Optimal Resource Allocation in Multi-Hop Networks: Contention vs. Scheduling

R. Badalamenti, L. Giarré and I. Tinnirello

***Abstract*— CSMA/CA (Carrier Sense Multiple Access/Collision Avoidance) is actually the most used method in ad-hoc networks for transmitting on a contending medium, even if it shows poor performance in presence of hidden nodes. To increase performance, we propose an algorithm that combines CSMA and TDMA (Time Division Multiple Access) approaches. The adopted solution consists of grouping contending nodes in non-interfering subsets and granting a different numbers of time slots to different groups, while using the CSMA to manage medium access among nodes belonging to the same subset. An optimization procedure to assign the time slots to each subset of nodes and to find an equilibrium between contention and scheduling is presented.**

I. Introduction

CSMA/CA (Carrier Sense Multiple Access/Collision Avoidance) is actually the most used method in ad-hoc networks for transmitting on a contending medium. CSMA protocols, using carrier sense and random backoff mechanisms for controlling medium access, present poor performance in presence of *hidden nodes*, i.e. nodes that are not able to sense each other but are in radio visibility with a common receiving node. The phenomenon is more and more severe as the network traffic load increases and approaches the saturation throughput of the links. How to improve CSMA channel utilization in non-fully connected networks is actually an open problem. In literature several solutions can be found. An approach is to use full duplex communication network, in which a node receive and transmit simultaneously. Theoretically this method can double the throughput, but it presents as drawback the self-interference phenomena. Moreover, with this approach it is not simple to value how much the throughput can be increased. For a literature review we refer to [21] where stochastic geometry and random graphs methods for the analysis and design of wireless networks have been presented. To improve the network performance in terms of throughput, several distributed protocols have been recently proposed in [2], [3], [4], [5], [6]. They are all based on the idea of adapting the backoff distribution at each transmitter on the basis of the length of queues. In order to improve the channel utilization some optimization algorithms have been described in [8], [9], [10] for designing a good scheduling algorithm. In [7] it has been described a fully-distributed CSMA based MAC with a provable per-link order optimal delay performance for general wireless network topologies. The key part of this framework is the use of quadratic Lyapunov functions in conjunction with Lindley equation and Azuma's inequality in order to obtain an exponential decaying property in certain queueing dynamics.

Our solution proposed for improving CSMA performance was firstly proposed in [11] and [12], where TDMA has been used to schedule transmissions of potentially interfering nodes in different time slots, while carrier sense is adopted to solve contentions of nodes in radio visibility. These works have faced the problem in terms of map coloring. Map coloring is described in a vast literature such as [13], [14], [15], [16], [17]. All these works have proved how coloring network can improve the network performance thanks to a pre-allocation of channel resources. The basic idea is to combine the TDMA approach (for grouping the contending nodes in non-interfering sets) with the CSMA/CA approach (for managing the final access to the shared channel). This pre-allocation mechanism of channel holding times can significantly reduce the channel wastes due to hidden node collisions (and has been recently considered also in some standardization task groups working on mesh networks).

Hereafter we maintain the combined approach TDMA CSMA/CA, but conversely to other approaches, we present a solution where we do not assign a single time slot to each subset of nodes, but we provide an heterogeneous number of time slots that depends on the total traffic generated in each subset. The slot number allocation is based on an optimization criterion devised to maximize the mininum throughput perceived in the network.

The rest of paper is organized as follows. In section II we formulate the problem. In section III we present the system model. In section IV we explain the idea behind the algorithms. In section V we present the algorithm for deciding the best partition and the optimization algorithm for assigning resources while, section V-B provides numerical results. Finally we draw some conclusions in section VI.

