# Data, Methods, and Technology

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This section of the *Journal of Real Estate Literature* focuses on innovative use of real estate related data and methodologies. This is also a forum for the discussion of new data sources and standardization techniques in the real estate industry. If you have any suggestions or submissions to this section, please contact Sofia Dermisi.

# Multilevel Methodology Approach for the Construction of Real Estate Monthly Index Numbers

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***Abstract***
In this paper, we evaluate price indices and hedonic price indices for Italian real estate data using multilevel models. The methodology is based on a random coefficient panel data model. We propose a Laspeyres-type price index and hedonic prices indexes based on some characteristics of the sold properties. The multilevel hierarchical analysis has the advantage of allowing the appraisal analysis for groups, and identified in the same sample data the hierarchical structures of market segmentation according to the parameters of the real estate segment. It allows getting a lot of regression functions as the number of groups identified. Obviously, this depends on the sample size and the variability between groups. Specifically, if the data are also grouped by date, the model allows an analysis of the time series which makes possible the calculation of index numbers and the overall monthly index numbers of real estate properties, consistent with collected data.

In their general meaning, the index numbers are useful indicators to make predictions, to make decisions, and to study price movement trends in the various sectors of the economy. These are frequently used in the analysis of time series and in particular the historical study of long-term trends, seasonal variations, and cyclical developments (Freud and Wilson, 1997). In real estate, the construction of index numbers of real estate prices in different market segments provides important information about real estate trends, investment profitability, and capital appreciation/depreciation (Del Giudice and d'Amato, 2008).

For real estate, the index number time series is exhaustive if expressed on a fixed basis. These real estate numbers express the percentage change of a variable over time compared to a fixed period called the base period. The index numbers presented in this paper are direct indices, which are based on real estate market price surveys and are related to specific properties typologies. These monetary indices are developed on annual basis and based on the systematic and continuous collection of market prices (transaction based). The price index number calculation is based on simple price index methods, which determine the unitary market price position indices (average) in the segment between one index and the next. In their application, it is considered that, while the surface characteristics are involved into the calculation of the unit price, the other real estate characteristics are considered under equal conditions, even if they may be different. The property's characteristics can vary over time due to construction or maintenance activities. Therefore, normally the application is based on the use of a type of property in a specific location both defined in such a way as to identify a market segment (Case and Quigley, 1991).

The hedonic price index analysis is based on the hedonic price methods that use the real estate market's price as the dependent variable (Bailey, Muth, and Nourse, 1963). To understand the multilevel model, note *direct* hedonic models consider the real estate characteristics and time characteristics, the latter in the form of dichotomous variables, in correspondence to successive dates; *indirect* hedonic models are based on regression equations constructed in relation to the various time periods (cross-section) (Diewert, 1976, 2001, 2004). The multilevel model does not consider time variables in the regressions (such as direct models) or build independent equations, one for each time period considered (such as indirect models), but is based on a data hierarchical structure. Typically, the multilevel model is used in presence of data with a hierarchical structure and separates the effects of variables at different levels according to the various groups present in the structure of the data. In practice, it maintains general information (and variability) carried by the entire sample in formulating the functions for each level of analysis (Ciuna, 2007).

In short, the index numbers we present are monetary index numbers, calculated on a monthly basis and referenced to a fixed base (1995); these numbers are direct indices and transaction based. The models of real estate price index numbers are part of simple price index methods, based on the average price calculated in relation to the commercial market (rough average price).

The hedonic price index numbers were calculated using a regression model at different levels (multilevel), applied for the first time to the time series of real estate prices. The multilevel model was constructed in order to consider in the first level the property and in the second level the date of sale. The test aims to assess whether the model is able to consider the temporal variations as required by hedonic index numbers. Multilevel models are not new: they were well-developed in the early 1990s, and have been applied to geographical-oriented property hedonic studies ever since. With respect to the application of multilevel model to time series data, there is a large amount of literature. For example, Francke and Vos (2004) have done similar work and confirm that a multilevel model with time series components performs better and more usefully than a simple hedonic model for analyzing small housing sub-market segments rather than large housing sub-markets. There are other models that consider the spatial and temporal effects of hedonic models, such as the spatial-temporal autoregressive model (STAR) (Pace, Barry, Clapp, and Rodriquez, 1998), two-stage spatial-temporal autoregressive model with Bayesian heteroscedasticity correction (2BSTAR) (Sun, Tu, and Yu, 2005) and geo-statistical model (Dubin, Pace, and Thibodeau, 1999), by considering spatial-temporal correlation among the residuals and trying to solve the heteroscedasticity problem. There are other studies that have used multilevel methodology on real estate data: one study on asking prices in Cardiff by Orford (2000) and Djurdjevic, Eugster, and Haase's (2008) work about Swiss rents.

Leishman (2009) affirms that multi-level models have considerable potential as a tool both to identify the sub-market structure, and for detecting temporal change in the delimitation of sub-markets. Leishman, Costello, Rowley, and Watkins (2013) suggest that the more granular multilevel specification enhances empirical performance and reduces the incidence of non-random spatial errors.

## METHODOLOGY

The multilevel model is applied to a sample of $N$ properties (units of level 1) grouped with respect to $T$ months of sale (units of level 2). Let $y_{it}$ be the price of a sold property $i$ ($i = 1, \ldots, N$) at period $t$ ($t = 1, \ldots, T$). The hedonic multilevel model, considering the hierarchical structure of the data, is given by:

$$y_{ij} = \beta_{0,it} + \sum_{k=1}^{K} X_{k,it}\beta_{k,it} + \varepsilon_{it}, \tag{1}$$

where $X_k$ represents the property's characteristics and $\varepsilon_{it}$ is a remainder term. We suppose that the intercept $\beta_{0,it}$ and slope coefficients $\beta_{k,it}$ are random:

$$\beta_{0,it} = \beta_0 + u_{0,t}, \tag{2}$$

$$\beta_{k,it} = \beta_k + u_{k,t}, \tag{3}$$

where the terms $u_{0,t}$ and $u_{k,t}$ are random variables representing the effect of belonging to the *t-th* month of sale, respectively, for the intercept and the coefficient of the independent variables.

Equation (1) can be presented then in the form:

$$y_{it} = \beta_0 + \sum_{k=1}^{K} X_{k,it}\beta_k + \left(u_{0,t} \sum_{k=1}^{K} X_{k,it}u_{k,t} + \varepsilon_{it}\right), \tag{4}$$

in which the response variable $y_{it}$ is expressed as the sum of a fixed part and a random part (in parentheses). In the case of a single-layer model, the residual of the first level is the usual residual term of a linear model.

The error terms (in parentheses) are subject to the standard assumptions: (1) first-level errors are normally distributed with zero mean and constant variance; and (2) the random coefficients of the second level are uncorrelated between the groups but can be correlated within each group and have a multivariate normal distribution with mean zero and constant variance-covariance matrix.

It is further assumed that the random parameters of the first and second levels are stochastically independent. The coefficients ($\beta_0$, $\beta_k$) in equation (4) are random variables with a multivariate normal distribution with mean zero and constant variance-covariance matrix. That is, each group has its own regression function and all the functions are connected through their parameters derived from common hyper-distributions. This concept is equivalent to assuming that all groups in the data are a random sample from a hypothetical group's population. When the units are grouped, in this case the properties by month of sale, the total variability can be decomposed in the variability due to the group and in that between the groups. In the case of a random parameter model, the relationship between the dependent variable and the explanatory variables can vary between the groups in different ways, that is, it can be a heterogeneity of the regressions between the different groups.

