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Non-uniform sediment transport estimation in non-equilibrium situations: case studies

D. Termini^{a*}

Dip. di Ing. Civ., Ambientale, Aereospaziale, dei Materiali (DICAM) – University of Palermo - Viale delle Scienze – 90128 Palermo (Italy)

Abstract

Quantitative estimate of sediment transport in alluvial channels is one of the most important task in river engineering. Even today, numerical models of sediment transport processes are confronted with some difficulties, often of conceptual nature. One of these difficulties is the simulation of non-uniform sediment transport in non-equilibrium situations, which requires the characterization of the ability of the alluvial system to immediately overcome the variations of the sediment boundary conditions. In this work a 1-D numerical model, which includes a new expression of the so-called “adaptation coefficient”, has been applied to test its capability to simulate the transient bed profiles. Specifically, the model has been applied to predict bed-level changes due to sediment overloading and sediment cut-off. The model’s application to literature study cases (used by other researchers to assess coupled models) has shown that it gives reasonable results and, thus, it appears suitable for practical applications

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1. Introduction

Human activities and/or natural processes could determine an unbalance between water and sediment discharges. Such a disturbance may lead to river-bed variations which develop at different spatial and time scales. In order to establish corrective measures, it is important to simulate both short-term and long-term river-bed variations. River-bed transient processes are of particular interest because they may produce several practical

* Corresponding author.

E-mail address: donatella.termini@unipa.it

consequences. It should be taken into account that the time scales associated with morphological processes and flow field could have or not comparable magnitudes so that slow or fast transients may occur.

Many one-dimensional (1-D) models have been developed to predict bed-level changes due to variation of sediment boundary conditions. These models solve the continuity and momentum equations for water and sediment. According with the method of solution employed, such models can be grouped into three categories: decoupled models (Thomas and Prashum, 1977; Karim and Kennedy, 1982; Chang and Hill, 1976; van Niekerk et al., 1992), where the equations for the water-flow and for sediment are solved separately in a uncoupled manner; semi-coupled models (Struiksmas et al., 1985; Shimizu and Itakura, 1989) which couple the governing equations for flow and sediment together through an iterative procedure so that, at each time step, they solve first the governing equations for water and, then, the obtained results are used to solve the equations for sediment; coupled models (Borah et al., 1982; Lyn and Godwin, 1987; Ruel et al., 1989; Saiedi, 1994; Hu and Cao 2009), where the equations for water and for sediment are solved simultaneously.

As previous researchers show (see as an example Cao et al., 2007; Termini, 2011b) the applicability of decoupled and coupled approaches depends on the ratio between the time scale of bed deformation and the time scale of flow depth.

Although the coupled solution is more stable than decoupled one, the implementation of a coupled model for non-uniform sediment transport is rather complicated (Wu et al., 2004). Furthermore coupled models do not account explicitly for the complex interplay of all factors producing the bed roughness and sediment transport changes, in time and in space.

Models could be classified also on the basis of the treatment of the near-bed water-sediment processes. This classification is determined by the difference between single-phase models, which can be used for low sediment concentration, and two-phase models, especially preferred for high sediment concentration. Two-phase models solve the conservations of mass and momentum for both phases by analyzing the forces of the fluid on sediment particles (Drew, 1983). But, these models are computationally more expensive than single-phase models.

The point is that the interpretation of near-bed processes is particularly important during transients (Barkdoll and Duan, 2008).

Cao et al. (2007) suggested that, for shallow flows with high sediment concentrations, fully coupled models are normally required. Very recently, two-phase models have been efficiently applied to solve fast transients problems (Greco et al., 2012). For deep flows at low concentrations, and/or in slow transients situations (Termini, 2011b), decoupled or semi-decoupled models are mostly justified (Saiedi, 1994; Wu et al., 2004; Vasquez et al., 2005). Also, recently, 3-D decoupled model has been used for practical cases (Jia et al., 2005) On the other side, Cui et al. (1996) verified that the performance of the two models was identical even for Froude numbers near unity and even when boundary conditions varied strongly in time.