II. Network Model

A. Network Structure

We consider a single channel network made of a set V of nodes randomly distributed over a given area. Each node $i \in V$ can communicate only with a subset V_i of adjacent nodes. We say that i is (radio) *visible* only to the nodes in V_i. Differently, i is *hidden* to the remaining nodes in $V \setminus V_i$. We assume that radio visibility is symmetric. We represent the network structure through an edge labeled graph $G = (V, E)$. Specifically, the nodeset V includes all the nodes i of the network and the edgeset E includes all the pairs of adjacent nodes (i, j) that are in mutual

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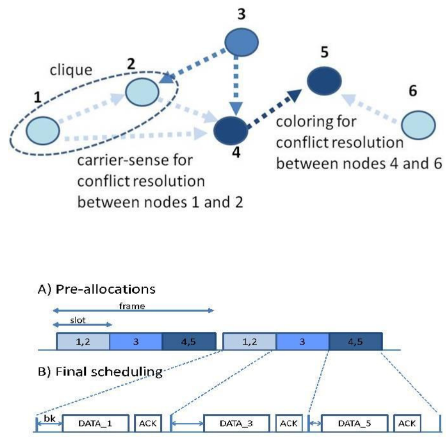

Fig. 1. An example of medium access in a network with 5 nodes and a frame composed of 3 slots.

radio visibility. To access the channel we consider a mixed approach in which medium is partly scheduled and partly contended. We divide the network in subnets, and consider time divided into elementary allocation units called slots. We schedule transmissions granting an advantageous number of slots to each subnet. Within a subnet nodes contend medium performing CSMA/CA.

B. Resource Allocation

We assume that the system is time slotted and we model the traffic source at each node in terms of per-slot packet probability. Specifically, we consider that each node i has a fixed probability λ_i to generate a packet in each slot. In order to avoid interactions with the routing protocol, we consider only one-hop packet deliveries. Packets are destined to a randomly selected node among the neighbor ones. For isolated nodes, i.e. nodes without neighbors, the traffic is assumed to be broadcast. The channel slot is pre-allocated to a subset of nodes, and the slot allocations are maintained on a per-frame basis: being x the total number of allocation slots, a sequence of x consecutive slots is a channel frame in which, slot by slot, the same sorted list of nodes are enabled to transmit. Figure 1 shows an example of medium access in a network with 5 nodes, in which a channel frame of 3 slots is considered. In the first slot, where only station 1 and 2 can access the medium, station 1 wins the contention (i.e. extracts the lowest backoff delay). The second slot is used by station 3 only, while the third slot is reserved to the contention between stations 4 and 5. The reason for pre-allocating channel slots to a subset of stations (thus grouping in independent sets the stations allowed to transmit simultaneously) is the mitigation of the hidden nodes problem. For example, if stations 1 and 3 are hidden to each other (as shown in the figure) and wish to transmit to station 2 (which is able to hear both the stations), the previous allocation avoids any collision possibility. Conversely, transmissions originated by stations 1 and 2, which are in reciprocal visibility, are separated by the CSMA/CA protocol. We formally define the problem of slot allocations in what follows.

In the present paper we assume that the reader is familiar with CSMA/CA protocols, which regulate the final channel access within an allocated channel slot, see [1] for a recent survey on it. Although most CSMA/CA protocols use a slotted backoff scale for efficiency reasons and for implementation limits (since the carrier sense cannot be instantaneous), we assume that backoff values are uniformly extracted in a continuous range $[0, b]$, thus implying that collisions cannot be originated by the extraction of two identical backoff values.

In order to implement a slot allocation mechanism, two basic functionality have to be provided in the network: i) a mechanism for inferring the network topology; ii) a mechanism for keeping a common time reference among the nodes. For both aspects, we consider that an independent signaling channel is available (managed by a random access scheme) and nodes in radio visibility can exchange control information (e.g. the list of neighbor nodes). We also assume that nodes do not have data storage constraints, while processing capabilities may depend on the specific network scenario.

In the above context, the main problems to be tackled in the present paper are the following.

Problem 1: Determine the best partition of the network i.e.: the best number of subsets in which divide the network and how divide nodes in the subnets.

Problem 2: Determine the best number of time slots to grant to each subnet in each frame.

III. System Model

The correlation between nodes in a mesh network (either due to spatial reasons, or to traffic routes) has a more important effect on the final network performance than the specific evolution of the backoff counter. Therefore, we propose a system model based on the following simplifying assumptions:

- The time for the backoff countdown and medium sensing is negligible;
- The probability to have the minimum residual backoff counter is uniform for all the contending stations, while the probability to have exactly the same residual backoff expiration time is zero.