In the model in equation (4), both the intercept and the regression coefficient are variable at the group level and both can be divided into two parts: a constant, or average coefficient, and a part that varies at the group level and measure the distance from the average coefficient. The method used to estimate the parameters is the maximum likelihood method. The Wald test was used to test the hypotheses on the parameters of the fixed part of the model (Goldstein, 2003):

$$T(\beta_k) = \frac{\hat{\beta}_k}{s.e.\ (\hat{\beta}_k)}; \tag{5}$$

under the null hypothesis, the test has approximately a $t$-distribution with degrees of freedom ($df$), which depend on the structure of multilevel analysis. To test the random part of the model, the $D$ deviance was used (Goldstein, 2003).

Multilevel analysis has been used to calculate price index numbers and hedonic price index numbers (Hox, 1995; Rasbash et al., 1995). The index of hedonic prices for each characteristic was calculated by dividing the hedonic price calculated by the multilevel analysis for each month by the hedonic price referenced to the base month, and multiplying by a hundred. The index of hedonic price is presented as follows:

$${}_0I_t = \frac{\beta_{kt}}{\beta_{k0}} \cdot 100, \tag{6}$$

with $k = 1, \ldots, K$ for the $k$ characteristics and with the usual meaning of the $\beta$ coefficients of multilevel analysis.

The Laspeyres price index was calculated by dividing the equation of the multilevel price for each month to the multilevel function of the price referred to the base month, and multiplying by a hundred. From equation (1) considering $k$ explanatory variables, the index is as follows:

$${}_0I_t^L \frac{\sum_{k=1}^{K} \beta_{0t} + \beta_{kt} \cdot x_{k0}}{\sum_{k=1}^{K} \beta_{00} + \beta_{k0} \cdot x_{k0}} \cdot 100, \tag{7}$$

with the usual meaning of the $\beta$ coefficients of multilevel analysis. The coefficients $\beta_{00}$ and $\beta_{k0}$ refer to the month chosen as the base. The index weights the marginal price of each characteristic at time $t$ with fixed amounts for a property with average characteristics.

## The Real Estate Data Sample

The real estate valuations are based on the market analysis and the sales data. In some real estate markets like Italy, the data required for the application of many statistical techniques are not available because of the lack of transparency in price formation, of the deficiency of competitiveness, and of the strong viscosity, which differentiates markets from one another.

Note that in Italy, no national or local statistics illustrating the situation in the housing market are available. Official quotations about the trend of the housing market are provided by agencies such as the Land Registry; however, they merely provide intervals of values and are not supported by a solid statistical or appraisal evaluation.

In this study, the real estate data were acquired from the Observatory of Real Estate Market, a regional real estate market observatory instituted by University of Calabria. It is one of the few databases containing information on real market prices in Italy, although on a local scale, and was developed initially for teaching purposes. We examine an urban area that has uniform characteristics in the quality of the housing stock, the types of properties examined, and with a similar population. The market segment is defined with respect to sales, the residential use for real estate units in multi-story buildings, in condominiums built in the 1990s.

Our sample included 833 real estate sale transactions. All real estate units were in the used market. The sample period was 1995 to 2013. The units are physically, technically, economically and functionally similar. Their use is exclusively for housing one. The motivations of purchase and sale are usually attributable to transfer. The shape of the market is restricted due to monopolistic competition, in which franchise agents are the most common form of intermediary.

From the survey forms associated with each real estate transaction, information about real estate features, deemed relevant in the mechanism of price formation, were acquired.

The data sample, in this case for the geographic area considered, approximates the population of observable data corresponding to sales contracts. Few remaining data relate to the practice of concealment of the real estate data and had to be disregarded. The accuracy of multilevel functions is linked to the availability of reliable market prices.

The geographical area of the survey is the municipality of Cosenza in Italy. In the sample, all transactions are single sales. For each unit in the sample, the main characteristics and the price (total price) were detected. The characteristics considered were: the sale contract date (*DAT*), measured in months; the commercial surface (*SUR*), measured in square meters; the number of toilets (*TOI*), measured in terms of number; the level of the plan (*LEV*), measured with an ordinal scale, indicating the level in which the apartment is placed (1st, 2nd, 3rd, …); the state of maintenance (*MAI*): measured with a scale score, 0 was attributed to indicate a state of poor maintenance, 1 for mediocre maintenance, 2 for adequate maintenance, 3 for a discrete state, 4 if the maintenance status is good, 5 if the maintenance condition is excellent; and the number of fronts with windows (*FRW*), which indicates the number of fronts with openings (windows and balconies).

The commercial surface is calculated according to commercial practice by adding to the interior surface the balconies surface at the rate of 50%, the attic surface at the rate of 30%, the basement surface in proportion of 25%, and the garage surface at a 40% ratio. Sample statistics are given in Exhibit 1.

## Results

The model used to value the hedonic index numbers of the real estate characteristics is a multilevel random parameter 2, in which the random covariates are expressed by the constant, the commercial surface, the number of toilets, the floor level of the apartment, and the level of maintenance. The estimated model is explained by a fixed part represented by the following components: intercept, commercial surface, number of toilets, floor level, state of maintenance, and the number of fronts with windows; and random components of level 2 for the intercept, the commercial surface, the number of toilets, the floor level, and the state of maintenance (which explain the variability of the characteristics coefficients for each month of sale). The fixed parameters concern all the characteristics in the model and do not consider the random components of the second level.

The random component of the intercept (*CON*) measures the distance of the intercept of the individual groups (month) by the media intercept. The average coefficient of the fixed part is equal to −22,897.94 € with a standard error of 5,705.574 €. The random component of the second level represents the distance of the average price of the commercial surface (*SUR*) for each month from the related average price. The average coefficient is equal to 885.41 €/sqm with a standard error of 44.335 €/sqm, for the number of toilets (*TOI*), the average coefficient is equal to 8,093.88 €/n with the standard error of 2,338.243 €/n, for the floor level (*LEV*) the average coefficient is equal to 3,136.98 €/lev with a standard error of 603.528 €/lev. For the maintenance level (*MAI*), the average coefficient is equal to 2,818.51 €/n with a standard error of 842.792 €/n. For the fronts with windows (*FRW*), the fixed parameter represents the constant marginal price for each month not being statistically significant the 2nd level component. For this characteristic, the fixed coefficient is 2,609.66 €/n.

The Wald test is successful for all the features included in the model. The multilevel function, considering the fixed parameters and the second level residuals, is as follows:

$$\begin{aligned} V_{it} = {} & \beta_{0t} \cdot CON + \beta_{1it} \cdot SUR_{it} + \beta_{2t} \cdot TOI_{it} \\ & + \beta_{3t} \cdot FLO_{it} + \beta_{4t} \cdot MAI_{it} + 2{,}609.66 \cdot FRW_{it}, \end{aligned} \tag{8}$$

