the following: a situation with a certain symmetry evolves in such a way that, in the absence of an asymmetric cause, the initial symmetry is preserved. In other words, a breaking of the initial symmetry cannot happen without a reason, or an asymmetry cannot originate spontaneously. So, the notion of symmetry breaking in physics makes its appearance, which historically dates back to some works made in 19th-century solid state physics.

To be precise, the study of symmetry breaking goes back to Pierre Curie, who formulated three principles about crystal symmetry on the basis of some of his notable crystallographic research made in the 1890s. Following (Radicati, 1985), he was primarily concerned, not with the symmetry of the dynamical equations, but rather with that of their solutions, i.e. with the symmetry of the physical states. This phenomenological approach led him naturally to emphasize the role of *asymmetry* rather than that of *symmetry*. A symmetry can be *exact*, *approximate* or *broken*. *Exact* means unconditionally valid; *approximate* means valid under certain conditions; and *broken* can mean different things, depending on the object considered and its context. According to Curie, *symmetry breaking* has the following role: for the occurrence of a phenomenon in a medium, the original symmetry group of the medium must be lowered (broken, in today's terminology) to the symmetry group of the phenomenon (or to a subgroup of the phenomenon's symmetry group) by the *action* of some cause. In this sense symmetry breaking, or asymmetry, is what creates a phenomenon. Following (Radicati, 1985), Curie was in a better position to appreciate the role of symmetry breaking as a necessary condition for the existence of phenomena. Generally, the breaking of a certain symmetry does not imply that no symmetry is present, but rather that the situation where this symmetry is broken is characterized by a lower symmetry than the original one. In group-theoretic terms, this means that the initial symmetry group is broken into one of its non-trivial subgroups. It is therefore possible to describe symmetry breaking also in terms of relations between transformation groups, in particular between a group (the unbroken symmetry group) and its non-trivial subgroups. As has been clearly illustrated by Stewart & Golubitsky (2006), starting from this point of view, a general theory of symmetry breaking can be developed by tackling such questions as which non-trivial subgroups can occur?, when does a given subgroup occur? See also Stewart & Golubitsky (2006), and the next section. The notion of symmetry breaking is emblematic to explain the paradigmatic dialectic contraposition between symmetry and asymmetry.

Symmetry breaking was first explicitly studied in classical physics with respect to physical objects and phenomena. This is not surprising, since the theory of symmetry originated with the visible symmetry properties of familiar spatial figures and everyday objects. As has been said above, it was Curie's works on crystal symmetry that opened this new perspective. However, it is with respect to the laws that symmetry breaking has acquired special significance in physics. There are two different types of symmetry breaking of the laws, *explicit* and *spontaneous*, the case of spontaneous symmetry breaking being the more interesting from a physical as well as philosophical point of view. *Explicit symmetry breaking* (ESB) refers to a situation where the related dynamical equations are not manifestly invariant under a certain symmetry group. This means, in the Lagrangian or Hamiltonian formulation, that the Lagrangian or Hamiltonian operator of the system contains one or more terms (like *anomalies*) which are explicitly breaking the symmetry. *Spontaneous symmetry breaking* (SSB) instead occurs in a situation where, given a symmetry of the equations of motion, solutions exist but are not invariant under the action of this symmetry often without any explicit asymmetric input[5] (hence the attribute 'spontaneous'). A situation of symmetry breaking can be first illustrated by means of simple cases taken from classical physics: an emblematic and meaningful example is given by the phenomenology of the well-known *Euler's elastic bar*. In this case, the actual breaking of the symmetry may then easily occur by effect of a (however small) external cause, and the stick bends until it reaches one of the infinite possible stable asymmetric equilibrium configurations. In substance, what happens in the above kind of situation is the following: when some parameter reaches a critical value, the lowest energy solution respecting the symmetry of the theory ceases to be stable under small perturbations and new asymmetric (but stable) lowest energy solutions appear. The new lowest energy solutions are asymmetric but are all related through the action of the symmetry transformations (which are asymmetric relations in the Matte Blanco sense). Therefore, it seems that for reaching equilibrium solutions of a given dynamically unstable problem, it is needed to break the initial symmetry of unstable and unrelated states, so obtaining solutions with less symmetry (asymmetry) but more stability; furthermore these latter equilibrium states, meant as asymmetric solutions of the given dynamical problem, are related among each other by (asymmetric) relations given by the action of a transformation group. In quantum physics, instead, SSB is applicable only to infinitely extended systems (like unconscious, according to Matte Blanco). These last considerations might also turn out to be useful in regard to further formal considerations about an object's relationship.

Historically, the concept of spontaneous symmetry breaking first emerged in the condensed matter physics of the first half of 20th century, and was later transferred to quantum field theory (QFT) in the 1960s, above all in relation to weak interactions. The notion of symmetry breaking still waits to be laid out within a unified formal treatment because of its variegated nature and the different contexts in which such a notable mechanism is involved. Herein, we shall consider only the quantum context in which it is better

[5] In this regard, Strocchi (2012) states that in the development of theoretical physics, the standard way of describing a broken symmetry has been that of introducing an explicit non-symmetric term in the equations of motion. A real revolution occurred with the realization of a much more economical and powerful mechanism, called *spontaneous symmetry breaking*, by which symmetry breaking may be realized even if the equations of motion are symmetric. Furthermore, Strocchi (2012) points out that there also exist cases in which asymmetric inputs are needed.

known and studied from a formal viewpoint. The spontaneous breakdown of a global continuous internal symmetry gives rise to massless bosons (known as *Goldstone bosons*) according to a general QFT theorem, known as *Goldstone theorem*, stated by J. Goldstone in the early 1960s and valid only in the case of global continuous symmetries; moreover, other important elements are *locality* and infinite dimensionality (Strocchi, 2012). Subsequently, starting from the previous notion of *dynamical symmetry breaking* (DSB) related to the creation, via Higgs mechanisms, of massive gauge vector bosons from symmetry-violating vacuum expectation values of Higgs scalar fields, a more general mechanism was proposed, today also known as the *Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism* (hereafter briefly called *Higgs phenomenon*), according to which, when a *global* internal gauge symmetry is promoted to a *local* gauge symmetry, then Goldstone bosons disappear and gauge bosons acquire mass, this taking place often without explicitly breaking the gauge invariance of the theory. Following (Strocchi, 2012), in the Haag-Kastler algebraic approach to quantum field theory, the essence of a spontaneous symmetry breaking for an infinitely extended system, with a *locality* condition related to the algebra of local fields (i.e., local operators) $\mathcal{F}$ which, in turn, contains, as subalgebra, the $\mathbb{C}^*$-algebra of observable fields $\mathcal{F}_{obs}$ associated with the given system, is as follows. From the pioneering works by E.P. Wigner, the physical observables of such an algebra can be obtained only through a suitable Hilbert space representation $\pi$ of $\mathcal{F}_{obs}$. There is a symmetry group $G$ acting on a ray Hilbert space $\mathcal{H}$ to give a group of unitary operators $U(g)$ of the latter. This gives rise to a group of algebraic automorphisms of $\mathcal{F}_{obs}$ through the algebraic map $g \to \alpha_g$ given by $\alpha_g(A) = U(g)AU(g)^{-1}$ for each $A \in \mathcal{F}_{obs}$, which induces an action of the group $G$ on $\mathcal{F}_{obs}$ via the assigned Hilbert space representation $\pi$, in turn induced by the group of unitary operators $\{U(g); g \in G\}$ via the Wigner's theorem. Nevertheless, in the case of an infinitely extended system, there may be automorphisms $\alpha_g$ which are not described by unitary operators provided by a representation $\pi$ of $\mathcal{F}_{obs}$, that is to say, $g$ exists as a symmetry at the algebraic level, but it is not a symmetry of the realization of the system provided by the representation $\pi$ in $H_\pi$, that is, it is not implemented by unitary operators in $H_\pi$. In this case, we say that the symmetry $g$ is *broken* in $H_\pi$. Therefore, there is a substantial level detachment between the algebraic and the functional stance of a symmetry when this is broken. This fact, that is to say disjoint realizations of the physical system induced by inequivalent representations of $\mathcal{F}_{obs}$, is the main essence of the mechanism of symmetry breaking. It is just this inequivalence of, at least, two distinct representations to give rise asymmetry, for infinitely extended systems: in this case, the breaking of color symmetry mentioned in (Strocchi, 2000) and made possible thanks to the comparison with the environment or background, is very meaningful to clarify the idea. From quantum cromodynamics case studies, it is also known that highest symmetry levels imply a flattening of reality, so that asymmetry – arising from symmetry breaking – is a fundamental element to detect this, as well as to account for diversity.

As has already been said above, symmetry breaking raises a number of deep philosophical questions, such as the one asking about the evidence for the (hidden) symmetry underlying the directly observable asymmetry. SSB allows symmetric theories (like those concerning unconscious) to describe asymmetric reality (of consciousness), assuming the salient fact that an observed asymmetry requires the *action* of a cause which can be an explicit breaking of the symmetry of the laws or asymmetrical initial conditions or SSB. This last consideration is very similar to the aim of the Curie principle that, when extended to include the case of SSB, is substantially equivalent to the methodological principle according to which an asymmetry of the phenomena must come from the explicit or spontaneous breaking of symmetry of fundamental laws. This might be called an *extended Curie principle*. Following (Strocchi, 2000), roughly we might say that the general mechanism of spontaneous symmetry breaking related to a physical system is characterized by symmetric interactions between its parts, but the environment in which the system stays is not symmetric (like the vacuum), this implying an asymmetric behaviour of the above interactions. In general, the lowest energy states lie at the bases of this asymmetry. However, in agreement with (Strocchi, 2000), it is just the comparison between, at least, two different realizations that highlights the paradigmatic duality symmetry-asymmetry through the symmetry breaking mechanism, this being just one of the possible philosophical aspects inherent the general symmetry breaking phenomena, to which we shall refer later.

### *C. From mathematics: on groupoids*

The groupoid structures have a wide range of applications, ranging from pure and applied mathematics to physics and computer science. We refer to (Brown, 1987), (Weinstein, 1996), (Buneci, 2003) and (Harary et al., 1965) for more information. In what follows, we are interested in a particular groupoid structure which is the most suitable one to formalize in an elementary way what is our object of discussion. Roughly speaking, a *groupoid* is simply an algebraic system with a partial binary relation. A more precise but succinct definition is that a groupoid $G$ is a small category in which every morphism is an isomorphism. Thus $G$ has a set of morphisms, which we shall call just *elements* of $G$, a set $Ob(G)$ of *objects* or *vertices*, together with two functions, say $s,t: G \to Ob(G)$, $i: Ob(G) \to G$ such that $si = ti = 1$. The functions $s,t$ are sometimes called the *source* and *target* maps respectively. If $a,b \in G$ and $ta = sb$, then a $product$ or $composite$ $ab$ exists such that $s(ab) = sa$, $t(ab) = tb$. Further, this product is associative; the elements $ix$, $x \in Ob(J)$, act as identities; and each element $a$ has an inverse $a^{-1}$ with $s(a^{-1}) = ta$, $t(a^{-1}) = sa$, $aa^{-1} = isa$, $a^{-1}a^{-1}a = ita$.

