MR2684422 Deville, Robert; Rodrguez, José Integration in Hilbert generated Banach spaces. Israel J. Math. 177 (2010), 285306, 46Exx (46J10)

It is known that each McShane integrable function is also Pettis integrable, while the reverse implication in general is not true. The equivalence of McShane and Pettis integrability depends on the target Banach space X and has been proven: by R. A. Gordon [Illinois J. Math. 34 (1990), no. 3, 557567, 26A42 (28B15 46G10 49Q15)], and by D. H. Fremlin and J. Mendoza [Illinois J. Math. 38 (1994), no. 1, 127147, 46G10 (28B05)] if X is separable, by D. Preiss and the reviewer [Illinois J. Math. 47 (2003), no. 4, 11771187. 28B05 (26A39 26E25 46G10)] if $X = c_0(\Gamma)$ (for any set Γ) or X is super-reflexive, by the second author of the present paper [J. Math. Anal. Appl. 341 (2008), no. 1, 8090, 46G10 (28B05 46B99 47B10)] if $X = L^1(\nu)$ (for any probability measure ν).

Here the authors show that the McShane and Pettis integrability coincide for functions taking values in a subspace of a Hilbert generated Banach space. This result includes all previous known ones concerning the above mentioned equivalence. The used approach relies heavily on some special properties of the Markushevich bases of those Banach spaces.

They also give a ZFC example of a scalarly negligible function which is not McShane integrable.

Moreover they prove that, whenever the target Banach space is super-reflexive generated, the Birkhoff integrability lies strictly between Bochner and McShane integrability.

Reviewed by (*L. Di Piazza*)