**Exhibit 1**
**Sample Statistics**

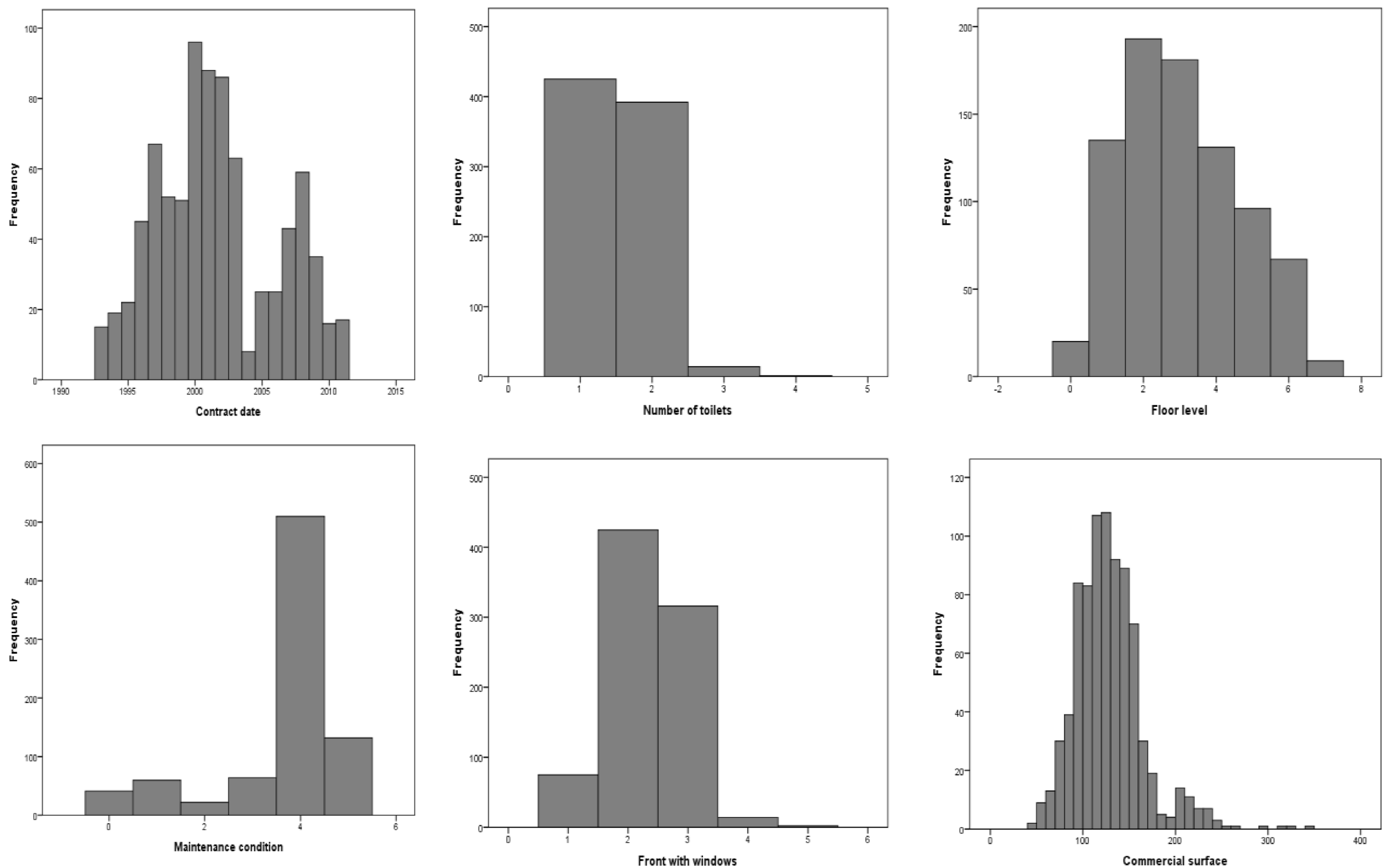
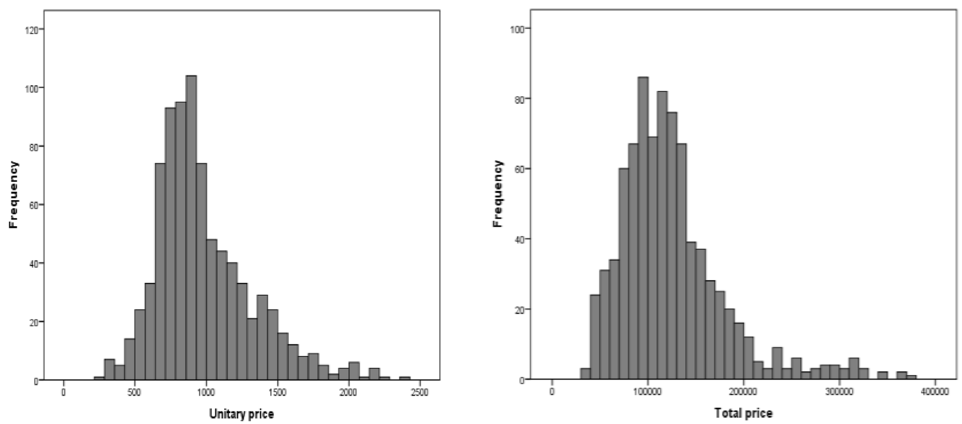


**Exhibit 1** (continued)
**Sample Statistics**




| Real Estate Features | Average/Maximum Frequence |
|---|---|
| Commercial Surface | 127,58 sq |
| Number of Toilets | 1 |
| Floor Level | 2 |
| Maintenance Condition | 4 |
| Front with windows | 2 |
| Unitary Price | 980,87€/sq |
| Total Price | 123.779,77€ |

where:

$$\beta_{0it} = -22.897,94 + u_{ot}, \; [€]. \tag{9}$$

$$\beta_{1it} = 885,41 + u_{1t} + e_{1it}, \left[\frac{€}{\text{sqm}}\right]. \tag{10}$$

$$\beta_{2t} = 8.093,88 + u_{2t}, \left[\frac{€}{n}\right]. \tag{11}$$

$$\beta_{3t} = 3.136,98 + u_{3t}, \left[\frac{€}{\text{lev}}\right]. \tag{12}$$

$$\beta_{4t} = 2.818,51 + u_{4t}. \left[\frac{€}{n}\right]. \tag{13}$$

The coefficients of the function in equation (8) measure the variation in the total average price varying one unit of the characteristic considered, keeping all other variables constant. The results of the multilevel analysis provide the residual of the second level ($u_{1t}$), which represents the effect, on the characteristic's marginal price, linked to the month of sale. The residuals of the second level estimated for the date of sales (months) are statistically significant for the constant, the commercial surface, the number of toilets, the level of the floor, and the level of maintenance, which means that the contribution of these characteristics to the market price varies depending on the month of sale. The residuals of the second level must be added to the fixed parameter to obtain the monthly coefficients. In practice, the coefficient (marginal price) of the characteristics consists of two components: the fixed part and the second-level residual, one for each month from January 1994 to December 2013.

The Laspeyers index has been calculated in equation (7) and refers to a virtual property whose characteristics are considered an average of those of the whole sample (Exhibit 1). It has been made a comparison between price index numbers based on a rough price and those referred to as a Laspeyers index (Exhibit 2).

The rough prices' index numbers trend, estimated by the simple price method, show an extremely irregular progress, with a strong growth trend by June 2006. The values are also significantly higher than those related to the Laspeyers indices, which conversely show a more harmonious trend and a more modest growth (Exhibits 2–4).

Results obtained for hedonic price index numbers are reported in Exhibits 5–10. The monthly trend of floor level feature shows an irregular trend, with a significant overall growth for the entire period investigated, much higher than that related to the other real estate characteristics.

The appreciation of real estate properties located at higher levels was significant. The floor level is considered to be a proxy variable that is able to capture other important real estate characteristics such as panorama, brightness, quiet, and clean air, which are really appreciated by ordinary market operators.