According to the first assumption, all the contending stations complete their backoff count-down sequentially (as determined by their backoff counters) but with negligible time intervals between their transmission grants. In other words, the time interval Δ between consecutive transmission grants is assumed equal to 0 when it accounts for consecutive backoff countdown, and equal to P for completing the slot when all the stations are in a transmitting or frozen state. When Δ is equal to P, at the end of the slot all the stations with non-empty queues switch synchronously to a contending state. The second assumption implies that visible stations never collide and that the start of a new slot is a regeneration instant, since all the stations with traffic are in a contending state at the beginning of the slot with the same residual backoff distribution.

A. Transport Model

Under the previous assumptions and that all the stations are saturated we can evaluate per-node throughput determining transmission events and occurrence probability as seen in [18]. The idea is characterizing the occurrence probability of different *transmission events*. A transmission event is the collection of the state of each node at a generic time slot, where the node state is considered equal to 1 if the node is transmitting during the time slot, and equal to 0 otherwise.

Let A be the set of all the possible values $\boldsymbol{a}$ of transmission events, $a(i)$ the binary transmitting/frozen state of node i in the transmission event $\boldsymbol{a}$, and $p_A(\boldsymbol{a})$ the total probability of event $\boldsymbol{a}$. For a given transmission event $\boldsymbol{a}$, the number of nodes able to receive successfully a frame sent by node i is $S_i(\boldsymbol{a}) = \sum_{j=1}^{n} a(i) g_{ij} \prod_{k=1}^{n}(1 - g_{jk} a(k))$, where we consider that the same frame can be received by all the 1-hop neighbors j for which none of the relative 1-hop neighbors k is active. We consider the time frame divided in N time slots and we grant α_i time slots to the subnet i, so the nodes in this subnet have transmission opportunity only in α_i time slots every N. This implies that for computing the average transmission throughput we have to consider a scale factor α_i/N It follows that the average transmission throughput S_i for each node i (in terms of packets/slot) can be obtained as:

$$\begin{aligned} S_i &= \frac{\alpha_i}{N} \sum_{\boldsymbol{a} \in A} p_A(\boldsymbol{a}) S_i(\boldsymbol{a}) \\ &= \frac{\alpha_i}{N} \sum_{\boldsymbol{a} \in A} p_A(\boldsymbol{a}) \sum_{j=1}^{n} a(i) g_{ij} \prod_{k=1}^{n} (1 - g_{jk} a(k)) \end{aligned} \quad (1)$$

B. Channel Access Model

In order to implement our hybrid TDMA/CSMA access scheme, we organize the channel access time in periodic frames composed of N time slots. Time slots are not exclusively allocated to a single node, but rather are shared among the set of nodes belonging to the same subnet. Contention is still used for arbitrating the channel accesses among the subnet nodes.

We define *partition of a network* the partition of the nodeset V. A partition of V is a set of subsets of V such that:

- All sets in V are pairwise disjoint: $V_i \cap V_j = \emptyset \ \forall i \neq j$
- The union of all the sets forms the whole set $V = \bigcup V_i$
- None of the sets in V is empty: $V_i \neq \emptyset \ \forall i$

Our goal is to find both the best partition of the network and the best resources allocation to improve the network performance in terms of throughput with respect to either pure TDMA or pure CSMA based approaches.

IV. Network Partitioning

Network paritioning substantially affects the network performance. The number of subsets in which we divide nodes decides how much the medium access will be scheduled or with contention. In fact if we consider only one subset, that is the entire network, in all the time slots all nodes will contend according to CSMA, so this is the case of total contention. Conversely if we consider a partition formed by n subsets in a network formed by n nodes, all the subsets are formed by one node only. In this case, each node will transmit without contention and the system is entirely scheduled. All the division in a number between 1 and n gives medium access partially scheduled and partially with contention and our goal is to find the number of subsets with the best performance. After solving the problem on how many subsets need to be selected, we have to decide how to divide nodes in subsets. Network performance may change considerably in different placement of nodes. For example, because hidden nodes degrade CSMA performance, we would like to assign hidden nodes to different subsets, so that they will never collide because they can access the channel in non-overlapping time intervals.