In this work, attention is paid to transients sediment transport phenomena created in the alluvial reach under steady flow conditions when the sediment transport rate changes with time. It is taken into account that the alluvial stream is unable to immediately overcome the variation of sediment boundary conditions and a spatial distance (spatial lag or adaptation length) is required to reach the equilibrium transport capacity (Armanini and di Silvio, 1988; Ruel et al., 1989). The applicability of a 1-D semi-coupled approach (Termini, 2003, 2011a,b) to simulate the river-bed variations and transient bed profiles is verified. To account for the spatial delay of the bed-load sediment transport rate, the model has been improved by including a new expression (previously developed – see Termini, 2011b; 2012a) of the so-called adaptation coefficient. The model allows the simulation of the sediment interchange between the bed and the stream and the variation of sediment size distribution in space and in time. The two-layer bed schematization and the inclusion of friction factor in formulation of sediment transport rate, allows user to take adequately into account for the effect of different bed roughness conditions and the effect of sediment properties in the hydraulic resistance formulation. Furthermore, the model includes the separate treatment of bed-load and suspended load.

The present paper complements previous works conducted by Termini (2011; 2012a, b). The details on the derived spatial lag equation are described in Termini (2011). The sensitivity analysis of model's results to the adaptation coefficient can be found in Termini (2012a); in Termini (2012b) the model's applicability to simulate

local scouring process has been verified with the aid of experimental data appositely collected at the laboratory of DICAM (University of Palermo).

In the present work, the results by applying the model are compared with literature experimental results for sediment overloading (aggradation) and sediment cut-off (degradation), which represent rapid changes in sediment discharge at the boundaries. In particular, experimental cases which have been also efficiently simulated by other researchers using coupled models have been selected for the model's application.

Nomenclature

A	radius of
B	position of
C	further nomenclature continues down the page inside the text box
b	function
c	dimensionless Chèzy friction factor
C_k	vertically averaged concentration of suspended sediment of size class k
d_k	mean diameter of size class k
d_{50}	median sediment diameter
D_k	sediment deposition rate.
E_k	resuspension rate
F_k	fractional representation of size class k
g	gravitational acceleration
h	water depth
k	size class
K_x	longitudinal dispersion coefficient for suspended sediment
h	water depth
L	length of the spatial domain
P_k	probability of size class d_k to stay on the bed
$P_{s,k}$	probability to suspension for size class d_k
q	flow rate per unit width
$q_{sb,k}$	actual specific (per unit width) volumetric bed-load
$q^*_{sb,k}$	equilibrium specific volumetric bed-load sediment transport rate
t	time
T	time scale
u'	vertical flow velocity fluctuation
z_b	bed level referred to an horizontal plane
x	distance in flow direction
α_k	function
γ_s	specific weight of grains in the fluid
Δx	space step
Δt	time step
η_k	is the dimensionless bed shear stress
λ	sediment porosity x distance in flow direction
λ_c	reduction of c factor
ϕ_k	adaptation coefficient
σ	standard deviation of the distribution of P_k
σ_s	root square error of the vertical velocity fluctuation
τ	total bed shear stress
$\tau_{cr,k}$	critical shear stress for size class k

1. Mathematical model

The equations of solving equations are briefly described herein. More details can be found in previous works (Termini, 2003; 2011).

1.1. Hydrodynamic Equations

In unsteady condition the flow motion in a wide rectangular channel is governed by the following partial differential equations:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (1)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{q^2}{h} \right) + gh \frac{\partial h}{\partial x} + gh \frac{\partial z_b}{\partial x} + \sqrt{g} h \frac{q^2}{c h^{7/3}} = 0 \quad (2)$$

where h is the water depth, q is the flow rate per unit width, z_b is the bed level referred to an horizontal plane, t is time, x is distance in flow direction, g is the gravitational acceleration, c is the dimensionless Chèzy friction factor.

1.2. Sediment transport Equations

The bed is divided into two layers where the surface active layer has thickness δ_a . Both layers contain sediments of all size fractions. In the stream (h), the sediments are transported in suspension and the sediment motion is essentially due to turbulence and to particle fall velocity.

The sediment size distribution is schematized in N discrete k -th size classes identified by the mean diameter, d_k , of the two limiting diameters of each class.

The following equations are solved for sediment:

$$\frac{\partial q_{sb,k}}{\partial x} + \varphi_{s,k} = -(1 - \lambda) F_k \frac{\partial(z_b + \delta_a)}{\partial t} \quad (3)$$

$$\frac{\partial(C_k h)}{\partial t} + \frac{\partial(C_k q)}{\partial x} = \frac{\partial}{\partial x} \left(h K_x \frac{\partial C_k}{\partial x} \right) + \varphi_{s,k} \quad (4)$$

with:

$$\frac{\partial q_{sb,k}}{\partial x} = \phi_k (q_{sb,k}^* - q_{sb,k}) + \alpha_k \frac{\partial q_{sb,k}^*}{\partial x} \quad (5)$$

$$\varphi_{s,k} = (E_k - D_k) \quad (6)$$

$$\sum_{k=1}^N F_k = 1 \quad (7)$$

which respectively represent the conservation of sediments of k -th size class transported as bed load (Eq. (3)), the conservation of sediments of k -th size class transported in suspension (Eq. (4)), the spatial variation of local bed-load sediment transport (Eq. (5)), the exchange between bed load and suspended load for each k -th size class (Eq. (6)), the conservation of the total grain size distribution (Eq. (7)).

