Application of Dual Boundary Element Method in Active Sensing

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Abstract. In this paper, a boundary element method (BEM) for the dynamic analysis of 3D solid structures with bonded piezoelectric transducers is presented. The host structure is modelled with BEM and the piezoelectric transducers are formulated using a 3D semi-analytical finite element approach. The elastodynamic analysis of the entire structure is carried out in Laplace domain and the response in time domain is obtained by inverse Laplace transform. The BEM is validated against established finite element method (FEM).

Introduction

In engineering application, structural integrity is of critical importance. In recent years, structural health monitoring (SHM) has emerged as a favoured approach in ensuring the safety of structures. Among the available instruments for SHM applications, piezoelectric transduction patches have attracted much interest for real-time in-service monitoring purposes. They have been used widely for detection of damages and impacts for both metallic and composite materials.

The development of feasible SHM techniques requires knowledge from various areas. These include, but are not limited to, structural mechanics, fracture and damage mechanics, transducer mechanisms, signal processing and optimisation theory. Among these aspects, a reliable mathematical model for predicting the behaviour of the structures under inspection would provide valuable assistance in the design process. An extensive review into the available models of smart structures bonded with piezoelectric transducers can be found in [1].

In this work, the analysis of smart structures with bonded piezoelectric transducers is achieved with BEM. The derivation of the 3D models for piezoelectric actuators and sensors takes into account the full electro-mechanical behaviour and the relevant boundary conditions. The piezoelectric models are purposely expressed in terms of the relationships between voltage and BEM variables, i.e. displacement and traction, in order to allow for direct coupling with the host structure, which is modelled with BEM. The boundary integrals are solved for a number of Laplace parameters and the corresponding time history is found by inverse Laplace transform. The sensor signals obtained from BEM and FEM simulations show promising agreement.

Model of Piezoelectricity

Consider a piezoelectric transducer, whose top and bottom surfaces are perpendicular to the electric poling direction, i.e. x_3-direction, as shown in Fig.1. By taking into account the elastic and the electric relationships, and the electro-mechanical behaviour of piezoelectricity, the following expression, which relates the properties of the top and the bottom surfaces of the piezoelectric transducer, is deduced such that [2]

$$\begin{bmatrix} \tilde{\boldsymbol{u}}_t \\ \tilde{\boldsymbol{V}}_t \\ \tilde{\boldsymbol{\sigma}}_t \\ \tilde{\boldsymbol{D}}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{uu}(h) & \boldsymbol{L}_{uV}(h) & \boldsymbol{L}_{u\sigma}(h) & \boldsymbol{L}_{uD}(h) \\ \boldsymbol{L}_{Vu}(h) & \boldsymbol{L}_{VV}(h) & \boldsymbol{L}_{V\sigma}(h) & \boldsymbol{L}_{VD}(h) \\ \boldsymbol{L}_{\sigma u}(h) & \boldsymbol{L}_{\sigma V}(h) & \boldsymbol{L}_{\sigma\sigma}(h) & \boldsymbol{L}_{\sigma D}(h) \\ \boldsymbol{L}_{Du}(h) & \boldsymbol{L}_{DV}(h) & \boldsymbol{L}_{D\sigma}(h) & \boldsymbol{L}_{DD}(h) \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{u}}_b \\ \tilde{\boldsymbol{V}}_b \\ \tilde{\boldsymbol{\sigma}}_b \\ \tilde{\boldsymbol{D}}_b \end{bmatrix} \tag{1}$$

where the subscripts t and b indicate the top and the bottom surfaces, and h is the thickness of the material. In eq (1), the vectors $\tilde{\boldsymbol{u}}$, $\tilde{\boldsymbol{V}}$, $\tilde{\boldsymbol{\sigma}}$ and $\tilde{\boldsymbol{D}}$ are the nodal values of mechanical displacements $\tilde{\boldsymbol{u}}_1$, $\tilde{\boldsymbol{u}}_2$ and $\tilde{\boldsymbol{u}}_3$, voltage in x_3-direction $\tilde{\boldsymbol{V}}_3$, mechanical stresses $\tilde{\boldsymbol{\sigma}}_{13}$, $\tilde{\boldsymbol{\sigma}}_{23}$ and $\tilde{\boldsymbol{\sigma}}_{33}$, and electric displacement in x_3-direction $\tilde{\boldsymbol{D}}_3$. Also, it is worth noting that the dynamics of the piezoelectric transducer has been included in eq (1) in terms of Laplace parameter.