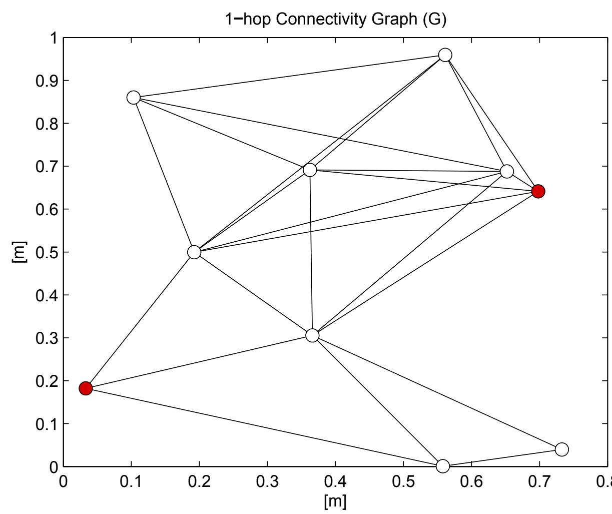

Fig. 2. Network Topology

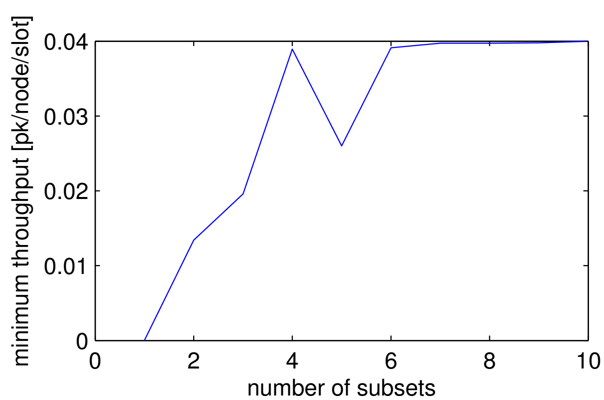

Fig. 3. Minimun Throughput

Consider for example a simple network topology with three nodes in a row. With pure CSMA, it is very likely that the edge (non-visible) nodes will achieve an almost zero throughput due to the fact that when the first edge node starts its transmission, the opposite one will decrement its backoff to zero before the end of the ongoing transmission (thus resulting deterministically in continuous collisions).

Similar phenomena arise in more general topologies, as the one shown in figure 2. For this network, we considered the minimum throughput achieved in the network as the number of slots in the hybrid TDMA/CSMA access scheme varies from 1 to 6. Obviously, the case with one slot only refers to the pure CSMA approach. In all the other cases, nodes are grouped into subnets.

If the partition is limited to two subnets only, being 10 the total number of network nodes, in principle we have 2^{10} possible combinations for organizing the node groups, which are reduced to one half of this number if we consider that throughput performance are invariant to the grouping order. In other words, the performance is the same if nodes 1 and 2 are in subset 1 and nodes 3 and 4 in subset 2 or if nodes 1 and 2 are in subset 2 and 3 and 4 in subset 1. For each possible combination over the total 2^9 possible ones, we can then solve an optimization algorithm in order to decide how many slots allocate to the first and second subnet in a frame.

We can repeat the same reasoning considering the network divided in three subnets e go on. An optimization algorithm that determines the optimal value of subsets in which we can divide the network is NP-Hard [19] [20], and increasing the number of subsets, full exploration becomes unthinkable because of the huge number of possible combinations.

For this reason, we have faced the problem according to an heuristic approach finding some logical rules for dividing the nodes in subnets. If for some nodes is not possible to decide the destination group because the proposed rules cannot be applied, we explore all possible decisions by considering as freedom degree only the decisions about the nodes for which the heuristic logic cannot decide. This allows to dramatically reduce the exploration space in comparison to the whole combination set. In figure 3 we see the minimum throughput for the reference exemplary network, that has been obtained varying the numbers of subnets determined by our heuristic for a frame with 6 total slots. We can see that with CSMA we have a minimum throughput of 0 packets/node/slot in nodes colored in red; when we introduce some scheduling these nodes will be assigned in different subsets and all nodes achieve a non-null throughput

Heuristic decision logic. We base our heuristic decision logic on the following considerations. A first rule to be applied in the subdivision of nodes is trying to put hidden nodes in different subsets, so that they do not interfere. If the number of subnets is not enough to separate all the hidden nodes, we decide the assignment of the pending nodes by considering all the possible decisions. For example, if we have to divide nodes in two subsets and a given node is hidden both to a node in the first group and a node in the second group, the node is considered as a freedom degree and all possible decisions (assignment to the first group or assignment to the second group) need to be explored by means of the throughput model described in section III-A. In general, if the total number of pending nodes is p, we need to consider 2^{p-1} possible combinations.