**Exhibit 2**
**Index Numbers**

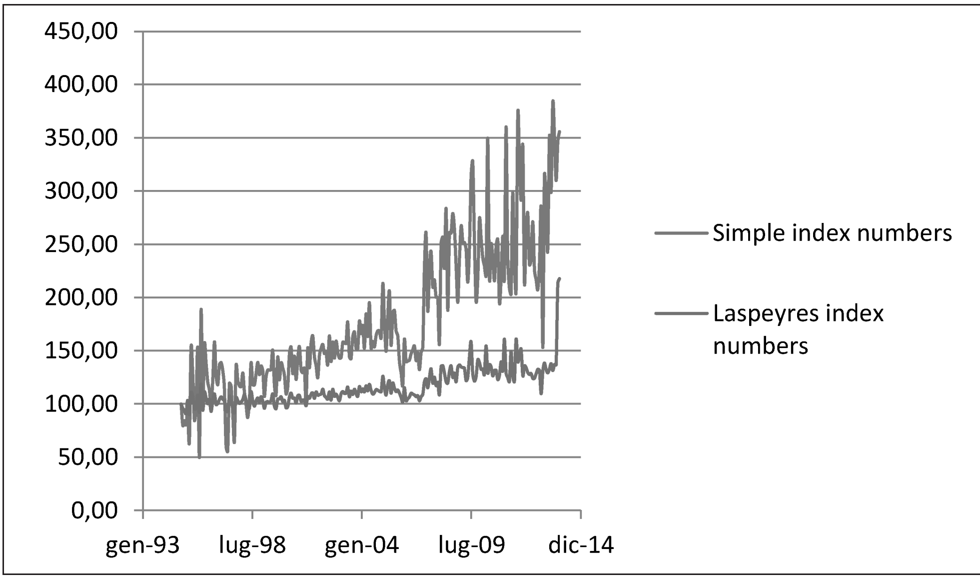



Exhibit 5 also shows the other real estate characteristics trends; they exhibit similar trends and values, and have an overall increasing tendency. The surface variable and the location factor (the constant value in the equation) indicate higher values compared to the services and the maintenance variables, confirming that location and dimensional variables are the most appreciated characteristics in the real estate market.

## Appraisal Validation of the Multilevel Model

Our multilevel model is a purely statistical model, based on the collection of data and their analysis. We have used statistical tests to check all the variables. The multilevel statistical model applied in real estate valuations must be validated to make sure it has achieved the required quantitative standards set for its use. This is done through the study of ratios (ratio study), in which the values appraised by the model are compared to the prices observed in the market. It is an extra statistical validation of valuations based on international valuation standards (IAAO, 1999) for mass appraisal.

In the ratio study, we compare the appraised values, $V$, with market prices, $S$, defining the elementary ratio for each property. The ratio for the generic apartment $i$, with $i = 1,2, \ldots, k$, with the appraised value $V_i$ and the market price $S_i$ will be equal to $V_i / S_i$.

The performance of the valuation of the model is measured with respect to: (1) the valuation level represented by the appraisal error $V_i - S_i$; (2) the uniformity of the valuation represented by the variability of the appraisal error; and (3) the mean

**Exhibit 3**
**Monthly Index Numbers (based on average prices)**

| Year | Monthly Index Numbers | | | | | | | | | | | | Annual Index |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | January | February | March | April | May | June | July | August | September | October | November | December | Numbers |
| 1995 | 100.00 | 79.86 | 84.30 | 80.29 | 91.44 | 64.24 | 152.64 | 128.19 | 84.68 | 97.34 | 152.64 | 49.88 | 97.13 |
| 1996 | 188.41 | 94.44 | 155.94 | 138.36 | 120.78 | 113.02 | 99.10 | 122.33 | 158.27 | 126.27 | 118.05 | 136.65 | 130.97 |
| 1997 | 138.78 | 130.83 | 115.20 | 60.28 | 55.67 | 119.24 | 117.04 | 91.03 | 65.01 | 135.71 | 121.94 | 116.98 | 105.64 |
| 1998 | 116.24 | 129.04 | 111.84 | 99.51 | 87.19 | 103.14 | 138.63 | 118.31 | 117.84 | 128.16 | 139.00 | 127.40 | 118.03 |
| 1999 | 135.70 | 130.87 | 106.97 | 125.20 | 131.63 | 131.75 | 132.34 | 150.43 | 117.68 | 108.34 | 144.08 | 122.58 | 128.13 |
| 2000 | 138.38 | 133.56 | 127.05 | 109.32 | 115.59 | 146.93 | 153.41 | 128.47 | 144.79 | 122.87 | 142.80 | 153.72 | 134.74 |
| 2001 | 131.84 | 129.16 | 133.33 | 102.11 | 152.13 | 133.90 | 156.91 | 163.98 | 144.31 | 135.83 | 124.62 | 145.43 | 137.80 |
| 2002 | 152.19 | 155.76 | 147.32 | 149.38 | 136.62 | 163.97 | 139.80 | 158.15 | 142.98 | 158.96 | 148.47 | 143.06 | 149.72 |
| 2003 | 143.52 | 157.93 | 157.59 | 156.93 | 176.92 | 144.30 | 142.42 | 162.71 | 167.58 | 152.04 | 151.57 | 177.77 | 157.61 |
| 2004 | 165.94 | 173.59 | 152.29 | 184.51 | 163.27 | 195.04 | 152.49 | 158.72 | 153.54 | 164.21 | 168.62 | 168.32 | 166.71 |
| 2005 | 162.02 | 213.07 | 178.66 | 149.43 | 180.96 | 205.74 | 155.13 | 185.71 | 187.78 | 168.67 | 163.58 | 139.96 | 174.23 |
| 2006 | 128.91 | 117.87 | 160.58 | 139.67 | 140.39 | 141.11 | 146.93 | 150.61 | 154.28 | 140.61 | 148.02 | 132.14 | 141.76 |
| 2007 | 148.46 | 153.62 | 224.86 | 260.40 | 187.50 | 225.36 | 243.24 | 209.94 | 216.55 | 200.22 | 199.53 | 156.68 | 202.20 |
| 2008 | 250.58 | 256.71 | 227.63 | 283.18 | 187.99 | 260.93 | 260.36 | 278.82 | 263.73 | 228.72 | 195.44 | 236.44 | 244.21 |
| 2009 | 267.23 | 251.30 | 251.64 | 244.83 | 214.31 | 246.20 | 312.78 | 327.13 | 250.82 | 196.16 | 218.39 | 273.75 | 254.55 |
| 2010 | 255.17 | 236.66 | 229.93 | 220.61 | 349.64 | 218.53 | 250.10 | 237.52 | 215.69 | 248.09 | 254.25 | 194.91 | 242.59 |
| 2011 | 218.70 | 257.53 | 218.10 | 360.15 | 238.62 | 208.71 | 203.25 | 298.13 | 230.99 | 206.29 | 371.09 | 328.26 | 261.65 |
| 2012 | 291.39 | 342.29 | 213.59 | 265.01 | 279.14 | 230.98 | 235.12 | 271.08 | 225.90 | 216.68 | 207.46 | 232.51 | 250.93 |
| 2013 | 283.88 | 153.12 | 313.47 | 287.12 | 244.12 | 351.48 | 298.83 | 383.00 | 351.59 | 309.80 | 345.97 | 355.75 | 306.51 |

