
# BOOK OF ABSTRACTS

$63^{rd}$ Workshop: Variational Analysis and Applications

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**Nonsmooth dynamical systems through concrete examples**

*Adly Samir* (University of Limoges, France)

Nonsmooth dynamical systems is a part of differential inclusions. With the emergence of many engineering applications, nonsmooth dynamical systems have played a central role in the understanding of many phenomena. The mathematical formulation of the unilateral dynamical system involved inequality constraints and necessarily contains natural nonsmoothness. The nonsmoothness could originate from the discontinuous control term, or from the environment of the system e.g. impact with dry friction in mechanical systems. Due to the lack of smoothness, classical mathematical methods are applicable only to a limited amount and require naturally extensions for both analytical and numerical methods by using modern tools of variational analysis. In this talk, we will review the following class of problems:

i) Linear Complementarity Systems and their applications in electrical circuits

ii) Evolution Variational inequalities and their applications in mechanics

iii) Moreau's sweeping process

**Weak sharp solutions and minimum and maximum principle sufficiency properties for nonsmooth Variational Inequalities**

*Al-Homidan Suliman* (King Fahd University of Petroleum and Minerals, Saudi Arabia)

This Talk is based on our two recent papers [1, 2]. We introduce the concept of weak sharp solutions for nonsmooth variational inequalities and give its characterization by using equilibrium version of Ekelend's variational principle. We also study of minimum and maximum principle sufficiency properties for nonsmooth variational inequalities by using gap functions. Several characterizations of these two sufficiency properties are provided. Two open questions are given at the end.

**References:**

[1] S. Al-Homidan, M. AlShahrani and Q.H. Ansari, *Weak shrap solutions for equilibrium problems in metric spaces*, To appaer in J. Nonlinear Convex Anal. (2015).

[2] S. Al-Homidan, M. AlShahrani and Q.H. Ansari, *Minimum and Maximum principle sufficiency properties for nonsmooth variational inequalities*, Submitted for publication (2015).

**Non-pivot Hilbert spaces and use of wireless information: weighted quasi variational inequalities**

*Barbagallo Annamaria* (University of Naples Federico II, Italy)

Co-author: Stéphane Pia

Given a urban network we consider a model in which the user prefers the path with a minor traffic density for the same path cost. Nevertheless in a realistic situation in addition to the information related to the traffic density it could be crucial to take into account which is the effective flow on a route and modify the constraint set accordingly reducing the demand if the flow tends to increase. In such a model it needs to use weights do not depend on the equilibrium solution in order to graduate the path cost. For this reason we introduce non-pivot Hilbert spaces and observe that a system of wireless communications allows us to obtain information in real time about traffic congestion in the paths. In this setting we present a new mathematical tool to study the cited realistic situation: weighted quasi-variational inequalities. Moreover we establish some existence and regularity results for such inequalities. At last a numerical example is provided.

**Keywords:** Weighted quasi-variational inequalities, Non-pivot Hilbert spaces, Weighted elastic traffic equilibrium problem

**References:**

[1] A. Barbagallo and S. Pia, *Weighted quasi-variational inequalities in non-pivot Hilbert spaces and applications*, Journal of Optimization Theory and Applications, vol. 164, 2015, 781–803.

**On families of $c$-convex $c$-antiderivatives and their envelopes**

*Bartz Sedi* (University of British Columbia, Canada)

Within the general settings of $c$-convexity theory given partial data regarding a $c$-subdifferential we consider the family of all $c$-convex $c$-antiderivatives that comply with the given data. Our family contains its envelopes which are then optimal $c$-convex $c$-antiderivatives and has a dual family. We present duality relations and construct the optimal antiderivatives explicitly. As time permits we review applications and interpretations in several fields of variational analysis such as economical aspects of optimal transport theory the field of Lipschitz extensions and in the theory of representation of $c$-monotone mappings. Our discussion sheds new light even when restricted to the classical case of convexity and monotonicity that is whenever $c$ is the coupling between a real vector space and its dual. The presentation is initiated with a review of the basics of $c$-convexity theory.

**Some remarks on the mountain pass theorem**

*Bonanno Gabriele* (University of Messina, Italy)

Some remarks on the Ambrosetti-Rabinowitz theorem are pointed out and multiple critical points theorems are presented. As a consequence multiple solutions to nonlinear differential problems are established.

**A new insight into the semivectorial bilevel optimization problem: the Riemannian setting**

*Bonnel Henri* (University of New Caledonia, France)

Co-authors: Leonard Todjihounde, Constantin Udriste

A semivectorial bilevel problem is characterized by a scalar optimization problem to be solved by the leader (the upper level) and a multiobjective optimization problem to be solved by several followers (the lower level). The followers are acting in a cooperative way inside the greatest coalition and they choose a Pareto solution with respect to a given ordering cone. We consider the Riemannian manifold framework for both upper and lower level. For the so-called "optimistic problem" when the followers choice among their best responses is the most favorable for the leader we give optimality conditions. Also for the so-called "pessimistic problem" when there is no cooperation between the leader and the followers and the followers choice may be the worst for the leader we present an existence result.

**On the solving of nonsmooth convex optimization problems with complex structures**

*Bot Radu Ioan* (University of Vienna, Austria)

In this talk we address the solving of nonsmooth convex optimization problems with complex and intricate structures by actually solving the corresponding system of optimality conditions which involves mixtures of linearly composed Lipschitz single-valued and parallel-sum type monotone operators. We formulate iterative schemes that process the set-valued maximally monotone operators via backward steps and the single-valued ones via explicit forward steps and analyze their convergence behavior. The performances of the proposed algorithms are illustrated by numerical experiments on real-life problems arising in machine learning and image and video processing.

**Keywords:** nonsmooth convex optimization, splitting algorithms, numerical experiments

### Convergence rates for forward-backward dynamical systems associated with strongly monotone inclusions

*Csetnek Ernö* (University of Vienna, Austria)

We present convergence rates of the trajectories generated by implicit first and second order dynamical systems associated to the determination of the zeros of the sum of a maximally monotone operator and a monotone and Lipschitz continuous one in a real Hilbert space. The trajectories converge with exponential rate to a zero of the sum provided the latter is strongly monotone. We derive from here convergence rates for the trajectories generated by dynamical systems associated to the minimization of the sum of a proper convex and lower semicontinuous function with a smooth convex one provided the objective function fulfills a strong convexity assumption. In the particular case of minimizing a smooth and strongly convex function its values converge along the trajectory to its minimum value with exponential rate too.

**Keywords:** dynamical systems, strongly monotone inclusions, continuous forward-backward method, convergence rates, convex optimization problems

### Financial Networks: new theorems and analysis of contagion

*Daniele Patrizia* (University of Catania, Italy)

Co-author: Antonino Maugeri

In this lecture we present a financial economy and provide some significant laws such as the Deficit formula the Balance law and the Liability formula for the management of the world economy. We also consider the Lagrange dual formulation and the Lagrange variables called deficit and surplus variables. By means of these variables we study the possible insolvencies related to the financial instruments and their propagation to the entire system producing a financial contagion. Moreover we extend the previous model to the case when the financial volumes depend on time and on the expected solution in order to take into account the influence of the expected equilibrium distribution for assets and liabilities on the investments on all financial instruments. We derive the quasivariational formulation which characterizes the equilibrium of the dynamical financial model. Further we prove a general existence theorem for quasi-variational inequalities under general assumptions which is also applied to the financial model.