An equivalence relation $R$ on $X$ becomes a groupoid with $s,t: R \to X$ the two natural projections, and product $(x,y)(y,z) = (x,z)$ whenever $(x,y)$, $(y,z) \in R$. There is an identity, namely $(x,x) \in \Delta(R)$, for each $x \in X$. A special case of this groupoid is the *coarse* or *natural groupoid* $X \times X$, which is obtained by taking $R = X \times X$. This apparently banal and foolish example is found to play instead a key role in the theory and applications. At the opposite extreme to the coarse groupoid $X \times X$, is the *fine groupoid* on $X$ that can be considered as the diagonal equivalence relation on $X$, or alternatively as the groupoid $X$ consisting only of identities, namely the elements of $X$. This consideration of an equivalence relation as a groupoid also suggests the utility of groupoids for studying quotienting constructions, particularly in cases where the quotient set $X/R$ cannot carry the appropriate

structure. We are just interested in these last types of groupoid structures. Following the preprint of F. Latremoliere, given any set $X$ endowed with an equivalence relation $\sim$, we define a groupoid structure over $X$ by letting $\mathcal{G}_\sim = \{(x,y) \in X^2 : x \sim y\}$ together with the following multiplication

$$\circ : \{((x,y),(y,z)) \in \mathcal{G}_\sim^2; (x,y,z) \in X^3\} \to \mathcal{G}_\sim,$$
$$((x,y),(y,z)) \rightsquigarrow (x,z).$$

This operation is well-defined by the transitivity of the equivalence relation, which also gives the associativity property. Now, the symmetry of $\sim$ ensures that for all $(x,y) \in \mathcal{G}_\sim$, $(y,x)$ is also in $\mathcal{G}_\sim$, the latter being clearly the inverse of the former. Finally, the reflexivity is not required to define the structure; the set of units of this groupoid is the set of elements $x$ of $X$ such that $x \sim x$, so when $\sim$ is reflexive, then $\mathcal{G}^0 = \{(x,x) : x \in X\} \cong X$. Note that, for any $(x,y) \in \mathcal{G}_\sim$, $s((x,y)) = x$ and $r((x,y)) = y$, whilst $(x,y)^{-1} = (y,x)$. We shall name this groupoid the *groupoid graph* of the relation $\sim$. Also (Weinstein, 1996) considers groupoids as generalized equivalence relations.

## III. Some Elementary Formalization Attempts

### *A. Again on groupoids*

The above mentioned groupoid structures will make possible certain formalization attempts of some main aspects of Matte Blanco's bi-logic. The groupoid structure has been proved to be at the early bases of almost all the most basic commutative and non-commutative algebraic structures, so that they lie at the deeper roots of the general algebraic formalization. In particular, groupoid structures are also at the basis of graph and combinatorial structures (see (Živaljević, 2006) and references therein) as well as having applications in type theory (see (Barthe et al., 2003)); likewise, ordered groupoids are at the foundations of other algebraic structures, like groups and inverse semigroups (see (Gilbert & Miller, 2011)). Finally, groupoids have recently received remarkable attention also in non-linear dynamics of networks: in this regard, see (Golubitsky & Stewart, 2006), where a very interesting discussion of network synchrony, asynchrony and related symmetry breaking phenomena, in the context of groupoid formalism, is made[6]. The main feature and potentiality of a groupoid structure is just the partiality of its binary operation, that makes this structure quite versatile.

On the other hand, following (Guay & Hepburn, 2009), for some time mathematicians have argued for the more general groupoids as more suitable structures for treating symmetries. At the crossroads between groups and groupoids, lies a distinction between local and global symmetries. For physicists, the local-global symmetry distinction is aligned with the distinction between local and global transformations. For mathematicians, the distinction is based on whether part or the entire structure is conserved (the former requiring a groupoid and not a group representation). Therefore, groupoids are offering more flexible mathematical structures embracing a wider formal tool box to treat symmetry and related phenomena. Groupoids can be understood not only as generalized groups but we can also see them as generalized equivalence relations: to be precise, it is possible to prove the existence of a certain two-way correspondence between groupoids and equivalence relations: see (Guay & Hepburn, 2009) and (Weinstein, 1996) for more details. Meant as a generalization of equivalence relations between parts of an object, groupoids open the door to local symmetries. The group, instead, can only represent global symmetries, that is to say it is unable to put into relation parts of those (whole) objects which are involved in the group transformations. Groupoids will be those unifying formal structures which will comprise either local and global symmetries, both from physical and mathematical standpoint (see (Weinstein, 1996)). Thus, since we stress the fact that equivalence relations, according to Matte Blanco, characterize the unconscious, it is immediately realized that groupoids are the most suitable elementary formal structures to try to give a minimal theoretical framework of this psychoanalytic construct.

Finally, we consider ordered groupoids, that is to say groupoids equipped with a compatible partial order relation. Following (Lawson, 2005), every ordered groupoid is isomorphic to one constructed from a category acting in a suitable fashion on a groupoid arising from an equivalence relation. Lawson considers the simplest groupoids, those arising from equivalence relations and named *combinatorial groupoids*. Then, he constructs ordered groupoids from combinatorial ones plus some data, namely a partial order in turn inferred from a partial preorder relation that, quotienting, gives rise to a partial order. In turn, such a preorder relation may be induced by a suitable category action on the given combinatorial groupoid. Thus, the essential steps are as follows:

- A category $C$ acts on a combinatorial groupoid $H$.
- It induces a preorder $\preccurlyeq$ on $H$ whose associated equivalence relation is $\equiv$.
- The quotient structure $H/\equiv$ is a groupoid $J(C,H)$ on which the preorder induces an order.
- The groupoid $H/\equiv$ is ordered and every ordered groupoid is isomorphic to one constructed in this way.

This is the clear formal treatment given by Lawson, in which a final universality property is proved, namely, every ordered groupoid is isomorphic to one of the form $J(C,H)$ for some action of a category $C$ on a combinatorial groupoid $H$. Roughly, if the

[6] As regard possible links between Matte Blanco thought and synchronous/asynchronous logic, see also (Iurato, 2013).

above action of $C$ on $H$ is of the general type $(a, x) \to a \cdot x$ (e.g., it may provide some interpersonal relation) then we put $x \preccurlyeq y$ in $H$ if and only if there exists $a \in C$ such that $x = a \cdot y$, being then easy to prove that $\preccurlyeq$ is a preorder; then, $x \equiv y$ if and only if $x \preccurlyeq y$ and $y \preccurlyeq x$, is an equivalence relation on $H$. Finally, an order on $H/\equiv$ is given by setting $[x] \leq [y]$ if and only if $x \preccurlyeq y$, that is to say, if and only if there exists $a \in C$ such that $x = a \cdot y$, this last definition turning out to be well-posed. Finally, given $O(x) = \{a \cdot x; a \in C\}$ the orbit of $x$ under the given action of $C$ on $H$, then it is easy to prove that $[x] < [y]$ if and only if $y \notin O(x)$, that is to say $O(x) \cap O(y) = \emptyset$, with a consequent violation, with respect to the given action, of the inversion symmetry $(x, y) \to (y, x)$, which is the simplest example of a permutation symmetry breaking with respect to the diagonal. Therefore, a strict order arises from a simple inversion symmetry breaking, which basically expresses an asymmetry condition as a simple outcome of the inequivalence of at least two different representations. Furthermore, following (Hermann 1968, Chapter 2), every transitive action of a group/groupoid structure $G$ on a set $M$ induces on it a *homogeneous space* structure. It is possible to prove that the study of homogeneous spaces can, in principle, be reduced to the study of coset spaces $G/H$ when $H$ is a sub-group of $G$, hence to the study of pairs $(G, H)$ of the type group/groupoid-subgroup/subgroupoid, just the typical formal pair of algebraic structures involved in the above formulation of symmetry breaking phenomena. On the other hand, a strict ordering is, in a certain sense, a formal condition characterizing the asymmetry involved in every possible hierarchical structure which is a strictly ordered structure too. In conclusion, we can say that, in a certain sense, inversion symmetry breaking is a compatible mechanism that, for instance, via combinatorial groupoid or groupoid graph structure, is offering to formalize the passage from symmetry (essentially represented by an equivalence relation) to asymmetry (essentially represented by a strict order relation[7]). As has been said at the end of section II.C, a groupoid structure is naturally associated with every equivalence relation, where reflexivity is not necessarily required. Now, the properties of an equivalence relation approximately correspond, by means of a kind of cryptomorphism theorem, to Matte Blanco's generalization[8] and symmetry principles, whereas a preorder is simply a reflexive and transitive binary relation. An order [strict order] is an antisymmetric preorder that differs from an equivalence relation simply replacing the symmetry condition with an antisymmetric [asymmetric] one. Thus, with a breaking of inversion symmetry, we obtain an asymmetric relation. Therefore, via groupoid graph or combinatorial groupoids, it is possible to go from an equivalence relation to an order one, basically through a suitable inversion symmetry breaking.

*B. On the formal structure of kinship, on the Gregory Bateson double bind, and all that*

Following (White & Jorion, 1996), it was André Weil and Robert R. Bush to give, starting from *Murngin* system, a first attempt to formalize kinship by means of permutation group structures, exposed in a very interesting appendix to chapter XIV of the celebrated Claude Lévi-Stauss 1949 work *Les Structures Élémentaires de la Parenté*; see also (Kemeny et al., 1974). While the empirical cultural content remains to be filled in, the problem of formal representation of kinship is concerned with the relations or orderings between culturally given unions such as reproductive or other types of matings, not the cultural characteristics of the unions themselves. In graph theoretic terms, vertices and their labels, even while associated, are distinct. Therefore, ordered structures are at the early bases of every further attempt to formalize kinship. After this work of Weil and Bush, other formalizing attempts were attained by other scholars, mainly using graphs and other ordered structures. But, having seen that groupoids are at the basis of all these just mentioned structures and having seen too that not all binary relationships are allowed in any possible kinship structure theory, it seems quite natural to think groupoids (like combinatorial groupoids or groupoid graphs) as the most suitable structures to formalize kinship and, in general, interpersonal relationships, just due to partiality of its binary operation. In any case, every possible formal structure used so far to formalize kinship, is an a priori given ordered structure which may be thought as a formal framework of a social structure as it really appears, without giving any hint on the possible early origins of it, that is to say, how and whence it came about. On the other hand, according to (Lerner 1961, Model 2), thanks to the Weil and Bush work, the matrimonial rules of primitive societies are aimed to hinder the marriage between close kin even in the case in which the involved individuals weren't aware of being in a kin relationship; this was possible because there exist primitive[9] societies where familiar links are liable to be quickly forgotten. This means that implicitly (or unconsciously) they follow just these group rules, so that it is allowed to consider group/groupoid structures as tools suitable to formalize such questions, in a very precise manner. Not all marriage types are allowed, the marriages between brother and sister turning out to be automatically hindered, so that it is as if Oedipal rules unconsciously acted to shape kinship[10]. This is a notable fact because it is very amazing to see how a primitive society, through trial and error, has been able to perform formal structures of a certain complexity, like the group ones. Then, since psychology (see (Atkinson et al., 1996)) and social anthropology (see (Lévi-Strauss, 1975)) are nowadays even more oriented to

[7] Furthermore, the strict set inclusion is a (partial) strict order which allows to distinguish, for instance, between set and (proper) subset, hence to introduce a minimal hierarchical setting.