**Exhibit 4**
**Monthly Index Numbers (Laspeyers)**

| Year | Monthly Index Numbers | | | | | | | | | | | | Annual Index Numbers |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | January | February | March | April | May | June | July | August | September | October | November | December | |
| 1995 | 95.74 | 93.88 | 91.00 | 103.31 | 89.91 | 110.52 | 107.11 | 95.28 | 96.10 | 108.35 | 92.23 | 95.74 | 98.26 |
| 1996 | 118.58 | 94.04 | 110.93 | 105.52 | 100.10 | 104.38 | 93.00 | 102.95 | 109.30 | 99.43 | 100.40 | 104.14 | 103.56 |
| 1997 | 106.62 | 104.28 | 102.50 | 93.75 | 93.01 | 107.07 | 101.39 | 96.52 | 91.65 | 105.69 | 103.12 | 100.43 | 100.50 |
| 1998 | 101.83 | 104.19 | 108.66 | 103.83 | 99.00 | 95.87 | 111.37 | 103.02 | 97.86 | 102.82 | 105.54 | 101.39 | 102.95 |
| 1999 | 106.76 | 102.41 | 95.87 | 101.45 | 102.16 | 101.64 | 103.03 | 109.32 | 97.81 | 95.43 | 104.45 | 104.57 | 102.08 |
| 2000 | 106.65 | 102.03 | 103.34 | 96.29 | 97.43 | 106.42 | 110.17 | 105.30 | 105.54 | 100.87 | 107.65 | 107.90 | 104.13 |
| 2001 | 102.36 | 103.71 | 103.40 | 98.24 | 106.75 | 105.14 | 109.09 | 110.29 | 105.17 | 111.27 | 108.20 | 108.69 | 106.03 |
| 2002 | 109.40 | 113.83 | 107.97 | 106.12 | 104.58 | 111.20 | 103.52 | 113.75 | 107.21 | 109.13 | 106.04 | 103.00 | 107.98 |
| 2003 | 109.58 | 111.60 | 109.81 | 110.15 | 115.83 | 106.63 | 112.01 | 108.85 | 113.81 | 109.64 | 107.19 | 116.28 | 110.95 |
| 2004 | 111.40 | 114.16 | 111.21 | 116.95 | 112.29 | 118.34 | 112.76 | 109.18 | 110.64 | 113.48 | 112.78 | 111.77 | 112.91 |
| 2005 | 111.47 | 125.95 | 115.37 | 108.25 | 117.98 | 121.66 | 110.04 | 119.66 | 114.86 | 111.55 | 113.07 | 110.11 | 115.00 |
| 2006 | 106.03 | 101.95 | 115.31 | 103.03 | 104.76 | 106.48 | 109.96 | 109.15 | 108.34 | 106.81 | 107.68 | 102.93 | 106.87 |
| 2007 | 106.84 | 109.13 | 120.59 | 123.55 | 115.11 | 124.42 | 132.98 | 119.44 | 125.09 | 117.18 | 120.16 | 108.66 | 118.60 |
| 2008 | 135.05 | 135.86 | 129.12 | 120.69 | 119.53 | 130.49 | 135.72 | 124.78 | 128.92 | 120.75 | 134.47 | 136.60 | 129.33 |
| 2009 | 134.66 | 134.13 | 133.56 | 121.85 | 124.72 | 144.09 | 158.31 | 127.99 | 121.40 | 125.64 | 141.47 | 138.90 | 133.89 |
| 2010 | 132.43 | 132.82 | 127.67 | 154.46 | 129.19 | 136.84 | 135.07 | 126.25 | 131.28 | 131.43 | 122.76 | 126.70 | 132.24 |
| 2011 | 139.60 | 131.23 | 160.85 | 129.33 | 124.72 | 121.11 | 149.17 | 126.40 | 121.29 | 160.71 | 139.74 | 143.73 | 137.32 |
| 2012 | 151.36 | 126.42 | 135.63 | 133.98 | 129.52 | 127.62 | 128.51 | 124.04 | 124.17 | 128.34 | 132.51 | 131.31 | 131.12 |
| 2013 | 109.58 | 131.07 | 138.43 | 132.34 | 129.15 | 132.77 | 137.75 | 131.33 | 137.02 | 136.70 | 213.04 | 217.61 | 145.57 |

## Exhibit 5
## Hedonic Price Index Numbers

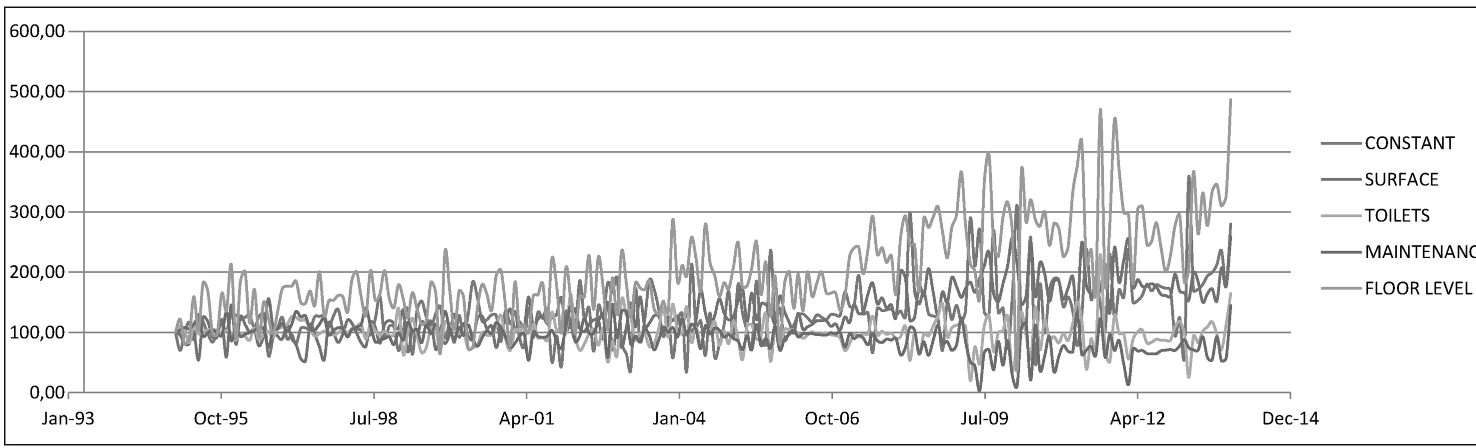


**Exhibit 6**
**Monthly Index Numbers Intercept (CON)**

| | Monthly Index Numbers | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | January | February | March | April | May | June | July | August | September | October | November | December | Annual Index Numbers |
| 1995 | 100.00 | 69.86 | 97.25 | 116.65 | 87.87 | 128.82 | 117.34 | 97.73 | 83.77 | 99.79 | 114.75 | 102.14 | 101.33 |
| 1996 | 128.79 | 128.63 | 121.62 | 124.90 | 128.17 | 107.79 | 126.29 | 130.91 | 61.14 | 90.64 | 107.53 | 124.58 | 115.08 |
| 1997 | 89.35 | 96.03 | 80.76 | 106.58 | 107.38 | 105.17 | 125.89 | 126.33 | 126.77 | 108.82 | 127.32 | 137.88 | 111.52 |
| 1998 | 106.56 | 95.95 | 105.17 | 104.47 | 103.77 | 95.63 | 96.69 | 121.58 | 159.87 | 92.75 | 110.13 | 108.86 | 108.45 |
| 1999 | 69.73 | 103.57 | 139.82 | 63.27 | 135.88 | 152.04 | 125.63 | 109.22 | 130.90 | 142.41 | 90.86 | 112.67 | 114.67 |
| 2000 | 83.50 | 127.99 | 94.64 | 130.46 | 184.01 | 156.14 | 118.33 | 120.94 | 129.00 | 127.85 | 84.84 | 136.31 | 124.50 |
| 2001 | 114.15 | 134.34 | 93.06 | 75.99 | 140.59 | 91.65 | 133.14 | 123.35 | 137.81 | 50.42 | 93.99 | 42.71 | 102.60 |
| 2002 | 133.47 | 113.21 | 92.94 | 185.80 | 122.96 | 140.58 | 68.57 | 144.02 | 125.67 | 181.97 | 166.23 | 189.39 | 138.73 |
| 2003 | 82.97 | 63.49 | 37.27 | 172.25 | 84.96 | 122.24 | 186.58 | 171.57 | 136.59 | 92.54 | 125.62 | 57.69 | 111.15 |
| 2004 | 117.76 | 130.60 | 35.11 | 212.03 | 110.75 | 119.32 | 61.60 | 131.77 | 58.05 | 80.17 | 112.71 | 167.63 | 111.46 |
| 2005 | 89.61 | 180.07 | 133.41 | 101.81 | 74.29 | 184.94 | 87.50 | 79.55 | 236.20 | 117.78 | 139.72 | 88.09 | 126.08 |
| 2006 | 100.17 | 112.25 | 128.45 | 130.54 | 124.03 | 117.52 | 128.91 | 125.08 | 121.26 | 129.55 | 128.96 | 128.00 | 122.89 |
| 2007 | 166.56 | 143.51 | 142.75 | 194.47 | 131.18 | 132.27 | 66.04 | 139.60 | 135.95 | 137.16 | 138.33 | 129.13 | 138.08 |
| 2008 | 135.56 | 125.19 | 298.78 | 168.75 | 147.13 | 167.68 | 131.38 | 127.05 | 122.72 | 69.75 | 147.54 | 121.31 | 146.90 |
| 2009 | 144.70 | 118.26 | 136.39 | 289.14 | 210.34 | 270.22 | 134.12 | 129.02 | 269.45 | 133.25 | 140.47 | 72.71 | 170.67 |
| 2010 | 159.46 | 310.64 | 117.64 | 108.76 | 257.67 | 46.93 | 180.07 | 98.91 | 170.64 | 189.30 | 187.11 | 142.57 | 164.14 |
| 2011 | 158.68 | 134.63 | 77.88 | 129.36 | 237.88 | 154.85 | 187.92 | 145.44 | 102.96 | 229.87 | 123.02 | 171.13 | 154.47 |
| 2012 | 158.37 | 238.23 | 149.92 | 152.96 | 161.60 | 179.94 | 168.99 | 178.92 | 175.61 | 173.00 | 170.39 | 94.40 | 166.86 |
| 2013 | 123.98 | 60.15 | 358.05 | 169.52 | 171.57 | 179.74 | 193.60 | 199.29 | 212.23 | 236.01 | 181.92 | 279.90 | 197.16 |

