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SIEVE OF ERATOSTHENES TO FIND NEW NUMBERS

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Abstract. Why is Math so hard for some children? We want to start from the vision and the teorethical dissertation in [16], Supported by educational and philosophical theories that here are presented, to use the concept of reification. A new element in *contemporary reason* is therefore the *re-emergence of the subject*, the reconsideration of the distinction between objective and subjective, between that which belongs to the subject and that which belongs to the object. We want to submit an experience of study with 10-year old children, at primary school. The aim is to help children in forming the abstract concept of prime numbers and to use this concept for build other concepts.

Key words. Objectivization and subjectivization, sieve of Eratosthenes.

Mathematics Subject Classification: Primary 97A30, 97C30; Secondary 97F30.

1 Mathematical concept development

In [16], the author outlines two main perspectives that influence the formation of mathematical concepts: procedural and structural perspectives. Historically, the formation of a mathematical concept took place according to different stages: the preconceptual, the operational and the structural stage. The preconceptual stage: mathematicians were getting used to certain operations on the already known concepts; the operational stage, during which a new kind of concept begun to emerge out of the familiar processes; the structural phase: the concept in question has eventually been recognized as a fully-fledged mathematical object

To sum up, the history of numbers was presented as a long chain of transitions from operational to structural conceptions: again and again, processes performed on already accepted abstract objects have been converted into compact wholes, or *reified* (from the Latin word *res – a* thing), to become a new kind of self-contained static constructs. Abstract notions can be conceived in two fundamentally different ways: *structurally* - as objects, and *operationally* - as processes.

Sfard individualise three stages in concept development *interiorization, condensation* and *reification*. At the stage of interiorization a learner gets acquainted with the processes which will eventually give rise to a new concept. These processes are operations performed on lower-level mathematical objects. Gradually, the learner becomes skilled at performing these processes.

The phase of condensation is a period of "squeezing" lengthy sequences of operations into more manageable units. At this stage a person becomes more and more capable of thinking about a given process as a whole, without feeling an urge to go into details. This is the point at which a new concept is "officially" born.
The condensation phase lasts as long as a new entity remains tightly connected to a certain process. Only when a person becomes capable of conceiving the notion as a fully-fledged object, we shall say that the concept has been reified. *Reification,* therefore, is defined as an ontological shift, a sudden ability to see something familiar in a totally new light. Reification is an instantaneous quantum leap: a process solidifies into object, into a static structure. Various representations of the concept become semantically unified by this abstract, purely imaginary construct. The stage of reification is the point where an interiorization of higherlevel concepts begins.
It seems that the structural approach should be regarded as the more advanced stage of concept development. I*n the process of concept formation, operational conceptions would precede the structural.* This statement is basically true whether historical development or individual learning is concerned.

2 Objectivization and subjectivization

Bacon had already stated that the error made by *ancient science* was to consider *objective knowledge* possible only by excluding the action of man from natural reality. Objective knowledge was interpreted as an *absolute view* of the world and as *reason and indisputable truth* in understanding phenomena which govern not only physical laws, but particularly a perspective view of world knowledge.
With the advent of the so-called *science of complexity* a new *thinking reasoning* originated, which places the dimensions of *logic* and *time* at the centre of *philosophical and speculative* interests*,* and particularly, has led to a different interpretation of the concept of time. Ilya Prigogine, in his interesting work "*From being to becoming"* deals with this theme*,* identifying a form of temporality which he defines *creative time*. His thinking represents a focal point in extrication from reflection on the phenomena which lead to the interpretation of the *concept of knowledge*.
Prigogine, in explaining his theory, essentially wanted to demonstrate that life does not simply consist of the execution of a *pre-determined programme*, but is defined within a recursive cycle as *creation of new*, as *invention*. We have before us a *creator time* which requires its own particular logic: the logic which comes closest to this kind of temporality is *the logic of non-linearity,* better defined as *logic of complexity*.
In this view, *contemporary scientific rationality* is a *reasoning* in which universalizing principles do not exist, neither a *globality of basic concepts* exists, nor a *universality of methods* applicable to all fields of knowledge. From this context and on these theoretical bases arises the need to think up new ways of interpreting *didactics* and the dynamics which characterize teaching theory, along with the necessity to re-qualify the cognitive and structural background on which to base a new *teaching professionalism*.
A new element in *contemporary reason* is therefore the *re-emergence of the subject*, the reconsideration of the distinction between objective and subjective, between that which belongs to the subject and that which belongs to the object. Every human being produces his own world as a result of being produced by this world (Maturana and Varela). A "subject reality" and an "object reality" do not exist; autopoiesis creates simultaneously the object and the *particular view* for which that object acquires, at that moment, a specific personal meaning.
From Piaget onwards, there has been the consideration that the *biological basis of the human organism* is the condition from which the *intelligent processes* flower and, at the same time, the