In Eqs. (3)–(7) λ is the sediment porosity, $q_{sb,k}$ is the actual specific (per unit width) volumetric bed-load sediment transport rate for size class k , C_k is the vertically averaged concentration of suspended sediment of size class k , K_x is the longitudinal dispersion coefficient for suspended sediment, D_k and E_k are, respectively, the sediment deposition and resuspension rates. F_k is the fractional representation of size class d_k in the active layer at the known time level, $q_{sb,k}^*$ is the equilibrium specific volumetric bed-load sediment transport rate for sediment of size class k .

According with Termini (2011b), ϕ_k is the so-called adaptation coefficient. This coefficient is estimated as:

$$\phi_k = \frac{a(t)|_x}{(q_{sb})_{x=L}} \quad (8)$$

where L is the length of spatial domain and $a(t)|_x$ depends on the median sediment diameter (d_{50}) as follows:

$$a(t)|_x = 2d_{50} e^{-t(0.0002d_{50})} \quad (9)$$

1.3. Closure equations

The following closure equations are also included:

- - to determine the total resistance factor, c , as:

$$\frac{1}{c^2} = \frac{1}{\bar{c}^2} + \frac{1}{c_A^2} \quad (10)$$

where

$$\bar{c} = 2.5 \ln \left[11 \frac{h}{k_s} \right] \quad (11)$$

($k_s=2d_{50}$ = equivalent bed roughness; d_{50} = median sediment diameter), c_A denotes the friction coefficient related to the bed-forms resistance factor (see in Yalin and da Silva, 2001 p. 50);

- - to determine the probability, $P_{s,k}$, to suspension for size class d_k . By assuming that the distribution of u' (vertical flow velocity fluctuation) near the bed surface follows the Gaussian distribution, it is:

$$P_{s,k} = \frac{1}{\sigma_s \sqrt{2\pi}} \int_{w_k}^{\infty} \exp\left(-\frac{u'^2}{2\sigma_s^2}\right) du' \quad (12)$$

where σ_s is the root square error of the vertical velocity fluctuation.

- - to determine the fractions of sediments left on the bed at the unknown time level $n+1$ as: $F_k^{n+1} = (F_k) P_k$, being P_k the probability of size class d_k to stay on the bed as:

$$P_k = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X_k} \exp\left(-\frac{X^2}{2}\right) dX \quad \text{with} \quad X_k = \frac{\zeta_k - 1}{\sigma} \quad (13)$$

where $\eta_k = \tau / \tau_{cr,k}$ is the dimensionless bed shear stress (τ =total bed shear stress; $\tau_{cr,k}$ = critical shear stress for size class k). σ is the standard deviation of the distribution, X is the variable of integration,

$$\zeta_k = \left(\frac{d_k}{d_{50}} \right)^{0.85} \quad (14)$$

is the “corrective factor” that takes into account the hiding effect;

- - to determine the equilibrium specific volumetric bed-load rate $q_{sb,k}^*$ as: $q_{sb,k}^* = s_k q_{sb,ku}^*$ where $q_{sb,ku}^*$ indicates the equilibrium specific bed-load transport rate for uniform sediment of size k and $s_k = F_k P_k \zeta_k$ is a weighting factor of the size class k in the active layer.

The system is then completed by other closure equations for variables $D_k, E_k, K_x, \lambda, q_{sb,ku}^*$, see Termini (2011) and (2012).

2. Numerical procedure and time and space steps

The algorithm solves the governing equations at each time step. Flow computation is performed first to provide the hydraulic parameters for sediment routing; then, the estimated flow conditions are used to compute the probability to be suspended and the probability to stay on the bed for each size class d_k , the new bed material size distribution, the sediment discharge and the new bed levels. Next, the new bed levels are used to adjust the flow equations.

The flow chart of the proposed procedure is reported in Fig. 1.