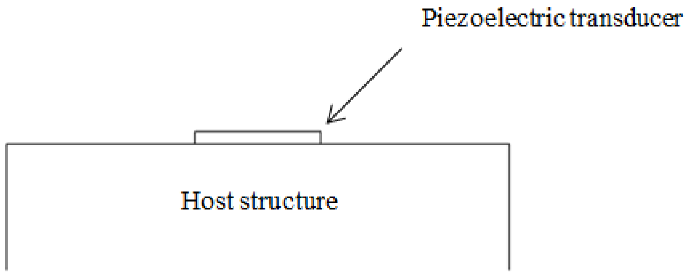

Fig. 1 Piezoelectric transducer bonded with host structure

Actuator Model. When deriving the model for piezoelectric actuators, a couple of boundary conditions are considered. First of all, since the top surfaces of piezoelectric transducers are not bonded to anything, they are stress free. Also, because electrical potential is a relative quantity, the bottom surfaces of piezoelectric transducers are often made the reference surface with zero potential. By applying these two boundary conditions, the model for piezoelectric actuators can be found as

$$\tilde{\boldsymbol{t}}_b = \boldsymbol{\Psi}_a \tilde{\boldsymbol{u}}_b + \boldsymbol{\Phi} \tilde{\boldsymbol{V}}_t \tag{2}$$

Sensor Model. When deriving the model for piezoelectric sensors, another two boundary conditions are used in addition to the ones formulated for the actuator model. The first condition comes from the fact that both surfaces of piezoelectric transducers are usually coated with metallic films to ensure that equi-potentiality is achieved on each surface. The second condition states that electric charge on the top surfaces of piezoelectric sensors is zero since they are not subject to external electric potential. With a total of four boundary conditions, the model for piezoelectric sensors can be represented by

$$\tilde{\boldsymbol{t}}_b = \boldsymbol{\Psi}_s \tilde{\boldsymbol{u}}_b \tag{3}$$
$$\tilde{\boldsymbol{V}}_t = \boldsymbol{\Theta} \tilde{\boldsymbol{u}}_b \tag{4}$$

Moreover, it can be seen that both the actuator and the sensor models are expressed in terms of voltage, displacement and traction, in order to couple directly with the host structure.

Boundary Element Method

The boundary integral equations, which govern the dynamics of an elastic isotropic body in Laplace domain, are given by [3]

$$c_{ij}(\boldsymbol{x}')u_j(\boldsymbol{x}') + \int_{\Gamma} T_{ij}(\boldsymbol{x}', \boldsymbol{x}, s)u_j(\boldsymbol{x})d\Gamma = \int_{\Gamma} U_{ij}(\boldsymbol{x}', \boldsymbol{x}, s)t_j(\boldsymbol{x})d\Gamma \tag{5}$$

$$\frac{1}{2}t_j(\boldsymbol{x}') + n_j(\boldsymbol{x}') \int_{\Gamma} T_{kij}(\boldsymbol{x}', \boldsymbol{x}, s)u_k(\boldsymbol{x})d\Gamma = n_i(\boldsymbol{x}') \int_{\Gamma} U_{kij}(\boldsymbol{x}', \boldsymbol{x}, s)t_k(\boldsymbol{x})d\Gamma \tag{6}$$

After carrying out the collocations and the integrations, a linear system of equations can be obtained as

$$\boldsymbol{H}(s)\tilde{\boldsymbol{u}}(s) = \boldsymbol{G}(s)\tilde{\boldsymbol{t}}(s) \tag{7}$$

By applying boundary conditions, eq (7) can be rearranged into

$$\boldsymbol{A}(s)\tilde{\boldsymbol{x}}(s) = \tilde{\boldsymbol{y}}(s) \tag{8}$$

Couple of Host Structure and Piezoelectric Transducers

Assuming that the piezoelectric transducers, as shown in Fig.1, are perfectly bonded to the host structure, eq (7) can be expanded into

$$\boldsymbol{H}_h\tilde{\boldsymbol{u}}_h + \sum_{K_a} \boldsymbol{H}_a^k\tilde{\boldsymbol{u}}_a^k + \sum_{K_s} \boldsymbol{H}_s^k\tilde{\boldsymbol{u}}_s^k = \boldsymbol{G}_h\tilde{\boldsymbol{t}}_h + \sum_{K_a} \boldsymbol{G}_a^k\tilde{\boldsymbol{t}}_a^k + \sum_{K_s} \boldsymbol{G}_s^k\tilde{\boldsymbol{t}}_s^k \tag{9}$$

where K_a and K_s are the numbers of actuators and sensors respectively, the superscript k indicates the kth transducer, and the subscripts a, s and h stand for the actuators, the sensors, and the rest of the host structure.