V. Frame Decision Optimization

In addition to the number of subsets in which we divide the network and to the grouping of nodes in these subnets, we have to decide how many time slots in the frame need to be allocated to each subset. In order to decide how to organize the frame, because of the size of the problem, it is not possible to explore all the combinations for each number of subsets. So, we implement the following optimization algorithm, that we run for each partition of the network.

A. Optimization problem

The optimization problem is aimed at solving the following issues: i) determine the best number N of subsets in which divide the network, ii) divide the nodes in subnets; iii) obtain the best number of time slots α_i to be granted to each subnet i in each frame.

We recall from eq. 1 that the throughput is related to the number N of slots and to the parameter α_i, then our problem can be solved with an algorithm that makes an optimal choice of N and α_i to improve the network throughput.

As possible solution we have considered the maximization of the throughput for each node achieving the worst performance in the network. In each subnet i the node presenting the minimum throughput x_i is chosen. As described in section III, the throughput is a portion $\frac{\alpha_i}{T}$ of the throughput calculated with the CSMA algorithm, so we can maximize, varying α_i, the minimum comparing all $\alpha_i * x_i$. The problem becomes a multiobjective integer problem max-min to be solved as follows:

$$
\begin{aligned}
& \max \min_{\alpha_i}(\alpha_1 * x_1, \alpha_2 * x_2, ..., \alpha_n * x_n) \\
& s.t \sum_{i=1,2,..,n} \alpha_i \leq T \\
& \alpha_i \geq 0 \quad i = 1, 2, ..n \\
& \alpha_i \text{integer}
\end{aligned} \tag{2}
$$

where T is the frame duration, and the sum of time slots α_i must be less or equal to T.

This optimization needs to be applied in all possible combinations in case of full exploration of the partitioning possibilities, as we have seen in the previous paragraph.

Because the solution of the above optimization problem which is multi-objective and integer is not viable, we recast the problem introducing another way to assign the frame to the nodes. In this approach we maximize the sum in the network of the minimum throughput of each subnet adding a fairness factor: we grant to each node a throughput grater or equal to a value that we choose as $\frac{1}{n}$ of the sum of minimum throughput. In this way we obtain the following optimization single-objective integer problem:

$$\max_{\alpha_i} \sum_{i=1,2,..,n} \alpha_i * x_i$$
$$s.t \sum_{i=1,2,..,n} \alpha_i \leq T$$
$$0 \leq \alpha_i \leq T \quad i = 1, 2, ..n \tag{3}$$
$$\alpha_i \text{integer}$$
$$\alpha_i x_i > \frac{1}{n} \sum_{i=1,2,..,n} x_i$$

B. Numerical results

We have carried out simulations in random networks formed by 25 nodes situated in an area of 1 m^2 with a range of visibility between nodes varying from $0,4$ m to 1 m. Varying the radius of visibility we can see how the *hidden nodes* influence the network performance: a small radius corresponds to a lot of hidden nodes, while a radius equal to 1 corresponds to a completely connected network, without *hidden nodes*. Simulations are carried out with MATLAB software and GUROBI extension for the integer optimization algorithm. Each curve in figure 4 is the result of an average over ten simulations.

The algorithm applied for each number of subsets is described schematically as follows:

0 From the knowledge of G and $G^2 - G$, divide nodes in subsets without assigning hidden nodes to the same subset.

i. Check if we have nodes that we can not assign to a subset according to the above rule.

ii. In case there are still not assigned nodes, explore all possible combinations of these nodes in subsets.

iii. For each combination (if all nodes have been divided at the first step, there is only one combination) apply the optimization algorithm.

iv. Check if there exist nodes presenting a throughput equal to zero, and then try all possible combinations repeating from step 2.

v. Evaluate the best configuration solving an optimization problem for each number of subnets in the partition.