**Keywords:** Lagrange multipliers, quasi-variational inequalities, financial models

### The $h$-principle and the equations of fluid dynamics

*De Lellis Camillo* (Universitaet Zuerich, Switzerland)

The incompressible Euler equations were derived more than 250 years ago by Euler to describe the motion of an inviscid incompressible fluid. It is known since the pioneering works of Scheffer and Shnirelman that there are nontrivial distributional solutions to these equations which are compactly supported in space and time. If they were to model the motion of a real fluid we would see it suddenly start moving after staying at rest for a while without any action by an external force. A celebrated theorem by Nash and Kuiper shows the existence of $C^1$ isometric embeddings of a fixed flat rectangle in arbitrarily small balls of the threedimensional space. You should therefore be able to put a fairly large piece of paper in a pocket of your jacket without folding it or crumpling it. In a series of joint papers with La'szló Sze'kelyhidi we pointed out that these two counterintuitive facts share many similarities. Indeed a suitable form of Gromov's $h$-principle (of which the Nash-Kuiper theorem is one famous geometric instance) holds for quite a few systems of partial differential equations in fluid-dynamics.

### A survey on the study of an economic equilibrium with variational inequality arguments

*Donato Maria Bernadette* (University of Messina, Italy)

Co-authors: Milasi Monica, Vitanza Carmela

In this talk a survey on the study of the general economic equilibrium problem in the framework of the variational inequality approach is presented. We first give the description of the main aspects characterizing the model and the equilibrium definition. Then, we focus on the developments and improvements obtained in this topic to obtain a wider applicability in the economic framework. In particular, the utility functions of agents are initially taken to be strongly concave and differentiable; subsequently, these assumptions are relaxed by assuming generalized concavity and non-differentiability. From a mathematical viewpoint, assuming different conditions on the utility functions permits to consider several quasi-variational inequalities, in which suitable multi-valued operators are involved. Finally, the existence of equilibrium points is achieved by using arguments of the set-valued analysis. The study of this equilibrium by means of the variational approach has been the main motivation leading to a class of generalized quasi-variational inequalities, whose existence

of solutions is obtained.

### Newton-type methods for nonsmooth equations and variational inequalities

*Dontchev Asen* (Mathematical Reviews (AMS)
and the University of Michigan, USA)

I will give a short survey on convergence results for various Newton-type methods including semismooth methods applied to generalized equations. In particular I will present generalizations of the Dennis-Moré theorem which characterizes the superlinear convergence of quasi-Newton methods for nonsmooth equations and variational inequalities. The talk is based on the paper [1] and the book [2].

**Keywords:** Newton method, variational inequality semismoothness, Dennis-Moré theorem.

**References:**

[1] R. Cibulka R., A.L. Dontchev, M.H. Geoffroy, *Inexact Newton methods and Dennis-Moré theorems for nonsmooth generalized equations*, SIAM J. Control Optim. 53 (2015), no. 2, 1003-1019.

[2] A.L. Dontchev, R.T. Rockafellar, Implicit functions and solution mappings. A view from variational analysis, Second edition. Springer, New York, 2014.

### Tame Variational Analysis

*Drusvyatskiy Dmitriy* (University of Washington, USA)

Over the past decade semi-algebraic geometry has had a pronounced impact on optimization and on variational analysis. Semi-algebraic functions – those whose graphs are representable by finitely many polynomial conditions – are common easy to recognize and are "pathology-free". The mere existence of polynomial descriptions endows semi-algebraic functions with powerful analytic properties there to be used. This analytic insight interplays nicely with all the usual notions of variational analysis such as selection theorems error bounds and Sard-type results. In this talk I will survey the main trends of semi-algebraic variational analysis assuming no prior familiarity of the audience with the subject.

**Keywords:** semi-algebraic, subdifferential, stratification, metric regularity, critical point

### On a unified approach to classical assertions in variational calculus

*Elster Rosalind* (Martin-Luther-University of Halle-Wittenberg, Germany)

The development of variational calculus is closely connected to a fundamental theorem of convex optimization called characterization theorem. This theorem will be used as a unified approach to some classical assertions in variational calculus. Let us mention here assertions with respect to Planck's uncertainty principle, the Euler-Lagrange equation, Dirichlet's problem, Ekeland's variational principle.

**Keywords:** characterization theorem, variational principles, variational inequalities

**References:**

[1] F. Giannessi (Ed.), Vector Variational Inequalities and Vector Equilibria, Kluwer Academic Publishers, (2000)

[2] B. Mordukhovich, N.M. Nam, An easy Path to Convex Analysis and Applications, Morgan&Claypool Publishers, (2014)

[3] D. Pallaschke, S. Rolewicz, Foundations of Mathematical Optimization, Kluwer Academic Publishers, (1997)

[4] R.T. Rockafellar, Convex Analysis.Princeton Landmarks in Mathematics, Princeton University Press, Princeton, NJ, (1997)

### Recent developments on stability for functional and geometric inequalities

*Figalli Alessio* (University of Texas at Austin, USA)

Geometric and functional inequalities naturally appear in several problems in the calculus of variations, partial differential equations, geometry, etc. Among the most classical inequalities, we recall the isoperimetric inequality, Sobolev and Gagliardo-Nirenberg inequalities, and the Brunn-Minkowski inequality. Although different, all these inequalities are intimately related.

The issue of finding the sharp constant in these inequalities, as well as the characterization of minimizers, is a classical and important question which is by now well understood. More recently, a lot of attention has been given to the stability issue:

Suppose that a function almost attains the equality in one of the previous inequalities. Can we prove, if possible in some quantitative way, that such a function is close (in some suitable sense) to one of the minimizers?

In the latest years several results have been obtained in this direction. The aim of this talk is to give an overview of some of these developments.

### A variational approach to second order optimality conditions in optimal control

*Frankowska Hélène* (CNRS and Université Pierre et Marie Curie, France)

Co-author: Hoehener Daniel

This talk is devoted to pointwise second-order necessary optimality conditions for the Mayer problem of optimal control theory. The control system under consideration involves arbitrary closed control sets $U(t)$ depending measurably on time. The first result says that with every optimal control it is possible to associate a matrix solution $W(.)$ of an adjoint matrix differential equation that satisfies a second-order transversality condition and a second-order maximality condition. Then a Jacobson like necessary optimality condition is stated for general control systems and singular or even "partially singular" optimal controls. Proofs are based on a variational differential inclusion involving second order tangents.

**Keywords:** optimal control, second-order necessary optimality conditions

**References:**

[1] H. Frankowska and N. Osmolovskii *Second-order necessary optimality conditions for the Mayer problem subject to a general control constraint*, in Analysis and Geometry in Control Theory and its Applications, vol. 12, Springer INDAM series, Springer Verlag, 2015.

[2] H. Frankowska and D. Hoehener *Jacobson type necessary optimality conditions for general control systems*, submitted.

### Measure type and $L^p$ Lagrange multipliers in elastic-plastic torsion

*Giuffré Sofia* (Mediterranea University of Reggio Calabria, Italy)

Aim of the talk is to present advanced results on the existence of Lagrange multipliers associated to the elastic-plastic torsion problem. The existence of Lagrange multipliers as a Radon measure is ensured for an elastic-plastic torsion problem associated to a nonlinear strictly monotone operator. A regularization of this result, namely the existence of $L^p$ Lagrange multipliers, is obtained under a strong monotonicity assumption on the operator. Finally the relationships between elastic-plastic torsion problem and the obstacle problem are investigated.

### Lagrange multipliers in constrained variational problems of continuum mechanics

*Gwinner Joachim* (Universität der Bundeswehr München, Germany)

Various constrained variational problems in continuum mechanics, like unilateral or bilateral contact in a steady-state regime and elasto-plasticity in a quasi steady-state regime, lead to the problem of existence and regularity of Lagrange multipliers. It is well-known that the Zowe-Kurcyusz [5] approach does not apply directly, because of the emptiness of the interior in $L^p$ spaces. In this talk we discuss several resorts.

One resort is the use of the quasi-relative interior as it is done in the work of R.I. Boţ and G. Wanka, see e.g. [1] and in the work of A. Maugeri with P. Daniele and co-workers see e.g. [2], resp. with F. Raciti [4]. Another resort, often used in the mixed approach of numerical analysis, is to work in the preimage space of the constrained problem.