## Exhibit 7
## Monthly Index Numbers of Commercial Surface (*SUR*)

| Year | Monthly Index Numbers | | | | | | | | | | | | Annual Index Numbers |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | January | February | March | April | May | June | July | August | September | October | November | December | |
| 1995 | 100.00 | 95.31 | 81.42 | 80.97 | 101.25 | 54.02 | 124.51 | 117.98 | 92.04 | 92.45 | 120.63 | 58.85 | 93.28 |
| 1996 | 145.46 | 80.10 | 125.94 | 113.12 | 100.31 | 104.40 | 77.34 | 96.35 | 155.85 | 108.64 | 93.58 | 89.07 | 107.51 |
| 1997 | 112.54 | 89.19 | 79.61 | 56.17 | 52.47 | 112.14 | 96.81 | 75.70 | 54.59 | 115.50 | 99.89 | 83.27 | 85.66 |
| 1998 | 97.10 | 120.35 | 115.07 | 98.84 | 82.62 | 76.82 | 116.49 | 108.30 | 81.88 | 113.17 | 119.55 | 115.26 | 103.79 |
| 1999 | 105.76 | 136.42 | 67.96 | 102.05 | 94.24 | 117.88 | 109.35 | 117.86 | 71.55 | 92.73 | 81.27 | 97.23 | 99.53 |
| 2000 | 130.16 | 91.83 | 114.95 | 93.24 | 88.45 | 123.50 | 126.81 | 106.57 | 110.45 | 74.95 | 103.40 | 119.84 | 107.01 |
| 2001 | 138.29 | 104.44 | 117.79 | 106.51 | 158.48 | 109.88 | 121.71 | 127.28 | 114.33 | 124.54 | 130.49 | 157.82 | 125.96 |
| 2002 | 105.25 | 119.18 | 139.29 | 106.24 | 101.66 | 113.01 | 120.38 | 119.59 | 89.46 | 149.75 | 71.99 | 104.38 | 111.68 |
| 2003 | 74.84 | 109.76 | 148.96 | 109.08 | 158.99 | 108.77 | 115.34 | 127.46 | 126.64 | 99.82 | 115.96 | 119.93 | 117.96 |
| 2004 | 127.02 | 112.95 | 130.42 | 99.97 | 120.39 | 170.20 | 132.49 | 107.80 | 128.38 | 155.61 | 131.93 | 120.68 | 128.15 |
| 2005 | 126.59 | 152.51 | 173.70 | 118.34 | 165.05 | 120.99 | 146.96 | 146.01 | 135.59 | 114.68 | 160.34 | 134.93 | 141.31 |
| 2006 | 116.82 | 98.71 | 131.56 | 104.96 | 109.11 | 113.27 | 119.17 | 119.18 | 119.19 | 109.89 | 113.97 | 104.01 | 113.32 |
| 2007 | 125.48 | 116.40 | 145.97 | 131.32 | 133.66 | 163.46 | 182.04 | 147.46 | 157.08 | 140.41 | 145.62 | 124.31 | 142.77 |
| 2008 | 201.46 | 189.73 | 120.50 | 124.94 | 177.28 | 161.22 | 205.70 | 166.84 | 127.97 | 167.01 | 149.61 | 191.15 | 165.28 |
| 2009 | 176.06 | 158.10 | 172.76 | 183.45 | 166.58 | 189.30 | 210.52 | 233.43 | 164.17 | 150.45 | 181.00 | 209.76 | 182.97 |
| 2010 | 257.50 | 214.34 | 147.05 | 160.20 | 249.52 | 159.45 | 215.66 | 197.11 | 151.79 | 184.31 | 184.99 | 155.31 | 189.77 |
| 2011 | 173.27 | 192.25 | 133.13 | 249.30 | 177.87 | 162.82 | 162.20 | 170.97 | 211.80 | 131.54 | 240.86 | 182.86 | 182.41 |
| 2012 | 220.21 | 254.40 | 174.94 | 187.53 | 176.32 | 176.54 | 180.77 | 175.22 | 162.85 | 161.23 | 159.61 | 196.53 | 185.51 |
| 2013 | 168.56 | 165.45 | 153.20 | 198.97 | 184.53 | 150.04 | 163.94 | 172.09 | 151.75 | 206.78 | 176.52 | 258.87 | 179.22 |