[8] Matte Blanco generalization principle shouldn't be literally meant as it is commonly stated, because its usual definitions could lead to the wrong possibility to clearly distinguish between set and subset in a hierarchical way, a thing which is impossible to do in the unconscious realm. Instead, such a generalization principle should be rather understood as expressing the ability to make equivalence classes and partitions. Only overcoming symmetry and generalization principles, it will be possible to reach communicative metalevels, the simplest of which having been those concerned with the various distinctions among element, subset, set and class arose, with the rising of type theory.

[9] The primitiveness condition is a primary working hypothesis which puts us closest to unconscious behaviour.

[10] This is coherent with what Lévi-Strauss claims about the origin of culture and society (hence consciousness), because he puts the Oedipus complex at the basis of the crucial passage from nature to culture. Moreover, from this standpoint, we might also identify a kind of anthropomorphic origin of group structures.

consider a familial nucleus as the primary structural key of society as well as the natural environment that much more influences the psychological growth of an individual, in agreement with this, we would want to put forward the hypothesis that the familial triadic structure is the key component which lies at the early basis of any possible further social formal structure in agreement with what Lévi-Strauss himself states in (Lévi-Strauss, 1975). Therefore, we want to focus on the family structure, believing that it is just its deep triadic structure that mostly contributes to the individual's basic psychological formation.

Now, according to Matte Blanco, the main feature of consciousness is just the passage from the symmetric logic to the asymmetric one, so that it is of extreme importance to shed light on this crucial step. The minimal cardinality of a non-trivial ordered groupoid is three for two chief reasons: first, to warrant a non-trivial transitive property, second because of the fashion with which an ordered groupoid is obtainable, that is to say by means of non-trivial action of a non-trivial category with at least two elements, on a non-trivial combinatorial groupoid with at least two elements. Now, in the 1950s, Gregory Bateson, Don D. Jackson, John Weakland and Jay Haley gave a possible (interpersonal) interpretation of schizophrenia based on the so-called *double bind* (see (Bateson 1972, Part III)), in which the familiar nucleus plays a fundamental role. Let us briefly discuss it. Bateson, likewise Matte Blanco, knew Bertand Russell and Alfred Norton Whitehead *type theory*, the first one that enabled to identify different levels of abstraction, pointing out the criticalities and potentialities of two crucial axioms of formal set theory, the *axiom of specification* and the *axiom of extensionality*, which allow to clearly distinguish between element, set and class, starting from the well-known Cantor *naïve set theory*. So, the way towards metamathematics had been opened. Taking into account this theory, as well as previous notable studies made by Rudolf Carnap, Ludwig Wittgenstein, Frieda Fromm-Reichmann and Benjamin Lee Whorf, first Bateson considered the possibility of the existence of many abstraction levels in human communication, with the logical distinction between different *types*[11]. For instance, in schizophrenia, there is often a confusion between literal and metaphoric levels of either her or his own and other messages (inability to metacommunicate). He also argues on the lack of the main formal properties characterizing an equivalence relation, that is to say reflexivity, symmetry and transitivity, which are almost totally disregarded by primary process[12]. There, indistinguishable are everyone from someone and no-one, as well as the whole from the part, which are features of conscious processes. Then, Bateson and his collaborators point out the chief fact that a schizophrenic is mainly unable to contextualize (to realize *metacommunicative frameworks*, according to Bateson), that is to say, he or she has serious problems in semantics and pragmatics tasks[13], as well as in discriminating the various logical types. Bateson and co-workers[14] bring this back to the presence of a double bind. The double bind will be retaken by the *Palo Alto school* of Paul Watzlawick on pragmatic communication (see (Watzlawick et al., 1967)).

The notion of *double bind* is quite original, and refers to the presence of a double and logically ambiguous link between mother and daughter/son which moulds two-way interpersonal relations centred around an emotively ambiguous and, at the same time, logically contradictory communication between mother and daughter/son, and that will engrave, in a negative manner, upon the next emotive states and logical abilities of the girl or boy, hindering the capacity to distinguish the logical status of the thoughts, if the paternal figure does not act as an intermediary[15], a kind of "breaker" coherently with what will be said later. Indeed, it is as if such a paternal figure acted by breaking the inversion symmetry between mother and daughter/son, re-establishing the right hierarchical position within the familial nucleus. The paternal figure (or paternal caregiver) acts to make inequivalent at least those two primary representations given by maternal and paternal figures, as a kind of symmetry breaking, according to what has been said in section II.B. The double bind plays a fundamental role in structuring the family[16]. Indeed, following (Jackson, 1954), from the clinical analysis of the triadic familial structure of six female patients, it turns out that in all these cases but a psychotic patient, the mother permitted a third person – like father – to be present between the double bind given by mother-daughter, acting as a sort of "role inverter": in fact, Jackson reports, for instance, that the father's closeness and overt interest in his daughter, was the reverse of the mother's in that it tended to decrease as the child grew older and, in some instance, was abruptly terminated at the menarche. These patients have been trained symbiotically to feed on triadic involvement. This is most apparent when they are interacting with only one person and must in fantasy involve a third as though they feel no "ego wholeness" without a collection of "part egos". One price paid for this need to interact in two directions at the same time is a multifaceted inferiority feeling. For example, these women equate head and penis, hence intellect and maleness. The parental interaction constituted a *nidus* for the development of the girl's

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[11] A very useful and interesting application of Russell-Whitehead type theory in informatics has been given by B.C. Pierce in (Pierce, 2002), with his *type system*.

[12] For instance, in (McCulloch 1965, pp. 40-44), it is reported the case of the intransitivity of psychological preferences, a kind of psychological basis for the well-known Kenneth Arrow theorem on social choices. See also (Harary et al., 1965) and references therein, about other interesting remarks on transitivity or not, concerning many examples of interpersonal relationships.

[13] Modern neurophysiologic research confirms this last fact, that is, schizophrenics have mainly impaired the basic semantic-pragmatic integration functions (see (Iurato, 2012) for a very brief review).

[14] The main components of this research group (*Palo Alto school*) were Gregory Bateson, Donald D. Jackson, John Weakland, Bill Fry, Paul Watzlawick, Janet Helmick Beavin and Jay Haley.

[15] In this, recalling the famous Lacanian *name-of-the-father* (see (Iurato, 2012)).

[16] The Bateson and co-workers theory is based on a set of controlled studies as well as on a wide series of observations, even if related only to schizophrenic disorders. First extensions to the general social context of this theory, going beyond the schizophrenic domain, have been proposed by Carlos E. Sluzki and Eliseo Veron. Nevertheless, the Bateson hypothesis on double bind has found experimental validation only within the familial group (see (Sluzki & Ransom, 1979)). All this would require further attention.

hysterical and phobic symptoms and acting out. Therefore, the paternal figure could be seen as *acting*[17] on the double bind to break the often dangerous inversion symmetry mother-daughter/son[18], distinguishing between, at least, two different realizations, so giving rise to an order relation; as regard symmetry of double bind, see (Watzlawick, 1967). On the other hand, psychoanalytic history (from Œdipal phase till to Lacan's theory[19]) has always stressed the primary and unavoidable role played by a paternal figure in breaking the dangerous symbiotic and aphasic reciprocity of child-mother. In agreement with Matte Blanco's bi-logic, *triadicity*, therefore, is a primary and essential structural element to have the primary presuppositions for a normal psychic development. There exists literature on the relationships between Matte Blanco and Bateson. The above supposition concerning symmetry breaking made by the paternal figure is supported by the interesting work of Klaus Fink (1993) about the bi-logic perception of time. Fink reports a clinical case of a young man (John) who had not a really present paternal figure but a dominant, incorporating and prevaricating maternal figure who caused a spread of symmetrical thought which entailed, amongst other things, a distortion of time perception, with an impairment of time ordering. Only through the transference intervention of the psychoanalyst, it was possible to try to restore a stronger paternal role, so recovering the right time perception, hence stemming maternal symmetric thought drives. John gradually re-acquired his own sense of individuality and awareness, a right time perception, a normal balanced relationship between physical and psychological time[20], all elements, these, which gave rise to a right phenomenological psychology relationship between *here-and-now* and *there-and-then*, roughly between psychic (or internal) and external reality. On the other hand, also Bateson and co-workers had already pointed out about the possibility to implicitly change rules only under transference setting[21]. Therefore, according to Fink, a correct reality-testing just consists in putting (also by means of transference intervention) the right relationships between symmetrical and asymmetrical thinking. Again according to Fink, the resolution of psychic conflicts has been considered as a sort of *catastrophic point* (also in the René Thom sense) by W. Bion, as well as an *uncanny point* by Freud himself, to highlight the criticality of the turning-point element marking the equally crucial transition point from symmetrical to asymmetrical thought. Fink's considerations also justify our view of this passage as a kind of symmetry breaking development. On the other hand, Freud too considered time as one of the main features of the passage from primary to secondary process, related to delaying of desire's satisfiability.

Taking into account what has been said in section II.B, if one considers the passage from symmetric to asymmetric thought as due to a kind of inversion symmetry breaking, besides that as an overcoming of Bateson and co-workers double bind mother-daughter/son through father action, then it is possible to account for the rising of ordering, hence hierarchical structures. In particular, taking into account what has been said about Higgs mechanism, the SSB allows the transition from a *global* to a *local* symmetry with a generation of massive bosons (asymmetry), which might be interpreted as an unavoidable presence of symmetry interstices into the asymmetry, a thing quite analogous to the bi-modal presence of symmetric and asymmetric thought. Therefore, although a global symmetry has broken into local symmetries, these latter never utterly disappear (because symmetry breaking in any case involves non-trivial groups), consistent with Matte Blanco according to whom conscious thought may include only few asymmetrised pieces of symmetry. On the other hand – see for instance (Wetterich, 2005) and (Ellis et al., 1992) – at the early origins of space and time and their difference, there could be basically symmetry breaking phenomena. The unavoidable presence of symmetry interstices into the asymmetry might also be put into comparison with the so-called *paraconsistent logics* in which not all contradictions (due to symmetric thought) of a formal system have serious implications or relevance for the whole system; these contradictions may be relegated into neighbourhoods, or circumscribed regions, in such a way that they cannot influence the

[17] For a technical treatment of the theory of symmetry breaking via group action, see, for example, (Albertson & Collins, 1996) as regard graph theory, and (Field, 1996) as regard bifurcation symmetry breaking theory.