The time and space scales, which represent physical constraints of the spatial and temporal resolutions, need to be properly defined in order to simulate the process evolution. According to Termini (2011b), to simulate the transient bed profiles the spatial and temporal resolutions have been determined by taking into account both physical and numerical constraints of the observed processes. Termini (2011b) verified that to ensure the stability and convergence requirements in transient bed phenomena simulation, the numerical constraints should be: $0.01 \leq \Delta x / L \leq 0.10$ and $\Delta t / T \leq 0.05$ (where L is the length of the spatial domain and T is the time scale of transient process).

2.1. Boundary and initial conditions

For water computation, upstream boundary condition is given by the flow rate. The downstream boundary condition is given by the rating curve. For sediment routing, the bed-load discharge is specified at upstream boundary, for all size fractions. When also suspended load is present, the inflow suspended-load concentration (for all size fractions) must be given at upstream boundary and a zero sediment concentration gradient has to be imposed at downstream boundary. Initial conditions include the channel geometry and the sediment size distributions at each spatial step Δx .

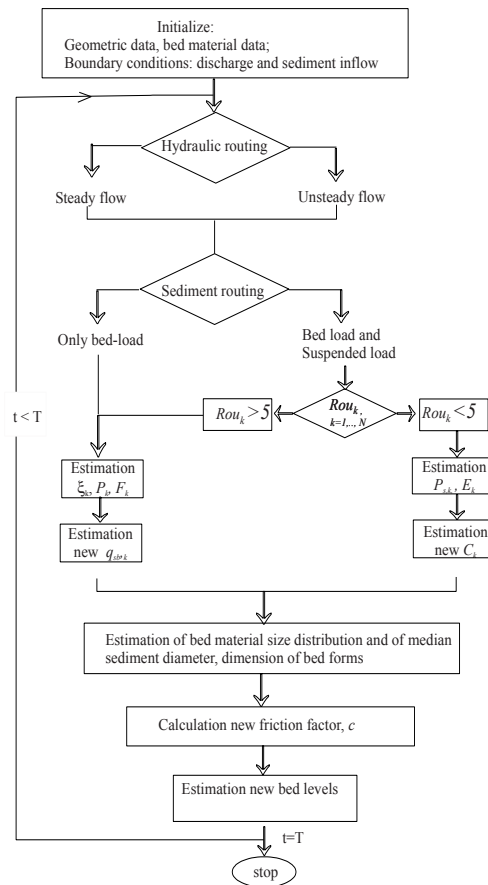


Fig. 1. Flow chart.

3. Applications

To verify the applicability of the model, the computed results are compared with literature experimental results. In particular, literature experimental results for rapid sediment overloading and sediment shut-off (respectively aggradation and degradation cases), which have been also efficiently simulated by other authors using coupled models, have been used.

3.1. Transient water and bed profiles test case: sediment overloading

For this application the flume experiment of Soni et al. (1980), which has been extensively studied in literature and used to assess the applicability of coupled models (among others Kassen and Chaudhry, 1998; Cao et al., 2002). The flume was rectangular 0.2 m wide and 30 m long. A constant flow discharge, q , and uniform flow depth, h , (in equilibrium with sediment discharge, qs) were considered. After the establishment of such conditions the sediment supply rate, Δqs , was increased at upstream end of the flume at a constant rate. The sediment was characterized by diameter of 0.32 mm. The initial conditions are listed in Table 1 (from Cao et al., 2002). In Fig. 2 the results of numerical simulation ($\Delta x=0.5$ m and $\Delta t=5$ sec) are compared with experimental data.

As Fig. 2 shows, the computed results compare well with measured ones.

Table 1. Initial conditions: aggradation test case Soni et al. (1980).

$q(m^2/s)$	$h (cm)$	$S (%)$	$\Delta qs (%)$
0.036	8.6	2.25	3.5

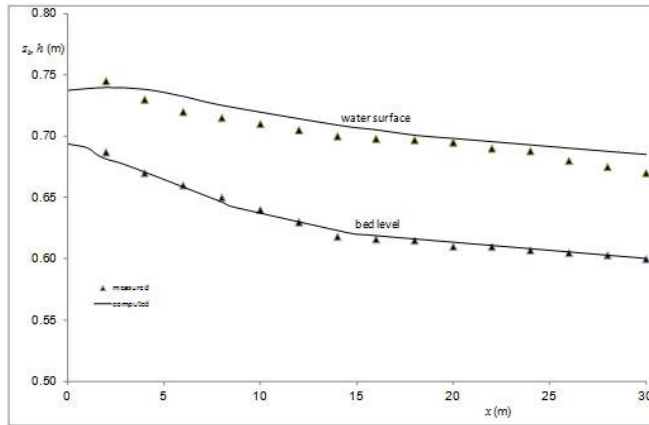


Fig. 2: Transient profiles: aggradation test case Soni et al. (1980)