We exploit these ideas in the study of various nonsmooth boundary value problems in two-fold saddle point formulations [3].

**References:**

[1] Boţ, R. I., Csetnek, E. R. and Wanka, G., *Regularity conditions via quasi-relative interior in convex programming*, SIAM J. Optim., 19, (2008), 217-233.

[2] Daniele, P., Giuffrè, S., Idone, G., and Maugeri, A., *Infinite dimensional duality and applications*, Math. Ann., 339, (2007), 221-239.

[3] Gwinner, J., *Three-field modelling of nonlinear nonsmooth boundary value problems and stability of differential mixed variational inequalities*, Abstr. Appl. Anal., (2013), Art. ID 108043.

[4] Maugeri, A. and Raciti, F., *Remarks on infinite dimensional duality*, J. Global Optim., 46, (2010), 581-588.

[5] Zowe, J. and Kurcyusz, S., *Regularity and stability for the mathematical programming problem in Banach spaces*, Appl. Math. Optim., 5, (1979), 49-62.

### On some relations between probability functions and variational analysis

*Henrion René* (Weierstrass Institute Berlin, Germany)

Given a parameter-dependent random inequality system a probability function assigns to the parameter the probability of satisfying this system. Such functions play a fundamental role in defining so-called probabilistic or chance constraints as they arise in numerous engineering problems (e.g. hydro reservoir management under

uncertain inflows avoidance of random obstacles in robotics). Probability functions exhibit some inherent nonsmoothness even if the input data (random inequality system density of random distribution) are smooth. Hence the application of tools from nonsmooth and variational analysis comes as a natural consequence. The talk discusses subdifferentiabilty of probability functions and the estimation of the maximal increasing slope of the optimal value function in probabilistic programming.

**Keywords:** probability functions, subdifferentials, probabilistic constraints

### A variational characterization of the Monge point of a tetrahedron

*Hiriart-Urruty Jean-Baptiste* (University of Toulouse, France)

Co-authors: Hadjisavvas Nicolas, Laurent Pierre-Jean

*"... nihil omnino in mundo contingint, in quo non maximi minimive ratio quapiam eluceat"*, translated into "... nothing in all the world will occur in which no maximum or minimum rule is somehow shining forth", used to say L. Euler in 1744. This is cofirmed by numerous applications of mathematics in physics, mechanics, economy, etc. In this work, we show that it is also the case for the classical "centers" of a tetrahedron, more specfically for the so-called Monge point (= the substitute of the notion of orthocenter for a tetrahedron). To the best of our knowledge, this variational characterization of the Monge point of a tetrahedron, that we present here, is new. The corresponding paper is going to appear in Journal of Optimization Theory and Applications.

### Global error bounds for systems of convex polynomials over polyhedral constraints

*Huynh Van Ngai* (University of Quy Nhon, Vietnam)

In this talk we present some new results on the Lipschitzian/Holderian type global error bound for systems of finitely many convex polynomial inequalities over a polyhedral constraint. Firstly for systems of this type we show that under a suitable asymptotic qualification condition the Lipschitzian type global error bound property is equivalent to the Abadie qualification condition in particular the Lipschitzian type global error bound is satisfied under the Slater condition. Secondly without regularity conditions the Hölderian global error bound with an explicit exponent is investigated.

**Keywords:** subdifferential, error bound, polynomial, recession cone, recession function

**References:**

[1] Huynh Van Ngai, *Global error bounds for systems of convex polynomials over polyhedral constraints*, SIAM Journal on Optimization, Vol. 25, No. 1, 2015, 521-539

### Polyhedrality and metric regularity

*Ioffe Alexander* (Technion Israel Institute of Technology, Israel)

The regularity theory for variational inequalities over polyhedral sets developed in a series of papers by Robinson, Ralph and Dontchev-Rockafellar in the 90s has long become classics of variational analysis. But in the available proofs of almost all main results, fairly nontrivial as they are, techniques of variational analysis do not play a significant part. The key steps of the proofs are fully based on a number of deep facts of topology and algebra of matrices. (The only exception is probably the proof by Dontchev and Rockafellar of their "critical superface regularity criterion".)

A natural question therefore is whether it is possible to develop an adequate theory based mainly on techniques more traditional for variational analysis. This is not just a question of methodology or aesthetics, it is also about consistency of variational analysis as such: whether it has enough strength to solve problems simply and naturally formulated in its terms.

In the talk I shall present some arguments (a general approach, brief descriptions of proofs of some fundamental facts that don't use anything beyond elementary geometry of polyhedra and basic facts of the theory of metric regularity, certain new results) in favor of the positive answer to the question, as well as some problems that still remain open.

### Cost-minimizing mechanisms for a wholesale electricity market game

*Jofré Alejandro* (Universidad de Chile, Chile)

Co-authors: Figueroa Nicolás, Heymann Benjamin

We consider a short-term wholesale electricity market model with general networks, quadratic transmission losses and energy producers playing strategically. Previous works by Escobar and Jofré show how regulation mechanisms such as the case when prices correspond to the Lagrange multipliers of a centralized cost minimization program allow the producers to charge significantly more (market power) than marginal price. In this paper we consider an incomplete information setting where

the cost structure of a producer is partially unknown to both its competitor and the regulator, which corresponds to a MPEC with asymmetric information. We derive an optimal regulation mechanism, and compare its performance to the "price equal to Lagrange multiplier" mechanism from the numerical and analytical point of view.

### n-polyconvexity: a new generalised semiconvexity which contains both poly- and rank-one convexity

*Kabisch Sandra* (University of Surrey, UK)

Polyconvexity is a special case of Abstract Convexity in the sense that the pointwise supremum of polyaffine functions is polyconvex. The same cannot be said for rank-one convexity since rank-one affine functions are nothing but polyaffine functions. However we will show that it is possible to generalise the concepts used in Abstract Convexity to include cases like rank-one convexity. This generalisation of Abstract Convexity weakens the notion of elementary functions (e.g. polyaffine functions for polyconvexity) to allow them to be localised to specific points of the domain. For rank-one convexity this yields a new definition of rank-one affine functions that is different to the standard one not equivalent to polyaffine functions and which has the property that the pointwise supremum of the new rank-one affine functions is indeed rank-one convex (but not necessarily polyconvex). Furthermore the generalisation immediately shows much more room for other generalised convexity notions one of which is what we call n-polyconvexity. For $f : \mathbb{R}^d \times D \to \mathbb{R} \cup \{+\infty\}$ n-polyconvexity unifies polyconvexity and rank-one convexity in the following sense: If $n = d \wedge D := \min\{d\, D\}$ then n-polyconvexity is equivalent to polyconvexity and if $n = 1$ then 1-polyconvexity is equivalent to rank-one convexity. Additionally one gains the new convexities for $n = (d \wedge D - 1) \dots 2$ in weakening order. Therefore the aforementioned generalisation of Abstract Convexity provides a new formal framework for a different approach to the generalised convexity notions of polyconvexity rank-one convexity and even quasiconvexity that are commonly used in the Calculus of Variations and allows to gain new insights about them.

**Keywords:** generalised semiconvexity, abstract convexity, polyconvexity, rank-one convexity

### Optimization framework for the elasticity imaging inverse problem of identifying tumors

*Khan Akhtar* (Rochester Institute of Technology, USA)

This talk will present theoretical and numerical study of the elasticity imaging in-

verse problem of tumor identification in the soft tissue of the human body. Beyond the obvious merits of its applications, this problem also presents significant mathematical challenges. The near incompressibility inherent in the model of linear elasticity in the body gives rise to the "locking effect" and necessitates a unique treatment of both the direct and inverse problems. A general optimization framework for the identification of parameters in saddle point problems will be presented along with a new modified output least squares (MOLS) objective functional. The MOLS functional is convex, thus overcoming the non-convexity of the classical output least-squares (OLS) functional and the new framework will be shown to be capable of accommodating both smooth and discontinuous parameters. Generalized derivative formulas for the coefficient-to-solution map will also be discussed along with a complete convergence analysis. Both continuous and discontinuous numerical examples will be shown together with details regarding discretization, implementation issues, and preliminary performance comparisons against existing methods.