[18] On the symmetry or reciprocity character of double bind, as well as on other interesting remarks and considerations, see (Watzlawick et al., 1967, 6.43) and references therein. In this work, the general presence and relevance of double bind, as well as the need for overcoming it, in regard to all those systems having a certain intelligence autonomy (including animals), is highlighted.

[19] Indeed, the (symbolic) father mediation between mother and child is needed for making a separation between the imaginary register and the symbolic one.

[20] In this regard, it would also be possible to suppose that the crucial passage from symmetric to asymmetric thought might be related to the equally crucial relationships between physical time (*chrónos*) and psychological time (*kairós*), amongst which there is no a priori two-way correspondence. Psychology of time, from Paul Fraisse onward (see (Vicario, 1997)), says that these last two are independent amongst them. Perhaps, the acquisition of physical time perception, considered having an ontological dimension, might exert, according to Klaus Fink, a structuring action on psychological time, making easier or even inducing the passage from symmetric to asymmetric thinking. Fink's work has been confirmed by further studies on time distortion in the transference (see (Rose, 2011)), supporting the idea that psychic reality is strictly influenced by physical time, as experienced in psychoanalytic transference that, as known, above all reproduces the childish Oedipus conflicts, re-evoking the various familiar images. *En passant*, we remember that also the mathematician L. Brouwer claimed the primary importance of a sort of (Kantian) *temporal a-priori* in the mathematical reasoning, that is to say, he gave much attention to the basic role played by the temporal dimension. Likewise, some remarkable phenomenological-existentialistic trends of philosophy pay much attention to the role played by temporal dimension from a psychic standpoint. Finally, time perception seems to be impaired in schizophrenic patients (see (Bonnota et al., 2011)).

[21] And this could reflect also on Krikpe-Wittgenstein paradox and on a clarifying psychoanalytic revisitation of it. Following (Piattelli Palmarini 1987, I), Kripke provides a kind of collective solution to this paradox, that is to say, the reason (even if the usage of this term is paradoxical just in this context, unless we distinguish between implicit and explicit reasons – in this case, it would be better to speak of an *implicit* reason which will become *explicit* at the consciousness level) why we follow a rule should be a collective fact, not an individual one, as if it belonged to a sort of collective unconscious (like, for instance, the one described by C.G. Jung and C. Lévi-Strauss).

coherence of the system, without having trivializations of it. Thus, also in logic, with this last type of confinement operations, it is possible, as the saying goes, to conceive a kind of contemporaneous presence of symmetric and asymmetric thought elements, coherently with Matte Blanco bi-logic. Finally, what has been said so far is also coherent with many points of the rich thought of Jean Piaget on the genesis of consciousness in children.

*C. Other formal remarks*

Following the work of Klaus Fink (1993), we have pointed out the primary role played by transference intervention in setting up the right *ratio* between symmetric and asymmetric thinking. Also Bateson and co-workers pointed out the great force exerted by transference to implicitly change interpersonal rules which are the symbolic transposition of the familiar ones. The two main dimensions which characterize the transference, where the essence of childish Oedipus complex is re-evoked, are the actualization of past experience and the displacement towards the paternal figure. The paternal imago, in general, is the one that has major load in the transference intervention; other imago, like the maternal one, may also be involved in transference setting. Therefore, following Bateson and co-workers, we agree with the assumption that the paternal figure plays a very fundamental role in establishing the right interpersonal relationships (according to Freud, symbolically moulded on the familial ones) and in overcoming the pernicious double bind.

In the previous sections, we have tried to formalize, in an elementary fashion, the crucial passage from symmetric to asymmetric thinking, taking into account some elementary notions drawn from theoretical physics, and certain basic structures drawn from algebra. In particular, we have considered some groupoid structures which are offered to formalize certain notable aspects of Matte Blanco's bi-logic. Now, recent research on computational psychoanalysis made by Lauro-Grotto (2008) and Murtagh (2012) about structure of the semantic field, have realized that the main aspects of the unconscious realm, via Matte Blanco bi-logic, can be formalized by means of ultrametric structure and that the passage from symmetric to asymmetric thinking might be put into analogical comparison with the formal passage from ultrametric to metric. But as early Khrennikov (1998; 2002; 2007), even before, had extended p-adic dynamical systems and analysis to mathematical modelling of mental space and psychoanalysis. To be precise, an ultrametric is a metric which satisfies a stronger triangular inequality, namely the following

$$d(x,y) \leq max\{d(x,z), d(z,y)\}, \ \forall x,y,z.$$

Now, Lauro-Grotto (2008) claims that if one considers, following Matte Blanco, an ultrametric space model of unconscious, due to the intrinsic properties of such a type of formal space, there follows a notable restriction of semantic field because of the great number of clusters of concepts that become indistinguishable due to a loss of homogeneity in their inner structure, with a consequent loss of hierarchical ordering. Therefore, the Matte Blanco symmetric-asymmetric thinking duality could also be stigmatized by the formal duality ultrametric-metric. On the other hand, the inseparability of these last two notions, essentially given by the various isometric embedding theorems of ultrametric spaces into a Euclidean space (see (Murtagh, 2012) and references therein) could refer to the inseparability between symmetric and asymmetric thinking within the Matte Blanco framework. Finally, from this standpoint, Lauro-Grotto also claims the possible usefulness of the consideration of the *replica symmetry breaking* concept of spin glass theory, coherently with what has been said above about symmetry breaking phenomena. Moreover, the above considered groupoid structures might also be usefully and suitably implemented by these last metric structures to reach a most complete and general formal framework of computational psychoanalysis modelling the unconscious construct.

We conclude with a final but important remark. It could seem quite contradictory to use an order relation, like $\leq$, in defining a property that holds in the unconscious realm, such as the one expressed by the above ultrametric triangular inequality. Indeed, just in the unconscious realm, due to symmetry principle, it is unconceivable to think any order relation. But, again following Klaus Fink's work (see (Fink, 1993) and references therein), such an apparent contradiction may be easily clarified. In fact, as said above, in transference, it is possible to re-establish the right relationship between external and internal reality through the institution of the right correspondence between external and internal time perception. This was possible just thanks to transference action that allowed the patient to make him aware how something *conceived* symmetrically[22] (unconscious disorder) must be *judged* asymmetrically[23] (usage of an order relation $\leq$ in the ultrametricity condition); there are no other ways to do this[24] because, having seen the nature itself of the unconscious, it couldn't be otherwise; according to Fink, this is ultimately what a

[22] Like in apprehension.

[23] Like in the expressed thought.

[24] Following (Mitchell & Black, 1995), maybe only Lacan was able to give an original and expressive description of mode of manifestation of the unconscious. Indeed, the elusive-allusive-illusive modality, the incrustations of rhetorical figures, the kaleidoscopic erudition, the intentional ambiguity, the grandiose expression, the perverse echoes of past authors, the oblique irony, the disdain of logical sequence, the humour and sarcasm, are all forms of an affected and precious modality with which Lacan wanted to deliberately show, with his celebrated verbal eloquence, what perverse manners are used by the unconscious to manifest itself.

reality-testing should consist of. On the other hand, the Matte Blanco epistemological program consists just in searching to explain unconscious phenomenology by means of rational thought. Therefore, in this only apparently contradictory sense, the Lauro-Grotto formal view must be understood, based on ultrametric spaces[25]. In any case, this last type of question would deserve further attention[26]. *En passant*, just for what has been said above about rule changing in transference, we would also want to put forward the hypothesis according to which the celebrated Kripke-Wittgenstein paradox, inasmuch as it is strictly involved in the critical question around the origins of rules, could receive a useful clarification from the psychoanalytic epistemology standpoint.

## IV. Further Remarks

In this first part of the paper, we have mainly discussed, within the general context of the history of culture, the crucial passage from symmetric to asymmetric thinking within the Matte Blanco framework, trying to formalize it through some elementary formal algebraic structures, the groupoids, with respect to which critical comparisons between equivalence and order binary relations have been made possible. Some simple formal considerations have been then put forward, also taking into account some elementary basic notions drawn from theoretical physics. But the landmark point which we wish to highlight in this paper is how order (asymmetry) springs out of that utterly disordered and chaotic realm that is the unconscious. This has been summarized masterfully by the following beautiful Greek maxim due to Anaxagoras of Clazomenæ[27]

*«In the beginning, all the things were together;*
*then, it came the mind (ó νοῦζ), and set them in order».*

Hence, once again, is a great appreciation for the foresight of classical wisdom. Now, in the second part of this paper, we carry on with other psychoanalytic considerations and further formal attempts which also confirm what has been said above or are linked with them.

## V. Brief Outlines of the Sasso's Model

The recent, valuable work achieved by Giampaolo Sasso, and exposed in (Sasso, 1999; 2005; 2011), has been centred on the critical relationships between neurosciences and psychoanalytic theories, taking into account the outcomes of the latest *Infant Research*. From it, as well as following what has already been very briefly outlined above (and published in (Iurato, 2014)), some further brief discussions and suggestions concerning possible formalization attempts of certain main aspects of the Freudian psychoanalytic framework may be usefully worked out. In the meanwhile, throughout the paper, clear and continuous recalls of Husserlian phenomenology and its historical roots are easily identifiable, until we reach a sort of phenomenological psychoanalysis[28].

### *A. A brief sketch of the model: first elements and basic notions*

From his experience as psychoanalyst and psychotherapist, Sasso has cast the first bases for a new unitary framework of the object relationships in (Sasso, 1999), trying to take into the right account the recent developments achieved by neurosciences. Following (Sasso 1999, Introduction), the functionality of the brain is mainly due to the dynamical action of a set of certain non-localized *mobile functions*, variously dislocated around all the general network of the nervous system which enact different perceptive-sensorial zones of it through *cerebral maps*, whose globally unitary dynamicity is the chief characteristic of the mind that, thanks to the running of distinct cerebral zones, allows to analyse different properties of an intentioned *object*.

These functions basically represent flows of information, along neural lines, given by the known psychic *introjective* and *projective* mechanisms. These latter neural pathways are called *s-o lines* (or *s-o pairs*) and locally connect two elements, namely an element *s*, representing a mind element approximately located in the frontal, motor and premotor areas, and an element *o*, codifying a somatic-sensorial or a perceptive property of the object. Thus, neurophysiologically, a *s-o* line is a motor-perceptive path which connects a motor element *s*, aimed to reach the subject functions, with a perceptive element *o*, devoted to attain object functions. Along the human ontophylogenetic evolution, a very great amount of *s-o* lines arises, also with the support of the *neural redundancy* which neurobiologically characterizes the ontogenetic evolution of the human brain. The *s* elements (where *s* stands for subject), at first, explicate simple control and motor functions upon the perceptive *o* elements (where *o* stands for object), hence they locally couple for giving rise to a *s-o* pair, that is to say to an *object relationship*, which will move along *s-o* lines. This simple dynamics is the essence of the mobility feature of the mind, roughly carried out by the dislocation ensuing from the double mobility given by both *s* and *o* elements that locally reconnect to produce *s-o* pairs which accomplish the basic role to locally represent, according to a general *vicariance principle*, a distal or global process just by means of multiple and structural integration process of

[25] In this regard, see also what is said in section 5. of the preface, by Remo Bodei, to (Matte Blanco, 2000).