3.2. Transient water and bed profiles test case: sediment shut-off

Two cases have been considered. First, experiments on bed degradation due to the shut-off of upstream sediment supply performed by Surayanatayana (1969) have been considered. In particular, data from run 24 (found in Kassen and Chaudhry, 1998; Vasquez et al., 2005) have been used in the present work. The flume was 18.29 m long and 0.61 m wide. The bed was of uniform sand with median sediment diameter of 0.45 mm. The flow discharge was $Q=0.0119 \text{ m}^3\text{s}^{-1}$, the initial longitudinal channel slope was 0.007 and the initial water depth was 0.034 m. The numerical simulation has been conducted for a spatial step of $\Delta x=0.61 \text{ m}$ and a time step $\Delta t=200 \text{ sec}$. Zero sediment inflow and a constant water discharge have been imposed as upstream boundary conditions. Fig. 3 shows the comparison between computed bed and water profiles at $t=10 \text{ hours}$. From Fig. 3 reasonable agreement between measured and computed bed and water profiles can be observed.

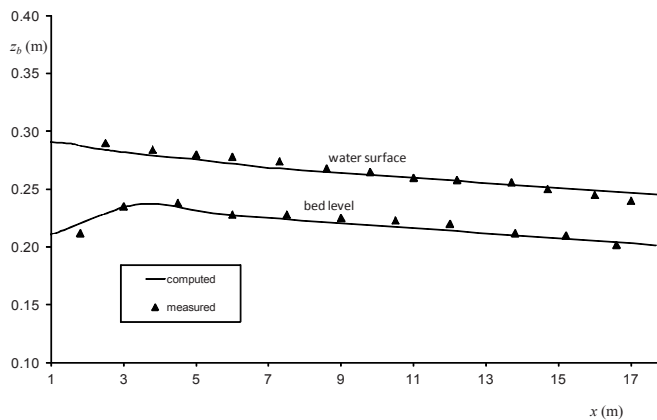


Fig. 3: Trasient profiles: degradation test case - Suryanarayana (1969)

Then, results on laboratory experiments conducted at the Water Research Laboratory (Australia) reported in Saiedi (1997), who also simulated the process applying a coupled model, have been considered. The channel was 18 m long and 0.61 wide. The sediment used was of uniform size with median diameter of 2 mm. In steady flow conditions (flow rate=0.137 m³/s, average flow depth=0.285 m and slope=0.001) the equilibrium sediment rate was cut off. The comparison between model's results (obtained for $\Delta x=0.18$ m and $\Delta t=0.2$ sec) and the measured bed profile at 75 min is reported in Fig. 4.

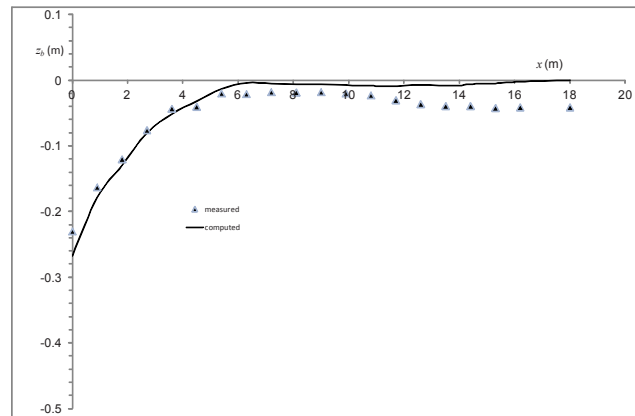


Fig.4: Trasient profiles: degradation test case - Saedi (1997)

Also in this case it can be observed that the model performs reasonably well on the considered application.

4. Conclusions

In this work a 1-D semi-coupled model is applied to simulate the river-bed variations and transient bed profiles. The model allows the simulation of the sediment interchange between the bed and the stream and the variation of sediment size distribution in space and in time. Furthermore the model includes a new expression (previously developed – see Termini, 2011; 2012) to estimate the spatial delay of the bed-load sediment transport rate. To assess the model's performance the computed results are compared with literature experimental results for sediment overloading (aggradation) and sediment cut-off (degradation), which represent rapid changes in sediment discharge at the boundaries. In particular, experimental cases, which have been also efficiently simulated by other researchers using coupled models, have been selected for the model's application. The comparisons have shown that the model performs reasonably well on all the considered applications. According with Termini (2011b), the spatial and temporal numerical resolutions, which exert an important role in simulating transients, have been selected on the basis of the space and time scales of simulated processes.

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