### Criteria of nonlinear metric subregularity

*Kruger Alexander* (Federation University Australia, Australia)

In this talk I am going to demonstrate how nonlinear metric subregularity of set-valued mappings can be treated in the framework of the theory of linear error bounds of real-valued functions. For this purpose the machinery of error bounds has been extended to functions defined on the product of (metric or normed) spaces. Several kinds of primal space and subdifferential slopes for real-valued functions and set-valued mappings will be discussed.

**Keywords:** error bounds, slope, metric regularity, metric subregularity, Holder metric subregularity, calmness

**References:**

[1] A. Y. Kruger, *Error bounds and metric subregularity*, Optimization, 64, (2015), 49-79.

[2] A. Y. Kruger, *Error bounds and Hölder metric subregularity*, Set-Valued Var. Anal., (2015), DOI 10.1007/s11228-015-0330-y.

### Optimal control with $\mathbf{L}^1$ constraints

*Kruse Florian* (University of Graz, Austria)

Co-authors: Kunisch Karl

Objectives featuring the $L^1$ norm have received much attention in optimal control with PDEs during the last decade. Yet little emphasis has been put on $L^1$ constraints. This talk addresses PDE-constrained optimal control problems in which the control is bounded both pointwise and with respect to the $L^1$ norm. We use first-order optimality conditions to develop a semismooth Newton method for these problems. Exploiting a new semismooth implicit function theorem we analyze the sensitivity of the Lagrangian multiplier associated to the $L^1$ constraint and prove local superlinear convergence of the method in an infinite-dimensional setting. The talk closes with a numerical example to demonstrate the efficiency of the approach.

**Keywords:** Optimal control with PDEs, sensitivity of multipliers, semismooth implicit function theorem, semismooth Newton methods

### Segmentation and inpainting of color images

*Leaci Antonio* (University of Salento, Italy)

Co-authors: Michele Carriero, Franco Tomarelli

We introduce and study a variational model for segmentation and inpainting of 2-dimensional color images. The model consists in the minimization of a functional dependent on second derivatives free discontinuity and free gradient discontinuity. The competitors are piecewise $C^2$ vector valued functions whose components represent the intensity of RGB channels.

**Keywords:** Free discontinuity problems, image analysis

**References:**

[1] M. Carriero, A. Leaci, F. Tomarelli, *Euler equations for Blake and Zisserman functional*, Calc. Var. Partial Differential Equations, 32, n.1 (2008), 81-110.

[2] M. Carriero, A. Leaci, F. Tomarelli, *A candidate local minimizer of Blake and Zisserman functional*, J. Math. Pures Appl., 96 (2011), 58-87.

[3] M.Carriero, A. Leaci, F. Tomarelli, *Image inpainting via variational approximation of a Dirichlet problem with free discontinuity*, Adv. Calc.Var. 7 (3), 267-295 (2014).

### On uniform strict minima for vector-valued functions

*Leśniewski Krzysztof* (Warsaw University of Technology)

Co-author: Ewa Bednaczuk

This talk is based on a joint work with Ewa Bednarczuk.
For scalar-valued functions the existence of sharp local minima are crucial for a number of issues e.g. stability of optimization problems convergence of algorithms metric regularity of subdifferential. Namely for a real-valued function $f$ defined on normed space $X$ a local minimum $x_0$ is sharp of order $m \geq 1$ if there exist a positive constant $\kappa > 0$ and a neighbourhood $V$ of $x_0$ such that

$$f(x) \geq f(x_0) + \kappa \|x - x_0\|^m \quad \text{for all } x \in V.$$

In the case of vector-valued functions the counterpart of sharp local minima (called strict local minima) has been introduced in [3] and exploited in [1] to study stability of vector optimization problems. We introduce the concept of uniform strict minima (local and global) of order $m \geq 1$ for vector-valued functions. We study their continuity properties and we exploit the concept of uniform strict minima to investigate metric subregularity of subdifferentials in the case of $K-$convex mappings. We study metric subregularity of two subdifferentials. We show that the subdifferential for vector-valued $K-$convex functions as defined in [4] is metrically subregular at $(\bar{x}\bar{a}) \in \text{gph}\partial f$ if

$$f_{\bar{a}}(x) \notin f_{\bar{a}}(\bar{x}) + cd^2(x(\partial f)^{-1}(\bar{a})\mathbb{B}_Y - K \quad \text{for } x \neq \bar{x}\ x \in V \tag{1}$$

for some neighborhood $V$ of $\bar{x}$ and constant $c > 0$. The analogous result for the subdifferential defined in [2] requires additional assumption.

**Keywords:** uniform strict minima, K-convex mappings, metric subregularity, domination property

**References:**

[1] E. Bednarczuk, Stability analysis for parametric vector optimization problems, Dissertationes Mathematicae, vol. 442, (2007), Warszawa.

[2] G. Isac, V. Postolica, The best Approximation and Optimization in Locally Convex Spaces, Aproximation and Optimization, vol. 2, (1993).

[3] B. Jiménez, Strict efficiency in Vector Optimization, Journal of Mathematical Analysis and Applications, vol.265, (2002), p. 264-284.

[4] C. Zalinescu, C. Tammer, A. Gopfert, H. Riahi, Variational Methods in Partially Ordered Spaces, Springer, 2003, New York Berlin Heidelberg.

## Regularity and non-existence results for critical singular subelliptic problems

*Loiudice Annunziata* (University of Bari, Italy)

We present regularity and non-existence results for semilinear subelliptic critical problems of the type

$$-\mathcal{L}u = \psi^{\alpha} \frac{|u|^{2^*(\alpha)-2}u}{d(\xi)^{\alpha}} \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega$$

where $\mathcal{L}$ is a sub-Laplacian on a Carnot group $\mathbb{G}$ with homogeneous dimension $Q$, $\Omega$ is an open subset of $\mathbb{G}$, $0 \in \Omega$, $0 < \alpha < 2$, $p_\alpha = 2(Q-\alpha)/(Q-2)$ is the related critical exponent, $d = d(\xi)$ is the natural gauge norm on $\mathbb{G}$ and $\psi := |Xd|$, where $X$ is the subelliptic gradient associated to $\mathcal{L}$. The variational formulation of the problem is based on subelliptic Hardy-Sobolev type inequalities. We prove non-existence results for some classes of domains of $\mathbb{G}$, by means of suitable Rellich-Pohozaev type identities on groups. Our starting point is a deep analysis of regularity of solutions, which has no analogue in the Euclidean elliptic setting, since it involves asymptotic estimates on higher layer derivatives of solutions.

**Keywords:** subelliptic critical problems, singular nonlinearities, Carnot groups

**References:**

[1] A. Loiudice, *Critical growth problems with singular nonlinearities on Carnot groups*, Nonlinear Anal., in press.

[2] A. Loiudice, *$L^p$-weak regularity and asymptotic behavior of solutions for critical equations with singular potentials on Carnot groups*, NoDEA Nonlinear Differential Equations and Applications 17 (2010), 575-589.

[3] E. Lanconelli, F. Uguzzoni, *Non-existence results for semilinear Kohn-Laplace equations in unbounded domains*, Comm. Partial Differential Equations 25 (2000), 1703-1739.