## Exhibit 8
## Monthly Index Numbers of Services (*TOI*)

| | Monthly Index Numbers | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | January | February | March | April | May | June | July | August | September | October | November | December | Annual Index Numbers |
| 1995 | 100.00 | 110.50 | 103.64 | 82.46 | 109.28 | 97.45 | 100.68 | 104.24 | 105.31 | 102.36 | 99.97 | 103.52 | 101.62 |
| 1996 | 98.52 | 87.02 | 99.58 | 93.31 | 87.03 | 107.31 | 87.49 | 96.82 | 101.15 | 91.65 | 104.83 | 108.50 | 96.93 |
| 1997 | 113.09 | 126.19 | 124.74 | 119.45 | 120.29 | 104.27 | 93.52 | 97.73 | 101.95 | 100.91 | 95.17 | 99.21 | 108.04 |
| 1998 | 105.48 | 99.23 | 106.01 | 109.58 | 113.14 | 88.95 | 113.41 | 102.71 | 94.53 | 101.76 | 94.68 | 96.13 | 102.13 |
| 1999 | 139.50 | 63.02 | 97.64 | 123.17 | 94.46 | 66.32 | 74.39 | 107.08 | 104.54 | 70.30 | 116.69 | 111.79 | 97.41 |
| 2000 | 105.93 | 97.34 | 100.04 | 72.13 | 74.44 | 79.32 | 99.57 | 95.20 | 96.21 | 109.44 | 128.34 | 89.48 | 95.62 |
| 2001 | 69.50 | 104.69 | 102.02 | 104.63 | 56.85 | 112.84 | 90.83 | 92.35 | 86.93 | 133.54 | 103.21 | 110.77 | 97.35 |
| 2002 | 98.84 | 162.77 | 95.60 | 70.11 | 80.79 | 97.60 | 104.51 | 78.57 | 105.54 | 50.79 | 84.85 | 61.31 | 90.94 |
| 2003 | 155.02 | 128.03 | 124.72 | 87.82 | 98.96 | 97.31 | 76.47 | 80.71 | 108.47 | 140.60 | 92.03 | 146.77 | 111.41 |
| 2004 | 100.17 | 99.07 | 142.39 | 83.34 | 108.58 | 79.94 | 107.71 | 103.28 | 117.63 | 78.60 | 102.61 | 81.18 | 100.37 |
| 2005 | 118.67 | 89.21 | 54.44 | 110.45 | 112.65 | 88.67 | 95.34 | 131.58 | 52.13 | 103.52 | 70.40 | 104.41 | 94.29 |
| 2006 | 104.04 | 103.67 | 98.31 | 89.96 | 95.43 | 100.90 | 96.69 | 97.95 | 99.20 | 95.82 | 94.62 | 90.66 | 97.27 |
| 2007 | 69.72 | 81.71 | 97.04 | 91.86 | 96.83 | 98.04 | 126.76 | 95.94 | 101.77 | 96.69 | 98.51 | 89.54 | 95.37 |
| 2008 | 92.90 | 109.53 | 53.27 | 98.94 | 63.12 | 97.31 | 94.00 | 110.30 | 126.60 | 147.79 | 92.84 | 107.31 | 99.49 |
| 2009 | 111.90 | 112.53 | 109.83 | 20.04 | 75.76 | 47.32 | 109.77 | 127.52 | 67.93 | 99.25 | 102.92 | 126.21 | 92.58 |
| 2010 | 35.55 | 84.48 | 133.41 | 95.12 | 56.36 | 142.16 | 71.05 | 120.64 | 89.70 | 92.75 | 82.42 | 97.74 | 91.78 |
| 2011 | 86.53 | 114.92 | 150.78 | 120.06 | 38.79 | 89.60 | 74.44 | 229.12 | 67.92 | 50.66 | 128.18 | 97.96 | 104.08 |
| 2012 | 96.84 | 55.44 | 88.07 | 100.85 | 103.90 | 82.19 | 83.64 | 88.45 | 86.75 | 86.76 | 86.77 | 108.77 | 89.03 |
| 2013 | 117.86 | 97.68 | 25.16 | 93.62 | 82.95 | 101.83 | 109.02 | 117.48 | 96.90 | 71.12 | 113.32 | 164.36 | 99.27 |

## Exhibit 9
## Monthly Index Numbers of Floor Level (*LEV*)

| Year | Monthly Index Numbers | | | | | | | | | | | | Annual Index Numbers |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | January | February | March | April | May | June | July | August | September | October | November | December | |
| 1995 | 100.00 | 121.57 | 81.19 | 89.63 | 159.05 | 103.42 | 181.99 | 168.28 | 97.36 | 96.42 | 165.04 | 136.03 | 125.00 |
| 1996 | 212.76 | 89.37 | 183.34 | 200.52 | 115.55 | 171.36 | 91.90 | 150.27 | 123.03 | 97.96 | 135.66 | 172.71 | 145.37 |
| 1997 | 176.57 | 176.78 | 184.53 | 149.36 | 148.28 | 168.57 | 144.21 | 200.52 | 127.85 | 152.27 | 153.20 | 161.15 | 161.94 |
| 1998 | 159.14 | 133.37 | 186.64 | 200.52 | 159.09 | 128.17 | 202.39 | 153.81 | 164.06 | 202.48 | 156.50 | 140.89 | 165.59 |
| 1999 | 178.92 | 154.94 | 124.53 | 165.60 | 140.03 | 113.78 | 119.55 | 183.19 | 165.75 | 64.99 | 234.66 | 171.11 | 151.42 |
| 2000 | 105.08 | 168.90 | 158.85 | 72.95 | 75.78 | 142.27 | 179.16 | 163.36 | 136.24 | 194.04 | 203.04 | 144.18 | 145.32 |
| 2001 | 77.49 | 183.83 | 110.44 | 113.41 | 99.91 | 160.77 | 162.18 | 181.36 | 116.78 | 223.13 | 176.75 | 99.05 | 142.09 |
| 2002 | 208.23 | 153.57 | 97.87 | 159.47 | 159.07 | 227.64 | 138.19 | 225.50 | 174.85 | 104.45 | 190.25 | 129.91 | 164.08 |
| 2003 | 235.88 | 179.29 | 124.69 | 182.39 | 173.68 | 171.34 | 184.81 | 138.72 | 130.39 | 151.67 | 133.42 | 287.06 | 174.45 |
| 2004 | 183.88 | 211.17 | 193.64 | 257.60 | 223.41 | 168.68 | 279.45 | 216.14 | 194.09 | 160.04 | 182.21 | 172.47 | 203.57 |
| 2005 | 207.94 | 249.37 | 175.57 | 177.63 | 206.49 | 250.79 | 163.02 | 216.71 | 126.46 | 193.58 | 80.06 | 179.54 | 185.60 |
| 2006 | 200.52 | 139.71 | 196.71 | 135.99 | 200.52 | 159.28 | 181.62 | 200.52 | 165.52 | 164.98 | 165.66 | 136.87 | 170.66 |
| 2007 | 159.86 | 226.34 | 239.95 | 241.53 | 197.26 | 237.74 | 293.14 | 229.56 | 240.52 | 216.10 | 227.04 | 163.09 | 222.68 |
| 2008 | 258.32 | 293.36 | 244.76 | 245.55 | 166.92 | 289.78 | 274.00 | 291.38 | 308.77 | 267.35 | 223.52 | 274.35 | 261.50 |
| 2009 | 295.65 | 366.49 | 278.21 | 214.42 | 202.20 | 156.94 | 346.75 | 397.04 | 247.77 | 225.87 | 291.29 | 316.31 | 278.24 |
| 2010 | 266.78 | 36.24 | 365.56 | 283.06 | 320.14 | 287.08 | 276.25 | 300.29 | 244.86 | 280.60 | 273.06 | 226.18 | 263.34 |
| 2011 | 241.64 | 332.67 | 374.39 | 415.86 | 212.63 | 237.68 | 165.82 | 470.33 | 179.76 | 277.11 | 453.90 | 370.28 | 311.01 |
| 2012 | 298.87 | 295.00 | 179.53 | 304.62 | 309.13 | 245.54 | 251.02 | 281.95 | 240.19 | 200.52 | 227.28 | 276.32 | 259.16 |
| 2013 | 294.00 | 183.02 | 205.51 | 366.58 | 263.63 | 331.63 | 277.34 | 333.28 | 345.65 | 309.90 | 326.94 | 486.43 | 310.32 |