[26] In any case, it seems that all these last types of considerations are part of a more general and wider epistemological discussion inherent in the origin, meaning and validity of the contradiction principle, its possible levels, types and interpretations, in the light of the critical relationships between mythological and rational thought.

[27] See the well-known Diogenes Lærtius, *Lives of Eminent Philosophers*, Volume II, Chapter III.

[28] This is, besides, not fully unfounded historically because, throughout the Freudian work, close parallels, meaningful comparisons and immediate analogies jump to the eye if one points out what primary role the *object* has played, and related notions (like object relationship or object cathexis, intentionality, and so on), in both contexts.

various primary *s*-*o* lines. This last principle, which allows a flexible functional dislocation or mobility, formalizes the well-known Freudian processes of displacement and condensation (see (Sasso 1999, Chapter 2, Section 2.4)).

Therefore, an object representation, within this model, is mainly based on a structural assemblage of primary *s*-*o* lines, each of which establishes an inseparable constraint between the representational structure of the *o* elements and the subject function of the *s* elements. The system of the *s*-*o* relations is already present since the prenatal phase and its potential strength to structurally organize is meaningful in regard to the *intentionality* of the child to early establish object relationships. The structuring of this rich system of relations through suitable reorganization and reconnection of its elements, is not an easy task that will be at the basis of the general psychic development. In doing this, the mother will play a truly fundamental role, since she is the first and closest human figure who the child approaches, hence the first object. Such an interaction takes place, by resonance[29], between an autonomous projective-introjective (*P*-*I*) endogenous dynamics, connate with the child and continuously turned on by the reticular formation, and the action of the mother who alternatively stimulates the functional properties of the *s* element and of the elements *o*, contributing to their reconnection into the *s*-*o* lines.

The neural information so stimulated, goes through the *s*-*o* lines in either directions *s*→*o* (roughly comparable with a *bottom-up* process) and *s*←*o* (roughly comparable with a *top-down* process) forming a net with capacity of rapid reconnection. Therefore, a *s*-*o* line should be more properly written as follows[30] $s \leftrightarrows o$ since it is the superposition of two unidirectional flows, namely *s*→*o* and *s*←*o*, each of which provides a *proto-object relationship*. The flow *s*←*o* integrates, into the perceptive element *o*, the information coming from environment, and its codification as internal information is the basis of the introjective processes. The flow *s*→*o*, instead, sprung out of the internal information already codified into the motor element *s*, and is aimed at anticipating and completing the external information, underlying a projective process. The continuous construction of the *s*-*o* line network, which constitutes the basis of the *P*-*I* endogenous dynamics[31], at first starts from the stimulation of the own reticular formation of the cerebral trunk through the combinatorial action of basic elementary functions ψ (d'après Freud) which provide ancestral innate abilities of reconnection of the elements *s* and *o*, so giving rise to the first nuclei of the *Self* that will relate with objects. The mother is at first support for the set of elements *o* of the child's net, then she contributes to the various identificatory meanings to be assigned to the information conveyed by the *s*-*o* lines through the directions *s*→*o* and *s*←*o*, through an integrative modulation and coordination by maternal introjective information (*Im*). From a functional viewpoint, a link of the type *s*←*o* assigns an object property *o* to the subjective element *s*, so giving rise to an identificatory constraint of *introjective* type (*I*); instead, a link of the type *s*→*o* entails that a subjective function *s* is assigned to an object property *o*, so giving rise to an identificatory constraint of *projective* type (*P*). When these latter pathways undertake a preferential role, they provide stable introjective and projective properties. If the information given by maternal object (*Im*) is massively present, then introjective proto-object influences of the type *s*←*o* prevail, whereas, if *Im* lacks, then projective proto-object developments of the type *s*→*o* prevail.

The maternal introjective information *Im* greatly influences, above all through the procedural memory structures, the growing up of the representational network given by the articulating bundle of the *s*-*o* lines, through a continuous re-organization of the endogenous and exogenous flows involved in the first autonomous *P*-*I* endogenous dynamics, and that constitute the proto-object relationships from which derives the Freudian drive structure and that will take object nature just by means of the action of *Im*, so giving rise to the Freudian object relationship structure. The crucial point is the right interplay between the constructive endogenous *P*-*I* dynamics and the exogenous object maternal influence *Im*, the former having to projectively regulate, at the primary proto-object level, the introjective modulation by the latter, to give rise to advanced object relationships. The above mentioned basic interaction (*P*-*I*) – *Im*, will be re-enacted and restored, in the right terms, during the crucial transferral-countertransferral relation involved in a psychoanalytic setting, which operates through linguistic nets.

### *B. A brief sketch of the model: further developments*

In (Sasso, 2011), the author retakes what was covered in his previous works (Sasso, 1999; 2005) to try to clarify a possible origin of consciousness[32]. Sasso again stresses the primary role played by the child's personal *P*-*I* endogenous oscillatory dynamics $s \leftrightarrows o$ which will drive the fundamental psychoanalytic identificatory development by means of the building of the representational system of the child which, in turn, will be greatly ruled and moulded, again through a periodic oscillatory dynamical mechanism, by the available introjective *Im* influences of the mother whose unconscious representations, already quite well-rooted in her, will converge into the incoming consciousness of the child just thanks to this endogenous dynamics with which the child naturally interacts and to whom is phylogenetically predisposed. So, the psychic development of the child begins at once with the institution of identificatory relations from her or his early stages, the mother greatly influencing this frail process with her own consciousness

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[29] The author, in (Sasso 2005, Introduction), considers the brain very similarly, although in a simplistic manner, to a composite musical instrument which runs through a complex resonance which is made active from reciprocal interactions.

[30] Already J. Piaget used a similar notation to mean the indissoluble active and dialectic relation between subject *S* and object *O*, from which the knowledge springs out of.

[31] This *P*-*I* endogenous dynamics should be meant as a sort of 'internal breath' of the cerebral system, almost analogous to a physiological one.

[32] In what follows, if not otherwise specified, we shall always refer to (Sasso, 2011), which is the main reference for this second part of the present paper.

states. The primary psychic development is centred upon procedural (or implicit, hence unconscious) memories, which remain inaccessible to the next accrual of the declarative (or explicit) memories, hence with denied access to the psychoanalytic exploration that just makes use of declarative memory functions only. This implies a basic distinction between two chief types of repression mechanisms, namely a deeper *non-removed primary repression* (correlated with a *dynamical unconscious*) and a *removed repression* (correlated with a *removed unconscious*), a distinction, this, already sketched but left incomplete by Freud himself, which, instead, can now be quite well clarified and formalized within the Sasso's model.

Into the *s*-*o* line network, one should distinguish a particular subset of it, namely the one formed by those lines of the type $s_i$-$o_i$ characterizing the internal psychic states related to the *representational* (or *perceptive*) *internal objects* $o_i$. The formation of the latter is needed for giving a wider and even more articulated reticular formation by means of a continuous going inwards and onwards along the initial line *s*-*o*, due to the intermediary presence of an internal object, say $o_1$, inside it, that is *s*-$o_1$-*o*, with the formation of another level line, *s*-$o_1$, but in a higher level than that of *s*-*o*, which, in turn, will require another element *s*, say $s_1$, to settle the new higher level line $s_1$-$o_1$, and so forth. This is the typical iterative process with which the *s*-*o* line system grows up and develops, the appearance of the various intermediary internal elements $o_i$ depending on the sensorial stimulation or excitation, whilst the consequent production of the elements $s_i$, needs to maintain the related stability of the system, and greatly depends on the *Im* action. In such a fashion, the basic reticular formation evolves, with even more complexification of its structure, from its lowest bases, placed into the most ancient cerebral structures, through the sub-cortical and limbic structures toward the high cortical ones. The various elements *o* will concentrate into the posterior perceptive-sensorial cerebral zones to give rise to the *object function*, whilst the consequent control elements *s* will converge into the anterior control cerebral zones to give rise to the *subject function*, and to control the former ones.

Initially, the various internal representational elements $o_i$ are distributed amongst many different cerebral zones, just this variegation standing at the basis of the human capability to identify and to represent the various distinct properties of an external object *o* whose phenomenological individuation is therefore characterized by the series of such representational objects $o_1$, $o_2$, …, $o_i$, … which are the codified perceptive information originated (or is the trace left) by *o*, hence from the external environment. But the control of these internal objects must be ruled by the representational system given by the (frontal) elements *s*, so that, for instance, when an external object *o*, which had already given rise to a perceptive internal object, say $o_i$, tries to recover the initial line *s*-*o* to be re-evoked, now it first meets $o_i$ along this pathway, but the new intermediary line $o_i \leftarrow o$ is a non-phenomenological elaboration since it does not provide any phenomenological consciousness content due to the absence of any *s* element. To this end, therefore, a new element *s* is needed to restore a consciousness constraint of the type *s*-*o*, so giving rise to a phenomenological consciousness content or intentional act through the following new elaboration $s_i \leftarrow (o_i \leftarrow o)$. Often, the external information is firstly worked out by those many cerebral zones devoted to the codification of the various possible perceptive elements $o_i$, amongst which, nevertheless, homologous bonds of the type *o*-*o* are possible, which however remain unconscious because their elaboration takes place mainly into procedural memories. But little by little such homologous links arise, they stimulate the representational system of the elements *s* (which, in any case, starts from albeit minimal innate equipment of unitary representational patterns), choosing one of its elements, say $s_i$, that will control and regulate the sensory-motor-perceptive element[33] $o_i$ through a one-way re-entering projective line $s_i \rightarrow o_i$ (providing a *representational access*[34], with which the sensorial-perceptive flows enter into the brain) codifying the information of $o_i$ by means of the internal restoring of the one-way introjective line $s_i \leftarrow o_i$ (providing a *representational restoring*, with which the frontal zones of the brain drive and control the sensorial-perceptive information), just promoted by $s_i \rightarrow o_i$, which reproduces, although in a less sensitive fashion with respect to the original or initial way[35] $s \leftarrow o$, the environmental information and that is needed for a sufficiently normal psychic development to prevent a projective excess of investment not adequately counterbalanced by introjective responses; hence, once such an introjective re-entries $s_i \leftarrow o_i$ assumes a stable asset, then $o_i$ institutes a constraint on the frontal zone that therefore acquires a property of the object *o*, codified by the set of internal objects $o_i$, so determining an introjective effect. The mother action always operates through introjective lines of the type $s_i \leftarrow o_i$, so that her influence superposes with the other introjections.

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[33] Which takes place in the perceptive-sensorial cerebral zones.

