FE-Meshless multiscale non linear analysis of masonry structures

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In masonry structures, the most relevant kinematical and mechanical phenomena take place at a scale which is small if compared to the dimensions of the structure. On the other side, the structure is governed, in its peculiar overall response, by its global geometrical and morphological configuration. In literature, two different scales of interest are distinguished, directly linked to as many theoretical approaches: the mesoscopic approach and the macroscopic approach. The mesoscopic approach considers the heterogeneous materials and their interfaces individually [1, 2], but many difficulties arise in the mesh creation and a fine discretization of the structure has to be used, which leads to prohibitive computational costs. The macroscopic approach considers the structure constituted by a fictitious homogeneous and continuous material. The multiscale techniques belong to the second approach and couple different scales of interest by means of apposite transition laws capable to exchange information between different consecutive scales [3-5]. In this work a multiscale first order computational homogenization technique is applied to simulate masonry structures. A unit cell (UC) is identified. The UC is assumed constituted by a block surrounded by mortar joints, which are simulated by zero-thickness interface models. The material of the block is assumed indefinitely elastic while the interface laws are expressed in the framework of elastoplasticity.

The scale transition between macroscale and mesoscale is based on the Hill-Mandel principle. By imposing the equilibrium over the entire structure, the displacement $\mathbf{u}_M$ and strains $\boldsymbol{\varepsilon}_M$ fields at the macroscopic level are derived. The macroscopic stress field $\boldsymbol{\sigma}_M$ associated to the strain field $\boldsymbol{\varepsilon}_M$ is instead obtained averaging the UC reactions r over the volume Ω_{UC}. To this end, the response of the UC is evaluated by solving a boundary value problem, that in this work are assumed to be of Taylor-Voigt type:

$$\mathbf{u}_m = \boldsymbol{\varepsilon}_M \mathbf{x} \qquad on \ \Gamma_{UC} \tag{1}$$

where $\mathbf{u}_m$ are the prescribed displacements for the point of position $\mathbf{x}$ located on the boundary Γ_{UC} of the UC. Once the solution of the boundary value problem is obtained, the macroscopic stress $\boldsymbol{\sigma}_M$ is calculated according to the following equation:

$$\boldsymbol{\sigma}_M = \frac{1}{2|\Omega_{UC}|} \int_{\Gamma_{UC}} (\mathbf{r} \otimes \mathbf{x} + \mathbf{x} \otimes \mathbf{r}) d\Gamma \tag{2}$$

The interface constitutive laws are developed in the framework of elastoplasticity for non standard materials. The elastic domain is defined by two convex limit surfaces intersecting in a non-smooth fashion: the Coulomb bilinear limit surface and a tension cut-off. Non-associative flow rules are derived to express the displacement discontinuities at the interfaces.

The solution of the UC boundary value problem is generally approached in an approximated way making use of the finite element method. In the present study the numerical solution of the mesoscopic model is obtained by means of a meshless strategy [6]. The UC is divided in five integration domains: the first domain corresponds to the volume occupied by the block, the other four domains are the interfaces. The displacement field inside each sub-domain is obtained from the nodal displacement values by a Moving Least Square approximation. The influence of a node on a point on the UC is defined by a weight function depending on the distance between the sampling point and the node.

The proposed model has been implemented on a research oriented finite element analysis program to run 2D simulations on masonry structures.

The solution can be separated in an elastic and a plastic phases. To find an elasto-plastic solution the interface constitutive laws are rewritten in a discrete way and integrated for a given incremental strain history. The solution over a single time step is obtained employing a Backward Euler integration scheme, separated in an elastic predictor stage and a plastic corrector stage. The governing equations for the UC at the elasto-plastic step are finally expressed in matrix form as follows:

$$\begin{bmatrix} \mathbf{K} & \mathbf{-G} \\ \mathbf{G^T} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{U}_m \\ \Delta\mathbf{R} \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{F}_p \\ \Delta\bar{\mathbf{U}}_m \end{bmatrix} \tag{3}$$

where $\mathbf{K}$ and $\mathbf{G}$ are two matrices depending on the UC geometry, $\Delta\mathbf{U}_m$ is the increment of nodal displacements, $\Delta\mathbf{R}$ the increment of nodal reactions, $\Delta\mathbf{F}_p$ the elasto-plastic forces evaluated in the correction stage, $\Delta\bar{\mathbf{U}}_m$ the imposed displacements on boundary nodes.

A macroscopic elastic tangent stiffness matrix can be evaluated at each integration point and a procedure is performed to localize the plastic zones at the macroscale, starting from the results at the mesoscale.

The FE-Meshless multi-scale computational strategy has been applied to simulate experimental tests available in literature in plane-stress conditions. The classical finite element analysis is run at the macroscale, while the meshless procedure is applied at the mesoscale on the UC. Two iterative Newton-Raphson procedures have been used during the analyses: one for the macroscale finite element procedure, one for the solution of the UC in the plastic phase. The convergence criterion at the mesoscale is considered satisfied when the difference between the elasto-plastic forces between two successive iterations is less than a tolerance value.

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