flowering of these processes is the condition for recognizing a biological base: the tree of knowledge is the tree of life, feeding back on itself.
For Piaget, the passage from *psyche* to *logic* is the passage from *irreversible* to *reversible* and is linked to the discovery of reversibility: it is the same type of relationship that Piaget establishes between *biological structure* and *stages of intelligence*. Using the model of Piaget's theory it is possible to affirm that *cognitive strategies* are the capacity for *self-regulation* (intellectual ability) through which the mind faces the environment. The pillar of this conception is found in the passage from *intellectual ability* to *cognitive strategies*: it is possible to solve a problem for which we do not know *objective rules* for solving it, because there is the possibility for the mind to feed back infinitely upon itself, transforming and intersecting the very rules that have formed it. In brief, while intellectual abilities are learned in situations of objectivization of the teaching-learning processes and are translated into objectively verifiable applications, the possibility of learning *cognitive strategies* is linked to a nonlinear logic. These are taught through nonlinear didactics (designed to favour autoreferentiality) in a field of subjectivization of educative processes.

3 Our experience and results

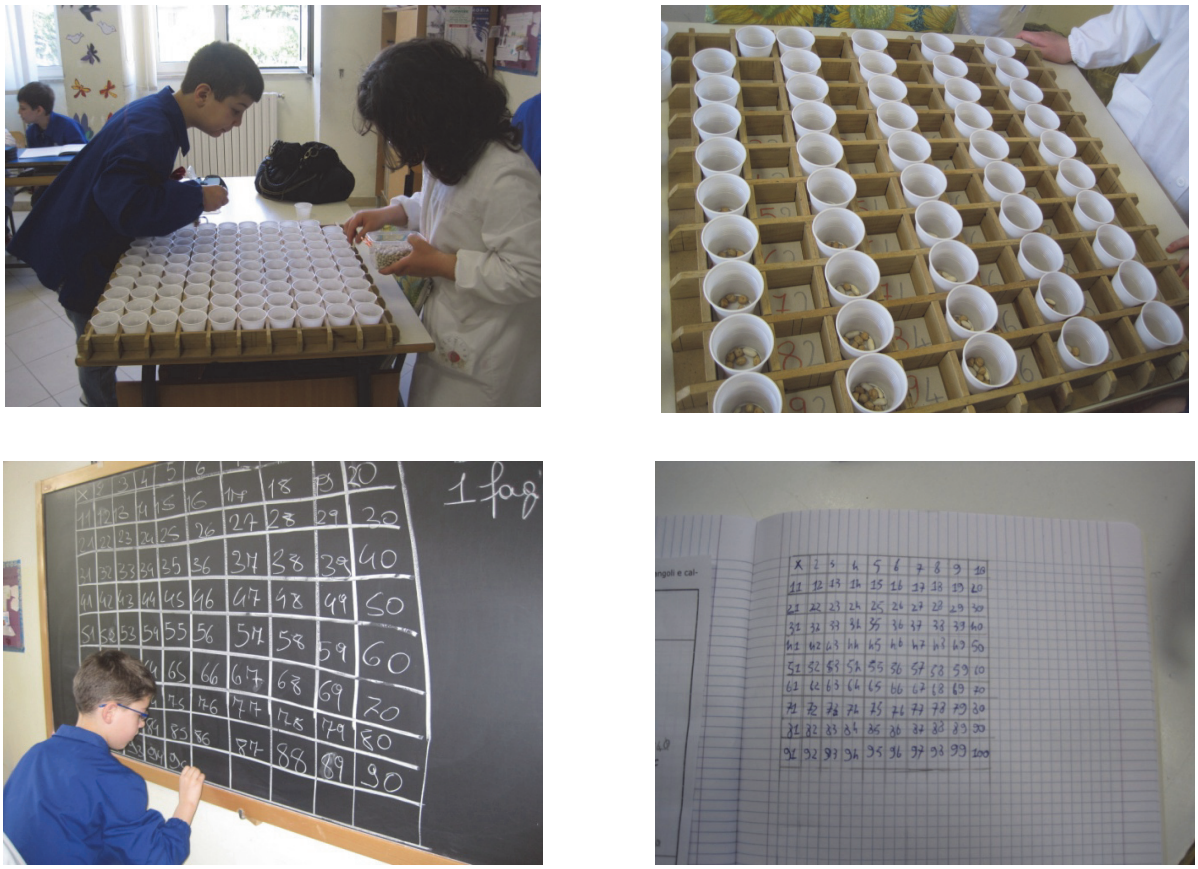
We conducted an experimental study with 49 10-year old children, at primary school. The aim was to help children in forming the abstract concept of prime numbers. The children have also improved their knowledge on concepts already known but not yet well established as: divisor, multiple, divisibility rules, and so on. The teacher turned several questions to the children:

The sieve in class.