[34] Which takes place in the frontal, motor and premotor cerebral zones and allows to access to the representational information content given by $o_i$. These lines $s \rightarrow o$ are always running in the brain, also without perceptive inputs, otherwise the cerebral activity would turn out, this also explaining why the child has normally needed this for a right counterbalanced supply of introjective feedback. A kind of limbic switchboard is assumed to be mainly responsible for the continuous activation of the various peripheral cortex zones to catch information. This might be considered as the neurophysiological counterpart of *human intentionality*. The restoring of a constraint of the type *s*-*o* implies the different (topological) *localization* given both by a frontal element *s* and a parietal-occipital element *o*: the former approaches the latter, revealing a consciousness element which is not localized in the element *s* but *elsewhere* (this recalling the Lacanian *Other*), that is, in the element *o*, which is needed for its phenomenological occurrence. According to the original Freudian thought, the various *object relationships* arose by relationships amongst different mental $s_i$-*s* representations of the objects *o*, that otherwise might not have phenomenological content.

[35] Because every internal object $o_i$ codifies, in general, only partial properties with respect to the ones codified by the original perceptive elaboration $s \leftarrow o$. It is very difficult to succeed to reach the first original perceptive element *o*, because the various perceptive internal objects $o_i$ are 'encapsulated' amongst them according to J. Fodor. The psychic mechanism that tries to pursue this, is findable, for instance, in the hallucinatory and dreaming phenomena, in the vivid imagination of the child, and in the artistic skills (in which the so-called 'eidetic' memory plays a very fundamental role).

In any way, the elements involved in any line $s \leftrightarrows o$ are inseparable from each other, and this serves to highlight the indissoluble intertwining between the object's properties *o* and the subject's intentions *s*, this unitary coupling giving rise to an irreducible and primitive elementary nucleus of consciousness, i.e. a *qualia*. The indivisibility of the constraint $s \leftrightarrows o$, allows to introduce the fundamental property of *binding* of the phenomenological constraint *s*-*o*, which comprises different functionalities, like the *conjunction binding* of projection's signals coming from distinct cerebral zones and the *dynamical binding* of synchronous signals activated in the same zone or in the bordering one; furthermore, a distinction between *spatial* and *temporal binding* is considered as well. A chief theoretical problem of the binding is the possible synchronization of oscillatory responses with regard to spatially separated regions of the cerebral cortex, issue which requires the presupposition of a stimulating intrinsic cerebral dynamical switchboard to build up the oscillatory fibre of the $s \leftrightarrows o$ lines, and upon which the bending relies. From what has been just said above, we may state a general principle[36] of the *joined elaboration* concerning the two main elements *s* and *o* of any primary phenomenological constraining line $s \leftrightarrows o$, according to which any phenomenological consciousness content exclusively springs out from the joined elaboration of an element *o* with an element *s*. Nevertheless, as we have seen above, there may exist as well links between homologous elements, for instance of the type *o*-*o* or *s*-*s*, but without giving a consciousness phenomenological content.

Therefore, together to the formation of the representational internal objects $o_i$, a related set of representational elements $s_i$ is also available little by little that representational accesses $s_i \leftarrow o_i$ arise, where each element $s_i$ is into correspondence with the original (frontal) representational element *s* through a non-phenomenological homologous (frontal) bond of the type *s*-*s*, that is to say $s_i \rightarrow s$, recalling that $s_i$ and $o_i$ are memory stored elements[37]. Thereafter, once that a frontal element $s_i$ is primed, an attempt at restoring the original frontal representation *s* takes place, this being possible, due to the above principle of the joined elaboration, only re-evoking the corresponding perceptive object *o*, hence one of its partial internal objects $o_i$, so restoring a phenomenological constraint of the type *s*-*o*, to be precise $s_i$-$o_i$, until up the completion of this process with the frontal non-phenomenological elaboration $s_i \leftarrow s$ in such a manner as to have a phenomenological perceptive elaboration of the type $(s_i \leftarrow s) \leftarrow (o_i \leftarrow o)$; and all this, of course, requires a non-zero temporal interval to be realized, in this case being a fundamental prerequisite to ascertain what type of time notion we are using (see (Iurato, 2014) for a very brief mention about physical and psychic time distinction[38]). Thus, to summarize, the original perceptive object *o* will be re-evoked by means of a series of partial internal objects $o_i$ each of which, in turn, pushing on the original representation *s* of the original perceptive object *o*, gives rise to a series of (frontal) representational elements $s_i$ such that a relation of the type $s_i \leftarrow s$ holds, each of these last homologous bonds not having any phenomenological content if it is considered without to the homologous bond $o_i \leftarrow o$.

Notwithstanding that they are void of any consciousness content, both the above mentioned frontal homologous elaborations $s_i \rightarrow s$ and $s_i \leftarrow s$ play a very fundamental role for the human psychic development. Indeed, as a main characteristic, they may run either with partial consciousness states or even with absence of phenomenological consciousness, that is to say, in an unconsciously manner. Therefore, it is clear too what fundamental role they may play for understanding the possible mechanisms with which unconscious does work. Thus, the elaboration of elements of the type *s*, notwithstanding takes place for a main object-environmental control purpose, may be phenomenological present either in a partial manner or even wholly absent. Furthermore, ideal, intuitive, attentive, self-reflexive and early states of consciousness may be brought back to the action of the homologous bonds of the type *s*-*s* with respect to the set of representational objects *o*, the former undergoing certain basic *hierarchical ordering* operations producing meaning[39], coming from the personal live experience and from the related live environment[40], which will allow to structurally build up the representational system of the *s*-*o* constraints, having the primary property to identify unitary patterns. This latter ordering processes will play a very fundamental role for human consciousness, because them are also related with the possible orders coming from the dynamics and topology of the cerebral maps.

### *C. A brief sketch of the model: advanced notions*

The above mentioned system of double representations *s*-*s* and *o*-*o*, is able to explain as well what the main distinction between a primary non-removed unconscious and a secondary removed unconscious consist of. The former mainly concerns deregulations of the representational system of the frontal lines *s*-*s*, with damages, in general due to a basic lacking of an adequately and enough maternal interaction child-mother in forming homologous bonds between elements $s_i$ and *s*, hence with a basic impossibility to have phenomenological contents provided by relations of the type $(s_i \leftarrow s) \leftarrow (o_i \leftarrow o)$, in this case, generally writing $s \nleftarrow$ to denote the simple fact that there exist persistent and consistent inhibitory representational frontal elements *s* hindering hierarchical orderings

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[36] Which has deep roots into the Husserlian phenomenology.

[37] Both elements *s* and *o* have representational nature in the human brain, so we might speak of a *double representational* nature of the system of basic constraints *s*-*o*.

[38] In turn, just this issue might be puts at the basis of a clarifying inquiring examination or of a deepening study, about one of the crucial problem of psychology, just that concerning nature and properties of the *physical* and *psychic* time, which might turn out to be of some usefulness also from a computational cognitive science standpoint.

[39] That is to say, having semantic functions.

[40] In this basic interaction, a truly fundamental role being played by *mirror neuron* systems, which allow the mother-child primary resonance interaction to come about, hence it is the primary basis for human intentionality.

into the *s*-*s* system with gaps in the abstract representational *mental states* $s_i$-*s*. The latter, instead, mainly relies on the inability to establish stable *s*-*o* phenomenological relationships due to the fact that frontal elements *s*, with the occurrence of suitable re-entering mental processes *s*-*s*, hinder the reaching of sensor-perceptive occipital-parietal objects $o_i$, so not allowing the formation of a consciousness content; in this case, we simply write $s \nleftarrow \hat{o}$ to mean the general fact that the representational frontal element *s* hinders the approaching of consciousness by the perceptive element $\hat{o}$.

Now, after having outlined the fundamental role played by the endogenous dynamics *s*-*o* (with its functional correlate in the *P*-*I* dynamics) in resonant periodic interaction with the mother, we can consider the next two main stages of the psychic development of the child, that is to say, the basic *identificatory system* and the next *mentalization*[41] process. The latter has mainly due to the structuration of the *s*-*s* line system, whereas the former requires something more. Indeed, along the primary psychodynamic development of the child, roughly codified by the well-known oral-anal-genital sequence, we observe here a convergence of identificatory traits due to both familial figures (i.e., mother and father), at first that of the mother then the one of the father, so giving rise to all that complex series of *identificatory constellations* but which, in general, turns out to be quite unstable. According to E.Z. Tronick, the first consciousness states are the outcome of the running of an *asymmetric*[42] *dyadic system* centred on the early interrelation between the child and the mother, from which starts a *dyadic expansion* of consciousness states which spring out from a projective-introjective (*P*-*I*) oscillatory dynamics, having a basic periodic recursive character[43], through which a remarkable transfer of consciousness states may take place, from mother to child, just thanks to an innate *intentional need* by the child due to the childhood attachment models. Therefore, the child intentionality is inherent with the *P*-*I* endogenous dynamics, directly oriented to maintain and to control the object relation, gradually engaging either the primary and secondary associative nets involving the elements *s*. With different action and running in dependence on the involved cerebral lobe, the introjective *I* and projective *P* oscillatory flows revolve around the elements $o_i$ upon which engrave with a dynamics of the general type $P \rightarrow o_i \leftarrow I$, where the introjective flows *I*, in normality conditions, excite bodily or somatic zones[44] in such a manner that is locally regulated and counterbalanced (as a kind of a restraining 'protective shield') by the re-entering projective flows *P*, so giving rise to a Freudian energetic cathexis which 'engrave' the stimulation (or excitement) into a certain element of the psychic apparatus, so giving rise, in turn, to that rich and variegated frontal representational grid of human mind given by the elements *s*. Steady imbalances of such a two-flow dynamics give rise to pathological psychic illnesses. On the other hand, the above mentioned asymmetric aspect of the basic intentional function involved in the crucial dyadic system mother-child discussed above, gives rise to a kind of implicit hierarchical structure into the consciousness dyadic state. Furthermore, all the discussions above, should be made for both the right and left cerebral lobes, for instance making use of a double sign ± to appoint one of these to the *s* and *o* elements, with related homologous and heterogeneous combination lines, for distinguishing these two zones.

A very important issue concerns the *topology* of an object relationship. The repression should not be understood as a kind of *area transfer* from one psychic place to another one but rather as a non-topological *local dynamical change* of the cathexis itself in such a manner it may be under the action of different psychic agencies. According to the Sasso's model, this may be explained as the *access* of an element *s* upon an element *o* through a *s*-*o* lines that modify its *local elaboration*. Therefore, the Freudian unconscious should not be meant as a set of topic agencies but rather as dynamically characterized by a continuous distribution of the various accesses *s*-*o*. Already Freud himself, in his work *The Unconscious* of 1915, considered the question whether the repression is characterized by *topical* displacement and condensation mechanisms (*topical hypothesis*) or rather it is characterized by a kind of *local state* or *phase transition* which takes place in the same locality, regarding the same material (*functional hypothesis*). Initially, Freud chose the first hypothesis, but next he decided upon the second one, considering as a basic repression mechanism, the one consisting in a sort of *cathexis subtraction* of either the unconscious mnestic *vestiges* (d'après Murtagh – see (Murtagh, 2014)) or conscious mnestic traces, hence as a local state change of the cathexis: in other words, a certain subtraction of energy weakens the cathexis itself, with a consequent changing or transformation of it. Nevertheless, thought within Sasso *s*-*o* model, these two Freudian hypotheses are not in contradiction with each other, because both should be seen as mainly due to changes of networks, in which either the object cathexis and countercathexis differently acts, distributing itself amongst the primary and the secondary nets; in turn, a different situation takes place in dependence on the cerebral lobe under consideration. Therefore, this multiplicity of phase (or state) changes, differently distributed amongst many distinct cerebral levels, basically characterizes the crucial link unconscious-conscious within the *s*-*o* model.