## Stability in linear optimization under perturbations of the left-hand side coefficients

*López Cerdá Marco Antonio* (Alicante University, Spain)

Co-authors: Daniilidis Aris, Goberna Miguel Ángel, Lucchetti Roberto

This talk deals with the stability of linear optimization problems with finitely many variables and an arbitrary number of constraints when only left hand side coefficients can be perturbed. The coefficients of the constraints are assumed to be continuous functions with respect to an index which ranges on certain compact Hausdorff topological space and these properties are preserved by the admissible perturbations. More in detail we analyze the continuity properties of the feasible set the optimal set and the optimal value as well as the preservation of desirable

properties (boundednesss uniqueness) of the feasible and the optimal sets under sufficiently small perturbations.

**Keywords:** linear optimization, stability theory, convex analysis

### On the Voronoi mapping

*Martínez-Legaz Juan Enrique* (Universitat Autonoma de Barcelona, Spain)

Co-authors: Goberna Miguel Ángel Goberna, Vera de Serio Virginia N.

Given an arbitrary closed set T in the Euclidean space whose elements are called sites and a particular site s; the Voronoi cell of s consists of all points closer to s than to any other site. The Voronoi mapping of s associates to each set T containing s the Voronoi cell of s with respect to T. The Voronoi cells are solution sets of linear inequality systems so they are closed convex sets. In this talk which is based on joint work with M. A. Goberna and V. N. Vera de Serio I will discuss the Voronoi inverse problem which consists in computing for a given closed convex set F containing a given site s those sets T that contain s and are such that the Voronoi cell of s with respect to T coincides with F. Set theoretical properties of the Voronoi mapping will also be discussed and explicit formulae for the inverse mapping will be provided.

### Variational and topological methods for nonlocal equations

*Molica Bisci Giovanni* (University of Reggio Calabria, Italy)

In the last years an always increasing interest has been shown towards nonlocal fractional problems (see among others the papers [1], [3], [4], [10] and the book [9]). Moving along this direction the aim of this talk is to present some results on the existence and the multiplicity of weak solutions for nonlocal fractional equations whose simple prototype is

$$\begin{cases} (-\Delta)^s u = f(xu) & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega \end{cases}$$

where $s \in (01)$ is fixed $(-\Delta)^s$ is the fractional Laplace operator $\Omega \subset \mathbb{R}^n$ $n > 2s$ is an open bounded set with continuous boundary and the nonlinearity $f$ satisfies suitable growth assumptions. At this purpose we employ variational and topological methods (see [2], [5], [6], [7], [8]).

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### Second-order characterizations of tilt and full stability in nonlinear programming under weakest qualification conditions

*Mordukhovich Boris* (Wayne State University, USA)

The importance of tilt and full stability of local minimizers has been well recognized from both theoretical and numerical perspectives of optimization and this area of research has drawn much attention in the literature especially in recent years. This talk concerns the study of these notions for classical nonlinear programs with equality and inequality constraints in finite dimensions described by twice continuously differentiable functions. Based on advanced techniques of variational analysis and generalized differentiation we derive complete pointbased second-order characterizations of tilt-stable as well as fully stable (in both Lipschitzian and Holderian senses) local minimizers entirely in terms of the initial program data under new qualification conditions which are the weakest ones for the study of these notions. We also discuss relationships of these notions with the existence of critical multipliers which are largely responsible for slow convergence of primal-dual algorithms

in numerical optimization. The talk is based on joint work with Helmut Gfrerer (Johannes Kepler University Linz Austria).

**Keywords:** nonlinear analysis, tilt and full stability, second-order characterizations, generalized differentiation

### Duality gap function in infinite dimensional Linear Programming

*Nguyen Vinh Thi* (Vietnam Academy of Science and Technology, Vietnam)

Co-authors: Do Sang Kim, Nguyen Nang Tam, Nguyen Dong Yen

In this paper we consider the concept of duality gap function in infinite dimensional linear programming. Basic properties of the function and two theorems on its behavior are obtained by using duality theorems with interior conditions. As illustrations for the results parametric versions of an example due to D. Gale and parametric linear programs on spaces of continuous functions are considered. The notions of Riemann-Stieltjes integral and function of bounded variation have been shown to be very useful for our investigations.

**Keywords:** Infinite dimensional linear programming, duality theorems, duality gap function, interior point

### Strategic gaming analysis for cement industry: a bilevel approach

*Oggioni Giorgia* (University of Brescia, Italy

Co-authors: Allevi Elisabetta, Conejo A.J., Riccardi Rossana, Ruiz Carlos

This paper investigates the equilibria reached by a number of strategic producers in the cement sector through a technological representation of the market. We present a bilevel model for each producer that characterizes its profit maximizing behavior. In the bilevel model the upper-level problem of each producer is constrained by a lower-level market clearing problem representing cement trading and whose individual objective function corresponds to social welfare. Replacing the lower level problem by its optimality condition renders a Mathematical Program with Equilibrium Constraints (MPEC). Then all strategic producers are jointly considered. Representing their interaction requires solving jointly the interrelated MPECs of all producers which results in an Equilibrium Problem with Equilibrium Constraints (EPEC). A parametric analysis on production costs and demand fluctuations has been conducted to detect the possible market trends.

**Keywords:** Cement industry, Equilibrium Problem with Equilibrium Constraints (EPEC), Mixed-Integer Linear Programming (MILP), strategic game

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## Necessary optimality conditions for an extended weak minimum in optimal control problems with Volterra-type integral equations on a variable time interval

*Osmolovskii Nikolai* (Kazimierz Pulaski University of Technology and Humanities in Radom, Poland)

Co-authors: Andrei Venediktovich Dmitruk

We study an optimal control problem with Volterra-type integral equations considered on a non-fixed time interval subject to endpoint constraints of equality and inequality type mixed state-control constraints of inequality and equality type and pure state constraints of inequality type. The main assumption is the uniform linear-positive independence of the gradients of active mixed constraints with respect to the control. We formulate first-order necessary optimality conditions for an extended weak minimum the notion of which is a natural generalization of the notion of weak minimum with account of variations of the time. The conditions obtained generalize the Euler-Lagrange equation and transversality conditions for the Lagrange problem in the classical calculus of variations with ordinary differential equations.

**Keywords:** Volterra-type equation, extended weak minimum, Euler-Lagrange equation, state constraints, mixed state-control constraints, Hamiltonian, adjoint equation, transversality conditions, local maximum principle

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## Computation of the graphical derivative of the normal-cone mapping

*Outrata Jiří* (Czech Academy of Sciences, Czech Republic)

Co-authors: Helmut Gfrerer, Boris Mordukhovich, Hector Ramírez C.

The graphical derivative of a multifunction is one of the basic notions from modern variational analysis. It was introduced by J.-P.Aubin and became a cornerstone of many important developments. The graphical derivative of the normal-cone mapping plays a crucial role in local analysis of solution maps associated with a class of variational inequalities/generalized equations. It may be used for instance both in the derivation of optimality/stationarity conditions as well as in various stability issues. In this contribution we consider normal cones to sets $\Gamma$ that can be expressed as pre-images of a convex cone $\Theta$ in a $C^2$-mapping and distinguish between the following two situations:

(i) $\Theta = \mathbb{R}^s_-$ so that we have to do with an inequality system and

(ii) $\Theta$ is a general nonpolyhedral closed convex cone.

As prominent cones in (ii) one could mention the Lorentz (ice cream) cone and the semi-definite programming (SDP) cone. Concerning the case (i) we will verify the formulas for the graphical derivative $DN_\Gamma$ arising in [5] and [4] under substantially relaxed qualification conditions ([1]). For the case of general conic constraints we impose like in [2] the reducibility and nondegeneracy assumptions but remove the restrictive requirement on the convexity of $\Gamma$. Further we specify a general local condition on the curvature of $\Theta$ under which $DN_\Gamma$ admits a simple expression in terms of the respective critical cone ([3]). This condition can be ensured e.g. via the extended polyhedricity. These results enable us among other things to test the strong metric subregularity of a class of generalized equations with constraint sets of the form of $\Gamma$ and can be used within the so-called implicit programming approach to the numerical solution of a class of mathematical programs with equilibrium constraints.