Due to the continuity of displacements and tractions at the interfaces between the host structure and the piezoelectric transducers, eq (9) can be rewritten as

$$\boldsymbol{H}_h\tilde{\boldsymbol{u}}_h + \sum_{k_a} \boldsymbol{H}_a^k\tilde{\boldsymbol{u}}_a^k + \sum_{k_s} \boldsymbol{H}_s^k\tilde{\boldsymbol{u}}_s^k + \sum_{k_a} \boldsymbol{G}_a^k\boldsymbol{\Psi}_a^k\tilde{\boldsymbol{u}}_a^k + \sum_{k_s} \boldsymbol{G}_s^k\boldsymbol{\Psi}_s^k\tilde{\boldsymbol{u}}_s^k = \boldsymbol{G}_h\tilde{\boldsymbol{t}}_h - \sum_{k_a} \boldsymbol{G}_a^k\boldsymbol{\Phi}\tilde{\boldsymbol{V}}_t \tag{10}$$

In equation (10), it can be seen that the first three terms on the left hand side and the first term on the right hand side assemble eq (7), and the other terms are contributed by the presence of piezoelectric transducers. Therefore, eq (10) can be rewritten in the form of equation (8) such that

$$\boldsymbol{A}\tilde{\boldsymbol{x}} + \sum_{k_a} \boldsymbol{G}_a^k\boldsymbol{\Psi}_a^k\tilde{\boldsymbol{u}}_a^k + \sum_{k_s} \boldsymbol{G}_s^k\boldsymbol{\Psi}_s^k\tilde{\boldsymbol{u}}_s^k = \tilde{\boldsymbol{y}} - \sum_{K_a} \boldsymbol{G}_a^k\boldsymbol{\Phi}\tilde{\boldsymbol{V}}_t \tag{11}$$

Equation (11) is solved by using an adaptive cross approximation (ACA) algorithm, whose detail and applications can be found in [4, 5].

Aspects of Numerical Modelling

Diagnostic Signal. For active sensing, five-cycle Hanning-windowed sinusoidal tonebursts are often used as the diagnostic signals. The time history of such a toneburst with a central frequency of 100 kHz and a peak voltage of 10 V, and its power spectrum in Laplace domain, are shown in Fig.2. It can be observed that the frequency components of such a toneburst are spread over the range of $2f_c$, where f_c is the central frequency.

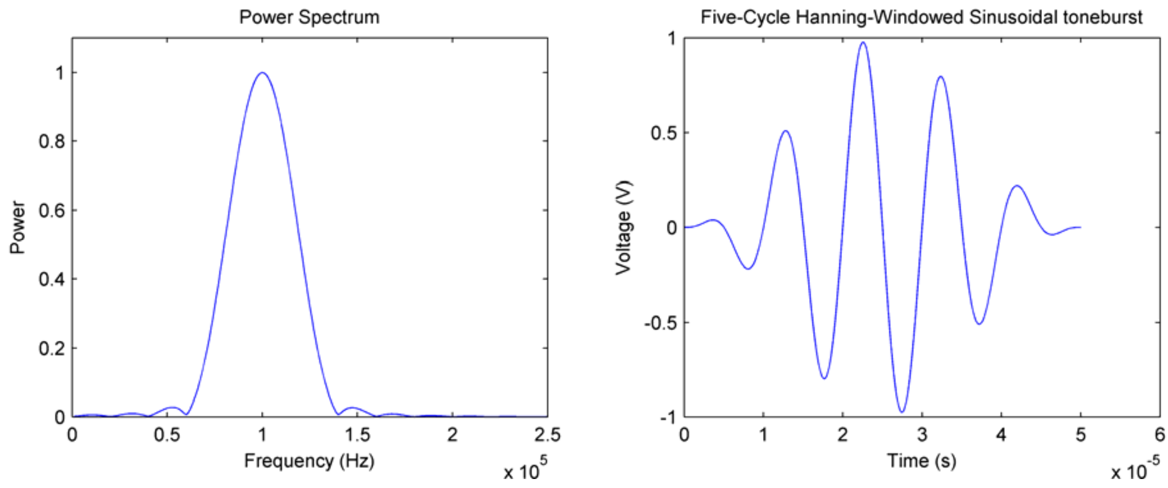

Fig. 2 Piezoelectric transducer bonded with host structure

Inverse Laplace Transform. In the field of BEM, the most often used approach for inverse Laplace transform is Durbin's Method [6]. The frequency increment of such method is given by

$$\Delta f = \frac{1}{T} \tag{12}$$

where T is the time period of interest. Consequently, the number of Laplace parameter required for the previously introduced diagnostic signals is given by

$$L = \frac{2f_c}{\Delta f} = 2f_c T \tag{13}$$

Computational Expenses. Benedetti et al [4] reported that for discretisation using 8-node quadrilateral elements, the element size needs to be smaller than the wavelength of the fundamental solution, which is given by

$$\lambda = \sqrt{\frac{E}{2\rho(1+v)}}\frac{1}{2f_c} \tag{14}$$

From eqs (13) and (14), it can be seen that as the frequency of the diagnostic signal increases, the number of elements and the number of Laplace parameters also need to increase accordingly.