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[41] That is to say, the comprehension of the mental states of other people's, which ensues from the suitable regularization between the two main systems of the maternal introjective flows $s \leftarrow o$ (*maternal intention*) and of the projective re-entries $s \rightarrow o$ controlling the representational grid (*child intentionality*) of the elements *o*. This intentionality comparison is of primary interest because, otherwise, the child would not be able to distinguish between the own intentionality and any other one if, at least, the intentionality of the mother did not make its appearance. Also this last fact refers to Husserl's phenomenology. The child intentionality increases as the interactions coming from mother and environment enhance, but the first founding nucleus is given by the recursive oscillatory-dynamical interactions with the mother.

[42] This asymmetry implicitly implies the presence of a first form of a hierarchical structure into the dyadic consciousness states.

[43] And the mirror neuron systems as a primary neurophysiological support.

[44] That is to say, the *source* of an object cathexis of the general type *source* $\xrightarrow{object}$ *drive destination*.

The very early interactions of the child with the mother and the related environment, essentially take place for innate physiological reasons, whilst the fundamental *identificatory development* takes place only later, thanks to a disaggregative condition of the previous stages of the child psychic development (mainly ruled by the mother action) which entails an *instability of identity* that will promote and will drive all the next psychic development comprising four main sequential steps: the first one given by the primary child-mother tuning in; the second one given by the identificatory development (studied by psychoanalysis); the third one converging into the various attachment models, and fourth and last one given by the mentalization process, which takes place when the primary cerebral net starts to differentiate into other higher levels. This last process provides distinct secondary networks, from the primary one which was mainly ruled by the primary relation maternal object, departing from this with the acceptance of new object relationships extending the initial introjective-projective dynamics of the *dual* type to the more complex one having of *group* type, with the occurrence of a paternal figure. This incoming group development, even due to the above mentioned dynamical instability, naturally characterizes that multiple dynamics of the cerebral mind, ever undergoing to continuous transformations and re-organization procedures all featured by hierarchy-producing associative-dissociative coordination processes, providing a differentiated set of various object relationships. The associative character regards the convergence integration of the primary cerebral net, moulded by the mother one and having a fundamental sensory-motor nature, during the initial basic child-mother interaction, whereas the dissociation feature has to do with the divergence proliferation of the secondary cerebral nets, which underlies any psychic splitting (like the *disavowal* mechanism) process[45], and that, therefore, has need of a re-organization for a basic psychic unity, although an essentially dissociated nature characterizes the psyche of every human being. In this latter step, the (mind-producing) mentalization will operate to coordinate the introjective-projective flows within the secondary cerebral net framework, in such a manner as to regulate the identitary nuclei which spring out from the primary net (having sensory-motor source) of the *dual* relation child-mother to the secondary nets due to the action of the *group* function upon the first dual nucleus. Thus, as a crucial point of the mature psychic development, the so-called *Œdipus situation* arises, forcing the child to come out from the initial *dual* relationship with the mother, pushing the child to find her or his own identity. All this is quite similar to the symmetry breaking like phenomena, outlined in the first part of this paper and mainly due to a paternal figure action, regarding the famous child-mother *double bind*, which includes many aspects and notions similar to either the optimal and pathogen mother influences provided by the primary child-mother *P-I* dynamical relation. Also belonging to this class of relations, characterized by symmetry breakdown like phenomena, might be considered the so-called Otto Rank's *double* (of 1914) and the related transformational function which gives rise to the *identificatory* function by means of the main passages from primary to secondary nets, with consequent intentionality diversifications.

In the Sasso *s-o* model, the psychoanalytic identificatory process, which implies a consciousness growing up, might be formalized as a *bundle of identificatory traits* each of which is the coordination of a finite sequence of elements[46] of the type $\langle s_i\text{-}s\text{-}o\rangle_1, \ldots, \langle s_i\text{-}s\text{-}o\rangle_n$, whose every element is formed by an *intentional nucleus* $s_i$-*s* that acts upon an object *o* to give rise to a consciousness content $s_i$-*s*-*o* on the basis of the reciprocal and perpetual child's comparative interrogation between what the mother has firstly implemented into her or him by primary interaction child-mother with the related environments, the above mentioned instability of this dual link leading to be opened to new identifications (corresponding to the passage from the primary to secondary cerebral nets). The various non-phenomenological identitary nuclei $\langle s_i\text{-}s\rangle_j$ will be organized, by the mind, into the secondary cerebral nets according to a certain hierarchical order of them which starts from the original founding nucleus of the primary cerebral net to develop into the secondary ones. Therefore, the mind is not the identity in itself but rather is the hierarchical ordering of the various intentional nuclei $\langle s_i\text{-}s\rangle_i$ distributed amongst the primary and secondary cerebral nets; the consciousness, in turn, is not the mind but rather is the phenomenological representational contents $\langle s\text{-}o\rangle$ from time to time available to the non-phenomenological nuclei $\langle s_i\text{-}s\rangle_j$ which form the mind coordination. The main feature of this process is the fact that we really do not perceive the *pattern* but its unitary (Gestalt) *hierarchical categorical configuration* that emerges from the action of a non-decreasing *temporal function* (to account for the basic and unavoidable consciousness temporal dimension) that regulates and controls the integrative processes of the ever evolving multilevel cerebral networks mainly according to a combinatorial dynamics. Therefore, we again highlight the fact that the temporal dimension plays a very fundamental role for consciousness and its integration, as already pointed out in the first part of this paper, giving a unitary and cohesive character to consciousness due to the primary founding nuclei of the *Self* (primary consciousness). We once again stress the truly unavoidable fundamental hypothesis on the consciousness origin and structure, according to which it is centred on the constraints *s-o* meant as a founding *ontological unity* of the consciousness itself, through which a sensorial elaboration *o* is liable to be linked with a motor elaboration *s* (in this, a preeminent role being played by mirror neuron systems – see also (Lauro Grotto, 2014)) to give rise to an elementary consciousness content *s-o* (a *qualia*) whose dynamics is promoted and ruled through the primary interaction child-mother given by the *P-I* oscillatory dynamics $s \leftrightarrows o$ which is needed for the development and integration of the composite cerebral network. The consciousness then evolves trying to integrate and coordinate the manifold of the various constraints *s-o* along the parallel evolution of the manifold of the various cerebral

[45] The recent *theories of the multiple structure of the Self* claim attention on the dissociated states of the human psyche, on a basic inhomogeneity of the processes which concur to form consciousness. Freud himself always pointed out the fundamental *disunity of the Ego*, in either normal and pathological cases. All that confirms many points exposed in (Iurato, 2013a,b).

[46] This sequence may also give an explanation to the *psychic group* nature of the psychoanalytic setting studied by René Kaës, who has also stressed, starting from the previous work made by Kurt Lewin, Émile Durkheim and Freud himself (about the mass psychology), on the unavoidable role played by the psychic group feature for the human inter-subjectivity which has an irreducible basic group structure meant in a precise manner.

networks whose instability is mainly due to the phylogenetic *neural redundancy* that pushes to produce high level cerebral networks (amongst which the one related to the language).

## VI. Toward Some First Formalization Attempts

In the previous sections, we have seen what fundamental role may play the sensory-motor *bundle* $\mathfrak{F}_{s-o}$ of all the possible phenomenological constraints $s \leftrightarrows o$, each of which may be considered as a kind of algebraic maps of the type $s \rightarrow o$ and $s \leftarrow o$, together its main properties, amongst which are the *locality* one and the *vicariance principle*. All that plays a crucial role in the dawning and in the development of the human psyche, through the interplay between the child *P-I* endogenous dynamics and the introjective maternal influence *Im*. Thus, we may surely state that the bundle $\mathfrak{F}_{s-o}$ in the child is, above all, the outcome of the decisive two-way dynamical interaction of periodic oscillatory type *P-I*$\leftrightsquigarrow$*Im*, respectively between the *P-I* endogenous dynamics – ruling the primary identificatory processes of the child – and the so-called *mother-child tuning in*, so that the following pair $\mathcal{P}_c = \{\mathfrak{F}_{s-o}, \textit{P-I}\leftrightsquigarrow\textit{Im}\}$, made by the bundle of functions $\mathfrak{F}_{s-o}$ and by the oscillatory dynamical system *P-I*$\leftrightsquigarrow$*Im* structuring $\mathfrak{F}_{s-o}$, gives rise to a formal basis, say $\mathcal{P}_c$, for the psychic development of the child. These latter simple claims may imply, from a general history of science standpoint, some possible brief elementary formal remarks such as the following ones.

### *A. On bundles, logic and dynamical systems*

In mathematics and theoretical physics, a very notable role is played by the wide class of the so-called *fibred structures*, amongst which there are *bundles*, *fibre bundles* and *fibre spaces* (see (Husemoller, 1975) and (Souriau, 1964)), whose notions first arose out of questions posed in the 1930s about issues on the topology and geometry of manifolds, and whose first rigorous formulations date back to the 1950s. We shall refer to a very simpler basic notion, that of bundle of functions, from which some of the other ones may be built up. Following (Bruhat 1961, Section 1.1) (see also (Godement 1959, Section 3.1), (Quan 1969, Chapter I, Section II.3) and (Cabras et al. 1991)), given two abstract sets[47] *X* and *Y*, a *bundle* $\mathfrak{F}(X,Y)$ *of functions* from *X* to *Y*, is a class of sets of functions, say $\mathfrak{F}(U,Y)$, of the type $f: U \subseteq X \rightarrow Y$ for each $U \subseteq X$, such that, for every covering $\{U_i\}_{i\in J}$ of *U*, we have $f_{|U_i} \in \mathfrak{F}(U_i,Y)\ \forall i \in J$, this last property expressing a *locality condition*. Moreover, following (Borceux, 1989), the basic idea underlying the notion of bundle is just that of *locality*, that is, the existence of elements locally defined and having enough properties of gluing and restriction. But, what is important is the fact that a bundle defined between topological spaces, besides having local elements, has too a *local logic* through which one may establish the truth values of every local property by means of the construction of an appropriate *Heyting algebra* on it, recalling that every Boole algebra is a Heyting algebra as well. Therefore, if properly contextualized, it would not be fully out of place to consider what has just been said above referred to $\mathfrak{F}_{s-o}$, the fact that the consciousness is mainly the result of the integration, into unitary and temporal-hierarchically ordered patterns called *qualia* (or primitive *psychic qualities*), of the various emerging phenomenological outcomes at most unconsciously coming out from a multiple system of centres placed into different cerebral networks, together with the various *binding* and *hierarchical ordering* operations basically ruled by a primary temporal dimension, which might be suitably formalized within this context or framework.