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### Numerical methods for hemivariational inequalities modelling a delamination problem

*Ovcharova Nina* (Universität der Bundeswehr München, Germany)

We present efficient numerical models for nonconvex nonsmooth variational problems with applications to contact mechanics. As a model we consider a delamination problem. The variational formulation of this problem leads to hemivariational inequality with a nonsmooth functional defined on the contact boundary. The problem is first regularized and then discretized either by FEM or BEM. Directly approach by applying numerical methods of nondifferentiable optimization is also considered.

**Keywords:** hemivariational inequality, delamination problem, regularization, FEM, BEM

### Global Continuity of Solutions to Quasilinear Equations with Morrey Data

*Palagachev Dian* (Politecnico di Bari, Italy)

We will present some recent results which regard global regularity of the weak solutions to Dirichlet problem for quasilinear operators whose prototype is the *m-Laplacian* operator. Precisely we consider the problem

$$\begin{cases} u \in W_0^{1m}(\Omega) \\ \text{div } \big(\mathbf{a}(xuDu)\big) = b(xuDu) \quad \text{weakly in } \Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ $n \geq 2$ is a bounded domain with generally non-smooth boundary $m \in (1n]$ and $\mathbf{a}\colon \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ and $b\colon \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ are Carathéodory maps. We will assume *corcivity* of the differential operator studied together with *controlled growths* of $\mathbf{a}$ and $b$ with respect to $u$ and $|Du|$. As for the $x$-behaviour

of the nonlinear terms it is controlled by means of measurable functions belonging to suitable *Morrey spaces.* Under very general assumptions on the non-smooth boundary of $\Omega$ expressed in terms of *variational $P$-capacity* with $P \in (1m)$ global boundedness and Hölder continuity of the weak solutions will be discusses.

**Keywords:** Quasilinear coercive operators; m-Laplacian; Regularity; Controlled growths; Natural growths; Variational capacity; Morrey space

### Nonlinear mixed variational-like inequality under weakly relaxed $\eta - \alpha$ monotonicity extended to equilibrium problem in Banach Spaces

*Pany Gayatri* (IIT Bhubaneswar, India)

Co-author: Sabyasachi Pany

In this paper nonlinear mixed variational-like inequality under weakly relaxed $\eta-\alpha$ monotonicity is studied in Banach space. Existence of the solution to the problem is established using KKM technique. An iterative algorithm is proposed using auxiliary principle technique which involves formulation of an auxiliary minimizing problem and then characterizing it by an auxiliary variational inequality. Solvability of the auxiliary variational inequality problem is established and convergence of the iterates to the exact solution is proved. Results obtained are extended to corresponding equilibrium problem.

**Keywords:** weakly relaxed $\eta-\alpha$ monotonicity, KKM technique, auxiliary principle technique, iterative algorithm, equilibrium problem

### Convex integral functionals of stochastic processes

*Pennanen Teemu* (King's College London, UK)

Co-author: Ari-Pekka Perkkio

We give dual representations for convex integral functionals on the linear space of regular processes. This space turns out to be a Banach space containing many more familiar classes of stochastic processes and its dual is identified with the space of optional Radon measures with essentially bounded variation. Combined with classical Banach space techniques our results allow for systematic treatment of a large class of optimization problems from optimal stopping to singular stochastic control and financial mathematics. This is joint work with Ari-Pekka Perkkio.

**Variational analysis of integral functionals**

*Penot Jean-Paul* (Sorbonne Universités, Université Pierre et Marie Curie, France)

Integral functionals form an important class of functionals in analysis. They play an important role in integral equations and they lay at the foundations of the calculus of variations. A major challenge consists in finding criteria in order to describe the subdifferential of the integral functional in terms of the subdifferential of the integrand. Some inclusions are available under various assumptions. In order to obtain equalities and characterizations it is sensible to use a notion of regularity. Here the word "regularity" is taken in the sense that two subdifferentials coincide. The concepts used by F.H. Clarke [C], R.T. Rockafellar [R2 R3] and J.-P. Penot [P1] (under the terms of "regularity" "protodifferentiability" and "softness" respectively) are examples of such appearances. They can be completed by a number of different notions. For especial classes of functions such regularity properties are automatic; that is the case for convex or approximately convex functions and for functions of class $C^1$. A link with duality can be pointed out. It uses the concept of generalized Legendre function. Such results can be seen as extensions of the pioneering studies of convex integral functionals made by R.T. Rockafellar [R1] and more recently by R.T. Rockafellar and P. Wolenski.

**Keywords:** integral functional, regularity, subdifferential, variational analysis

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### The set-valued H-subdifferential in the Heisenberg group

*Pini Rita* (University of Milan-Bicocca, Italy)

In this talk we present some recent extensions of results of Convex Analysis to the sub-Riemannian setting of the Heisenberg group $\mathbb{H}$. We show that like in the classical case the notion of horizontal subdifferential (H-subdifferential) which takes into account the horizontal structure of $\mathbb{H}$ turns out to be a key tool in order to recover some known results in this different setting: among others the characterization of H-convexity the Rockafellar integration theorem and the Minty theorem.

### A new neighboring optimal guidance for aerospace vehicles

*Pontani Mauro* (University "Sapienza" Rome, Italy)

This research is focused on the general-purpose perturbative guidance algorithm termed variable-time-domain neighboring optimal guidance which is capable of driving a generic dynamical system such as a ship or an aerospace vehicle along a specified nominal optimal path. This goal is achieved by minimizing the second differential of the objective function (related to fuel consumption) along the perturbed trajectory. This minimization principle leads to deriving all the corrective maneuvers in the context of an iterative feedback guidance scheme. Original analytical developments based on optimal control theory and adoption of a variable time domain constitute the theoretical foundation for three relevant features that characterize the guidance algorithm proposed in this work: (a) a new efficient law for the real-time update of the time of flight (b) a new effective termination criterion and (c) a new formulation of the sweep method. This closed-loop perturbative guidance is rather general avoids the usual numerical difficulties related to the occurrence of singularities for the gain matrices and is exempt from the main disadvantages of similar algorithms proposed in the past. For illustrative purposes the guidance algorithm at hand is first applied to the classical Zermelo's problem which involves

a ship that crosses a position-dependent current in minimum time. Then two special aerospace maneuvers are considered: (i) the minimum-time-to-climb path of a commercial aircraft and (ii) minimum-time lunar ascent and descent trajectories. In the first application trajectory deviations due either to atmospheric density fluctuations or to displacements from the nominal initial conditions or arising from the imperfect knowledge of the instantaneous thrust are included in the dynamical simulations. For lunar paths the nominal trajectories are two-dimensional while the corresponding perturbed paths are three-dimensional. Also in this case trajectory deviations related to propulsion perturbations or to displacements from the nominal initial conditions are taken into account. Extensive Monte Carlo tests are performed and unequivocally prove the effectiveness and remarkable accuracy of the variable-time-domain neighboring optimal guidance algorithm for all of the applications of interest.

**Keywords:** Perturbative guidance, neighboring optimal guidance, second-order conditions for optimality, optimal space trajectories

### Mathematical and numerical challenges in modelling the cardiovascular system

*Quarteroni Alfio* (Ecole Polytechnique Fédérale de Lausanne, Switzerland)

To face the complexity of the numerical simulation of cardiovascular flows, several reduction strategies can be adopted at the modeling level. We review some recent results on the geometric multiscale model and address new methods based on the reduced basis paradigm for the solution of parametrized problems and in the context of optimal control problems. A few examples of clinical interest will be illustrated.