Numerical Validation

The sensor signals from BEM and FEM simulations are compared. Abaqus®/Standard with implicit integration is used to perform the FEM analysis. Although it is known to be computational expensive, it is the only solver in which piezoelectric elements are available.

Fig.3 shows the BEM mesh of the aluminium beam used for numerical validation. The beam, which has a dimension of 200 mm × 30 mm × 30 mm, is bonded with one actuator and one sensor, both of which measure 10 mm × 10 mm × 1 mm. The piezoelectric transducers, which are marked in red, are 5 cm apart from the centre of the beam. The material properties of the aluminium and the piezoelectric ceramic can be found in [7].

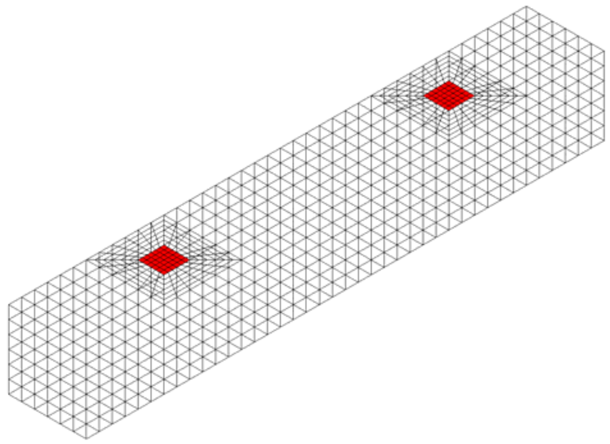

Fig. 3 BEM mesh of specimen

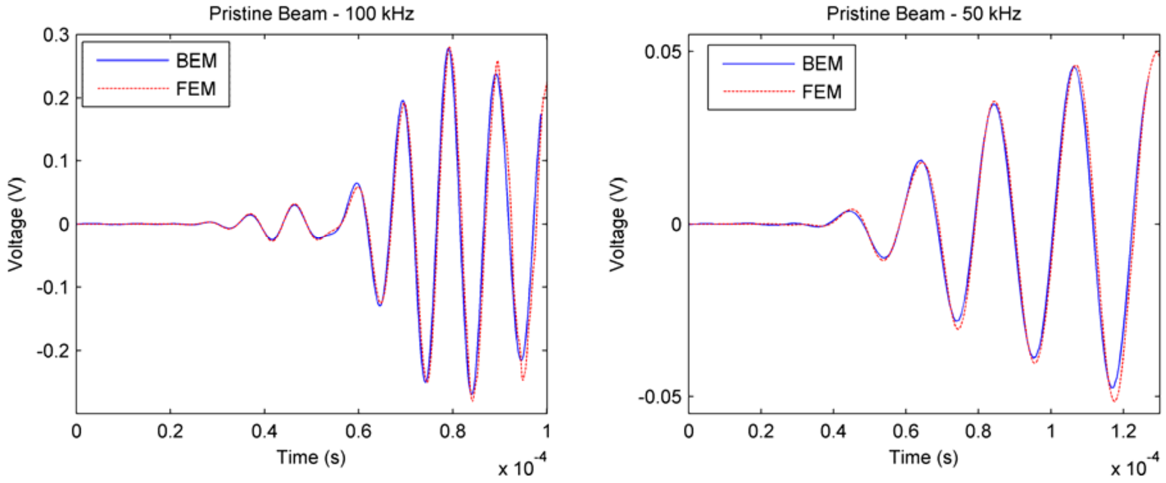

Fig. 4 Sensor signals

Fig.4 shows the comparison of the sensor signals, obtained from BEM and FEM simulations, for diagnostic signals with a peak voltage of 100 V and two different central frequencies – 50 kHz and 100 kHz. For both diagnostic signals, the agreements are excellent. However, this is more or less expected since the fundamental principles of the two models are essentially the same. The slight difference in the signals could very well be the result of accumulated numerical inaccuracy when performing matrix operations.

Conclusion

In this paper, a BEM for modelling smart structures bonded with piezoelectric transducers is introduced. The generalised 3D model for piezoelectricity comes from the basic elastic, electric and piezoelectric relationships, and the specific models for actuators and sensors are found by enforcing the relevant boundary conditions. The piezoelectric models are coupled with the host structure via BEM variables. The elatsodynamic analysis is carried out in Laplace domain. Outstanding agreement between the sensor signals obtained from BEM and FEM simulations is seen for examples involving active sensing. This paper aims at providing an alternative mathematical model for smart structures in SHM applications.

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