**Exhibit 10**
**Monthly Index Numbers of Maintenance Level (*MAI*)**

| Year | Monthly Index Numbers | | | | | | | | | | | | Annual Index Numbers |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | January | February | March | April | May | June | July | August | September | October | November | December | |
| 1995 | 100.00 | 103.96 | 107.80 | 105.93 | 113.72 | 121.33 | 93.98 | 99.38 | 103.76 | 99.53 | 94.64 | 134.06 | 106.51 |
| 1996 | 86.22 | 107.30 | 93.37 | 96.49 | 99.61 | 103.16 | 108.69 | 104.77 | 75.17 | 102.57 | 105.73 | 114.85 | 99.83 |
| 1997 | 102.59 | 113.94 | 134.63 | 125.42 | 127.17 | 105.29 | 103.62 | 113.30 | 122.99 | 96.23 | 102.99 | 107.62 | 112.98 |
| 1998 | 105.49 | 92.41 | 104.67 | 108.86 | 113.04 | 140.57 | 110.28 | 84.49 | 87.22 | 89.59 | 92.27 | 79.65 | 100.71 |
| 1999 | 99.54 | 86.81 | 116.53 | 102.68 | 104.97 | 82.70 | 113.15 | 97.74 | 113.70 | 104.25 | 134.84 | 105.34 | 105.19 |
| 2000 | 88.85 | 107.81 | 94.90 | 112.31 | 88.46 | 76.95 | 90.84 | 102.92 | 97.52 | 116.04 | 107.74 | 92.95 | 98.11 |
| 2001 | 74.87 | 81.92 | 92.65 | 97.79 | 53.56 | 100.47 | 93.19 | 91.66 | 93.62 | 103.21 | 86.16 | 73.86 | 86.91 |
| 2002 | 109.96 | 142.39 | 85.33 | 93.83 | 123.19 | 98.95 | 98.79 | 118.76 | 128.38 | 58.46 | 146.20 | 91.03 | 107.94 |
| 2003 | 135.19 | 133.42 | 80.49 | 93.75 | 83.86 | 105.23 | 94.03 | 71.06 | 88.09 | 108.56 | 100.08 | 107.78 | 100.13 |
| 2004 | 92.53 | 122.20 | 95.35 | 113.52 | 104.32 | 72.73 | 110.63 | 97.26 | 107.27 | 103.61 | 92.00 | 96.08 | 100.62 |
| 2005 | 89.83 | 85.11 | 70.63 | 93.67 | 70.91 | 113.00 | 78.25 | 89.61 | 73.54 | 110.55 | 70.25 | 88.88 | 86.19 |
| 2006 | 96.24 | 103.61 | 92.26 | 97.59 | 97.72 | 97.85 | 96.11 | 95.75 | 95.38 | 99.28 | 97.29 | 99.62 | 97.39 |
| 2007 | 75.33 | 98.01 | 89.61 | 95.02 | 90.36 | 79.49 | 96.85 | 86.16 | 83.16 | 88.65 | 87.08 | 89.54 | 88.27 |
| 2008 | 62.40 | 74.76 | 108.08 | 103.25 | 64.78 | 84.21 | 61.82 | 86.73 | 111.65 | 81.86 | 84.64 | 70.04 | 82.85 |
| 2009 | 79.31 | 123.89 | 84.44 | 50.88 | 43.93 | 2.87 | 65.95 | 71.25 | 37.92 | 84.75 | 45.27 | 86.05 | 64.71 |
| 2010 | 28.24 | 10.92 | 106.36 | 114.19 | 20.68 | 111.87 | 36.60 | 61.06 | 93.09 | 34.26 | 59.66 | 77.86 | 62.90 |
| 2011 | 68.60 | 68.32 | 133.05 | 63.40 | 70.42 | 76.67 | 61.67 | 123.02 | 58.36 | 96.21 | 69.82 | 86.43 | 81.33 |
| 2012 | 46.68 | 13.31 | 72.39 | 68.78 | 69.81 | 64.64 | 64.39 | 64.39 | 69.31 | 70.33 | 71.35 | 69.98 | 62.11 |
| 2013 | 77.50 | 87.69 | 74.63 | 70.69 | 70.32 | 92.50 | 58.50 | 54.35 | 92.69 | 52.60 | 57.23 | 144.45 | 77.76 |

**Exhibit 11**
**Results of the Ratio Study for the Data Sample**

| Indices | | | | | |
|---|---|---|---|---|---|
| $\tilde{V/S}$ | $V/\bar{S}$ | $\bar{V}/\bar{S}$ | COD | COV | PRD |
| (1) | (2) | (3) | (4) | (5) | (6) |
| 1.015 | 1.025 | 0.997 | 11.545 | 14.940 | 0.9998 |

absolute percentage difference. Statistically, the core measures provide an indication of the overall level of assessments for a group of properties. The measures of the appraisal level are: the median $\tilde{V/S}$, the mean $V/\bar{S}$, and the weighted mean $\bar{V}/\bar{S}$, which express the mean ratio of the group of properties weighted by market prices.

The measures of the uniformity of the estimate are: the range of variation calculated from the difference between the maximum and the minimum ratio of the group of properties. The coefficient of dispersion (COD) is calculated based on deviations from the median of the ratios. Technically the COD is calculated by first identifying the difference between each individual sales ratio and the median ratio. These differences are added (without reference to negative or positive numbers) and divided by the total number of sales in the group (this is the average deviation). Divide the average deviation by the median ratio and multiply this result by 100. The lower the COD, the more uniform are the assessments. A high COD indicates a lack of equality and uniformity among individual assessments.

The coefficient of variance (COV) is calculated based on deviations from the mean ratio. Technically defined as the standard deviation divided by the mean, times 100, a high COV indicates greater variety in individual ratios. The closer the COV is to 0, the more stable the sales group (provided there are a sufficient number of sales).

The price-related differential (PRD) is an index for measuring the regressivity or progressivity of assessment. It is calculated by dividing the mean ratio by the weighted ratio. According to the standard on ratio studies (IAAO, 1999), the valuation tests were carried out on the sample of market data (Exhibit 11).

The valuation standards fix the thresholds of acceptability of the index of the ratio study. The percentage standard deviation for the average residential property should be less than 10% to 15%. For the study sample, the percentage difference is equal to 11.75%. For measurements of the estimation level (columns 1–3 in Exhibit 11), the test provides an indication of the overall level of assessment for the property in question.

The COD for residential real estate should be less than 15% in the older and heterogeneous areas, and less than 10% of residences in the newest and quite similar areas. The COV, calculated based on deviations from the mean of the ratios, is exceeded for the full sample of data. The PRD, calculated as the ratio between the average and the weighted average ratio, should be close to 1, in particular between

0.98 and 1.03. The standards for the PRD are not absolute when samples are small or when there are wide variations in prices. The standard indicates that the level of the ratios of a group of properties must be within 5% of the overall ratio of all groups considered. The level of the overall ratio should be within 10% of the level of 100% (0.90 ÷ 1.10).

It is not necessary that the analyst knows in detail or is capable of explaining the algorithm of the statistical model or the complexity of its statistical and mathematical formulas. It is necessary instead that the analyst is able to describe the process of development and to verify that the results are constant and faithfully reflect the behavior of the market for the property being assessed (Appraisal Standards Board, 2006).

## Conclusion

In this paper, we propose a multilevel model for improving the spatial-temporal nature of the of hedonic price index construction, in the case of residential apartments in the city of Cosenza, Italy. It is well-documented that price indices play an important role in gaining a better understanding of the housing market and investigating issues of societal relevance (such as analyses of housing affordability or whether or not housing bubbles exist). Accurate price indices that capture the spatial-temporal dynamics of the housing market can be used as a method of valuing property, updating the asset's value, and calculating investment performance. We contribute to the literature by applying a multilevel hedonic model for the first time using historical time series data (January 1995 to December 2013) in order to determine the implicit marginal prices of various housing characteristic variables over time. The results from the multilevel analysis are straightforward and show the model's ability to the variables' effects and their variations over time, and may be useful to the community that deals with constructing better price indices.

The index numbers of real estate prices play an important role for the full understanding of the real estate market and the causes of price changes. These indices are useful in monitoring the property's profitability and at the same time recognizing possible anomalous phenomena.

The construction of price indices of residential apartments in the city of Cosenza, Italy is possible due to the availability of a large number of real estate transactions. The sample was residential apartment sales data from multi-story buildings in a condominium in Cosenza.

The hierarchical structure of the property data allowed us to perform a regression analysis on several levels to determine the implicit marginal prices of the variables with respect to time. The analysis considered some property features, commercial surface, sale date, services number, floor level, maintenance level, and number of fronts with windows.

We calculated unit prices index numbers referenced to the commercial surface and hedonic price index numbers of real estate features, the latter using a multilevel model, applied for the first time to the time series of hedonic housing prices. The index numbers are direct index numbers, monthly and fixed base.

The multilevel analysis applied to historical data has shown the model's ability to express the effects of property characteristics and their variation over time, as measured by the index numbers. The model has advantages over direct hedonic models including saving degrees of freedom, for the absence of time variables, and over the indirect hedonic models, related to the precarious stability of regression models constructed for different periods of time.

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