### On some regularity properties for the derivatives of local minimizers of variational integrals

*Ragusa Maria Alessandra* (University of Catania, Italy)

Let us show some regularity properties for the derivatives of local minimizers of variational integrals of the form

$$\mathcal{A}(u,\Omega) \;=\; \int_{\Omega} F(x,u,Du)dx$$

where $\Omega$ is a domain in $R^m$ and the integrand has the following shape

$$F(x,u,Du) \;=\; A(x,u,\, g(x)\, h(u)\, Du\, Du).$$

Is not assumed the continuity of $A$ and $g$ with respect to $x$. We suppose that $A(\cdot, u, t)/(1+t)$ and $g(\cdot)$ are in the class $L^\infty \cap VMO$.

A "local minimizer" of the functional $\mathcal{A}$ is a function $u \in W^{1,p}_{loc}(\Omega, R^n)$ which satisfies

$$\mathcal{A}(u;\, supp\, \varphi) \leq \mathcal{A}(u+\varphi;\, supp\, \varphi)$$

for every $\varphi \in W^{1,p}_0(\Omega, R^n)$.

**Keywords:** Primary 35R05, Secondary 35B65, 35N10, 46E30, 35J50.

### A general nonconvex multiduality principle

*Riccardi Rossana* (University of Brescia, Italy)

Co-authors: Bonenti Francesca, Martínez-Legaz Juan Enrique

In this work we achieve a multiduality result for nonconvex problems. This leads to an optimality condition which is both sufficient and necessary without any convexity assumption. It also allows us to unify some duality principles including a new result which generalizes Toland-Singer duality and another new duality property of perspective functions.

**Keywords:** Nonconvex optimization, Multiduality, Toland-Singer duality.

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### Convex hull-like property and supported images of open sets

*Ricceri Biagio* (University of Catania, Italy)

This lecture will be focused on the following result:

THEOREM. *Let $\Omega \subset \mathbf{R}^n$ be a bounded open set and let $f : \Omega \to \mathbf{R}^n$ be a continuous function.*

*Then at least one of the following assertions holds:*
*(a) For every continuous and quasi-convex function $\psi : \mathbf{R}^n \to \mathbf{R}$ there exists $\tilde{x} \in \partial\Omega$ such that*

$$\limsup_{x \to \tilde{x}} \psi(f(x)) = \sup_{x \in \Omega} \psi(f(x)) \ .$$

*(b) There exists a non-empty open set $X \subseteq \Omega$ with $\overline{X} \subseteq \Omega$ satisfying the following property: for every continuous function $g : \Omega \to Y$ there exists $\tilde{\lambda} \geq 0$ such that for each $\lambda > \tilde{\lambda}$ the set $(g + \lambda f)(X)$ is supported at one of its points.*

Here is a consequence: if $n = 2$ and $h : \Omega \to \mathbf{R}$ is a non-negative function for each $u \in C^2(\Omega) \cap C^1(\overline{\Omega})$ satisfying in $\Omega$ the Monge-Ampère equation

$$u_{xx}u_{yy} - u_{xy}^2 = h$$

one has

$$\nabla u(\Omega) \subseteq \mathrm{conv}(\nabla u(\partial\Omega)) \ .$$

### Affine variational inequalities based on cones

*Robinson Stephen* (University of Wisconsin-Madison, USA)

This lecture will summarize several results obtained in recent years about affine variational inequalities (AVI) based on polyhedral convex cones. We will suggest reformulation of these AVI as operator equations with independent variables in the graph of the normal-cone operator of the cone in question and develop a basic regularity condition for good behavior. We will then present various properties of these AVI all of which employ this condition. These properties cover areas such as duality reducibility and inversion among others as well as the usefulness of these AVI in approximating nonlinear variational inequalities for purposes such as computation (Newton's method).

**Keywords:** affine variational inequalities, regularity

### Stochastic variational inequalities in a dynamical framework

*Rockafellar Tyrrell R.* (University of Washington - SeattleStock, USA)

Most of the research on stochastic variational inequalities has concentrated on models in which information about the uncertain future is revealed only once. Such models are inadequate to cover multistage stochastic programming where information comes in stages that offer repeated opportunities for recourse decisions. That feature can be brought into stochastic variational inequalities by adapting them to a constraint of nonanticipativity. In that way not only stochastic programming but multistage multiagent games with uncertainty can be covered.

**Keywords:** stochastic variational inequalities, nonanticipativity

### How to make the most out of evolving information: function identification using epi-spline technology

*Royset Johannes* (Naval Postgraduate School, CA, USA)

Analysts in every field face the challenge of how to best use available data to estimate performance quantify uncertainty and predict the future. The data is almost never "just right" but rather scarce excessive corrupted uncertain evolving and incomplete. External information derived from experiences established laws and physical restrictions offer opportunities to remedy the situation and should be utilized. We address these problems within a framework that identifies a function that according to some criterion best represents the given data set and satisfies constraints derived from the data as well as external information. Epi-splines provide the linchpin that allows us to handle shape restrictions information growth and approximations. The framework is illustrated with applications from the areas of energy queueing models dynamical systems surface reconstruction and financial engineering.

**Keywords:** Epi-splines, infinite-dimensional optimization

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### Perturbation and regularization of linear-quadratic optimal control problems with bang-bang solutions

*Schneider Christopher* (University of Jena, Germany)

Co-authors: Alt Walter, Seydenschwanz Martin

We analyze the behavior of solutions for a class of convex linear-quadratic optimal control problems with control appearing linearly depending on parameters and a regularization term. Assuming that the optimal control of the reference problem is of bang-bang type we show that the solutions are calm functions with respect to the perturbation and regularization parameters. The result is applied to $L^1$-regularization and implicit Euler discretization. Numerical experiments confirm the theoretical findings and demonstrate the usefulness of implicit methods and regularization.

**Keywords:** bang-bang control, perturbation, regularization, $L^1$-optimization, Euler discretization

### Regulation of pollution in variational inequality models

*Scrimali Laura* (University of Catania, Italy)

Academic corporate and political interest in sustainable economies has increased considerably in recent years. This can also be seen by the number of papers published. The purpose of this paper is to offer a conceptual framework to deal with some pollution control problems. First we present both a noncooperative and cooperative game framework for modelling and analysing the evolutionary pollution control problem. Specifically we examine the situation in which different countries aim to determine the optimal investment allocation in environmental projects and the tolerable pollutant emissions so as to maximize their welfare. We state the equilibrium conditions underlying the model and provide an equivalent formulation in terms of an evolutionary variational or quasivariational inequality. Then by means of infinite dimensional duality tools we prove the existence of Lagrange multipliers which play a fundamental role in order to describe countries' decision-making processes. We also discuss the existence of solutions. Finally we discuss a sustainable supply chain coalitional game. The supply chain network structure consists of three layers of decision-makers (providers manufacturers and retailers) in a duopolistic market in the case when prices and shipments evolve in time. We establish an evolutionary variational inequality formulation of the coalitional game and study the existence of solutions.

**Keywords:** Variational inequality, duality theory, pollution

### $r_L$–density and maximal monotonicity

*Simons Stephen* (University of California, Santa Barbara, USA)

$r_L$–density is a concept that can be applied to subsets of $E \times E^*$ where $E$ is a nonzero real Banach space. Every closed $r_L$–dense monotone set is maximally monotone but there exist maximally monotone sets that are not $r_L$–dense. The graph of the subdifferential of a proper convex lower semicontinuous function on $E$ is $r_L$–dense. The graphs of certain subdifferentials of certain nonconvex functions are also $r_L$–dense. (This follows from joint work with Xianfu Wang.) The closed monotone and $r_L$–dense sets have a number of very desirable properties including a sum theorem under both natural and unnatural constraint conditions so $r_L$–density satisfies the ideal calculus rules. We also give a generalization of the Brezis–Browder theorem on linear relations.