In figure 4 the simulation results are depicted. In y axis we represent the aggregated throughput, that is the sum of the node's throughput in a subnet. In the figure we represent the maximum value of aggregated throughput for each number of subsets in the partitions. In x axis we see the number of subsets in which the network is divided, starting from 1 subnet, that is the entire network, in which medium access is completely contended between nodes, up to 25 subnets, in which each subnet is formed by one node, corresponding to an entirely scheduled access. Dividing the network in subnets it can be shown that it is possible to obtain a throughput improvement with respect both to an entirely contention access and to an entirely scheduling access. Considering the curve with radius of visibility 0.4 m and dividing the network in 2 subsets we obtain an aggregated throughput of 2.6 pk/node/slot, while with CSMA we obtain 1.35 and with TDMA 0.12. The minimum throughput obtained dividing in 2 subnets is 0.0426, while with CSMA is 0.0063 and with TDMA is 0.04. The maximum throughput obtained dividing in 2 subnets is 0.2, while with CSMA is 0.1 and TDMA is 0.04. So we can see that dividing the network in two subnets we improve all these values.

In table V-B we show the comparison between the optimal values of throughput obtained with our algorithm and with the throughput obtained, respectively, with CSMA end TDMA for all the considered radius of visibility. These values are obtained as average of ten simulation with random topologies.

We note that the effectiveness of the hybrid CSMA/TDMA scheme improves as the visibility radius becomes smaller, because in these conditions the impact of hidden nodes is more relevant.

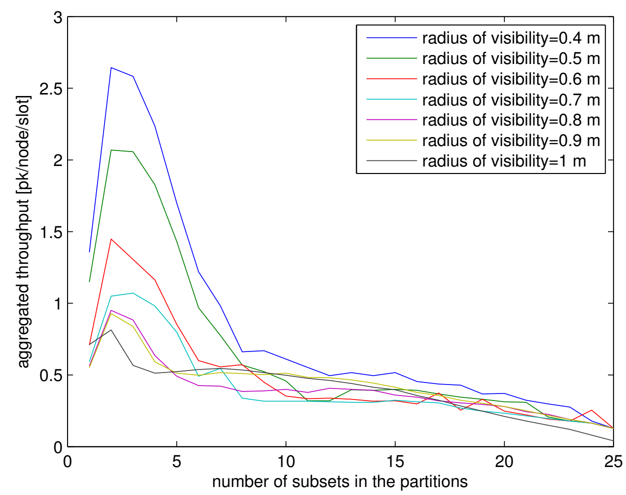

Fig. 4. Aggregated throughput varying node's subset

VI. CONCLUSION

CSMA/CA shows poor performance in presence of hidden nodes. A form of coordination among nodes can improve the network performance. In this paper, we propose a novel algorithm presenting through an integer optimization algorithm an equilibrium between contention and scheduling for allocating optimally resources in multi-hop networks. The idea is to split nodes in subsets transmitting in different time slots. Nodes inside a subnet contend medium according CSMA algorithm. We have showed simulation results proving that this method improves the network performance in terms of throughput both respect to CSMA and TDMA algorithms. We are still working on finding better algorithm for dividing nodes in subnets in order to further improve network throughput.

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radius	opt alg	CSMA	TDMA
		thr aggr	
0.4	2,64	1,35	0,13
0,5	2,06	1,14	0,12
0,6	1,44	0,70	0,12
0,7	1,07	0,59	0,12
0,8	0,95	0,56	0,12
0,9	0,92	0,55	0,12
1	0,81	0,71	0,04
		thr min	
0,4	0,04	0,00	0,04
0,5	0,03	0,00	0,04
0,6	0,02	0,01	0,04
0,7	0,02	0,01	0,04
0,8	0,02	0,01	0,04
0,9	0,01	0,01	0,04
1	0,03	0	0,04
		thr max	
0,4	0,21	0,10	0,04
0,5	0,10	0,07	0,04
0,6	0,08	0,03	0,04
0,7	0,16	0,04	0,04
0,8	0,06	0,04	0,04
0,9	0,06	0,04	0,04
1	0,05	0,04	0,04

TABLE I

COMPARISON BETWEEN THE OBTAINED (MAX, MIN, AGGREGATED) OPTIMAL VALUES OF THROUGHPUT WITH THE ONE OBTAINED WITH, RESPECTIVELY, CSMA END TDMA CHANGING OF THE RADIUS OF VISIBILITY

International Conference on , vol., no., pp.199,204, 21-23 Nov. 2012 doi: 10.1109/ICCS.2012.6406138

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