- ✓ What is it?
- ✓ What is its utility?
- ✓ In your opinion what are the "natural sieves"?
- ✓ What can we do with a sieve in mathematics?
- ✓ Why do we call Sieve of Eratosthenes a sieve in mathematics?

The teacher asked them if they had already heard of Eratosthenes; All said they did not know who he was, so she lead to reflect on the fact that they had heard of the scientist during the history lesson; they studied the history of Egypt and its important library.

It reads a difficulty by children in considering the same subject in the context of different disciplines.Have all numbers the same properties?
The children built the Sieve of Eratosthenes by using chickpeas, peas and beans.



Some stages of the experience

Then the teacher asked them which means a number is a multiple of another number and she called 10 children with the names „one“, „two“, ..., „ten“ and other two children with the names „unità“ („units“) and „decina“ („tens“).
The teacher said to wait to „one“ child, while the „two“ child had to delete on the sieve all all numbers divisible by two; the “three” child had to delete on the sieve all all numbers divisible by three and so on. Before these operations, they had to remember the respective divisibility rules.
We can say that this first part of our experience corresponds to the *interiorization stage*. A learner gets acquainted with the processes which will eventually give rise to a new concept. These processes are operations performed on lower-level mathematical objects.
Subsequently, the teacher asked the children to control what the numbers were left on the sieve. He said them to write the numbers and to call those “prime numbers”. Gradually, the learner becomes skilled at performing these processes. So, This is the phase of condensation: the period of "squeezing" lengthy sequences of operations into more manageable units. At this stage children become more and more capable of thinking about a given process as a whole, without feeling an urge to go into details. This is the point at which a new concept is "officially" born.

Finally the children calculated primes up to 600. Children become capable of conceiving the notion as a fully-fledged object, so we We reached the stage of *Reification,* and it is possible to use this new concept for new procedures.

References

[1.] G. ACONE, *L'identità teorica della pedagogia tra complessità e postmodernità,* Pedagogia e vita, La Scuola, Brescia 2005.

[2.] G.T. BAGNI, *La introducción de la historia de las matemáticas en la enseñanza de los números complejos. Una investigación experimental en la educación media superior*, *Revista Latinoamericana de Investigación en Matemática Educativa*, 2001.

[3.] BROUSSEAU, *Fondaments et méthodes de la didactique des Mathématiques - Recherches en didactique des mathématiques*, 1986.

[4.] M. CAPPUCCIO, *Neurofenomenologia,* Mondatori, Milano 2006.

[5.] B. D'AMORE, *Elementi di didattica della matematica*, Pitagora, 1999;

[6.] H. MATURANA and F.J. VARELA, *Autopoiesi e cognizione,* Marsilio, Padova 1985.

[7.] H. MATURANA and F.J. VARELA, *L'albero della conoscenza*, Garzanti, Milano 1987.

[8.] G. MINICHIELLO, Ricerche sul problema della razionalità in pedagogia, Edisud, Salerno 1999.

[9.] G. MINICHIELLO, Elementi di didattica generale, Edisud, Salerno 2003.

[10.] F. PALLADINO, L. LOMBARDI, N. Palladino, *Algoritmi elementari del calcolo aritmetico e algebrico*, Pitagora editrice , Bologna, 2005.

[11.] J. PIAGET, *Biologia e conoscenza,* Einaudi, Torino 1983.

[12.] J. PIAGET and A. SZEMINSKA, *La genesi del numero nel bambino*, La Nuova Italia, Firenze, 1976.

[13.] K.R. POPPER, Conoscenza oggettiva. Un punto di vista evoluzionistico, Armando, Roma 1975.

[14.] I. PRICOGINE, Dall'essere al divenire, Einaudi, Torino 1986.

[15.] RADFORD, *On Psychology, Historical Epistemology and the Teaching on Mathematics: Towards a Socio-Cultural History of Mathematics*, For the Learning of Mathematics, 1997.

[16.] A. SFARD, *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing*, Cambridge University Press, 2008

[17.] A. SFARD, *On the duale nature of mathematical conceptions:: reflections on processes and objects as different sides of the same coin*, Educational Studies in Mathematics, Volume 22, Number 1, 1991.

[18.] VYGOTSKIJ, *Pensiero e linguaggio*, Giunti&Barbera, Firenze, 1966.

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