**Keywords:** Monotone set, Subdifferential, Sum theorems, Brezis-Browder theorem

## An $\eta$-approximation method for multiobjective semi-infinite programming problem

*Singh Yadvendra* (Banaras Hindu University, India)

Co-author: Shashi Kant Mishra

In this paper we consider a mutiobjective semi-infinite programming problem involving differentiable invex functions. We constructed an $\eta$-approximated mutiobjective semi-infinite programming problem associated with the original problem. The equivalence between original problem and its associated $\eta$-approximated mutiobjective semi-infinite programming problem is established. In addition we propose Mond-Weir type $\eta$-approximated dual mutiobjective semi-infinite programming problem and establish weak strong and strict converse duality theorems related to primal and dual problem. Examples are given to illustrate the obtained results.

**Keywords:** semi-infinite multiobjective programming problem, invexity, optimality

**References:**

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### An epiperimetric inequality for the thin obstacle problem

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I will discuss some recent results on the thin obstacle problem (also known as the scalar Signori problem). This is a classical problem in the calculus of variations and, in its simplest form, it consists in minimizing the Dirichlet energy among all functions that are constrained to have a positive trace on a hyperplane.

Despite the long tradition and the great effort of several authors in the last years, many questions concerning the regularity of the solutions remain still open. In this talk, our focus is on the rate of converge of the solutions to their unique blowup limits near certain points of the free boundary. Following the pioneering work by G. Weiss on the classical obstacle problem, we prove an "epiperimetric inequality" a la Reifenberg, which closely resembles the one firstly shown in the '60s in the study of the regularity of minimal surfaces.

### Minimax theorems for $\Phi-$convex functions with the intersection property

*Syga Monika* (Warsaw University of Technology, Poland)

$\Phi-$convexity a form of abstract convexity was first introduced by Ky Fan [1] and next investigated by Pallaschke and Rolewicz [4] Rubinov [5] Singer [7] and many other authors. $\Phi-$convexity has a number of important applications e.g. in nonsmooth analysis (Rubinov [5]) in mathematical economics (Rubinov and Iofee [3]) and in optimal transportation problems (Rüschendorf [6]). The present talk is

devoted to minimax theorems for functions which are $\Phi-$convex with respect to at least one variable. We introduce the concept of the intersection property. This property is proved to be a sufficient as well as a necessary condition for the minimax equality to hold for $\Phi-$convex functions. We define also jointly convexlike functions for which the intersection property can be simplified siginificantly. We also present a number of examples showing among others that the intersection property allows us to obtain minimax equality for functions with the level sets which are not necessarily connected.

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### A unified approach to uncertain optimization


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Co-authors: Klamroth Kathrin, Köbis Elisabeth, Schöbel Anita


Most optimization problems involve uncertain data due to measurement errors unknown future developments and modeling approximations. For companies these uncertainties could be future demands that have to be predicted in order to adapt the production process. In risk theory assets are naturally affected by uncertainty due to market changes changing preferences of customers and unforeseeable events. Consequently it is highly important to introduce uncertain parameters to optimization problems. Different approaches regarding uncertain optimization problems have been concentrated on in the literature. First stochastic optimization assumes

that the uncertain parameter is probabilistic. The second approach is called robust optimization which expects the uncertain parameter to belong to a set that is known prior to solving the optimization problem. The focus lies on looking at the worst case hence no probability distribution is needed. Other approaches to deal with uncertainty concern online optimization and a posteriori approaches uncluding parametric optimization. In this talk we consider scalar uncertain optimization problems. We show that it is possible to apply methods from vector optimization in general spaces set-valued optimization and scalarization techniques for developing a unified characterization of different concepts of robustness and stochastic optimization also for the case of infinite uncertainty sets. Furthermore we use our interpretations as vector and set-valued optimization problems as well as our expression via nonlinear scalarization in order to derive new concepts of robustness.

**Keywords:** Robust Optimization, Nonlinear Scalarization, Vector Optimization, Set-valued Optimization, Stochastic Optimization

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### On extended versions of Dancs-Hegedus-Medvegyev's fixed point theorem

*Théra Michel* (University of Limoges, France)

Co-author: Truong Q. Bao

This presentation is based on a recent joint work with Truong Q. Bao. It will provide an extension of Dancs-Hegedus-Medvegyev's fixed point theorem which not only unify several recent generalized versions of this theorem due to Khanh and Quy, the preorder principles established by Qiu, and the results obtained by Bao et al, but also further extend them to the setting of quasi-metric space. This feature allows us to obtain new applications to Ekeland's variational principle and Caristi's fixed point theorem.

### Some types of sets which are not sequentially normally compact

*Truong Vu Xuan* (Vietnam Academy of Science and Technology)

Co-author: Yen Nguyen Dong

Sequential normal compactness (SNC for short) is a crucial concept in Variational Analysis. This paper investigates some specific cases in which a set does not have the SNC property. We show that the homeomorphic image of an open set in a finite dimensional normed space via a Fréchet differentiable mapping into an infinite dimensional Banach space is non-SNC at any point satisfying a regularity assumption. An analogue of the result is valid for closed sets and nondifferentiable mappings. In addition a detailed analysis of the pathological set suggested by J.M. Borwein, S. Fitzpatrick and R. Girgensohn (2003) which has a non-closed Mordukhovich normal cone is proposed. The difference between the Bouligand-Severi tangent cone and the weak Bouligand-Severi tangent cone is also considered.

**Keywords:** Mordukhovich normal cone, sequential normal compactness, differentiable mapping homeomorphism, closed set, Bouligand derivative, contingent cone

### Bi-metric Hoelder regularity and stability of bang-bang optimal control

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First we argue that in some cases it is reasonable to modify the usual definition of metric regularity (MR) by considering two metrics in the image space: the weaker one for the Lipschitz-like property in the definition of MR the stronger - for defining the neighbourhoods in which the Lipschitz property holds. A Lyusternik-Graves-type theorem is valid for this modification. Then the bi-MR is extended to the Hoelder (instead of Lipschitz) case and some general results are presented. The theory is applied to optimal control problems for linear systems where the concept of bi-metric Hoelder regularity turns out to be a relevant tool for investigation of stability and approximations. A number of results of this kind will be presented.

### Fixed point iterative schemes for variational inequality problems

*Vetro Calogero* (University of Palermo, Italy)

Co-author: Toscano Elisa

In a wide class of evolutionary processes, the problem of computing the solutions of an initial value problem is encountered. Here, we consider projected dynamical

systems in the sense of [2] and references therein. Precisely, a projected dynamical system is an operator which solves the initial value problem:

$$\frac{dx(t)}{dt} = \Pi_{\mathbb{K}}\left(x(t), -F(x(t))\right), \quad x(0) = x_0 \in \mathbb{K},\, t \in [0, +\infty[, \tag{P}$$

where $\mathbb{K}$ is a convex polyhedral set in $\mathbb{R}^n$, $F : \mathbb{K} \to \mathbb{R}^n$ and $\Pi_{\mathbb{K}} : \mathbb{R} \times \mathbb{K} \to \mathbb{R}^n$ is given as follows $\Pi_{\mathbb{K}}(x, -F(x)) = \lim_{t \to 0^+} \frac{P_{\mathbb{K}}(x - tF(x)) - x}{t}$, by using the directional derivative in the sense of Gâteaux of the projection operator $P_{\mathbb{K}} : \mathbb{R}^n \to \mathbb{K}$.

The set of critical points of (P) coincides with the set of solutions of a variational inequality problem (VIP) in the sense of [5]. In view of this equivalence, established in [3], we study the extension and application of fixed point iterative schemes (see [1], [4]) to a (VIP), via admissible perturbations of projection operators in real Hilbert spaces. As a sample model, we prove convergence theorems for Krasnoselskij-type fixed point iterative schemes.

**Keywords:** Hilbert space, Krasnoselskij-type iterative scheme, Projected dynamical system, Projection operator, Variational inequality problem

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