Following (Thom 1980, Chapter VII, Section 7.3.A-C), within the general dynamical system framework (above all, catastrophe theory), between two dynamical systems presenting recurrence (like metabolic ones), resonances will always appear, the stability of the corresponding dynamical system resulting from the topological product of the two given systems depending on the ratio of the corresponding resonances. In particular, between two metabolic systems in a free interaction promoted by a weak perturbation with stochastic direction, an exchange of information always takes place. In general, the topological product of two dynamical systems put into free interaction between them (like two oscillators that enter into resonance), is structurally unstable and has a wide and variable range of possible resonances in competition amongst them, the related stability depending on the nature of the initial perturbation. In the topological product-system, each factor-system loses its individuality, converging into a new unstable and mixed dynamical system (*resonance system*) whose initial instability gradually goes towards a more stable regime through the reaching of a resonance state chose amongst the possible ones. By the way, we note that the mathematical framework suitable for the treatment of this last issue involves a fibration of the product topological space; finally, we also note that René Thom, in (Thom 1980, Chapter XIII, Appendix 1), provides a very interesting formal model of the a memory from a dynamical system point of view, which might have as many interesting applications from a computational psychoanalysis standpoint.

In (Thom 1985, Chapter X, Section 10.2), a lucky formal geometrical model, centred around the dynamical notion of resonance, is proposed to try to formalize the notion of *signification*. The *resonance* is one of the key notions of modern dynamical system theory and, from what has already been said above, it may be put at the basis of the first most elementary attempts to formalize communication transmission phenomena. On the other hand, already E.C. Zeeman, in his celebrated 1965 paper *Topology of the Brain*, considered the totality of our own cerebral activities as forming a topological dynamical system, amongst other things predicting and foreshadowed many results which would be later confirmed by the subsequent mirror neuron research. Later works of Zeeman confirmed the usefulness of general topological dynamics in modelling the brain, whose chief dynamical feature seems just be that oscillatory nature of its phenomena (see (Zeeman, 1976) and references therein). The above mentioned basic interaction between the *P-I* endogenous dynamics and the maternal introjective influence *Im*, might be laid out within the above sketchily described framework of the theory of dynamical systems, for instance formally identifying, in the activation promoted by the

[47] In general, *X* is, at least, a topological space.

cerebral reticular formation of the cerebral trunk, the dynamical action of an *attractor*. Obviously, the dynamical system approach to cognitive systems is by now well-stated but, following the brief lines outlined in this note, it might deserve interesting further perspectives on the computational psychoanalytic side. Furthermore, also in the early history of neural computation, after the pioneering works by J.J. Hopfield, several formal architectures were set up between the 1970s and the 1980s, considering some dynamical system notions and concepts, like that of resonance and self-organization: to be precise, the *adaptive resonance theory*, mainly developed by S. Grossberg and G. Carmeking, considers the hypothesis that the brain autonomously runs in such a manner as to organize it according to certain recognition codes, whilst T. Kohonen developed the idea according to which neurons self-organize themselves for variable adaptive purposes through the institution of *self-organizing maps*.

*B. On symmetry breakdown, generative structures, and other*

On the other hand, modern differential geometry techniques have allowed to work out symmetry breaking formalism in the most general fashion through fibre bundle tools within gauge theory framework: in this regard, see (Bleecker 1981, Chapter 10, Section 10.3), (Percacci 1986, Chapter 4, Section 4.4) and (Derdzinski 1992, Chapter 11). Moreover, the real essence of symmetry breaking relies on functional analysis of operator spaces, like Hilbert spaces. Indeed, following (Strocchi, 1981; 1999) and references therein, roughly we may say that, from a formal viewpoint, the spontaneous symmetry breaking mechanism for infinite dimensional classical systems involves maps between Hilbert space sectors in relation to actions of certain related symmetry groups (e.g., the group of internal symmetries of a certain mathematical entity – like an equation – and the related stability group). All these formal considerations might be suitably reworked in relation to $\mathfrak{F}_{s-o}$, where a crucial point is the comparison between different representations of non-phenomenological intentional nuclei $<s_i\text{-}s>_i$ to give rise to phenomenological contents when we approach an object *o*. On the other hand, in (Thom 1980, Chapter 10, Section 10.3, Remark 10.3.11), the author stresses which formal analogies, similarities and common points may exist between symmetry breaking phenomena – via the Higgs mechanism – and catastrophe theory of dynamical systems, this last theory being the most suitably formal framework in which possibly to put $\mathcal{P}_c$. Furthermore, in (Thom 1972, Section II.1), the author provides a very interesting discussion of the possible formal relationships existing between (macroscopic) broken symmetry phenomena and the resonance ones, bearing, as a main example, the case of two periodic linear oscillators in interaction between them whose structural instability may be overcome by means of a (macroscopic) symmetry breaking phenomenon with the consequent appearance of a resonance[48]. In conclusion, from these very simple remarks, we would want to put forward possible and plausible useful comparisons and transpositions between the symmetry breaking phenomena theory, as taking place within the general framework of dynamical systems (d'après the pioneering works of R. Thom, I. Prigogine, L. Von Bertalanffy, M.A. Brazier, and others), and the formal system $\mathcal{P}_c$ where some main features have been identified, amongst which are the fibre structure of its elements, the basically resonant oscillatory nature of its dynamics, some main topological aspects (amongst which the locality and the functional hypothesis of the object relationships) and the hierarchical ordering character provided by an innate temporal dimension.

The latter should be meant as a primitive psychic function rooted into the primary and most ancient structures of the *Self*, which regulates and controls all the running of $\mathcal{P}_c$. The temporal dimension belongs to the set of *generative structures* which are considered, from a more properly structuralistic viewpoint, as those early, primitive (structural) archetypical bases of the human psyche upon which to build up the next psychic development of every human being. In fact, following Yuri I. Manin, as early as the human individual has taken consciousness of the own Ego in front of a world that changes, the notion of time has gradually undertaken even a more prominent role along the course of the history of culture. Since the primitive epoch, the human being has always had some method to measure time, usually by means of recurrent phenomena. The time originally makes its phenomenological appearance as a consciousness' flow characterized by the awareness both of the transiency of the present and of the content of the various thought entities. Furthermore, following too (Miller 1983, Chapter 1), since the known pioneering Piaget works on child psychic development, the psychological bases of mathematical thinking have been studied from a structuralistic point of view. Precisely, certain primary biological mechanisms have been ascertained to be at the basis of certain elementary formal structures, called *generative structures* (so named after N. Bourbaki), from which all the other ones constructively follow according to a well-defined *architecture des mathématiques*. These generative structures are classified into three main classes, respectively containing the *algebraic*, the *topological* and the *ordered* structures. According to this view, the psychological bases for the ordered generative structures is the primitive idea of *time*, both discrete and continuous, while the topological generative structures have their psychological bases in the idea of *closeness* which, in turn, find their roots in the biological mechanisms of the organization of the perceptive space around a privileged centre, namely the human body. In concluding this brief note, we would want to further stress the following fact that, according to us, plays a very fundamental role in the psychic development of human psyche, namely, the truly founding and primary role played by the triadic familial structure in structuring and developing the human psyche of the child.

## VII. FINAL REMARKS

*Computational psychoanalysis* officially is born with the works of the Special Session (A5) kindly hosted by the 13th IEEE ICCI*CC-2014 *International Conference on Cognitive Informatics & Cognitive Computing* held in London, UK, on August 18-20,

[48] With the change of the two periods in such a way to get a rational ratio, the choosing of the phase of the periodic composite process obtained in this way being a further symmetry breaking phenomenon.

2014 (see the related Proceedings quoted in (Iurato, 2014)). As a truly new discipline, it started, on the basis of the previous primary work made by Ignacio Matte Blanco in working out a formal ground to the Freudian psychoanalysis, with the early basic works and contributions due to Andrei Khrennikov, Rosapia Lauro Grotto and Fionn Murtagh (in chronological order). As one of the main purposes, it basically tries to understand what are the primary formal structures and running mechanisms of the unconscious, with the ambitious aim to possibly implement them into the wide computer science framework. On the other hand, recent progress in cognitive informatics has also pointed out what primary role may be played by unconscious phenomena. In fact, textually following (Wang et al., 2006), we have that

> *«According to the Layer Reference Model of the Brain* […] *the cognitive processes of the brain can be categorized into six layers and then two subsystems. The subconscious subsystem of the brain is inherited and fixed; while the conscious subsystem is acquired and highly plastic. It is noteworthy that the subconscious life functions determine the majority of human behaviours and cognitive processes and this might be overlooked in psychology and cognitive science* […]. *Although Sigmund Freud focused on the psychological effects of sex-related desires of human beings, he probably over simplified a whole set of other subconscious life functions* […]. *Therefore, a study on the subconscious behaviours of the brain and their mechanisms may be the key to understand how the brain works»*.

On the other hand, in this paper (above all, in the first part), we have very briefly outlined, in passing, the possible use of certain elementary algebraic structures (like the groupoid ones) to formalize some basic notions and concepts of psychoanalysis. In pursuing this, we have spoken about some special binary relations, amongst which are order relations and partially defined binary relations involved in groupoid structures. Now, the latest results in cognitive informatics (see (Wang, 2008)) make use of new algebraic structures in which binary relations are involved, amongst which are the so-called *concept algebra* which puts into relation generic elements called *concepts*. Roughly, a *concept*, defined on an arbitrary set $\mathcal{O}$, is an algebraic system of the type $(O, A, R^c, R^i, R^o)$ where $O \subseteq \mathcal{P}(\mathcal{O})$ (= power set of $\mathcal{O}$), $A$ is a non-empty set of attributes, $R^c$ is a set of internal relations, $R^i$ is a set of input relations, and $R^o$ is a set of output relations. Concept algebras, furthermore, are closely related to lattice structures. Therefore, it is evident[49] what possible generalizations might be achieved when one implements, into a concept, partially defined binary relations as well as orderings. Concept algebra provides a rigorous mathematical model as well as a formal semantics for object-oriented class modeling and analyses. Moreover, concept algebra provides a denotational mathematical means for algebraic manipulations of abstract concepts. Concept algebra is an abstract mathematical structure for the formal treatment of concepts and their algebraic relations, operations, and associative rules for composing complex concepts, and they can be used to model, specify, and manipulate generic "to be" type problems, particularly system architectures, knowledge bases, and detail-level system designs in computing, software engineering, system engineering, and cognitive informatics. On the other hand, cognitive informatics has also turned its attention to sociological context (see (Wang, 2010), coherently with one of the lines of thought followed in this paper. Therefore, the various formal aspects treated in (Wang, 2010) might be usefully contextualized with what has been said in this paper about social-anthropological features. In conclusion, computational psychoanalysis and cognitive informatics have promising and interesting relationships which deserve further analysis and deepening.

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[49] And this might be, for example, pursued in another place.

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